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Cobordism Conjecture in AdS

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McNamara and Vafa conjectured that any pair of consistent quantum gravity theories can be connected by a domain wall. We test the conjecture in the context of the AdS/CFT correspondence. There are topological constraints on existence of an interface between the corresponding conformal field theories. We discuss how to construct domain walls in AdS predicted by the conjecture when the corresponding conformal interfaces are prohibited by topological obstructions.

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In [1], McNamara and Vafa conjectured that any proposed quantum theory of gravity with non-trivial cobordism classes in the space of configurations belongs to the Swampland. Their cobordism conjecture was motivated by the expectation that string theory is a unique theory of quantum gravity rather than a collection of many different independent theories. They argued that cobordism classes are conserved global charges and that the absence of global symmetry in quantum gravity [2–5] demands their triviality.

The cobordism conjecture predicts a domain wall for any pair of consistent quantum theories of gravity. In particular, any quantum gravity must allow an end-of-the-world brane. In this paper, we discuss them in the context of the AdS/CFT correspondence. If there is a conformal interface between the corresponding pair of CFT's, we can use it to construct such a domain wall. Similarly, a CFT with a conformally invariant boundary condition can be dual to an AdS gravity with an end-of-the-world brane. Such end-of-the-world branes and domain walls have been discussed in the AdS/CFT correspondence starting with [6–8] and [9, 10], respectively.

However, there are topological constraints on possible conformal boundaries and interfaces [11]. For example, in two dimensions, conformal invariance can be consistently imposed on an interface only when the gravitational anomaly given by the difference $(c_L - c_R)$ of left and right Virasoro central charges matches across the interface. Let us write the left/right-moving Virasoro generators on the two side of the interface as $(L_n^{(1)}, \tilde{L}_n^{(1)})$ and $(L_n^{(2)}, \tilde{L}_n^{(2)})$, respectively. A conformal interface must preserve their linear combination $(L_n^{(1)} - \tilde{L}_{-n}^{(1)} - L_{-n}^{(2)} + \tilde{L}_n^{(2)})$ for all $n \in \mathbb{Z}$. Such an interface exists only when the central charge for this combination given by $(c_L^{(1)} - c_R^{(1)} - c_L^{(2)} + c_R^{(2)})$ vanishes. We would like to see whether domain walls in AdS still exist when gravitational anomalies do not match, and if so in what sense.

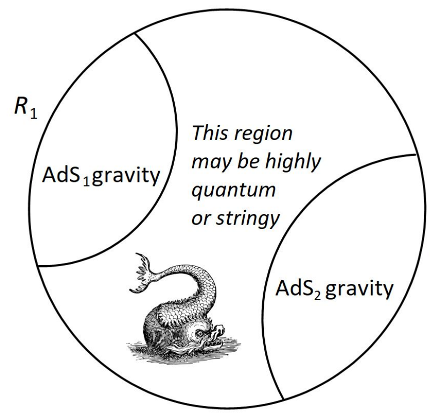

FIG. 1. dwQFT contains subsectors described by weakly-coupled gravity theories in subregions of AdS$_1$ and AdS$_2$. These subregions extend all the way to the boundaries of AdS$_1$ and AdS$_2$, respectively.

In order to address this question, we need to formulate properties of required domain walls more precisely. Consider a pair of quantum gravity theories in $(d+1)$-dimensional AdS's, which we denote by AdS$_1$ and AdS$_2$. According to the AdS/CFT correspondence, they are equivalent to conformal field theories in d dimensions, which we call CFT$_1$ and CFT$_2$, respectively. We say that there is a domain wall interpolating the two AdS gravity theories if there is a quantum field theory (which we denote by dwQFT) on $\mathbb{R}_{\rm time} \times \mathbb{S}^{d-1}$ with the properties described in the following paragraph.

Though dwQFT as a whole may not have a weakly-coupled gravity description in the bulk, we require that it has subsectors which can be approximated arbitrarily precisely by the gravity theories in AdS$_1$ and AdS$_2$. To formulate such a requirement mathematically, consider contractible and mutually disjoint regions $\mathcal{R}_1$ and $\mathcal{R}_2$ on the Cauchy slice $\mathbb{S}^{d-1}$ of dwQFT. We require that the algebra of local observables inside the domain of dependence of $\mathcal{R}_a$ $(a = 1, 2)$ can be described approximately by the AdS_a gravity theory in the entanglement wedge E_a for some region of the Cauchy slice of CFT_a. We also require that the approximation can be made arbitrarily precise by making the region $\mathcal{R}_a$ small. dwQFT with these properties gives a quantum mechanical description of the domain wall predicted by the cobordism conjecture since it interpolates the infinite-volume region E_1 described by the weakly coupled gravity theory in AdS_1 to the infinite-volume region E_2 described by the weakly coupled gravity theory in AdS_2, as shown in Figure 1.

