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Single-electron analysis of the space-charge effect in free-electron lasers

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An exact treatment of the space-charge effect in the single-electron analysis of a free-electron laser is presented to calculate its small-signal gain. With the inclusion of the repulsive force between electrons, it is found that the trajectory of an electron can be solved from a generalized equation which includes a space-charge term. The results show the gain is saturated with decreasing growth rate due to high electron density. The radiation frequency is found to increase with the electron density and approach the value at plasma resonance. The condition $\omega_p L/c = \pi$ clearly defines the boundary between the noninteracting and the collective regime of an electron beam, where w_p is the plasma frequency, L is the device length, and c is the light velocity in vacuum.

INTRODUCTION

In order to increase the gain and the output power of a free-electron laser, it is intended to use electron generators that can provide a high-density beam. However, it is known that at high electron density the space-charge field begins to influence the interaction between individual electrons and the radiation. This effect is usually neglected in a preliminary analysis of free-electron lasers, assuming that the current density is very small. Due to the potential use of high-density electron beams in future experiments, this problem has been discussed extensively in several theoretical investigations.[1-7] Most previous work has used the electron distribution function to obtain the growth rate of radiation from the dispersion relation or only the first-order correction to the no-space-charge gain.[4,5] Since the single-electron model deals directly with electron trajectories in the electromagnetic field, there has been doubt about its usefulness in solving the space-charge problem.[8]

In this paper, however, we shall show how to account for the space-charge effect exactly to obtain the small-signal gain of a free electron laser using the single-electron analysis. Since the laser oscillation involves only the low-gain process, the constant-field approximation is considered in the following analysis. The advantage of using the single-electron model in the problem of space-charge interactions is shown in its mathematical simplicity and extensive range of applicability. The analysis describes the wave-wave interaction from the single-particle point of view. It also shows how the radiation frequency of an oscillator changes with the electron density from the noninteracting to the collective (plasma) regimes.

THEORY

The classical single-electron model has been used successfully in the investigation of transverse[9] and longitudinal[10] free-electron lasers. The interaction between an electron and the radiation is described by the Lorentz force equations

$$\frac{d}{dt}(m\gamma\vec{v}) = e\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right), \tag{1}$$

$$\frac{d}{dt}(\gamma mc^2) = e\vec{v}\cdot\vec{E}, \tag{2}$$

where $\vec{v}$ and γmc^2 are the velocity and energy of the electron, and $\vec{E}$ and $\vec{B}$ are the total electric and magnetic fields in the interaction region.

The electron beam propagates in the z direction and through the axis of a helical magnet where the magnetic field is represented by

$$\vec{B}_m = B(\cos 2\pi z/l,\ -\sin 2\pi z/l, 0). \tag{3}$$

The spontaneous and stimulated radiations in this magnet are circularly polarized. Neglecting all the dependence on the transverse variable, the radiation field is represented by

$$\begin{aligned}\vec{E}_r &= E(\cos(\omega t - kz + \phi), -\sin(\omega t - kz + \phi), 0),\\ \vec{B}_r &= E(\sin(\omega t - kz + \phi), \cos(\omega t - kz + \phi), 0),\end{aligned} \tag{4}$$

where $\omega = kc$ is the radiation frequency, E is the field strength, and ϕ is the phase at the entrance of the interaction region. The total fields in (1) and (2) are then $\vec{E} = \vec{E}_r$ and $\vec{B} = \vec{B}_r + \vec{B}_m$. Under the constant-field approximation, the transverse component of Eq. (1) can be integrated exactly to obtain $v_\perp$, and the Lorentz equations are reduced to a simple equation describing the parallel motion of an electron

$$\begin{aligned}&\frac{d^2}{dt^2}\Delta z(t) = a\cos(\Omega t - \beta\Delta z + \phi),\\ &a = \frac{2e^2BE}{\gamma^2 m^2 c\omega(1 + e^2B^2/m^2c^2\omega^2)},\\ &\Omega = \omega - \beta v_0, \quad \beta = k + 2\pi/l,\\ &\Delta z(t) = z(t) - v_0 t.\end{aligned} \tag{5}$$

In (5), a is the interaction strength, Ω is the off-resonance parameter, β is the wave number of the first harmonic component of the radiation in a periodic structure with period l, Δz is the position deviation of an electron from the noninteraction value.

