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Low-Complexity Power Allocation for Network-Coded User Scheduling in Fog-RANs

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***Abstract*—Consider a Fog Radio Access Network (FRAN) in which a cloud base station (CBS) is responsible for scheduling user-equipments (UEs) to a set of radio resource blocks (RRBs) of Fog Access Points (F-APs) and for allocating power to the RRBs. The conventional graphical approach for solving the coordinated scheduling and power control problem in FRAN requires prohibitive computational complexity. This letter, instead, proposes a low-complexity solution to the problem under the constraint that all the scheduled UEs can decode the requested files sent by their associated RRBs/F-APs. Unlike previous solution that requires constructing the total power control graph, the proposed computationally efficient solution is developed using a single power control subgraph. Numerical results reveal a close-to-optimal performance of the proposed method in terms of throughput maximization for correlated channels with a significant reduction in the computational complexity.**

***Index Terms*—Coordinated scheduling, fog radio access networks, file streaming, network coding, power allocation.**

I. Introduction

THE diversity of users' demand combined with the growth of streaming applications, such as video-on-demand, skyrocketed the wireless data traffic by many folds [1]. A handful of popular content, such as multimedia streaming and video games, dominates most other data traffic. Therefore, higher throughput can be achieved by leveraging the cooperation and coordination between Fog Access Points (F-APs) in a centralized Fog Radio Access Network (FRAN). FRAN has been introduced to exploit both edge caching and Cloud RAN (CRAN) for carrying out data delivery effectively [2], [3]. Furthermore, combined with network coding (NC), FRAN offers a reliable and fast solution to streaming the requested data by efficiently utilizing its available radio resource blocks (RRBs), e.g., see [4], [5]. In this work, we consider a simple and efficient FRAN's design where the scheduling-level coordination takes place, i.e., each user-equipment (UE) is only scheduled to a single F-AP, but possibly to many RRBs in the same F-AP's frame [4], [5].

Instantly Decodable Network Coding (IDNC), a subclass of NC, has been shown to be promising for significantly maximizing throughput, minimizing data transmission time, and offloading the system's resources for real-time applications [4]–[7]. Besides its simple encoding/decoding operations, IDNC is further combined with rate adaptation, called Rate-Aware IDNC (RA-IDNC) [4], to enable the design of network models with cost and power-efficient UEs. We are interested in the joint optimization of selecting the power level of each RRB and IDNC files that are decodable to a significant set of UEs.

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The throughput maximization problem in FRANs/CRANs has been considered in several works, e.g., [8]–[11]. These solutions can be categorized into uncoded [8], [9], and RA-IDNC [10], [11] methods. As their name indicates, the uncoded approach does not utilize NC and assigns a single UE to each RRB. This requires a number of RRBs that is equal to the number of UEs in the network. However, it has been noticed that UEs tend to have a common interest in downloading the same files, especially videos, within a small interval of time [11]. By exploiting such UEs' common interest using NC, work [11] efficiently utilizes the RRBs to maximize the throughput by mixing files. This leads to serving multiple UEs simultaneously on the same RRB.

The aforementioned work [11] showed that the joint approach provides significant performance gain as compared to existing schemes. However, its complexity increases significantly with the number of NC combinations, F-APs, and RRBs, which may not be feasible for massive networks. Notably, the solution requires generating multiple power control subgraphs whose union gives the total power control graph. In this work, we develop a low-complexity solution that involves the construction of a single power control subgraph.

Motivated by the limitations of earlier works mentioned above, this work develops a low complexity solution that optimizes the power allocation of RRBs/F-APs. In particular, we demonstrate the trade-off between the throughput performance improvement and the achieved computational complexity reduction of our proposed scheme for perfectly correlated and un-correlated channels. Besides, our work develops an efficient power allocation algorithm for each network-coded UE scheduling. As a result, all the scheduled UEs can decode the requested files sent by their associated RRBs/F-APs. With a single power control subgraph and efficient power allocations of RRBs/F-APs, our solution offers similar performance of the scheme in [11] for perfectly correlated channels. For the un-correlated channels, our proposed scheme has certain degradation in the throughput performance as compared to the one in [11] (a maximum degradation of 27%). This degradation in the performance of the solution comes at the achieved significant reduction of computational complexity since it is not related anymore to the number of RRBs and their corresponding NC combinations.

II. System Model and Problem Formulation

A. System Model and Parameters

Consider an FRAN system consisting of a cloud base station (CBS) that is connected to K F-APs, denoted by $\mathcal{K} = \{1, 2, \cdots, K\}$, through low-rate fronthaul links. The F-APs are cooperating in streaming a frame of popular files $\mathcal{F} = \{1, 2, \cdots, F\}$ of F source files to single-antenna N UEs, denoted by the set $\mathcal{N} = \{1, 2, \cdots, N\}$. Popular files in this letter represent frames from a video-on-demand. The frame is fixed at any arbitrary scheduling epoch. The entire frame is available at the CBS, whereas F-APs can only store μF files, where $0 \leq \mu \leq 1$ is the caching ratio, i.e., $|\mathcal{H}_k| = \mu F$ is the files cached by F-AP k. At any arbitrary scheduling epoch, all files are assumed to be proactively cached at the F-APs. The CBS is responsible for making the NC scheduling and power allocation decisions and informing the decisions to the F-APs for execution. Different groups of UEs are interested in different subsets of $\mathcal{F}$.

