

Constraints on low-energy effective theories from weak cosmic censorship

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Abstract

We examine the weak cosmic censorship conjecture (WCCC) for the extremal charged black hole in the possible generalizations of the Einstein-Maxwell theory due to the fourth-derivative higher order corrections. Our derivation is based on Wald's gedanken experiment to destroy an extremal black hole. We find that WCCC no longer holds for all the possible generalizations. Thus, WCCC can serve as an additional constraint to the swampland conjecture. However, our constraint is independent of photon's self-interactions so that the precision measurement of quantum electrodynamics cannot constrain WCCC. For the higher-dimension operators induced by the one-loop correction for the minimally coupled spinor and scalar to gravity, our constraint is satisfied.

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1 Introduction and Sumamry

Weak cosmic censorship conjecture (WCCC) [1] has been proved recently in [2] for Einstein-Maxwell theory via Wald's gedanken experiment of throwing matters into the extremal or near-extremal Kerr-Newman black hole ¹. This ensures the naked singularity is always hidden behind the horizon and avoids the philosophical challenge to the validity of Einstein gravity. Especially, this result may also be related to the fact that the trajectory of constant entropy meets tangentially with the boundary of the extremality bound, which for a non-rotating black hole of mass M , charge Q is

$$m \geq \sqrt{\frac{2}{\kappa}}|q|. \quad (1)$$

where $m \equiv M/4\pi$, $q \equiv Q/4\pi$ and $\kappa = 8\pi G_N$.

However, from the effective theory point of view, the Einstein-Maxwell theory should be subjected to higher order corrections due to quantum effect. To be concrete, we consider the most general fourth-derivative higher order corrections to Einstein-Maxwell theory, namely,

$$I = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \Delta L \right) \quad (2)$$

where ²

$$\begin{aligned} \Delta L = & c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \\ & + c_4 R F_{\mu\nu} F^{\mu\nu} + c_5 R_{\mu\nu} F^{\mu\rho} F^\nu{}_\rho + c_6 R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \\ & + c_7 F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + c_8 F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu}. \end{aligned} \quad (3)$$

We will assume c_i 's are small and restrict our consideration to $\mathcal{O}(c_i)$. In [5] it was shown that the extremality bound is changed from (1) to the following

$$m \geq \sqrt{\frac{2}{\kappa}}|q| \left(1 - \frac{4}{5q^2} c_0 \right), \quad (4)$$

where

$$c_0 \equiv c_2 + 4c_3 + \frac{c_5}{\kappa} + \frac{c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2}. \quad (5)$$

¹See also [3, 4] for earlier attempts and discussions.

²We have neglected terms proportional to $\nabla^\mu F_{\mu\rho} \nabla_\nu F^{\nu\rho}$, as it does not affect the black hole metric or our parameter bound. Further note that terms like $(\nabla_\mu F_{\nu\rho})(\nabla^\mu F^{\nu\rho})$ and $(\nabla_\mu F_{\nu\rho})(\nabla^\nu F^{\mu\rho})$ can be recasted (up to some constant factor) into $\nabla^\mu F_{\mu\rho} \nabla_\nu F^{\nu\rho}$ plus existing terms in ΔL and an additional boundary term, upon using Bianchi identities, Ricci identities, and integrating by parts [6].

Despite the extremality condition is changed, by some argument ³ one can show that the trajectory of constant entropy still meets the boundary of extremality bound tangentially. However, it is not clear if WCCC holds or not in this case. This is what we will examine in this paper by adopting the method developed in [2, 7]. The key procedure can be sketched as follows. Firstly we obtain the first law of black hole mechanics based on the Noether charge method of [8] for constructing the following first-order variational identity for higher theory (2):

$$\delta\mathcal{M} - \Phi_{\text{H}} \int_{\mathcal{H}} \epsilon_{abcd} \delta j^a = - \int_{\mathcal{H}} \epsilon_{ebcd} \xi^a \delta T^e{}_a, \quad (6)$$

where \mathcal{M} is the ADM mass of black hole, $\Phi_{\text{H}} \equiv -(\xi^a A_a)|_{\mathcal{H}}$ electromagnetic potential on the horizon \mathcal{H} , with ϵ_{abcd} the spacetime volume form, ξ^a the time-like Killing vector, A_a the Maxwell gauge field, and δj^a and $\delta T^e{}_a$ are respectively the charge current and stress tensor of the matter falling into the horizon. The right-hand side should be positive if the matter obeys the positive energy condition. Secondly, based on the perturbative solutions of extremal black holes up to the first order of c_i 's for the higher theory (2), we will evaluate $\delta\mathcal{M}$, Φ_{H} and $\delta\mathcal{Q} \equiv \int_{\mathcal{H}} \epsilon_{abcd} \delta j^a$ which can be thought as the variation of black hole's charge caused by the in-falling matter, so that (6) turns into an inequality relating the variations of mass and charge of the extremal black hole due to the falling matters, which explicitly depends on c_i 's. Finally, compare this inequality with the condition for the extremal black hole of the higher theory (2), we arrive the main result of this paper, i.e., the condition to preserve the WCCC is

$$c_2 + 4c_3 + \frac{10c_4}{\kappa} + \frac{3c_5}{\kappa} + \frac{3c_6}{\kappa} \leq 0. \quad (7)$$

Our result implies that WCCC does not hold for all the higher order theories. This then raises an issue if WCCC should be taken as a criterion for a consistent theory of quantum gravity, to which (2) is considered as a low energy effective theory. If not, we should face the issue of dealing with the unphysical curvature divergence of the naked singularity. If yes, this is quite similar to the swampland conjecture [9, 10] introduced with the same purpose here. One concrete example of swampland is based on weak gravity conjecture (WGC) [10] which states that the gauge force must be stronger than gravity. This conjecture is motivated by the assumption of finiteness of the number of stable particles not protected by a

³See Appendix D for detailed arguments. Also, this issue becomes nontrivial when considering the non-extremal black hole as the second order variation is involved [2].

symmetry principle, which requires the the minimum of the ratio of mass to charge is less than one, i.e., $m < \sqrt{\frac{2}{\kappa}}|q|$. When invoking this conjecture to the extremal black holes, it implies the extremality condition cannot be $m = \sqrt{\frac{2}{\kappa}}|q|$. Instead, one shall expect the higher-order quantum corrections should turn it into $m < \sqrt{\frac{2}{\kappa}}|q|$, which then gives the following constraint [5]:

$$c_2 + 4c_3 + \frac{c_5}{\kappa} + \frac{c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2} \geq 0. \quad (8)$$

Recently the same constraint is obtained in [11] by requiring that the higher-order correction to the entropy of extremal black hole must be positive.