The simplest way to obtain the radiation gain is to calculate the energy loss of the electron beam in a single pass. According to the energy conservation, it is converted into the radiation energy if other loss mechanisms are negligible. The energy change of a single electron can be calculated from the work done by the radiation.

$$\Delta\epsilon \equiv \Delta(\gamma mc^2) = e\int_0^T \vec{v}\cdot\vec{E}\,dt\,. \tag{6}$$

$T \equiv L/c$ is the flight time of the electron. Since there is no longitudinal component of the electric field, the integrand contains only

$$v_\perp E_\perp = \frac{eEBl}{2\pi mc\gamma}\cos[\Omega t - \beta\Delta z(t) + \phi]\,. \tag{7}$$

The integration depends on the explicit form of $\Delta z(t)$ that is to be solved from the pendulum equation (5). In the small signal region, $\Delta z(t)$ is obtained up to first order in E:

$$\Delta z^{(1)}(t) = \frac{a}{\Omega^2}[(1-\cos\Omega t)\cos\phi - (\Omega t - \sin\Omega t)\sin\phi]. \tag{8}$$

Substituting (8) into (7) and (6), we can obtain its energy loss, which depends on ϕ. For an initially uniform beam, the ensemble average (over ϕ) of (6) is taken to find that average energy change. The averaging process eliminates the first-order term and leaves the energy loss proportional to the radiation energy ($\langle\Delta\epsilon\rangle_\phi \propto E^2$). For an initially monoenergetic beam, the average energy loss per electron is multiplied by the particle current density $|J/e|$ and divided by the radiation flow intensity $cE^2/4\pi$ to obtain the single-pass radiation gain of the device $[G(T)\equiv(I_{\text{out}} - I_{\text{in}})/I_{\text{in}}]$,

$$\begin{aligned} &G(T) = G_0 f(\theta)\,,\\ &G_0 = \frac{2e^3JlB^2L^3}{m^3\gamma^3c^7}\,,\\ &f(\theta) \equiv \frac{2-2\cos\theta - \theta\sin\theta}{\theta^3} = -\frac{\partial}{\partial\theta}\left(\frac{1-\cos\theta}{\theta^2}\right),\\ &\theta \equiv \Omega T\,. \end{aligned} \tag{9}$$

$f(\theta)$ represents the gain spectrum (Fig. 1). The maximum gain for given L is $0.135\,G_0$ at $\theta_{\max} = 2.6$. Since the electron distribution is assumed to be much narrower than the spectrum bandwidth, this corresponds to the limit of small cavity or homogeneous interaction and is known as a "no-space-charge gain."

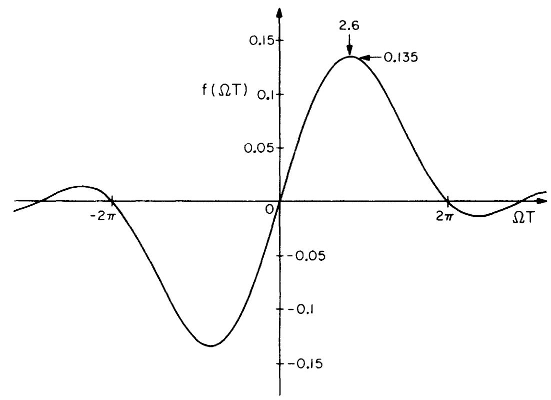

FIG. 1. No-space-charge gain $f(\Omega T)$, $\Omega = \omega - (k + 2\pi/l)v_0$.

Neglecting the space-charge field, Eq. (8) describes the trajectory of an electron which is assumed to pass the entrance of the interaction region $z=0$, when $t=0$, with phase ϕ. Such a periodic dependence on ϕ results in a nonuniform beam which generates a space-charge field. Equation (5) describes only the situation when this field is negligible compared to the ponderomotive force. In general, the space-charge field E_c should be included in the analysis, since the total field to which an electron is subjected is the sum of this field as well as the external applied field.

The space-charge field E_c is included in the equation as

$$\frac{d^2}{dt^2}\Delta z(t) = a\cos[\Omega t - \beta\Delta z(t) + \phi] + \frac{eE_{cz}}{m\gamma^3}\,. \tag{10}$$

The second term on the right side represents the contribution of the space-charge effect. γ^3 in the denominator comes from the relativistic consideration.