Each F-AP has a single antenna and can transmit a frame consists of Z orthogonal time/frequency RRBs, denoted by the set $\mathcal{Z} = \{1, 2, \cdots, Z\}$. The transmission power level of the k-th F-AP at the z-th RRB is denoted as P_{kz}. The received data rate (in bits/s/Hz) at the n-th UE from the z-th RRB and the k-th F-AP can be expressed as $R_{n,kz} = \log_2(1 + \frac{P_{kz}\gamma_{n,kz}}{1+\sum_{m=1,m\neq k}^{K} P_{mz}\hat{\gamma}_{n,mz}})$, where $\gamma_{n,kz}$ is the channel-gain-to-noise-ratio (CNR) between the k-th F-AP at the z-th RRB and the n-th UE, and $\hat{\gamma}_{n,mz}$ is the interfering-channel-gain-to-noise-ratio (INR) between the m-th F-AP at the z-th RRB and the n-th UE. From the above, we can see that the interference at the z-th RRB is experienced only from the z-th RRB on the other F-APs. Similar to [12], we consider non-frequency selective channels. If the RRB represents a time slot, the duration of the transmission frame is within the channel coherence time. Therefore, we have $\gamma_{n,kz} = \gamma_{n,k}$ and $R_{n,kz} = R_{n,k}$, $\forall n \in \mathcal{N}, k \in \mathcal{K}, z \in \mathcal{Z}$. For the sake of performance comparison, we also consider the case of frequency-selective channels in the numerical results. We consider that the reception of an uncoded/encoded file sent from the k-th F-AP is successful at the n-th UE if $R_k \leq R_{n,k}$.

At any arbitrary time epoch, UEs have downloaded some files and are interested in streaming a set of files. This can be represented by the side information of the n-th UE as follows:

- The *Has* set $\mathcal{H}_n$: Previously received files by the n-th UE.
- The *Lacks* set $\mathcal{L}_n = \mathcal{F} \backslash \mathcal{H}_n$: Non-requested files.
- The *Wants* set $\mathcal{W}_n \subset \mathcal{L}_n$: Files requested by the n-th UE in the current scheduling frame.

B. Problem Formulation

The problem of streaming a set of accessible files from the RRBs of the F-APs that maximizes throughput is equivalent to schedule UEs to the RRBs in the F-APs and adapt the power levels, where each UE can stream its requesting files from only one F-AP, but possibly from many RRBs in that F-AP.

Let κ_k denote a combination of some source files in $\mathcal{F}$ that is transmitted from the k-th F-AP. The set of the targeted UEs by the k-th F-AP is provided as $\tau_k = \{n \in \mathcal{N} \,||\kappa_k \cap \mathcal{W}_n| = 1 \,\&\, R_k \leqslant R_{n,k}\}$, where R_k is the minimum adopted transmission rate of the k-th F-AP. Let the binary variable $X_{n,k}$ be 1 if the n-th UE is assigned to the k-th F-AP. The throughput maximization optimization problem considered in this work can then be formulated as follows

$$\begin{aligned}
\mathcal{P}_1 : \max_{\substack{X_{n,k} \in \{0,1\} \\ \tau_k, \kappa_k, P_k}} & \sum_{k} \sum_{n \in \tau_k} X_{n,k} \left(\min_{n \in \tau_k} R_{n,k} \right) \\
\text{s.t. } & \sum_{k} X_{n,k} \leqslant 1, \ \forall n \in \mathcal{N}, \\
& \kappa_k \subseteq \mathcal{P}(\mathcal{H}_k), \ \forall k \in \mathcal{K}, \\
& 0 \leq P_k \leq P_{\max},
\end{aligned}$$

where the optimization is over the variables $X_{n,k}$, κ_k, R_k, and P_k. The first constraint ensures assigning the same UE to only one F-AP and the second constraint translates the caching limitation of each F-AP. The third constraint bounds the maximum transmit power of each F-AP.

The problem $\mathcal{P}_1$ is a mixed continuous and discrete optimization problem, thus it is NP-hard problem in general [11]. Solving it requires constructing all power control subgraphs and finding the maximum weight clique problem over these subgraphs. In order to seek an efficient and simple solution to $\mathcal{P}_1$, we use a graph representation. Specifically, $\mathcal{P}_1$ is shown to be equivalent to a maximum-weight vertex search problem as will be explained in the next section.

III. Network Coding Scheduling and Power Optimization

In this section, we first describe a single power control subgraph for a specific RRB. Then, we propose a low complexity, yet, efficient power allocation solution to the problem in $\mathcal{P}_1$.