It is interesting to see that (7) is quite different from (8), especially in (7) there is no term involving c_7 and c_8 , which are strengths of photon's self-interactions and can be severely constrained by precision test of quantum electrodynamics (QED). This implies that there is no severe constraint on WCCC by the precise measurement of QED, but it does on the WGC. On the other hand, the other c_i 's can be induced by one-loop corrections due to matter-gravity interactions. If they can be precisely measured in the future experiments, then (7) can serve as the constraints on these interactions and as an additional constraint to the swampland conjecture [9] besides (8). Below we will elaborate our derivation of (7) and then check some examples.

2 Charged black hole in higher theory

The field equations obtained by the variation of the action (2) with respect to A_μ and $g^{\mu\nu}$ are given respectively by

$$\nabla_\nu(F^{\mu\nu} - S^{\mu\nu}) = 0, \quad (9)$$

and

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu} = \kappa(\tilde{T}_{\mu\nu} + \Delta T_{\mu\nu}). \quad (10)$$

In the above $\tilde{T}_{\mu\nu} = F_{\mu}{}^\rho F_{\nu\rho} - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$ is the stress tensor of the Maxwell theory, and $\Delta T_{\mu\nu}$ and $S^{\mu\nu}$ are the corrections respectively to the stress tensor and Maxwell source field from the higher-dimension operators, and the details are given in Appendix A.

Next, we follow Ref. [5] to solve (9) and (10) for the $\mathcal{O}(c_i)$ corrections to the Reissner-Nordström black hole utilizing the spherical symmetry. A general static metric with spherical symmetry can be written as

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (11)$$

and also assuming the boundary condition that this metric approaches the Schwarzschild metric at $r \rightarrow \infty$. Then, after some manipulations the functions $\lambda(r)$ and $\nu(r)$ satisfy

$$\begin{aligned} e^{-\lambda(r)} &= 1 - \frac{\kappa M}{4\pi r} - \frac{\kappa}{r} \int_r^\infty dr r^2 T_t^t, \\ \nu(r) &= -\lambda(r) + \kappa \int_r^\infty dr r (T_t^t - T_r^r) e^{\lambda(r)}. \end{aligned} \quad (12)$$

We take the background to be the Reissner-Nordström black hole of Einstein-Maxwell theory,

$$\begin{aligned} e^{\nu(0)} = e^{-\lambda(0)} &= 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2}, \\ \frac{1}{2} F_{\mu\nu}^{(0)} dx^\mu \wedge dx^\nu &= \frac{q}{r^2} dt \wedge dr. \end{aligned} \quad (13)$$

The condition for the extremal black hole of the metric (13) is to saturate the extremality bound of (1), i.e.,

$$m = \sqrt{\frac{2}{\kappa}} |q| \quad (14)$$

which is also the location of the degenerate horizon.

Using the Maxwell source field $S_{\mu\nu}$ and stress tensor with the higher corrections $\Delta T_{\mu\nu}$ listed in Appendix A, we can solve the $\mathcal{O}(c_i)$ corrections to the Reissner-Nordström background. We have elaborated the solving procedure in Appendix B, and the corrected Maxwell gauge field is

$$A_t = -\frac{q}{r} + \frac{2q^3}{5r^5} \left(c_5 \kappa + 6c_6 \kappa - \frac{5c_6 \kappa m r}{q^2} + 8c_7 + 4c_8 \right). \quad (15)$$

The corrected metric is also obtained as listed in (50) of Appendix B, here we just show the tt-component,

$$\begin{aligned} e^\nu &= 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2} + c_2 \left(\frac{\kappa^3 m q^2}{r^5} - \frac{\kappa^3 q^4}{5r^6} - \frac{2\kappa^2 q^2}{r^4} \right) \\ &+ c_3 \left(\frac{4\kappa^3 m q^2}{r^5} - \frac{4\kappa^3 q^4}{5r^6} - \frac{8\kappa^2 q^2}{r^4} \right) + c_4 \left(-\frac{6\kappa^2 m q^2}{r^5} + \frac{4\kappa^2 q^4}{r^6} + \frac{4\kappa q^2}{r^4} \right) \\ &+ c_5 \left(\frac{4\kappa^2 q^4}{5r^6} - \frac{\kappa^2 m q^2}{r^5} \right) + c_6 \left(\frac{\kappa^2 m q^2}{r^5} - \frac{\kappa^2 q^4}{5r^6} - \frac{2\kappa q^2}{r^4} \right) \\ &+ c_7 \left(-\frac{4\kappa q^4}{5r^6} \right) + c_8 \left(-\frac{2\kappa q^4}{5r^6} \right) + \mathcal{O}(c_i^2). \end{aligned} \quad (16)$$

Note that in (16) there is no $\mathcal{O}(c_1)$ correction. From (16) we can solve the position of the degenerate horizon,

$$r_H = \frac{m\kappa}{2} + \frac{4c_2}{5m} + \frac{16c_3}{5m} + \frac{8c_4}{m\kappa} + \frac{4c_5}{5m\kappa} + \frac{4c_6}{5m\kappa} - \frac{64c_7}{5m\kappa^2} - \frac{32c_8}{5m\kappa^2}, \quad (17)$$

and derive the condition of extremal black hole in the higher theory. Moreover, from the corrected metric one can derive the new mass-charge relation for extremal black holes up to $\mathcal{O}(c_i)$ correction by finding the linearized solution to the double-root condition of $e^\nu = 0$. This then yields the condition by saturating the extremality bound of (4) [5], i.e.,

$$m = \sqrt{\frac{2}{\kappa}} |q| \left(1 - \frac{4}{5q^2} c_0 \right) \quad (18)$$

with c_0 given by (5).