If there is a conformal interface between CFT_1 and CFT_2, it naturally makes a dual to a domain wall between AdS_1 and AdS_2 [9, 10]. To see that it satisfies the conditions stated in the above paragraph, note that CFT correlation functions sufficiently away from the interface can be approximated by those without the interface and that the approximation can be made arbitrarily precise by taking their insertion points close to each other and away from the interface. Thus, if we take $\mathcal{R}_a$ to be sufficiently small compare to its distance to the interface, its entanglement wedge is well-approximated by that of the original AdS_a gravity.

However, a conformal interface does not always exist. For example, M5 branes wrapping a 4-cycle P_0 of a Calabi-Yau manifold with the second Chern class $c_2 \cdot P_0$ is described at low energy by a two-dimensional CFT with the gravitational anomaly,

$$c_L - c_R = -\frac{1}{2} c_2 \cdot q, \qquad (1)$$

where q is the magnetic flux charge [12]. If CFT_1 and CFT_2 have different values of $c_2 \cdot q$, the conformal interface conditions cannot be solved. Generally speaking, mismatch of gravitational anomalies signals difficulty in constructing an interface.

Even if a conformal interface does not exist, it is still possible to connect CFT_1 and CFT_2 at a junction of three conformal field theories, with an additional CFT_3 attached at the junction. To make the junction possible, CFT_3 must carry gravitational anomalies that cancel those of CFT_1 and CFT_2. In the above example, three sets of wrapped M5 branes can join at a junction if their anomalies given by (1) add up to zero. For a reason we will discuss below, such a junction is always possible with an appropriate choice of CFT_3. We can then stretch CFT_3 along the domain wall region between the AdS_1 and AdS_2 regions in Figure 1 and regard it as a part of degrees of the freedom of the region. Since CFT_3 can couple to CFT_1 and CFT_2 consistently at the junction, it should also be possible place it between their bulk duals, *i.e.*, AdS_1 and AdS_2. Even when the AdS_1 and AdS_2 gravities have different gravitational Chern-Simons terms generating different amounts of anomaly inflows into the domain wall region, the mismatch of anomalies can be cancelled by having CFT_3 degrees of freedom in the region.

If CFT_3 itself has large degrees of freedom and has an AdS dual, which we denote by AdS_3, it may be possible to construct a gravitational dual in terms of geometry with three asymptotically AdS regions as in [13]. In such a case, one can view that the AdS_3 region is emanating from the domain wall region in Figure 1. The new AdS_3 branch can be regarded as an effective description of the degrees of freedom localized between the AdS_1 and AdS_2 gravities.

A similar construction can be considered for end-of-the-world branes predicted by the cobordism conjecture. If an AdS gravity carries gravitational Chern-Simons terms, the corresponding CFT has gravitational anomalies and it cannot end on a conformal boundary. However, it is still possible to construct its conformal interface with another CFT with matching gravitational anomalies. This allows a construction of the predicted end-of-the-world brane by bending the CFT toward the bulk and using it as degrees of freedom localized on the brane. The anomaly inflow generated by the gravitational Chern-Simons terms can then be absorbed on the brane. One can even consider a totally transparent and topological interface, which connects CFT to itself. One can then bend the CFT on the other side of the interface and extend it toward the bulk AdS. The resulting bulk geometry is the pure AdS with the Dirichlet boundary condition along the end-of-the-world brane. This may not give a static configuration, but it is a consistent initial value condition on the Cauchy surface. This guarantees that there is an end-of-the-world brane for any AdS gravity. Similarly, there is a domain wall between any pair of AdS gravities since it can be regarded as an end-of-the-world brane for the pair, by the standard folding trick for a conformal interface. This shows that any potential topological obstruction against constructing a domain wall or an end-of-the-world brane can be absorbed by an appropriate brane.

A double Wick rotation of the configuration described in Figure 1 shows that the AdS_1 gravity can evolve into the AdS_2 gravity. On the CFT side, the evolution is described by an interface operator which maps the Hilbert space of CFT_1 to that of CFT_2. If we require that sufficient information can be transmitted from AdS_1 to AdS_2, the rank of the interface operator must be large. However, it is not always the case, and the rank can be as small as one. For example, if both CFT_1 and CFT_2 allow conformal boundaries, one can consider an interface operator in the form of a tensor product of the boundary operator for CFT_2 and the hermitian conjugate of the boundary operator for CFT_1. Such a totally reflective interface does not transmit information from AdS_1

to AdS_2.

In this paper, we discussed domain walls and end-of-the-world brane predicted by the cobordism conjecture in the context of the AdS/CFT correspondence. We find that a domain wall between a pair of AdS gravities can exist even when gravitational anomalies of the corresponding CFT's do not match and when there is no conformal interface between them, by considering a junction of three CFT's so that the anomaly mismatch can be absorbed by the third CFT. The third CFT can be regarded as a part of degrees of freedom of the domain wall. It may be necessary for the domain wall to carry large degrees of freedom, and their effective descriptions may involve new AdS branches emanating from the domain wall regions.

After completing this manuscript, we learned of work by Petar Simidzija and Mark Van Raamsdonk, who also studied domain walls connecting different AdS gravity theories from the point of view of conformal interfaces [14].

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