A. Power Control Subgraph

Let $\mathcal{M}_k$ denote the set of all possible associations between UEs and files that cached by the k-th F-AP, i.e., $\mathcal{M}_k = \mathcal{N} \times \mathcal{H}_k$, and s is an element in $\mathcal{M}_k$. For simplicity, $n(s)$ represents the n-th UE in the s-th association. The set of the possible IDNC file combinations $\mathcal{M}_{k,\text{IDNC}}$ is generated by combining two distinct associations $(s, s') \in \mathcal{M}_k$ if the corresponding UEs can be served simultaneously by one IDNC file which translates to one of the following conditions being satisfied:

- **C1**. $n(s) \neq n(s')$ and $f(s) = f(s')$.
- **C2**. $n(s) \neq n(s')$ and $f(s) \in \mathcal{H}_{n(s')}$ and $f(s') \in \mathcal{H}_{n(s)}$.

Let $\mathcal{A}_k$ be the set of all possible associations between IDNC file combinations and achievable capacities of the k-th F-AP, i.e., $\mathcal{A}_k = \mathcal{M}_{k,\text{IDNC}} \times \mathcal{R}$. For example, one possible association in $\mathcal{A}_k$ is (s, R) which represents the IDNC combination s and the rate R. Basically, each element in $\mathcal{A}_k$ contains combined associations of $s \in \mathcal{M}_k$ representing an IDNC combination s that is transmitted from F-AP k with its corresponding rate R_k. The set of all possible associations of the whole system is simply the union of the possible associations of all F-APs, i.e., $\mathcal{A} = \bigcup_{k \in \mathcal{K}} \mathcal{A}_k$.

The power control subgraph of the z-th RRB in the network is denoted by $\mathcal{G}_z(\mathcal{V}_z, \mathcal{E}_z)$ wherein $\mathcal{V}_z$ and $\mathcal{E}_z$ refer to the set of vertices and edges of this subgraph, respectively. A vertex $v \in \mathcal{V}_z$ is generated by merging all possible associations $s \in \mathcal{A}$ for the different F-APs under the system constraint that each UE schedules to only one F-AP, i.e., $k(s) \neq k(s')$ and $n(s) \neq n(s')$ for all $s \neq s' \in \mathbf{S}$. Thus, $\mathbf{S}$ is a feasible schedule. Therefore, each $v \in \mathcal{V}_z$ representing the associations $(s, s') \in \mathbf{S}$ satisfies the following local conditions **(LC)**.

- **LC1**. $k(s) = k(s')$ and $r(s) = r(s')$.
- **LC2**. $k(s) \neq k(s')$ and $\tau \cap \tau' = \varnothing$.

The weight of each vertex is the weighted sum-throughput of the represented associations, i.e., the weight of vertex v associated with the schedule $\mathbf{S}$ is

$$w(v) = \sum_{s \in \mathbf{S}} \min_{n(s) \in \tau_{k(s)}} R_{n(s),k(s)}. \tag{1}$$

The following theorem characterizes the solution of the joint coordinated scheduling and power control problem $\mathcal{P}_1$ in the power control subgraph.

Theorem 1. *Let $\mathcal{G}_z(\mathcal{V}_z, \mathcal{E}_z)$ be the power control subgraph for an arbitrary RRB z, and let $v = \{s_1, \cdots, s_K\}$ be the vertex with the highest weight among all other vertices in $\mathcal{G}_z$. The feasible schedule is the one that schedules same UEs to all RRBs in the same F-AP's frame and the corresponding sum throughput is given by $w(v)Z$, where $w(v)$ is given by (1).*

Proof. Let $\mathcal{C}^* = \{v_1, \cdots, v_K\}$ be the maximum weight clique of size Z, and let $\mathbf{C}$ is the set of all cliques in the total power control graph in [11]. Notably, each vertex of the maximum weight clique belongs to the different RRBs in the total power control graph. Without loss of generality, let us assume that $v_z \in \mathcal{G}_z$. Then, the maximum weight clique problem can be written as:

$$w(\mathtt{C}^*) = \max_{\substack{v_z \in \mathcal{G}_z \\ \mathtt{C} \in \mathbf{C}}} w(\mathtt{C}) = \max_{\substack{v_z \in \mathcal{G}_z \\ \mathtt{C} \in \mathbf{C}}} \sum_{z \in \mathcal{Z}} w(v_z) = \sum_{z \in \mathcal{Z}} w(v_z^*) = Z w(v_z^*), \tag{2}$$

where v_z^* is the vertex with the maximum weight in $\mathcal{G}_z$. The above equality holds because of the assumption that $\gamma_{n,kz} = \gamma_{n,k}, \forall n \in \mathcal{N}, k \in \mathcal{K}, z \in \mathcal{Z}$. In fact, by (1), the weight of each vertex in the power control subgraph depends solely on $R_{n(s),k(s)z(s)}$ of the scheduled UEs represented by that vertex. With the assumption of slow-varying channels, scheduling the same UEs to different RRBs will have the same $R_{n(s),k(s)z(s)}$, and thus the corresponding vertices will have the same weight. In mathematical terms, since $R_{n(s),k(s)k(s)} = R_{n(s),k(s)}$ for all the scheduled UEs, $w(v_z) = w(v_{z'})$. Specifically, the vertex with the maximum weight represents the same scheduling at the different RRBs. This concludes that the feasible schedule is the one that schedules UE $n(s)$ to all RRBs in the same F-AP's frame and the resulted total sum throughput is given by $w(v)Z$. ■