3 Gedanken experiments and weak cosmic censorship

We now consider Wald's gedanken experiments to destroy the quantum gravity corrected extremal black hole that we obtained in the last section. We throw charged matter into the black hole and test whether or not it will be overcharged. For simplicity, we assume that the in-falling matter finally turns the original extremal black hole (16) of the higher theory into a one-parameter family solutions of the same theory but with mass $m(\tau)$ and charge $q(\tau)$. This then turns the extremality condition (18) into a discriminant function

$$f(\tau) = m^2(\tau) - \frac{2}{\kappa} q^2(\tau) \left(1 - \frac{4}{5q^2(\tau)} c_0 \right)^2. \quad (19)$$

When $f(\tau) \geq 0$, the spacetime is a black hole, otherwise it becomes a naked singularity.

Assuming small perturbation due to in-falling matter, i.e., $m(\tau) = m + \tau\delta m + \mathcal{O}(\tau^2)$ and $q(\tau) = q + \tau\delta q + \mathcal{O}(\tau^2)$ with $\tau \ll 1$, where the mass and charge of the unperturbed extremal black hole of higher theory are denoted by m and q , respectively. Then, to the first order in τ , the discriminant function can be further reduced to $f(\tau) = 2\tau m \left[\delta m - \sqrt{\frac{2}{\kappa}} \left(1 + \frac{4c_0}{5q^2} \right) \delta q \right] + \mathcal{O}(\tau^2, c_0^2)$. Therefore, the perturbed spacetime is still a black hole, thus WCCC holds only if the first order perturbations satisfy

$$\delta m - \sqrt{\frac{2}{\kappa}} \left(1 + \frac{4c_0}{5q^2} \right) \delta q \geq 0. \quad (20)$$

3.1 The linear variational identity

Next we need to study the first law of black hole mechanics, based on which we can see if the superextremality condition (20) can be satisfied or not. To proceed we follow the Noether charge construction developed by Iyer and Wald [8] to derive the linear variation identity [2, 7] which takes the following form

$$\delta\mathcal{M} = - \int_{\Sigma} \epsilon_{abcd} \xi^a (\delta T^e{}_a + A_a \delta j^e). \quad (21)$$

Here $\delta\mathcal{M}$ is the variation of the ADM mass, ξ^a is the time-like Killing vector, and δj^e and $\delta T^e{}_a$ are the associated current and stress tensor of the in-falling matter passing through the hypersurface Σ . In Wald's gedanken experiment, Σ is chosen to be $\mathcal{H} \cup \Sigma_1$ where \mathcal{H} is the event horizon of the black hole and Σ_1 denotes a space-like hypersurface connecting to \mathcal{H} at the late time and then extending to infinity.

We briefly sketch how (21) is obtained in [2, 8]. Introduce the Lagrangian 4-form $\mathbf{L} = L\epsilon$ associated with a Lagrangian $L(\phi)$, where ϕ denotes (g_{ab}, A_a) and ϵ is the volume form associated with the metric. Then, the variation of \mathbf{L} yields

$$\delta\mathbf{L} = \mathbf{E}(\phi)\delta\phi + d\mathbf{\Theta}(\phi, \delta\phi) \quad (22)$$

where $\mathbf{E}(\phi) = 0$ is the Euler-Lagrangian equation, and $\mathbf{\Theta}(\phi, \delta\phi)$ is the symplectic 3-form potential. For an arbitrary vector ξ^a , one can construct the associated Noether current $\mathbf{J}_\xi = \mathbf{\Theta}(\phi, \mathcal{L}_\xi\phi) - i_\xi\mathbf{L}$, which, because \mathbf{J} is conserved, i.e., $d\mathbf{J}_\xi = 0$, can be rewritten as $\mathbf{J}_\xi = d\mathbf{Q}_\xi + \xi_a\mathbf{C}^d$ with the 3-form constraint $\mathbf{C}^d = 0$ when the equations of motion are satisfied. Assuming $\mathbf{E}(\phi) = 0$ and ξ^a is a Killing vector, i.e., $\mathcal{L}_\xi\phi = 0$, it is easy to show that $\delta\mathbf{J}_\xi = di_\xi\mathbf{\Theta}(\phi, \delta\phi)$ which is then combined with $\delta\mathbf{J}_\xi = d\delta\mathbf{Q}_\xi + \xi^a\delta\mathbf{C}_a$ to yield

$$\int_{\partial\Sigma=\infty} [\delta\mathbf{Q}_\xi - i_\xi\mathbf{\Theta}(\phi, \delta\phi)] = - \int_{\Sigma} \xi^a \delta\mathbf{C}_a \quad (23)$$

where $\partial\Sigma = \infty$ denotes the spatial infinity. If we assume ξ^a is the time-like Killing vector t^a for the non-rotating black hole, then

$$\delta\mathcal{M} := \int_{\partial\Sigma=\infty} [\delta\mathbf{Q}_\xi - i_\xi\mathbf{\Theta}(\phi, \delta\phi)] \quad (24)$$

and furthermore identify

$$(\delta\mathbf{C}_a)_{bcd} := \epsilon_{abcd} (\delta T^e{}_a + A_a \delta j^e), \quad (25)$$

we can then arrive at the linear variation identity (21). For simplicity, we assume all the matters fall into the black hole far earlier than the joint moment of \mathcal{H} and Σ_1 , thus we can replace \int_{Σ} in (21) by $\int_{\mathcal{H}}$, then (21) turns into (6).

To really utilize the power of the linear variational identity (6) to check the WCCC for the extremal black hole in the higher theory, we need to derive the explicit forms of \mathbf{Q}_ξ and \mathbf{C}_a and their variations. A canonical procedure to derive them is developed in [8], following which we derive the explicit results in Appendix C.

