# Self-quenching of fundamental phase and amplitude noise in semiconductor lasers with dispersive loss

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We show theoretically that the incorporation of a frequency-dependent loss mechanism in a semiconductor laser can lead, in concert with the amplitude-to-phase coupling, to major reductions of the fundamental intensity and phase noise. A loss dispersion of the wrong sign, on the other hand, leads to an increase of the noise and, at a certain strength, to instability.

The reduction of the linewidth of semiconductor (SC) lasers is a topic of continuing interest. It was proposed[1,2] and demonstrated[3,4] that a narrowing of the laser field spectrum can be accomplished by coupling the SC laser to external resonators properly detuned. In this Letter we show that the physical mechanism responsible for spectral narrowing is fundamentally the incorporation of a frequency-dependent loss (or gain) mechanism.

To analyze the effect of the frequency-dependent loss we use the Van der Pol oscillator laser model that is driven by spontaneous emission.[5,6] The optical field is taken as

$$E(t) = [A_0 + \delta(t)]\exp\{i[\omega_m t + \phi(t)]\}, \tag{1}$$

where $\delta(t)$ and $\phi(t)$ are the noise-driven amplitude and phase excursions, respectively. The loss is represented by a frequency-dependent photon lifetime $\tau_p$,

$$\frac{1}{\tau_p} = \frac{1}{\tau_{p0}} + 2C\dot{\phi}. \tag{2}$$

This is the key ensatz. It stipulates that the loss rate $\tau_p^{-1}$ adjusts instantaneously to the (instantaneous) frequency $\omega_m + \dot{\phi}(t)$. The results of the analysis are thus limited to frequencies below the inverse of the (longest) propagation time involved in the system or the response time of resonant elements. This limitation still allows us to explore the spectral regions of greatest interest, typically 0 to 100 MHz.

Solving, as in Ref. 6, for the steady-state amplitude $A_0$ and frequency $\omega_m$ leads one to the fluctuations' equations,

$$\dot{\delta} + \omega_1\delta + A_0 C\dot{\phi} = \frac{\Delta_i(t)}{2\omega_m}, \quad A_0\dot{\phi} - \omega_1\alpha\delta = -\frac{\Delta_r(t)}{2\omega_m}, \tag{3}$$

where $\Delta_i$ and $\Delta_r$ are, respectively, the real and imaginary parts of the Langevin noise sources representing the spontaneous emission, and

$$\omega_1 \equiv -\frac{A_0^2\omega_m\chi_i^{(3)}}{n^2}, \quad \alpha = \frac{\chi_r^{(3)}}{\chi_i^{(3)}}, \qquad \chi_i^{(3)} < 0, \omega_1 > 0, \tag{4}$$

with $\omega_m$ the average frequency and $\chi_i^{(3)}$ and $\chi_r^{(3)}$ the imaginary and real part of the third-order amplifying medium susceptibility so that $\alpha$ is the amplitude-to-phase coupling factor. A formal solution of Eqs. (3) with $\delta(0) = 0$, $\phi(0) = 0$ leads to

$$\delta(t) = \frac{1}{2\omega_m}\left\{\int_0^t \Delta_i(\lambda)\exp[-\omega_1'(t-\lambda)]d\lambda + C\int_0^t \Delta_r(\lambda)\exp[-\omega_1'(t-\lambda)]d\lambda\right\}, \tag{5}$$

$$\phi(t) = \frac{1}{2\omega_m A_0}\left\{-\int_0^t \Delta_r(\lambda)\exp[-\omega_1'(t-\lambda)]d\lambda + \frac{\omega_1}{\omega_1'}\int_0^t [\alpha\Delta_i(\lambda) - \Delta_r(\lambda)]d\lambda - \frac{\omega_1}{\omega_1'}\int_0^t [\alpha\Delta_i(\lambda) - \Delta_r(\lambda)]\exp[-\omega_1'(t-\lambda)]d\lambda\right\}, \tag{6}$$

$$\omega_1' \equiv \omega_1(1 + C\alpha), \tag{7}$$

where $\omega_1'$ is seen to play the role of the fundamental decay rate of the fluctuations. This rate increases for $C\alpha > 0$. Taking first the case of $\omega_1' > 0$, we can neglect the decaying terms, i.e., terms with $\omega_1'(t - \lambda)$, in the exponent of Eq. (6) when calculating the correlation functions. The result is

$$\langle\phi(t_1)\phi(t_2)\rangle = \frac{W}{4A_0^2\omega_m^2}\frac{(1+\alpha^2)}{(1+C\alpha)^2}\min(t_1, t_2),$$