B. Power Control Optimization

In this section, we solve the power control optimization problem for each vertex v in the power control subgraph that is associated with schedule $\mathbf{S}$. Let P_k^* denotes the optimal transmission power of the k-th F-AP. Our goal is to obtain sub-optimal F-AP power allocation vectors, denoted by $\{P_1^*, P_2^*, \cdots, P_K^*\}$, for a given network-coded UE scheduling $\mathbf{S}$[1]. The set $\mathbf{S} = \{\mathtt{S}_1, \mathtt{S}_2, \cdots, \mathtt{S}_K\}$ denotes a feasible schedule that is represented by a vertex v, and $\mathtt{S}_k$ denotes the set of associations representing scheduled UEs to receive files from the k-th F-AP. The power allocation problem is formulated as an optimization problem of maximizing the sum-rate of the F-AP of any vertex. As such, all the scheduled UEs can decode the files sent by their associated F-APs. The F-AP power allocations of a vertex v are obtained from the following optimization problem

$$\begin{aligned} \mathcal{P}_2: \quad & \max_{P_1, P_2, \cdots, P_K} \sum_{k=1}^{K} \min_{n \in \mathtt{S}_k} R_{n,k} \\ & \text{s.t.} \quad 0 \le P_k \le P_{\max}, \forall k = 1, 2, \cdots, K. \end{aligned} \tag{3}$$

[1]For notation simplicity, the RRB index z is omitted from the power and rate terms as we deal only with one RRB across all F-APs.

In this work, we derive sub-optimal yet efficient solution to $\mathcal{P}_2$ by searching the set of UEs having minimum rate, denoted by the minimum rate UEs (MRU), from all the possible combinations of UEs.

For the convenience of the ensuing analysis, we first define the properties and utility function of a valid MRU set. An MRU set, denoted by $\widehat{U}$, has the following properties:

***P*1**. $\forall m, n \in \widehat{U}$ where $m \neq n$, $(m, n) \notin \mathtt{S}_i$, $\forall i = 1, 2, \cdots, K$. This property suggests that that any two different UEs in the $\widehat{U}$ set must be associated with two different F-APs.

***P*2**. $\left|\widehat{U}\right| \le K$.

***P*3**. If $n \in \widehat{U} \cap \mathbf{S_k}$, $\forall \tilde{n} \in \mathtt{S}_k$ and $\tilde{n} \neq n$, $R_{\tilde{n},k} \ge R_{n,k}$.

The optimal rate vector of the $\widehat{U}$ set is obtained as the solution to the following optimization problem.

$$\mathcal{P}_3: \quad \left\{R^*_{\widehat{U}(1),1}, \cdots, R^*_{\widehat{U}(K),K}\right\} = \max_{\{P_k\} \in [0, P_{\max}]} \sum_{k=1}^{K} R_{\widehat{U}(k),K}.$$

A near-optimal solution to $\mathcal{P}_3$ can be obtained by satisfying the first-order optimality condition. In particular, by differentiating the objective function with respect to $\{P_1, P_2, \cdots, P_K\}$, K polynomial equations can be obtained. By solving these K polynomial equations and projecting the solution to the feasible space, a near-optimal solution to $\mathcal{P}_3$ can be readily obtained. Due to the brevity, the detailed analysis is omitted. The utility function of the MRU set, $\widehat{U}$, is defined as $\mathcal{R}_{\widehat{U}} = \sum_{k=1}^{K} R^*_{\widehat{U}(k),k}$.

Proposition 1. *For a given network-coded UE scheduling, let, K' MRU sets, i.e., $\widehat{U}_1$, $\widehat{U}_2$, $\widehat{U}_3$, $\cdots$, $\widehat{U}'_K$ be available with utility functions $\mathcal{R}_{\widehat{U}_1}$, $\mathcal{R}_{\widehat{U}_2}$, $\mathcal{R}_{\widehat{U}_3}$, $\cdots$, $\mathcal{R}_{\widehat{U}'_K}$, respectively. The near-optimal solution to $\mathcal{P}_2$ is obtained as*

$$\{P_1^*, P_2^*, \cdots, P_K^*\} = \arg\max \left\{\mathcal{R}_{\widehat{U}_1}, \mathcal{R}_{\widehat{U}_2}, \cdots, \mathcal{R}_{\widehat{U}'_K}\right\}. \tag{4}$$