If the change of the transverse space-charge field is assumed to be very small and neglected, the longitudinal space-charge field obeys the Poisson equation

$$\frac{\partial}{\partial z}E_{cz}(z,t) = \frac{e}{\epsilon_0}[N(z,t) - N_0]\,, \tag{11}$$

where $N(z,t)$ is the electron density at position z and time t; N_0 is the initial electron density. In order to follow the evolution of the electron density, we consider an infinitesimal section of the beam at z_0 when $t=0$. Its width is δz_0 and density is N_0. After time, t, it propagates to position z and develops into a section with width δz and density $N(z,t)$. If the electrons retain their relations in space during the propagation (the single-stream assumption), the density function obeys the equation

$$N(z,t)\delta z = N_0 \delta z_0 \qquad (12)$$

or, equivalently,

$$N(z,t) = N_0\left(\frac{\partial z}{\partial z_0}\right)^{-1}. \qquad (13)$$

In general, the position z is a function of t and z_0. It can be written as

$$z(z_0,t) = z_0 + v_0 t + \Delta z(z_0,t). \qquad (14)$$

Substituting (14) into (13) and assuming $(\partial \Delta z/\partial z_0)$ is small, we have

$$N(z,t) = N_0\left(1 - \frac{\partial}{\partial z_0}\Delta z(z_0,t)\right). \qquad (15)$$

Using (15) in (11), we obtain

$$\frac{\partial}{\partial z} E_{cz}(z,t) = -\frac{eN_0}{\epsilon_0}\frac{\partial}{\partial z_0}\Delta z(z_0,t). \qquad (16)$$

Equation (16) involves partial derivatives with respect to different variables: z and z_0. However, they are equivalent in the case where only the partial differentiation is concerned, and Δz is small compared to $(z_0 + v_0 t)$. This approximation is valid even in the strong signal regime. The integration of (16) over z_0 leads to

$$E_{cz}(z,t) = -\frac{eN_0}{\epsilon_0}[\Delta z(z_0,t) + h(t)]. \qquad (17)$$

The function $h(t)$ does not depend on z_0. Since E_{cz} becomes zero when Δz is uniform (i.e., independent of z_0), it is natural to identify $h(t)$ as the ensemble average of the position deviation $\langle\Delta z(z_0,t)\rangle_{z0}$. It is noted that it does not make any difference if we replace z_0 by ϕ to label electrons. We have thus found a way to relate the space-charge field to the dynamic variable of an electron Δz:

$$E_{cz}(z,t) = -\frac{eN_0}{\epsilon_0}[\Delta z(\phi,t) - \langle\Delta z(\phi,t)\rangle_\phi]. \qquad (18)$$

Physically, it means that the space-charge field experienced by an electron is proportional to its "net" position deviation.

SINGLE-PASS GAIN

We have shown that the space-charge term in (10) can be related to the single-electron position deviation through the key equation (18). Since no assumption was made concerning the electron density, the analysis which follows should apply to beams with arbitrary current density provided other conditions are satisfied. By combining Eqs. (10) and (18) we can write the force equation as

$$\frac{d^2}{dt^2}\Delta z + \omega_p^2[\Delta z - \langle\Delta z\rangle_\phi] = a\cos(\Omega t - \beta\Delta z + \phi),$$
$$\omega_p^2 = \frac{e^2 N_0}{\epsilon_0 m\gamma^3}. \qquad (19)$$

ω_p is the relativistic plasma frequency at the electron density N_0. To solve for Δz, we consider the perturbation expansion in the limit of small radiation field

$$\Delta z = \Delta z^{(1)} + \Delta z^{(2)} + \cdots, \qquad (20)$$

where $\Delta z^{(n)}$ is the nth order deviation proportional to E^n. Substituting (20) into (19) and considering the self-consistency in ϕ, we find $\langle\Delta z\rangle_\phi$ contains only even-order terms. Therefore, $\langle\Delta z\rangle_\phi$ is at most a second-order effect. The solution is