3.2 Parameter bounds from WCCC

The key ingredient for Wald's gedanken experiment to examine WCCC is to throw the positive-energy matter into the black hole in the finite-time interval, i.e., only through a compact region of \mathcal{H} , such that the original black hole turns into another linearly stable solutions at the late time. The late time solution is then characterized by the variation of mass and charge caused by in-falling matter, i.e., $\delta\mathcal{M}$ and $\delta\mathcal{Q} \equiv \int_{\mathcal{H}} \epsilon_{abcd} \delta j^a$. Since on the horizon both its normal vector n^a and the time-like Killing vector ξ^a become null, thus $\xi^a \propto n^a$, so that the null energy condition of the in-falling matter, $\delta T_{ab} n^a n^b \geq 0$ ensure the RHS of (6) is nonnegative. Namely, on the horizon one may write

$$\epsilon_{abcd} = -4n_{[c} \tilde{\epsilon}_{bcd]}, \quad (26)$$

where $\tilde{\epsilon}_{bcd}$ is the volume element on the horizon, then the RHS of (6) becomes $\int_{\mathcal{H}} \tilde{\epsilon}_{bcd} \delta T_{ea} \xi^a n^e \geq 0$. Thus, (6) turns into an inequality relating $\delta\mathcal{M}$ and $\delta\mathcal{Q}$,

$$\delta\mathcal{M} - \Phi_{\text{H}} \delta\mathcal{Q} \geq 0. \quad (27)$$

Here the electromagnetic potential $\Phi_{\text{H}} := -(\xi^a A_a)|_{\mathcal{H}}$ can be readily calculated by using the background A_a given in (15) as well as the location of horizon given in (17), the result is

$$\Phi_{\text{H}} = \sqrt{\frac{2}{\kappa}} \left(1 + \frac{4c'_0}{5q^2} \right) \quad (28)$$

where

$$c'_0 = -\frac{10c_4}{\kappa} - \frac{2c_5}{\kappa} - \frac{2c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2}. \quad (29)$$

However, the variational quantities $\delta\mathcal{M}$ and $\delta\mathcal{Q}$ for the black hole in the higher theory may not be the same as the ones for the Reissner-Nordström black hole, i.e., δM and δQ . Given the explicit forms in Appendix C, the corrections due to

the higher-dimension operators asymptotically approach infinity as r^{-h} with $h \geq 4$, thus they fall off quickly and have no contributions to the ADM mass \mathcal{M} , which is evaluated at the infinity. As a result, we find that

$$\delta\mathcal{M} = \delta M. \quad (30)$$

Similarly, using \mathbf{C}_a in Appendix C, after straightforward calculations we arrive ⁴

$$\delta\mathcal{Q} \equiv \int_{\mathcal{H}} \epsilon_{abcd} \delta j^a = \delta Q + \mathcal{O}(c_i^2). \quad (31)$$

Combine all the above, we can conclude that (27) gives

$$\delta m - \sqrt{\frac{2}{\kappa}} \left(1 + \frac{4c'_0}{5q^2} \right) \delta q \geq 0, \quad (32)$$

where we have expressed in terms of reduced quantities. This is the relation between δm and δq from black hole mechanics provided null energy condition is satisfied. Now if we require the condition (20) for the WCCC to hold so that an extremal black hole in the higher theory cannot be overcharged, then it is not hard to argue that we must have $c'_0 \geq c_0$. Explicitly we have

$$c_2 + 4c_3 + \frac{10c_4}{\kappa} + \frac{3c_5}{\kappa} + \frac{3c_6}{\kappa} \leq 0. \quad (33)$$

This is the key result of this paper, which gives the parameter bounds on the low-energy effective theory of quantum gravity by demanding that the low-energy theory preserves the weak cosmic censorship.

In this work we only consider the extremal black hole. It is then interesting to explore the WCCC for the near-extremal black hole under the condition that (33) is satisfied by following the same procedure in [2]. However, it should be technically far more involved as the the second order variational identities will involve the back-reaction of gravitational and electromagnetic fluxes and need the generalization of the work done in [7] to higher derivative theories. Moreover, for the extremal black hole, the surface gravity vanishes, it is not clear how the change of Wald entropy plays the roles in the first law of black hole mechanics, which manifests as the variational identity. This should also be answered when considering the WCCC for the near-extremal case.

⁴This can also be seen as follows. By the construction of source theory, $j_a = \nabla^b(F_{ab} - S_{ab})$ in which S_{ab} is $\mathcal{O}(c_i)$, and using (9) $F_{ab} = F_{ab}^{(0,j)} + S_{ab} + \mathcal{O}(c_i^2)$ where the superscript $(0, j)$ means to evaluate by plugging the background configurations (13) and keeping up to $\mathcal{O}(c_i)$ terms. We then arrive $j_a = \nabla^b F_{ab}^{(0,j)} + \mathcal{O}(c_i^2)$, and use the Gauss's law the integral $\mathcal{Q} = \int_{\mathcal{H}} \epsilon_{abcd} j^a = \int_B *F^{(0,j)} + \mathcal{O}(c_i^2)$. Then, $\delta\mathcal{Q} = \int_B (*F^{(0,j)} - *F^{(0)}) + \mathcal{O}(c_i^2) = \delta Q + \mathcal{O}(c_i^2)$, where δQ is the charge carried by the in-falling matter. Thus, (31) is obtained.

4 Discussion

With our new bound from weak cosmic censorship, it is then natural to ask how this bound works in the real world. At the low energy level, the leading order correction to the Einstein-Maxwell background is given by the graviton-photon-photon amplitudes with a scalar or spinor loop. For the minimally-coupled case, the one-loop effective actions for the Einstein-Maxwell background induced by spinor and scalar are given by [12, 13]

$$\begin{aligned}\mathcal{L}_{\text{spinor}} &\propto 5R F_{\mu\nu} F^{\mu\nu} - 26R_{\mu\nu} F^{\mu\rho} F^{\nu}_{\rho} + 2R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}, \\ \mathcal{L}_{\text{scalar}} &\propto (-5/2) R F_{\mu\nu} F^{\mu\nu} - 2R_{\mu\nu} F^{\mu\rho} F^{\nu}_{\rho} - 2R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma},\end{aligned}\tag{34}$$

where again we have neglected terms proportional to $\nabla^{\mu} F_{\mu\rho} \nabla_{\nu} F^{\nu\rho}$, as it does not affect the black hole metric or our parameter bound. Simply plugging the values of c_4 , c_5 and c_6 into the bound (33), we find the inequality hold for both theories. An important implication is then, the weak cosmic censorship conjecture not only holds for the Einstein-Maxwell theory, but may also hold at one-loop level. This could possibly mean the correctness of the conjecture in the real world!