Proof. Without loss of generality, we consider a simplified scenario having only two F-APs, and only two UEs scheduled to receive files from each F-AP. For such a special case, the optimization problem in $\mathcal{P}_2$ is reduced as

$$\begin{aligned} \mathcal{P}_1(a): \quad & \max_{P_1, P_2} \sum_{k=1}^{2} \min_{n \in \mathtt{S}_k} R_{n,k} \\ & \text{s.t.} \quad 0 \le P_k \le P_{\max}, \forall k = 1, 2. \end{aligned} \tag{5}$$

Here, $\mathtt{S}_k = \{\mathtt{S}_k(1), \mathtt{S}_k(2)\}$ is the set of the UEs scheduled to received files from the k-th F-AP where $k = 1, 2$. We can equivalently write $\mathcal{P}_1(a)$ as

$$\begin{aligned} \mathcal{P}_1(b): \quad & \max_{P_1, P_2, x, y} x + y \\ & \text{s.t.} \quad R_{\mathtt{S}_1(1),1} \ge x, R_{\mathtt{S}_1(2),1} \ge x, R_{\mathtt{S}_2(1),2} \ge y, \\ & R_{\mathtt{S}_2(2),2} \ge y, \quad 0 \le \{P_1, P_2\} \le P_{\max}. \end{aligned} \tag{6}$$

Note that for the given $\{P_1, P_2\}$, $\mathcal{P}_1(b)$ is a strict concave optimization problem, and accordingly it has a zero duality gap. The dual optimization problem to $\mathcal{P}_1(b)$ is given as $\min_{\boldsymbol{\lambda}, \boldsymbol{\mu} \ge 0} \max_{x,y} \mathcal{L}$ where $\mathcal{L}$ is defined as

$$\begin{aligned} \mathcal{L} = & x(1 - \lambda_1 - \lambda_2) + y(1 - \mu_1 - \mu_2) + \lambda_1 R_{\mathtt{S}_1(1),1} + \\ & \lambda_2 R_{\mathtt{S}_1(2),1} + \mu_1 R_{\mathtt{S}_2(1),2} + \mu_2 R_{\mathtt{S}_2(2),2} \end{aligned} \tag{7}$$

and where $\boldsymbol{\lambda}, \boldsymbol{\mu}$ are the Lagrangian multipliers. Obviously, at the optimality, $\lambda_1 + \lambda_2 = 1$ and $\mu_1 + \mu_2 = 1$ have to be satisfied.

Let, $\lambda_1 = \lambda \in (0,1)$ and $\mu_1 = \mu \in (0,1)$, and consequently, $\lambda_2 = 1-\lambda$ and $\mu_2 = 1-\mu$. Therefore, $\mathcal{P}_1(b)$ can be equivalently written as $\mathcal{P}_1(c)$ given at the top of the next page. We observe that $\mathcal{P}_1(c)$ contains both inner and outer optimization problems. For the inner optimization problem, the values of P_1 and P_2 are given, and as a result, we will obtain only one of the four possible cases: C1: $R_{\mathtt{S}_1(1),1} \leq R_{\mathtt{S}_1(2),1}, R_{\mathtt{S}_2(1),2} \leq R_{\mathtt{S}_2(2),2}$; C2: $R_{\mathtt{S}_1(2),1} \leq R_{\mathtt{S}_1(1),1}, R_{\mathtt{S}_2(1),2} \leq R_{\mathtt{S}_2(2),2}$; C3: $R_{\mathtt{S}_1(1),1} \leq R_{\mathtt{S}_1(2),1}, R_{\mathtt{S}_2(2),2} \leq R_{\mathtt{S}_2(1),2}$; and C4: $R_{\mathtt{S}_1(2),1} \leq R_{\mathtt{S}_1(1),1}, R_{\mathtt{S}_2(2),2} \leq R_{\mathtt{S}_2(1),2}$. Consequently, for the given P_1 and P_2, the inner optimization problem can be solved as

$$\min_{\lambda,\mu} \lambda R_{\mathtt{S}_1(1),1} + (1-\lambda) R_{\mathtt{S}_1(2),1} + \mu R_{\mathtt{S}_2(1),2} + (1-\mu) R_{\mathtt{S}_2(2),2}$$
$$= \begin{cases} R_{\mathtt{S}_1(1),1} + R_{\mathtt{S}_2(1),2} & \text{If C1 holds;} \\ R_{\mathtt{S}_1(2),1} + R_{\mathtt{S}_2(1),2} & \text{If C2 holds;} \\ R_{\mathtt{S}_1(1),1} + R_{\mathtt{S}_2(2),2} & \text{If C3 holds;} \\ R_{\mathtt{S}_1(2),1} + R_{\mathtt{S}_2(2),2} & \text{If C4 holds;} \end{cases}$$