$$\Delta z^{(1)} = \frac{eE/m\gamma^3}{\omega_p^2 - \Omega^2}\left[(\cos\Omega t - \cos\omega_p t)\cos\phi + \left(\sin\Omega t - \frac{\Omega}{\omega_p}\sin\omega_p t\right)\sin\phi\right] \qquad (21)$$

and

$$\Delta z^{(2)} = \frac{(eE/m\gamma^3)^2\beta}{4\omega_p(\omega_p^2-\Omega^2)^3} \times[(\omega_p+\Omega)^3\sin(\omega_p-\Omega)t - (\omega_p-\Omega)^3\sin(\omega_p+\Omega)t - 4\omega_p\Omega(\omega_p^2-\Omega^2)t], \qquad (22)$$

where, in $\Delta z^{(2)}$, only the part independent of ϕ is written explicitly.

The modulation of the electron position results in the modulation of the beam density which in turn can drive the radiation field according to Maxwell's equations. Since we are only interested in the energy gain of the radiation within the constant-field approximation, it is more convenient and straightforward to consider directly the energy exchange between the electron beam and the radiation. In the case where the Coulomb interaction is neglected, the energy loss of electrons is converted completely into the radiation energy. However, when Coulomb interactions are considered, the energy extracted from the beam must be distributed between the radiation and the space-charge field. Since only the increase in the radiation field is available as useful output, we must be able to calculate the increase in the space-charge-field energy and subtract it from the total energy lost by the beam.

The energy change of an electron in a single pass can be calculated from (6) with the integrand including the longitudinal space-charge field as well as the transverse radiation field

$$\vec{v}\cdot\vec{E} = v_\perp E_r + (v_0 + \Delta v_z)E_{cz}. \qquad (23)$$

If the ensemble average is taken before the integration is executed, we find immediately $\langle v_0 E_{cz}\rangle$ disappears:

$$\langle E_{cz}\rangle_\phi = -\frac{eN_0}{\epsilon_0}\langle \Delta z - \langle \Delta z\rangle_\phi\rangle_\phi \equiv 0 . \qquad (24)$$

We also find the energy loss due to $(\Delta v_z E_{cz})$ is

$$-\left|\frac{J}{e}\right| e\left\langle \int \Delta v_z E_{cz} dt\right\rangle_\phi = \left|\frac{J}{e}\right| \frac{e^2 N_0}{\epsilon_0}\left\langle \int \Delta v_z \Delta z\, dt\right\rangle_\phi$$
$$= \frac{c(eN_0)^2}{2\epsilon_0}\langle(\Delta z)^2\rangle_\phi$$
$$= \tfrac{1}{2}\epsilon_0 c\langle E_{cz}^2\rangle_\phi . \qquad (25)$$

To obtain the result in (25), we have neglected $\langle\Delta z\rangle_\phi$ in E_{cz} because it is at second order, which results in a third-order term in the energy exchange after multiplication by Δv_z. Physically, (25) shows that the energy loss due to $\Delta v_z E_{cz}$ is exactly equal to the space-charge energy. Therefore, the energy increase of the radiation comes exactly from the contribution of $v_\perp E_r$. With the explicit expression of $\Delta z^{(1)}$, we estimate roughly that the energy for the buildup of the space-charge field is only a very small part of the energy loss of the electron beam. Their ratio is $\sim\Omega/\omega$ or $\sim\lambda/l$, which is only 10^{-4} for the Stanford device.

Following a procedure similar to that used to derive the no-space-charge gain expression (9), we find that when we include the space-charge field the gain becomes

$$G(\theta,\theta_p) = G_0' \frac{\theta\theta_p^2}{(\theta_p^2-\theta^2)^2} \times\left[2 - 2\cos\theta_p\cos\theta - \left(\frac{\theta_p}{\theta}+\frac{\theta}{\theta_p}\right)\sin\theta_p\sin\theta\right], \qquad (26)$$

$$G_0' = G_0/\theta_p^2 , \quad \theta\equiv\Omega T , \quad \theta_p \equiv \omega_p T .$$

G_0' is a constant independent of θ and θ_p. This result is identical to the expression obtained by Gover and Livni.[11] It is interesting to note that the gain spectrum is almost the same in terms of either variable, θ or θ_p, although they have completely different physical meaning. θ_p indicates the electron density, while θ represents the velocity detuning from the resonance condition. The condition $\theta=\theta_p$ leads to a well-known phenomenon, "plasma resonance." However, it is approached for the first time from the single-particle point of view.