When the non-minimal coupling between matter and gravity is present, however, the bound (33) may subject to change under different situations. This is consistent with the fact that the combination of c -coefficients in our bound is not invariant under the field redefinition $g_{\mu\nu} \rightarrow g_{\mu\nu} + \delta g_{\mu\nu}$ [11], where

$$\delta g_{\mu\nu} = r_1 R_{\mu\nu} + r_2 g_{\mu\nu} R + r_3 F_{\mu\rho} F^{\rho}_{\nu} + r_4 g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}.\tag{35}$$

With a proper choice of the matter-gravity coupling, it is even possible that there yields no bound for the Wilson coefficients, and that the WCCC always holds. A further discussion is beyond the scope of this paper and we would like to explore it in the future.

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A Corrections to the Maxwell source and stress tensor

Here we list the details of the corrections to the Maxwell source field and stress tensor and , i.e., $S^{\mu\nu}$ in (9) and $\Delta T_{\mu\nu}$ in (10):

$$S^{\mu\nu} = 4c_4 R F^{\mu\nu} + 2c_5 (R^{\mu\rho} F_\rho{}^\nu - R^{\nu\rho} F_\rho{}^\mu) + 4c_6 R^{\mu\nu\rho\sigma} F_{\rho\sigma} + 8c_7 F_{\rho\sigma} F^{\rho\sigma} F^{\mu\nu} + 8c_8 F_{\rho\sigma} F^{\rho\nu} F^{\mu\sigma}, \quad (36)$$

and

$$\begin{aligned} \Delta T_{\mu\nu} = & c_1 (g_{\mu\nu} R^2 - 4R R_{\mu\nu} + 4\nabla_\nu \nabla_\mu R - 4g_{\mu\nu} \square R) + \\ & + c_2 (g_{\mu\nu} R_{\rho\sigma} R^{\rho\sigma} + 4\nabla_\alpha \nabla_\nu R_\mu^\alpha - 2\square R_{\mu\nu} - g_{\mu\nu} \square R - 4R_\mu^\alpha R_{\alpha\nu}) + \\ & + c_3 \left(g_{\mu\nu} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - 4R_{\mu\alpha\beta\gamma} R_\nu{}^{\alpha\beta\gamma} - 8\square R_{\mu\nu} \right. \\ & \quad \left. + 4\nabla_\nu \nabla_\mu R + 8R_\mu^\alpha R_{\alpha\nu} - 8R^{\alpha\beta} R_{\mu\alpha\nu\beta} \right) + \\ & + c_4 (g_{\mu\nu} R F^2 - 4R F_\mu{}^\sigma F_{\nu\sigma} - 2F^2 R_{\mu\nu} + 2\nabla_\mu \nabla_\nu F^2 - 2g_{\mu\nu} \square F^2) + \\ & + c_5 \left(g_{\mu\nu} R^{\kappa\lambda} F_{\kappa\rho} F_{\lambda\rho} - 4R_{\nu\sigma} F_{\mu\rho} F^{\sigma\rho} - 2R^{\alpha\beta} F_{\alpha\mu} F_{\beta\nu} \right. \\ & \quad \left. - g_{\mu\nu} \nabla_\alpha \nabla_\beta (F^\alpha{}_\rho F^{\beta\rho} + 2\nabla_\alpha \nabla_\nu (F_{\mu\beta} F^{\alpha\beta}) - \square (F_{\mu\rho} F_\nu{}^\rho)) \right) + \\ & + c_6 \left(g_{\mu\nu} R^{\kappa\lambda\rho\sigma} F_{\kappa\lambda} F_{\rho\sigma} - 6F_{\alpha\nu} F^{\beta\gamma} R^\alpha{}_{\mu\beta\gamma} - 4\nabla_\beta \nabla_\alpha (F^\alpha{}_\mu F^\beta{}_\nu) \right) + \\ & + c_7 (g_{\mu\nu} (F^2)^2 - 8F^2 F_\mu{}^\sigma F_{\nu\sigma}) + \\ & + c_8 \left(g_{\mu\nu} F^{\rho\kappa} F_{\rho\sigma} F^{\sigma\lambda} F_{\kappa\lambda} - 8F_\mu{}^\rho F_\nu{}^\sigma F_\rho{}^\kappa F_{\sigma\kappa} \right). \quad (37) \end{aligned}$$

Note that $F^2 = F_{\rho\sigma} F^{\rho\sigma}$ and $\square = \nabla_a \nabla^a$.

B Corrections to the Reissner-Nordström black hole

The functions $\lambda(r)$ and $\nu(r)$ of (11) are related to the components of Ricci curvature tensor $R_{\mu\nu}$ via

$$\begin{aligned} \frac{1}{2} (R_t^t - R_r^r) - R_\theta^\theta &= \frac{1}{r^2} \frac{d}{dr} \left[r(e^{-\lambda(r)} - 1) \right], \quad (38) \\ R_t^t - R_r^r &= -\frac{e^{-\lambda(r)}}{r} [\nu'(r) + \lambda'(r)]. \end{aligned}$$

To solve for λ and ν explicitly, we need an additional boundary condition. Assuming that at $r \rightarrow \infty$ the metric approaches the Schwarzschild solution, the results are

then given by

$$e^{-\lambda(r)} = 1 - \frac{\kappa M}{4\pi r} - \frac{1}{r} \int_r^\infty dr r^2 \left[\frac{1}{2} (R_t^t - R_r^r) - R_\theta^\theta \right], \quad (39)$$

$$\nu(r) = -\lambda(r) + \int_r^\infty dr r (R_t^t - R_r^r) e^{\lambda(r)}.$$

We further take the trace-reverse of Eq. (10) and obtain that

$$R_{\mu\nu} = \kappa \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right), \quad (40)$$

where T is the trace of the total energy-momentum tensor $T_{\mu\nu}$, and is given by $T = T_t^t + T_r^r + 2T_\theta^\theta$. Plugging the trace-reversed Einstein field equation into the integral expression (39), we get

$$e^{-\lambda(r)} = 1 - \frac{\kappa M}{4\pi r} - \frac{\kappa}{r} \int_r^\infty dr r^2 T_t^t, \quad (41)$$

$$\nu(r) = -\lambda(r) + \kappa \int_r^\infty dr r (T_t^t - T_r^r) e^{\lambda(r)}.$$

Once we know the diagonal components of the energy-momentum tensor, it will be straightforward to compute the corrections to the spherically symmetric static spacetime as induced by $T_{\mu\nu}$.