Essentially, we obtain maximum four possible power allocations for the F-APs as follows

$$P_1^{(\mathrm{C1})}, P_2^{(\mathrm{C1})} = \arg \max_{P_1,P_2 \in [0,P_{\max}]} R_{\mathtt{S}_1(1),1} + R_{\mathtt{S}_2(1),2} \quad (8a)$$

$$P_1^{(\mathrm{C2})}, P_2^{(\mathrm{C2})} = \arg \max_{P_1,P_2 \in [0,P_{\max}]} R_{\mathtt{S}_1(2),1} + R_{\mathtt{S}_2(1),2} \quad (8b)$$

$$P_1^{(\mathrm{C3})}, P_2^{(\mathrm{C3})} = \arg \max_{P_1,P_2 \in [0,P_{\max}]} R_{\mathtt{S}_1(1),1} + R_{\mathtt{S}_2(2),2} \quad (8c)$$

$$P_1^{(\mathrm{C4})}, P_2^{(\mathrm{C4})} = \arg \max_{P_1,P_2 \in [0,P_{\max}]} R_{\mathtt{S}_1(1),1} + R_{\mathtt{S}_2(2),2} \quad (8d)$$

The power allocations provided by (8a), (8b), (8c), and (8d) will be feasible if and only if the obtained power allocations satisfy the aforementioned C1, C2, C3, and C4 conditions, respectively. If more than one power allocations in (8a)-(8d) are feasible, a near-optimal F-AP power allocation will be the one that provides the maximum objective value in (8a)-(8d). Note that, in the considered scenario, maximum four possible MRU sets are possible, such as, $\widehat{U}_1 = \{\mathtt{S}_1(1), \mathtt{S}_2(1)\}$, $\widehat{U}_2 = \{\mathtt{S}_1(2), \mathtt{S}_2(1)\}$, $\widehat{U}_3 = \{\mathtt{S}_1(1), \mathtt{S}_2(2)\}$, and $\widehat{U}_4 = \{\mathtt{S}_1(2), \mathtt{S}_2(2)\}$. The optimal objective values in (8a), (8b), (8c), and (8d) become the utility functions of the MRU sets $\widehat{U}_1$, $\widehat{U}_2$, $\widehat{U}_3$, and $\widehat{U}_4$, respectively. Accordingly, the near optimal F-AP power allocations are obtained as $\{P_1^*, P_2^*\} = \arg\max \left\{\mathcal{R}_{\widehat{U}_1}, \cdots, \mathcal{R}_{\widehat{U}_4}\right\}$. Evidently, (4) holds for a simplified scenario consisting of two F-APs and two UEs per F-AP. With a straightforward extension, we can justify that for a general scenario consisting of K F-APs and N_1, N_2, $\cdots$, N_K scheduled UEs to these F-APs, the total number of possible MRU sets is bounded by $K' \leq N_1 N_2 \cdots N_K$. By inspecting the utility function of these MRU sets, we can readily identify the MRU set having the maximum utility function. Subsequently, the near optimal F-AP power allocations will be the power allocation of the MRU set of the maximum utility function. Essentially, (4) holds for any number of F-APs and scheduled UEs per F-APs. ■

By capitalizing the Proposition 1, we develop Algorithm 1 in order to determine the F-AP power allocations for a given network coded UE scheduling.

Algorithm 1 RRH Power Allocation Procedure

Input: Network coded UE scheduling, $\mathbf{S} = \{\mathtt{S}_1, \mathtt{S}_2, \cdots, \mathtt{S}_K\}$.
Initialize: $\mathcal{U} = \phi$, $K' = 1$;
repeat
  Construct a UE vector, $\widehat{U}'_K$, by selecting a single UE from each of the $\mathtt{S}_1, \mathtt{S}_2, \cdots, \mathtt{S}_K$ sets.
  Evaluate P3 to determine the rate vector of the constructed UE vector.
  If the property $\boldsymbol{P3}$ is satisfied, add $\widehat{U}'_K$ to $\mathcal{U}$, and store the corresponding power allocation vector.
  $K' = K' + 1$;
until $K' \leq |\mathtt{S}_1| \times |\mathtt{S}_2| \times \cdots |\mathtt{S}_K|$.
Using $\mathcal{U}$ as the collection of all the MRU sets, evaluate (4).
Output: $\{P_1^*, P_2^*, \cdots, P_K^*\}$ for a given NC UE scheduling.

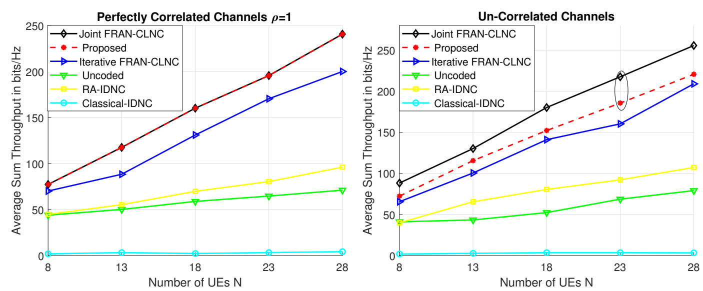

Fig. 1: Average sum throughput in bits/Hz versus a large number of UEs.

