Gover and Yariv Reply: The context of our Letter [1] is the interpretation of the quantum electron wave function (QEW) and its reality in the interaction with light and matter (the wave-particle duality). This has been a matter of debate since the inception of quantum theory [2,3]. In previous work, the reality of the QEW and the measurability of its shaping were studied in the context of *stimulated* interaction of single electron QEWs with *light* [4–6]. The transition from the quantum interaction regime of multisidebands electron energy photon-induced near-field electron microscopy (PINEM) spectrum (Refs. [2–12] in [1]) to the classical point-particlelike acceleration or deceleration regime was shown to take place in the regime where the QEW duration gets shorter than the optical period: $\Gamma = \omega\sigma_{\mathrm{et}} < 1$ [4–6]. Our Letter [1] extends these observations to the interaction of QEWs with *matter* and points out the feasibility of a new free-electron–bound-electron resonant interaction (FEBERI) effect with multiple optical-frequency modulation-correlated QEWs.

The feasibility of modulating the expectation value of Born's probability distribution of free QEWs

$$n(\mathbf{r}, t) = \overline{|\Psi(\mathbf{r}, t)|^2} \tag{1}$$

was demonstrated by Feist *et al.* [7], who showed that the PINEM sidebands energy modulation of a single QEW at the laser frequency ω_b leads after drift into a periodic spatiotemporal modulation of the ensemble-averaged QEWs probability density. The reality and measurability of this periodic spatial sculpting of the QEW density was demonstrated experimentally by stimulated interaction (acceleration or deceleration) of the modulated QEWs with a laser beam, synchronous with a harmonic of the QEW modulation frequency [8,9].

The semiclassical model of the FEBERI process in [1] is admittedly a crude model that is aimed only to direct attention to the feasibility of this new effect, which manifests the reality of the QEW and its modulation features in interaction with matter, in analogy to interaction with light. The interpretation of (1) as charge density may be valid under certain physical circumstances [10], but only in the sense of ensemble average of multiple identical QEWs. The FEBERI effect is an example of the wave-particle duality. While Garcia de Abajo's approach [13] has partial validity in the wavelike limit of the QEW, we show in a comprehensive quantum wave packet analysis [11] that the FEBERI effect takes place in the point-particlelike limit of the QEW, affirming the main claim in our Letter [1]. Thus, Garcia de Abajo's derivation does not disprove the validity of the FEBERI effect in the case of multiple modulation-correlated QEWs.

The main shortcoming of Garcia de Abajo's analysis is in neglecting the probability amplitude variation while integrating the Schrödinger equation in time, obtaining the quantum recoil of the QEW, but ignoring the dynamics of the two-level system (TLS). The approximation fails when the interaction time is short: $E_{2,1}t_{\mathrm{int}} < \hbar/2$. In a perturbative solution of the Schrödinger equation [11] and numerical computation, we get finite FEBERI TLS transitions of a general super-position state under this manifestly wave packet size-dependent condition (with $t_{\mathrm{int}} = \sigma_{\mathrm{et}}$the QEW width) that decay in the opposite limit. Other problems with [13] include the assumption of starting the TLS from ground level and the swift extension of the analysis to multiple electrons, thus missing our predicted effect of quadratic (N^2) buildup of the TLS transition probability, in analogy with classical superradiance of bunched particles beams [12,14]. This failure results from ignoring our clearly stated condition that the resonant FEBERI effect takes place only with *modulation-correlated* QEWs (even if their centroid timings are random).

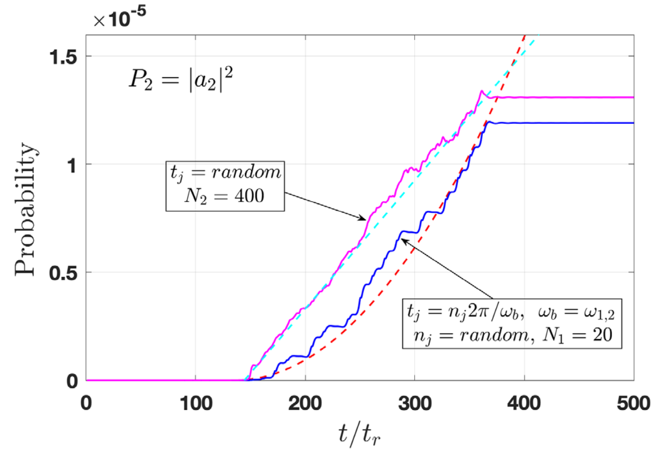

FIG. 1. FEBERI transition probability build-up of random-centroid QEWs ($\propto N$) and modulation-correlated QEWs ($\propto N^2$).

In [11] it is shown that, in the limit of short interaction time $t_{\mathrm{int}} < T_{2,1} = 2\pi/\omega_{2,1} = h/E_{2,1}$, the quantum model produces point-particlelike behavior of the QEW, with timing determined by the Born probability distribution (1) of the QEW. As in point-particle bunching (quantum klystron [15]), we extend this interpretation to a train of modulation-correlated QEWs, where modulation probability bunching spikes of attoseconds scale, much smaller than the transition period $T_{2,1}$, are achievable [7], and where, at modulation frequency resonance $\omega_b = \omega_{2,1}$, the timings of the bunching spikes t_j in the train are in phase with the transition frequency $\omega_{2,1}$. We show in Fig. 1 the quantum-model-based simulation results of [11] with the experimental parameters of [7]. The simulation curves display a transition probability buildup, which is linear in N for randomly injected QEWs and an N^2 scaling when the interaction time is determined by the Born quantum probability distribution, even though the QEW centroid timings are random.

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