We now take our background spacetime to be Reissner-Nordström black hole in four-dimension. That is,

$$e^{\nu^{(0)}} = e^{-\lambda^{(0)}} = 1 - \frac{\kappa M}{4\pi r} + \frac{\kappa Q^2}{32\pi^2 r^2}, \quad (42)$$

$$F_{\mu\nu}^{(0)} dx^\mu \wedge dx^\nu = \frac{Q}{4\pi r^2} dt \wedge dr.$$

Here $\nu^{(0)}(r)$ and $\lambda^{(0)}(r)$ refer to the metric components in the unperturbed black hole spacetime, and $F_{\mu\nu}^{(0)}$ is the background electromagnetic energy-momentum tensor. Considering the action in Eq. (2), we treat the corrections from higher-dimension operators as perturbations. For convenience, we also introduce a power counting parameter ε , and consider a one-parameter family of actions I_ε , which is given by

$$I_\varepsilon = \int d^4x \sqrt{-g} (L_0 + \varepsilon \Delta L). \quad (43)$$

The original action will be recovered after setting $\varepsilon = 1$. We then expand everything into powers series in ε . For instance,

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \varepsilon h_{\mu\nu}^{(1)} + \mathcal{O}(\varepsilon^2), \quad F_{\mu\nu} = F_{\mu\nu}^{(0)} + \varepsilon f_{\mu\nu}^{(1)} + \mathcal{O}(\varepsilon^2). \quad (44)$$

At order ε^1 , the stress energy tensor is given by

$$T_{\mu\nu}^{(1)} = \tilde{T}_{\mu\nu}[g^{(0)}, f^{(1)}, F^{(0)}] + \tilde{T}_{\mu\nu}[h^{(1)}, F^{(0)}, F^{(0)}] + \Delta T_{\mu\nu}[g^{(0)}, F^{(0)}]. \quad (45)$$

Noting that in order to compute the corrections to the metric, we need to calculate $T_{\mu}{}^{\nu}$ instead of $T_{\mu\nu}$. At order ε^1 , $T_{\mu}{}^{\nu(1)}$ is given by

$$T_{\mu}{}^{\nu(1)} = \tilde{T}_{\mu}{}^{\nu}[g^{(0)}, F^{(1)}] + \Delta T_{\mu}{}^{\nu}[g^{(0)}, F^{(0)}]. \quad (46)$$

From Eq.(9) we solve for the corrections to Maxwell equations, and obtain that the nonzero components of $f_{\mu\nu}^{(1)}$ are

$$f_{tr}^{(1)} = -f_{rt}^{(1)} = \frac{1}{32\pi^3 r^6} (c_5 \kappa Q^3 - 16\pi c_6 \kappa M Q r + 6c_6 \kappa Q^3 + 8c_7 Q^3 + 4c_8 Q^3). \quad (47)$$

This corresponds to the gauge field A_a given by

$$A_t = -\frac{q}{r} + \frac{2q^3}{5r^5} \left(c_5 \kappa + 6c_6 \kappa - \frac{5c_6 \kappa m r}{q^2} + 8c_7 + 4c_8 \right), \quad A_r = A_\theta = A_\phi = 0. \quad (48)$$

With the corrections to $F_{\mu\nu}$, we can solve for the corrected energy-momentum tensor $T_{\mu}{}^{\nu(1)}$. We then find the corrected metric tensor component to be

$$\begin{aligned} e^{-\lambda} &= 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2} + c_2 \left(\frac{3\kappa^3 m q^2}{r^5} - \frac{6\kappa^3 q^4}{5r^6} - \frac{4\kappa^2 q^2}{r^4} \right) \\ &+ c_3 \left(\frac{12\kappa^3 m q^2}{r^5} - \frac{24\kappa^3 q^4}{5r^6} - \frac{16\kappa^2 q^2}{r^4} \right) + c_4 \left(\frac{14\kappa^2 m q^2}{r^5} - \frac{6\kappa^2 q^4}{r^6} - \frac{16\kappa q^2}{r^4} \right) \\ &+ c_5 \left(\frac{5\kappa^2 m q^2}{r^5} - \frac{11\kappa^2 q^4}{5r^6} - \frac{6\kappa q^2}{r^4} \right) + c_6 \left(\frac{7\kappa^2 m q^2}{r^5} - \frac{16\kappa^2 q^4}{5r^6} - \frac{8\kappa q^2}{r^4} \right) \\ &+ c_7 \left(-\frac{4\kappa q^4}{5r^6} \right) + c_8 \left(-\frac{2\kappa q^4}{5r^6} \right), \\ e^{+\nu} &= 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2} + c_2 \left(\frac{\kappa^3 m q^2}{r^5} - \frac{\kappa^3 q^4}{5r^6} - \frac{2\kappa^2 q^2}{r^4} \right) \\ &+ c_3 \left(\frac{4\kappa^3 m q^2}{r^5} - \frac{4\kappa^3 q^4}{5r^6} - \frac{8\kappa^2 q^2}{r^4} \right) + c_4 \left(-\frac{6\kappa^2 m q^2}{r^5} + \frac{4\kappa^2 q^4}{r^6} + \frac{4\kappa q^2}{r^4} \right) \\ &+ c_5 \left(\frac{4\kappa^2 q^4}{5r^6} - \frac{\kappa^2 m q^2}{r^5} \right) + c_6 \left(\frac{\kappa^2 m q^2}{r^5} - \frac{\kappa^2 q^4}{5r^6} - \frac{2\kappa q^2}{r^4} \right) \\ &+ c_7 \left(-\frac{4\kappa q^4}{5r^6} \right) + c_8 \left(-\frac{2\kappa q^4}{5r^6} \right). \end{aligned} \quad (49)$$

In the above we have defined the reduced quantities $m = M/4\pi$ and $q = Q/4\pi$. Note that the R^2 -term in the action has no contributions to the equation of motion at leading order in ε . The contributions from $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\rho\theta}R^{\mu\nu\rho\theta}$ can be canceled out by choosing $c_2 = -4c_3$. This directly confirms that the Gauss-Bonnet term is a

topological invariant and does not influence the equation of motion. Due to the fact that only the tr - and rt -component of $F_{\mu\nu}$ are nonzero, the term $F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$ always have twice the contributions from $F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}$ towards the equation of motion [5].