C. Complexity Analysis

Here, we analyze the complexity of the proposed solution to be compared with the complexity of the existing algorithm in [11]. To that end, the computational complexity of ${}^{m}P_{k}$ Algorithm 1 that solves the power allocation problem (4) is $C_p = O\left(|\mathtt{S}_1| \times |\mathtt{S}_2| \times \cdots |\mathtt{S}_K|\right)$. Based on the analysis in [11], the complexity of generating all vertices in a single power control subgraph is $O({}^{|\mathcal{C}|}P_K)$ and finding the maximum weigh clique over that graph is also $O\left({}^{|\mathcal{C}|}P_K\right)$. Therefore, the overall complexity of our proposed solution is $O\left(\mathcal{C}_p\, {}^{|\mathcal{C}|}P_K\right)$. In contrast, The overall complexity of the solution in [11] is $O\left(C_p\, {}^{|\mathcal{C}|}P_K + ({}^{|\mathcal{C}|}P_K Z)^2\right) = O\left(({}^{|\mathcal{C}|}P_K Z)^2\right)$. This high complexity is due to generating all NC combinations, constructing all power control subgraphs and connecting their corresponding vertices by edges.

IV. Numerical Results

In this section, we compare the throughput maximization performance of the proposed low-complexity FRAN-cross layer NC (CLNC) algorithm against the following existing algorithms.

- Joint FRAN-CLNC that solves the throughput maximization by constructing all power control subgraphs [11].
- Iterative FRAN-CLNC that solves the coordinated scheduling and power control individually and iteratively.
- Optimal uncoded that assigns only one UE per RRB [8].
- RA-IDNC that assigns an equal rate to all F-APs [4].
- Classical IDNC that ignores the physical layer.

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$$\mathcal{P}_1(c): \quad \max_{P_1, P_2} \min_{\lambda\in(0,1),\mu\in(0,1)} \lambda R_{\mathtt{S}_1(1),1} + (1-\lambda) R_{\mathtt{S}_1(2),1} + \mu R_{\mathtt{S}_2(1),2} + (1-\mu) R_{\mathtt{S}_2(2),2}$$
$$0 \leq \{P_1, P_2\} \leq P_{\max}.$$

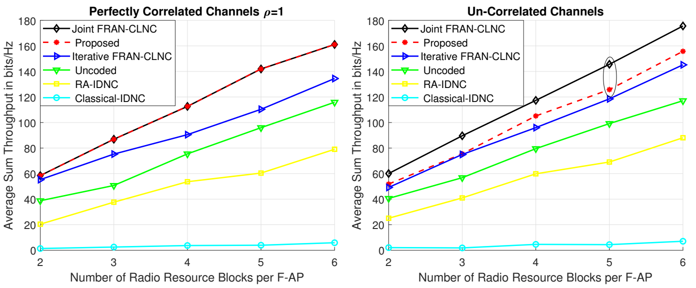

Fig. 2: Average sum throughput in bits/Hz versus the number of RRBs.

We consider a downlink FRAN system where the positions of 3 F-APs are fixed and UEs are distributed randomly within a hexagonal cell of radius 500m. The channels are assumed to be perfectly estimated. The noise power and the maximum power of the RRBs are set to $\sigma^2 = -168.60$ dBm/Hz and $P_{\max} = -42.60$ dBm/Hz, respectively. The bandwidth is 10MHz. For a fair performance comparison, we consider that each F-AP holds a subset of files in our proposed scheme and the scheme in [11]. Hence, the caching ratio μ is set to 0.6. The UEs' side information is drawn randomly. For implementing correlated channels, we use the following model to generate the channel gains: $\gamma_{n,kz} = (1-\lambda)\gamma_{n,k} + \Gamma_{n,kz}\lambda$, where $\gamma_{n,k}$ and $\Gamma_{n,kz}$ are channel gains that follow the SUI-3 terrain type B channel model. The channel $\gamma_{n,k}$ is equal for all RRBs in the F-AP's frame. $\Gamma_{n,kz}$ is induced to show the system's performance as a function of channel correlation variations across the RRBs, which has two extreme cases: 1) $\lambda = 0$: channel gains are the same across all RRBs, and 2) $\lambda = 1$: channel gains are different across all RRBs. For modeling the correlation between channels, we use: $\rho = \frac{\mathbb{E}[\gamma_{n,kz}\gamma_{n,kz'}]}{\sigma(\gamma_{n,kz})\sigma(\gamma_{n,kz'})} = \left(1 + (\frac{\lambda}{1-\lambda})^2\right)^{-1}$ [12]. Figs. 1 and 2 show the average sum throughput in bits/Hz versus a large number of UEs and the number of RRBs in FRAN settings, respectively. In Fig. 1, we have 2 RRBs per F-AP's frame and 28 files, while in Fig. 2, we use 6 UEs and 11 files.