$$\langle\phi^2(\tau)\rangle = \frac{W(1+\alpha^2)}{4A_0^2\omega_m^2(1+C\alpha)^2}|\tau| \equiv 2K|\tau|, \tag{8}$$

where we used[6]

$$\langle\Delta_r(t_1)\Delta_r(t_2)\rangle = WD(t_1 - t_2),$$

$$\langle\Delta_i(t_1)\Delta_i(t_2)\rangle = WD(t_1 - t_2),$$

$$\langle\Delta_r(t_1)\Delta_i(t_2)\rangle = 0,$$

$$W = \frac{4\hbar\omega_m^3\left(\frac{N_2}{\Delta N}\right)}{\epsilon V\tau_{p0}}, \tag{9}$$

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and $D(t)$ is the delta function of $t$. $\epsilon$ is the dielectric constant, $V$ is the resonator volume, and $(N_2/\Delta N)$ is the inversion factor. Use of standard noise formalism[6] leads to

$$\langle E^*(t)E(t+\tau)\rangle = A_0^2 \exp[-1/2\langle\phi^2(\tau)\rangle] \times \exp(i\omega_m\tau) + \text{c.c.}, \quad (10)$$

which, combined with Eq. (8) and after use of the Wiener–Khintchine theorem, leads to the following expression for the (optical) field spectral density:

$$W_E(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \langle E^*(t)E(t+\tau)\rangle \exp(-i\omega\tau)\mathrm{d}\tau + \text{c.c.} = \frac{A_0^2}{\pi}\int_{-\infty}^{\infty} \exp\{[-i(\omega-\omega_m) - K]|\tau|\}\mathrm{d}\tau = \frac{\frac{2}{\pi}A_0^2 K}{(\omega-\omega_m)^2 + K^2}. \quad (11)$$

The laser field spectral width is thus $(\Delta\nu)_{\text{laser}} = K/\pi$,

$$(\Delta\nu)_{\text{laser}} = \frac{h\nu_m\left(\frac{N_2}{\Delta N}\right)(1+\alpha^2)}{P_e\tau_{p0}^2(1+C\alpha)^2} = (\Delta\nu)_{\text{S-T}}\frac{1+\alpha^2}{(1+C\alpha)^2}, \quad (12)$$

where $P_e = (\epsilon A_0^2 V)/\tau_{p0}$ is the total power emitted by the electrons and $(\Delta\nu)_{\text{S-T}}$ is the Schawlow–Townes linewidth. The astounding result is that for $C\alpha \gg 1$, $(\Delta\nu)_{\text{laser}} \to (\Delta\nu)_{\text{S-T}}/C^2$ so that the simultaneous presence of amplitude-to-phase coupling ($\alpha \neq 0$) and dispersive loss ($C \neq 0$) leads not only to relief from the onerous $(1+\alpha^2)$ factor but even to linewidth values that are substantially smaller than the original Schawlow–Townes value with $\alpha = 0$. This is illustrated in Fig. 1, where, for example, the point $\alpha = -3$, $C = -30$ shows a reduction from the $(\Delta\nu)_{\text{S-T}}$ value by a factor of 900.

Turning our attention to the intensity fluctuations, from Eqs. (5) and (9) we obtain

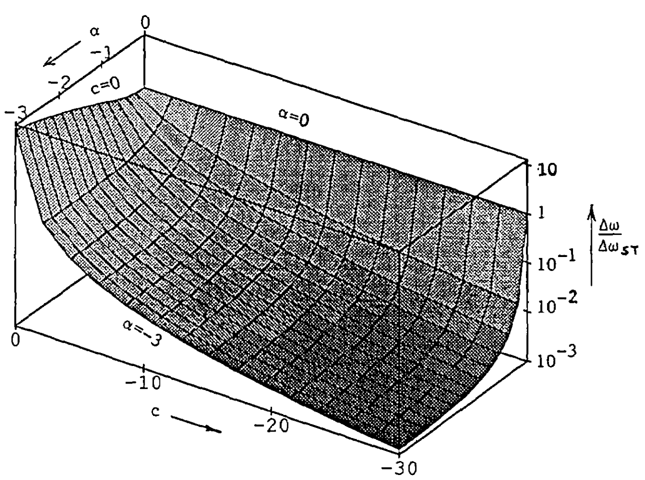


Fig. 1. Reduction factor $(\Delta\nu)_{\text{laser}}/(\Delta\nu)_{\text{S-T}} = (1+\alpha^2)/(1+C\alpha)^2$ as a function of $C$ and $\alpha$.