C Explicit forms of \mathbf{Q}_ξ and \mathbf{C}_a for the higher theory

The Lagrangian 4-form \mathbf{L} for the higher theory can be written as $\mathbf{L} = \mathbf{L}_0 + \sum_i c_i \mathbf{L}_i$. In this appendix, by following the canonical method developed in [8] we derive and present the Noether charge and constraint associated with each term in \mathbf{L} .

Variation of the Lagrangian 4-form \mathbf{L}_0 yields

$$\delta\mathbf{L}_0 = \delta g_{ab} \left(-\frac{1}{2\kappa} G_{ab} + \frac{1}{2} T_{ab}^{\text{EM}} \right) \epsilon + \delta A_a \left(\nabla_b F^{ba} \right) \epsilon + \mathbf{d}\Theta_0, \quad (51)$$

where $G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R$ is the Einstein tensor, and T_{ab}^{EM} is the electro-magnetic stress-energy tensor, which is defined by

$$T_{ab}^{\text{EM}} = F_{ac}F_b{}^c - \frac{1}{4}g_{ab}F_{de}F^{de}. \quad (52)$$

The symplectic potential can be written as

$$\Theta_0 = \Theta^{\text{GR}} + \Theta^{\text{EM}}, \quad (53)$$

where

$$\Theta_{abc}^{\text{GR}}(\phi, \delta\phi) = \frac{1}{2\kappa} \epsilon_{dabc} g^{de} g^{fg} (\nabla_g \delta g_{ef} - \nabla_e \delta g_{fg}), \quad (54)$$

$$\Theta_{abc}^{\text{EM}}(\phi, \delta\phi) = -\epsilon_{dabc} F^{de} \delta A_e. \quad (55)$$

Let ξ^a be any smooth vector field on the spacetime. We find that the Noether charges associated with the vector field are respectively,

$$(Q_\xi^{\text{GR}})_{ab} = -\frac{1}{2\kappa} \epsilon_{abcd} \nabla^c \xi^d, \quad (56)$$

$$(Q_\xi^{\text{EM}})_{ab} = -\frac{1}{2} \epsilon_{abcd} F^{cd} A_e \xi^e. \quad (57)$$

The equations of motion and constraints are given by

$$\mathbf{E}_0 \delta\phi = -\epsilon \left(\frac{1}{2} T^{ab} \delta g_{ab} + j^a \delta A_a \right), \quad (58)$$

$$C_{bcda} = \epsilon_{abcd} (T^e{}_a + j^e A_a), \quad (59)$$

where we have defined $T_{ab} = \frac{1}{\kappa} (G_{ab} - \kappa T_{ab}^{\text{EM}})$ as the non-electromagnetic stress energy tensor, and $j^a = \nabla_b F^{ab}$ is the charge-current of the Maxwell sources.

We similarly obtain the Noether charges and constraints for all higher-derivative terms. The results are presented below.

(1) \mathbf{L}_1 Variation of \mathbf{L}_1 yields

$$\delta \mathbf{L}_1 = \delta g_{ab} (E_1)^{ab} \boldsymbol{\epsilon} + \mathbf{d}\boldsymbol{\Theta}_1, \quad (60)$$

where we have defined

$$(E_1)^{ab} = \frac{1}{2} g^{ab} R^2 - 2R R^{ab} + 2\nabla^b \nabla^a R - 2g^{ab} \nabla_c \nabla^c R. \quad (61)$$

The Noether charge associated with the vector field ξ^a is

$$(Q_\xi^1)_{ab} = \epsilon_{abcd} \left(-4\xi^c \nabla^d R + 2R \nabla^d \xi^c \right). \quad (62)$$

The constraints are given by

$$C_{bcda} = -2\epsilon_{abcd} (E_1)^e{}_a. \quad (63)$$

(2) \mathbf{L}_2 Variation of \mathbf{L}_2 yields

$$\delta \mathbf{L}_2 = \delta g_{ab} (E_2)^{ab} \boldsymbol{\epsilon} + \mathbf{d}\boldsymbol{\Theta}_2, \quad (64)$$

where we have defined

$$(E_2)^{ab} = \frac{1}{2} g^{ab} R_{cd} R^{cd} + \nabla_c \nabla^b R^{ac} + \nabla_c \nabla^a R^{bc} - g^{ab} \nabla_d \nabla_c R^{cd} - \nabla^c \nabla_c R^{ab} - 2R^{ac} R^b{}_c. \quad (65)$$

The Noether charge associated with the vector field ξ^a is

$$(Q_\xi^2)_{ab} = \epsilon_{abcd} \left(4\xi^{[f} \nabla^{c]} R_f{}^d + R_f{}^d \nabla^f \xi^c + R_f{}^c \nabla^d \xi^f \right). \quad (66)$$

The constraints are given by

$$C_{bcda} = -2\epsilon_{abcd} (E_2)^e{}_a. \quad (67)$$

(3) L_3 Variation of L_3 yields

$$\delta L_3 = \delta g_{ab} c_3 (E_3)^{ab} \epsilon + \mathbf{d}\Theta_3, \quad (68)$$

where we have defined

$$(E_3)^{ab} = \frac{1}{2} g^{ab} R^2 + 2g^{ab} R_{cd} R^{cd} + 2R^{ab} R - 8R_{cd} R^{abcd} + 2\nabla^b \nabla^a R - 4\Box R^{ab}. \quad (69)$$

The Noether charge associated with the vector field ξ^a is

$$(Q_\xi^3)_{ab} = \epsilon_{abcd} \left(-4\xi^e \nabla_f R_e^{fcd} + 2R_{ef}{}^{cd} \nabla^f \xi^e \right). \quad (70)$$