From the above figures, we can observe that the throughput performance of the proposed solution outperforms the throughput performances of iterative, optimal uncoded, and RA-IDNC schemes for all network configurations under both scenarios, i.e., $\rho = 1$ and $\rho = 0$. In particular, the iterative scheme does not optimize the power levels of the F-APs for the whole graph space and only focuses on the outcome of the UE coordinated scheduling solution. While the uncoded solution ignores the UEs' side information and assigns a single UE to each RRB, the classical IDNC ignores the physical layer factors and adopts the rate of each RRB to the minimum achievable capacity of its assigned UEs. The RA-IDNC scheme inefficiently allocates rate among the RRBs/F-APs. Our proposed solution uses RA-IDNC that includes UEs' side information and the rates, and it optimizes the power levels of the F-APs by considering the whole search space of the graph. Compared to the joint FRAN-CLNC system, our proposed low complexity solution achieves the same performance for perfectly correlated channels, i.e., $\rho = 1$. When un-correlated channels are used, i.e., $\rho = 0$, the proposed algorithm has a certain degradation as compared to the joint FRAN-CLNC solution. In particular, compared to the joint FRAN-CLNC, the maximum performance reduction with our proposed scheme is roughly 27% for $N = 23$ and 19% for $Z = 5$, as depicted in Figs. 1 and 2, respectively. These reductions come at the benefit of reducing the complexity of our proposed scheme of $O\left(\mathcal{C}_p{}^{|\mathcal{C}|}P_K\right)$ as compared to the high complexity of the joint FRAN-CLNC of $O\left(({}^{|\mathcal{C}|}P_K Z)^2\right)$.

V. Conclusion

This letter proposed a low complexity yet efficient NC scheme that guarantees potential throughput maximization in FRANs. Unlike previous solutions that require constructing the total power control graph, a computationally efficient solution was developed over a single power control subgraph. The problem was solved as a maximum weight clique problem, which enables to develop an efficient power allocation algorithm. Presented numerical results illustrated that the proposed scheme achieves significant gains in UEs' throughput as compared with existing algorithms. Despite significant reduction of the computational complexity, our proposed solution offers same performance to the benchmark joint solution for correlated channels. For un-correlated channels, our proposed solution experiences performance degradation compared with benchmark joint solution.

References

[1] Cisco visual networking index: global mobile data traffic forecast update, 2017-2022, *White Paper,* Feb., 2019.

[2] N. Pontois, M. Kaneko, T. H. L. Dinh, and L. Boukhatem, "User pre-scheduling and beamforming with outdated CSI in 5G fog radio access networks," *2018 IEEE Global Communications Conference (GLOBECOM),* Abu Dhabi, UAE, 2018, pp. 1-6.

[3] Y. Shih, W. Chung, A. Pang, T. Chiu, and H. Wei, "Enabling low-latency applications in fog-radio access networks," in *IEEE Network,* vol. 31, no. 1, pp. 52-58, Jan./Feb. 2017.

[4] M.-S. Al-Abiad, A. Douik, and S. Sorour, "Rate aware network codes for cloud radio access networks," *IEEE Trans. on Mobile Comp.,* vol. 18, no 8, pp 1898-1910, Aug. 2019.

[5] M. S. Al-Abiad, M. J. Hossain, and S. Sorour, "Cross-layer cloud offloading with quality of service guarantees in Fog-RANs," in *IEEE Trans. on Commun.,* vol. 67, no. 12, pp. 8435-8449, Jun. 2019.

[6] S. Sorour and S. Valaee, "Completion delay minimization for instantly decodable network codes," *IEEE/ACM Trans. on Netw.,* vol. 23, no. 5, pp. 1553-1567, Oct. 2015.

[7] X. Wang, C. Yuen, and Y. Xu, "Coding based data broadcasting for time critical applications with rate adaptation", *IEEE Trans. on Vehicular Tech.,* vol. 63, no. 5, pp. 2429-2442, Jun. 2014.

[8] A. Douik, H. Dahrouj, T.-Y. Al-Naffouri, and M.-S. Alouini, "Coordinated scheduling and power control in cloud-radio access networks," *IEEE Trans. on Wireless Commun.,* vol. 15, no. 4, pp. 2523-2536, Apr. 2016.

[9] P. Seok-Hwan, S. Osvaldo, S. Onur, and S. Shlomo, "Joint precoding and multivariate backhaul compression for the downlink of cloud radio access networks," in *IEEE Trans. on Signal Proc.* 61, no. 22 (2013): 5646-5658.

[10] M. S. Al-Abiad, A. Douik, S. Sorour, and Md.-J. Hossain, "Throughput maximization in cloud radio access networks using network coding," in *Proc. of 2018 IEEE Intern. Conf. on Commun. Workshops (ICCWorkshops),* Kansas City, MO, 2018, pp. 1-6.

[11] M. S. Al-Abiad, A. Douik, S. Sorour, and M. J. Hossain, "Throughput maximization in cloud-radio access networks using cross-layer network coding," *IEEE Trans. on Mobile Comp.,* Early Access, pp. 1-1.

[12] A. Douik, H. Dahrouj, T. Y. Al-Naffouri and M. Alouini, "Low-complexity scheduling and power adaptation for coordinated cloud-radio access networks," in *IEEE Comm. Letters,* vol. 21, no. 10, pp. 2298-2301, Oct. 2017.