$$\langle\delta(t)\delta(t+\tau)\rangle = \frac{W}{8\omega_m^2\omega_1}\frac{(1+C^2)}{(1+C\alpha)}\exp(-\omega_1'|\tau|). \quad (13)$$

Use of $\Delta P = 2\epsilon V A_0 \delta(t)/\tau_p$ and the Wiener–Khintchine theorem yields

$$W_{\Delta P}(\Omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \langle\Delta P(t)\Delta P(t+\tau)\rangle \exp(-i\Omega\tau)\mathrm{d}\tau = \left(\frac{4}{\pi}\right)\frac{h\nu_m\left(\frac{N_2}{\Delta N}\right)P_e}{\tau_{p0}^2}\frac{(1+C^2)}{[\Omega^2 + \omega_1^2(1+C\alpha)^2]}. \quad (14)$$

The low-frequency spectral density of the power fluctuation $\Delta P$ is reduced relative to the case $C = 0$ by

$$\frac{W_{\Delta P}(0)(C \neq 0)}{W_{\Delta P}(0)(C = 0)} = \frac{(1+C^2)}{(1+C\alpha)^2}. \quad (15)$$

This reduction factor can approach a value of $\alpha^{-2}$ in the case $C^2 \gg 1$, $C\alpha \gg 1$. Since $\alpha$ is typically between 3 and 6, the reduction factor can become significant, since $C$ can be much larger than $\alpha$, though not so dramatic as the reduction $C^{-2}$ in linewidth. We note that here, as well as in the case of the laser linewidth, the noise reduction is brought about by the concerted action of loss dispersion ($C \neq 0$) and amplitude-to-phase coupling ($\alpha \neq 0$).

In the case $C\alpha \to -1$ it follows directly from the form of Eqs. (5) and (6) that the damping rate $\omega_1' \to 0$, so that intensity and frequency fluctuations do not decay. The laser field mode and its relation to instability and chaos are currently being investigated.

We have also considered the current modulation response of the laser by complementing Eqs. (3) (with $\Delta_r = 0$, $\Delta_i = 0$) with a third equation for the carrier density and a driving sinusoidal current.[6] The results show that the relaxation resonance frequency $\omega_R$, which is often taken as the maximum practical modulation frequency, increases for $C\alpha > 0$ by $(1+C\alpha)^{1/2}$, while the low-frequency portion of the response is unchanged. We also find that the attendant chirp broadening of the spectrum is reduced by a factor $(1+C\alpha)$.

To estimate the all-important $C$ parameter, we considered two cases. The first involves placing an atomic vapor absorption cell inside the resonator. Substantial values of $C \approx 10$ are indicated.

The second example involves the weak coupling of the semiconductor laser to an external high-finesse Fabry–Perot étalon. This configuration was demonstrated by Dahmani *et al.*[4] to lead to great (1000×) reduction in the laser linewidth. If we consider the external resonator, properly detuned, as a frequency-dependent loss element, it is an easy and short task to show that the $C$ parameter [see Eq. (2)] representing the extra loss coupled by the resonator into the SC laser is accounted for by

$$C = \frac{c\sqrt{\beta}(1-R_d)}{2\pi n L_d \Delta\nu_{\text{ex}}}, \quad (16)$$

where $\beta$ is the intensity spatial feedback factor (not including the SC laser facet reflectivity), $R_d$ is the SC laser facet reflectivity, $L_d$ and $L_{ex}$ are the SC laser length and Fabry–Perot cavity length, respectively, $n$ is the SC laser index of refraction, and $\Delta\nu_{ex}$ is the frequency width of the reflection peak of the external resonator. Using the reported values of $\beta \sim 10^{-5}$, $\Delta\nu_{ex} \sim 5 \times 10^6$ Hz, $L_d \sim 3 \times 10^{-2}$ cm, and $R_d \sim 0.3$, we calculate $C \sim 42$. The expected linewidth narrowing according to Eq. (12) is $\sim C^2 \approx 1750$. The observed narrowing of ~1000 is consistent with this estimate.

A recent detailed analysis by Laurent *et al.*[7] of the Dahmani *et al.* configuration using a coupled-cavity formalism as in Ref. 2 leads to a similar prediction. This shows that the operative physical principle in the configuration of Ref. 4 is the frequency-dependent loss.

A major challenge would be to incorporate a narrow-linewidth frequency-dependent loss in a monolithic fashion with the SC laser.

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