DISCUSSION

It is expected that the new expression for the gain (26) should reduce to (9) when the current density is very small. Indeed, if we let θ_p approach zero, we find

$$G^{(1)} = G_0'\theta_p^2 f(\theta) , \qquad (27)$$

where the superscript (1) indicates that the gain is proportional to the first power of the electron density. To obtain the lowest order correction to the collisionless gain, we expand (26) up to the order of θ_p^4 and find it to be

$$G^{(2)} = -G_0'\theta_p^4 g(\theta) ,$$
$$g(\theta) \equiv \frac{(24-6\theta^2)\cos\theta + (18\theta-\theta^3)\sin\theta - 24}{6\theta^5} . \qquad (28)$$

The result in (28) is identical to that obtained by Louisell[4] and Sprangle,[5] using the coupled Maxwell-Boltzmann equations. The fundamental spectrum $f(\theta)$ and the correction function $g(\theta)$ are shown in Fig. 2. Up to the first-order correction, the gain becomes smaller for $\theta < 4.6$. The nonuniform reduction results in an upshift of θ_{max}. The upshift is proportional to the electron density and can be written as

$$\Delta\theta_{max} = \frac{g'(\theta_{max})}{f''(\theta_{max})}\theta_p^2 . \qquad (29)$$

To demonstrate the phenomenon of gain saturation, the normalized gain $[G(\theta,\theta_p)/G_0'\theta_p^2]$ is plotted for different values of θ_p (Fig. 3). The reason why we normalize the gain with respect to the electron density (through θ_p^2) is to compare it with the gain in the no-space-charge situation, where it is proportional to the electron density. Therefore, the normalized gain for $\theta_p=0$ as shown in Fig. 3 corresponds to the case of the collisionless electron beam. In general, it is observed that the peak (normalized) gain decreases and shifts to the right with increasing electron density. Physically, the reduced gain is due to the repulsive force between the electrons which weakens the tendency of the electrons to bunch together. This reduces the beam alternating current which can couple to the electromagnetic field. The increase of θ_{max} with

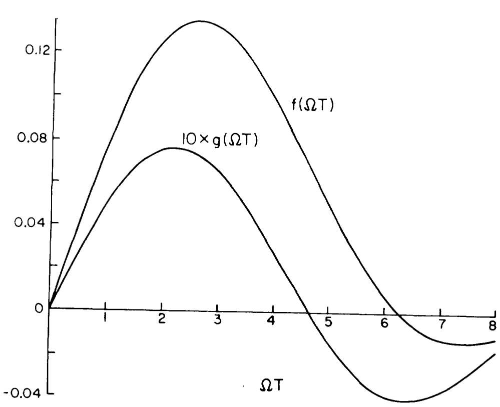

FIG. 2. The first-order space-charge correction $g(\Omega T)$ to the single-pass gain.

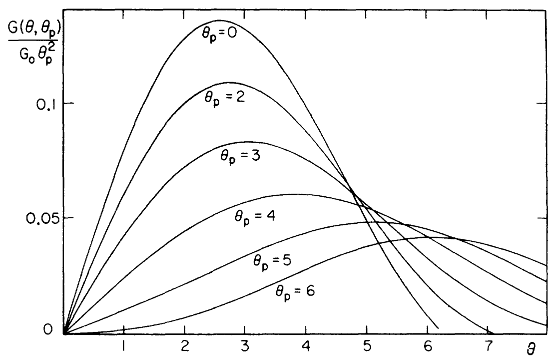

FIG. 3. Normalized gain $[G(\theta, \theta_p)/G_0'\theta_p^2]$ for different electron density (θ_p). The curve $\theta_p = 0$ corresponds to the case of no Coulomb interaction.

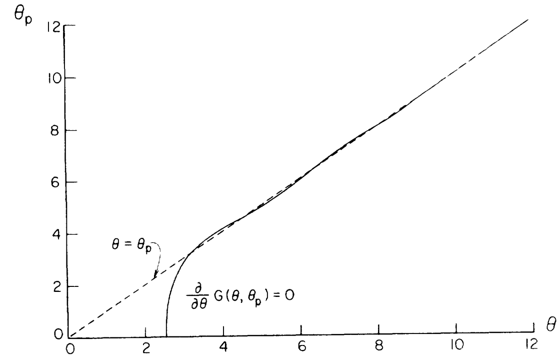

FIG. 4. The relation between the radiation frequency and the electron density in a free-electron laser is shown as the trace (solid curve) of maximum gain on the map (θ, θ_p). The dotted line indicates the condition for plasma resonance. The value $\theta_p = \pi$ serves as a critical boundary between the noninteracting ($\theta_p < \pi$) and the collective (or plasma $\theta_p > \pi$) regimes.

θ_p is due to increasing plasma frequency. In a practical device which is used as an amplifier, the radiation frequency is fixed by the input field. If the electron energy does not change (i.e., θ is a given constant), the normalized gain decreases very fast with the current density. If the electron energy is adjustable, we can choose θ to correspond to the value which yields the maximum gain. This reduces the effect of saturation. If the device is used as a laser oscillator, the radiation frequency adjusts itself automatically until the gain is maximum. It thus makes sense to study the effect of gain saturation by inquiring what happens to the peak gain as a function of the space-charge parameter θ_p.

The behavior of the maximum gain is easier to follow if we use an alternate expression for the gain

$$G(\theta, \theta_p) = \frac{G_0'\theta_p}{2}\left(\frac{1-\cos(\theta-\theta_p)}{(\theta-\theta_p)^2} - \frac{1-\cos(\theta+\theta_p)}{(\theta+\theta_p)^2}\right). \tag{30}$$

For given θ_p, the value of θ_{max} can be found from the solution of the equation

$$\frac{\partial}{\partial\theta} G(\theta, \theta_p) = \tfrac{1}{2} G_0' \theta_p[f(\theta+\theta_p) - f(\theta-\theta_p)] = 0\,, \tag{31}$$

where f is a function identical to the fundamental spectrum appearing in (9). It is obvious that $(\theta, \theta_p) \equiv (n\pi, m\pi)$ is always a solution of (31) whenever $(n \pm m)$ is an even integer number. The gain becomes a local maximum at these positions. Among those solutions, it can be observed that the solution $\theta = \theta_p$ leads to the overall maximum gain for given $\theta_p = m\pi$. The curve in Fig. 4 shows the trace of the maximum gain. It crosses the plasma resonance line $(\theta = \theta_p)$ whenever θ_p is a multiple of π, or, in general, is a solution of the equation $f(2\theta_p) = 0$.

The radiation frequency is determined in terms of the detuning parameter

$$\omega = 2\gamma^2 c\left(\frac{2\pi}{l} + \frac{\theta}{L}\right). \tag{32}$$

Therefore, the curve reveals clearly the transition of the radiation frequency from the noninteracting to the collective (plasma) regime. θ represents the deviation of the frequency from the "lattice frequency" $(2\pi c/l)$. At low electron density $(\theta_p \to 0)$, θ approaches the value of 2.6, where the field-interference process dominates. When θ_p begins to increase, θ approaches the value of θ_p very fast and starts to oscillate around the line $\theta = \theta_p$ with a decreasing oscillation amplitude

$$(\theta - \theta_p) \xrightarrow[\theta\to\infty]{} -\frac{3\sin 2\theta_p}{\theta_p^2}\,. \tag{33}$$

The point $\theta_p = \pi$ is then observed to serve as a critical boundary where the transition between two regimes occur. Above this point, the effect of plasma resonance dominates. The radiation frequency is then given by the sum of the lattice and plasma frequencies except for the small deviation (33).

We have shown the frequency shift due to the space-charge effect. The maximum gain along this curve is shown in Fig. 5 as a function of θ_p^2. In the limit of small electron density, the maximum gain is proportional to θ_p^2 ($G_{max} = 0.135\,G_0\theta_p^2$). When the beam density increases, the maximum gain begins to saturate with a smaller growth rate. However, there is no upper bound to the gain. In the limit of high electron density, the gain is proportional to the square root of the electron density ($G_{max} \to \frac{1}{4} G_0'\theta_p$).

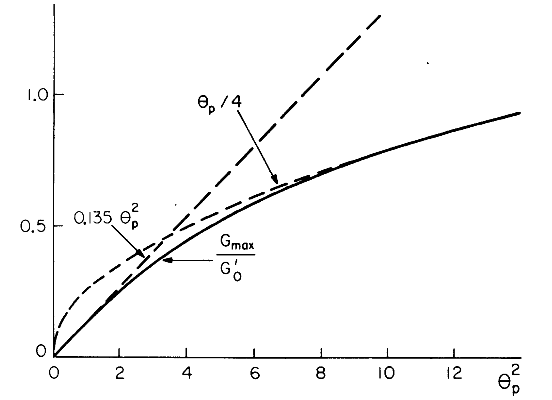

FIG. 5. The gain of a free-electron laser as a function of the electron density (θ_p^2). The gain grows linearly with θ_p^2 at low current densities and becomes saturated ($\rightarrow \frac{1}{4}\theta_p$) for higher densities.

We have performed all the quantitative analyses in terms of the dimensionless parameter θ_p. In order to get an appreciation of its value in a practical device, we write θ_p^2 as

$$\theta_p^2 = 7.382\,\frac{J(\mathrm{A/cm^2})L^2(m)}{\gamma^3}\,, \tag{34}$$

where J is the current density in the unit of amperes per centimeter squared. For the Stanford device used as an oscillator,[12] θ_p^2 is found to be about 0.1. Thus it is still far from the space-charge saturation.

If the pumping is strong, such that the gain is high enough, the increase of the field amplitude in the interaction region can no longer be neglected. In this case, a self-consistent treatment of the field is required, which is to be reported in detail elsewhere. However, the self-consistent method is important only when it is applied to a high-gain amplifier. For the analysis of a laser device using a resonator with high-reflectivity mirrors, the field amplitude increase per pass must equal the small mirror loss per pass, and the constant amplitude approximation is very good.

In summary, we have included the space-charge effect exactly in the single-electron analysis of free-electron lasers. The gain is found in (26) through the key equation (18). In the small signal and low gain region, the maximum gain is found to saturate at high electron densities and the radiation frequency approaches the condition of the plasma resonance.

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[1]N. M. Kroll and W. A. McMullin, Phys. Rev. A 17, 300 (1978).

[2]D. B. McDermott and T. C. Marshall, in *Physics of Quantum Electronics*, edited by S. F. Jacobs, H. S. Pilloff, M. Sargent III, M. O. Scully, and R. Spitzer (Addison-Wesley, Reading, Mass., 1980), Vol. 7.

[3]A. Gover, in *Physics of Quantum Electronics*, edited by S. F. Jacobs, H. S. Pilloff, M. Sargent III, M. O. Scully, and R. Spitzer (Addison-Wesley, Reading, Mass., 1980), Vol. 7.

[4]W. H. Louisell, J. F. Lam, and D. A. Copeland, Phys. Rev. A 18, 655 (1978).

[5]P. Sprangle and C. Tang, in Proceedings of the Fourth International Conference on Infrared and Near Millimeter Waves, Miami Beach, 1979.

[6]P. Sprangle, C. Tang, and W. M. Manheimer, Phys. Rev. Lett. 43, 1932 (1979); Phys. Rev. A 21, 302 (1980).

[7]P. Sprangle, C. Tang, and W. M. Manheimer, in *Physics of Quantum Electronics*, edited by S. F. Jacobs, H. S. Pilloff, M. Sargent III, M. O. Scully, and R. Spitzer (Addison-Wesley, Reading, Mass., 1980), Vol. 7.

[8]F. A. Hopf, P. Meystre, G. T. Moore, and M. O. Scully, in *Physics of Quantum Electronics*, edited by S. F. Jacobs, M. Sargent, and M. O. Scully (Addison-Wesley, Reading, Mass., 1978), Vol. 5.

[9]W. B. Colson, Ph.D. thesis, Stanford University, 1977 (unpublished).

[10]A. Yariv and C. Shih, Opt. Commun. 24, 233 (1978).

[11]A. Gover and Z. Livni, Opt. Commun. 26, 375 (1979).

[12]D. A. G. Deacon, L. R. Elias, J. M. J. Madey, G. I. Ramian, H. A. Schwettman, and T. I. Smith, Phys. Rev. Lett. 38, 892 (1977).