The constraints are given by

$$C_{bcda} = -2\epsilon_{ebcd} (E_3)^e{}_a. \quad (71)$$

(4) L_4 Variation of L_4 yields

$$\delta L_4 = \delta g_{ab} (E_4^g)^{ab} \epsilon + \delta A_a (E_4^A)^a \epsilon + \mathbf{d}\Theta_4, \quad (72)$$

where we have defined the equation of motions for g_{ab} and A_a respectively as

$$(E_4^g)^{ab} = \left[-R^{ab} + \frac{1}{2} g^{ab} R - g^{ab} \nabla^2 + \nabla^{(a} \nabla^{b)} \right] F^2 - 2RF^{ac} F_b{}^c, \quad (73)$$

$$(E_4^A)^a = 4\nabla_b (RF^{ab}). \quad (74)$$

The Noether charge associated with the vector field ξ^a is

$$(Q_\xi^4)_{ab} = \epsilon_{abcd} \left(F^2 \nabla^d \xi^c - 2\xi^c \nabla^d F^2 + 2RF^{cd} A_e \xi^e \right). \quad (75)$$

The constraints are given by

$$C_{bcda} = -2\epsilon_{ebcd} (E_4^g)^e{}_a - \epsilon_{ebcd} (E_4^A)^e A_a. \quad (76)$$

(5) L_5 Variation of L_5 yields

$$\delta L_5 = \delta g_{ab} (E_5^g)^{ab} \epsilon + \delta A_a (E_5^A)^a \epsilon + \mathbf{d}\Theta_5, \quad (77)$$

where we have defined the equation of motions for g_{ab} and A_a respectively as

$$(E_5^g)^{ab} = 2F^{(bc}F_c^dR^a)_d - F^{ac}F^{bd}R_{cd} + \frac{1}{2}F_c^eF^{cd}g^{ab}R_{de} \quad (78)$$

$$\begin{aligned} & - \nabla^{(a}F^{b)c}\nabla_dF_c^d - F^{cd}\nabla_d\nabla^{(a}F^{b)c} - F^{(bc}\nabla_d\nabla^a)F_c^d - F^{(bc}\square F^a)_c \\ & - \nabla^{(b}F_{cd}\nabla^dF^a)c - F^{cd}g^{ab}\nabla_{(d}\nabla_{e)}F_c^e - \nabla_dF^b{}_c\nabla^dF^{ac} \\ & + \frac{1}{2}g^{ab}\nabla_cF^{cd}\nabla_eF_d^e - \frac{1}{2}g^{ab}\nabla_dF_{ce}\nabla^eF^{cd}, \end{aligned}$$

$$(E_5^A)^a = 2\nabla_c\left(R^{bc}F^a{}_b + F^{bc}R^a{}_b\right). \quad (79)$$

The Noether charge associated with the vector field ξ^a is

$$(Q_\xi^5)_{ab} = \epsilon_{abcd}\left[-2\xi^eA_eF^{fc}R_f{}^d - 2\xi^cF^{f(e}\nabla_eF_f{}^d) + \xi^e\nabla^d\left(F^{fc}F_{ef}\right) + F_f{}^dF_e{}^f\nabla^{[c}\xi^{e]}\right]. \quad (80)$$

The constraints are given by

$$C_{bcda} = -2\epsilon_{ebcd}(E_5^g)^e{}_a - \epsilon_{ebcd}(E_5^A)^eA_a. \quad (81)$$

(6) L_6 Variation of L_6 yields

$$\delta L_6 = \delta g_{ab}(E_6^g)^{ab}\epsilon + \delta A_a(E_6^A)^a\epsilon + \mathbf{d}\Theta_6, \quad (82)$$

where we have defined the equation of motions for g_{ab} and A_a respectively as

$$(E_6^g)^{ab} = \frac{1}{2}F^{cd}F^{ef}g^{ab}R_{cdef} - 3F^{(ac}F^{de}R^b)_{cde} \quad (83)$$

$$- 2F^{(ac}\nabla_c\nabla_dF^b)d - 2F^{(ac}\nabla_d\nabla_cF^b)d - 4\nabla_cF^{(ac}\nabla_dF^b)d,$$

$$(E_6^A)^a = 4\nabla_d\left(F^{bc}R^ad{}_{bc}\right). \quad (84)$$

The Noether charge associated with the vector field ξ^a is

$$(Q_\xi^6)_{ab} = \epsilon_{abcd}\left[2\xi^eA_eF^{fg}R_{fg}{}^{cd} - 2\xi^e\nabla_f\left(F^{cd}F_e{}^f\right) + F^{cd}F_{ef}\nabla^f\xi^e\right]. \quad (85)$$

The constraints are given by

$$C_{bcda} = -2\epsilon_{ebcd}(E_6^g)^e{}_a - \epsilon_{ebcd}(E_6^A)^eA_a. \quad (86)$$

(7) \mathbf{L}_7 Variation of \mathbf{L}_7 yields

$$\delta\mathbf{L}_7 = \delta g_{ab}(E_7^g)^{ab}\epsilon + \delta A_a(E_7^A)^a\epsilon + \mathbf{d}\Theta_7, \quad (87)$$

where we have defined the equation of motions for g_{ab} and A_a respectively as

$$(E_7^g)^{ab} = \frac{1}{2}g^{ab}F^2F^2 - 4F^{ac}F^b{}_cF^2, \quad (88)$$

$$(E_7^A)^a = 8\nabla_b(F^{ab}F^2). \quad (89)$$

The Noether charge associated with the vector field ξ^a is

$$(Q_\xi^7)_{ab} = \epsilon_{abcd}\left(4\xi^e A_e F^{cd}F^2\right). \quad (90)$$

The constraints are given by

$$C_{bcda} = -2\epsilon_{ebcd}(E_7^g)^e{}_a - \epsilon_{ebcd}(E_7^A)^e A_a. \quad (91)$$

(8) \mathbf{L}_8 Variation of \mathbf{L}_8 yields

$$\delta\mathbf{L}_8 = \delta g_{ab}(E_8^g)^{ab}\epsilon + \delta A_a(E_8^A)^a\epsilon + \mathbf{d}\Theta_8, \quad (92)$$

where we have defined the equation of motions for g_{ab} and A_a respectively as

$$(E_8^g)^{ab} = \frac{1}{2}g^{ab}F_c{}^d F_d{}^e F_e{}^f F_f{}^c - 4F^{ac}F^{bd}F_c{}^e F_{de}, \quad (93)$$

$$(E_8^A)^a = -8\nabla_d(F^a{}_b F^b{}_c F^{cd}). \quad (94)$$

The Noether charge associated with the vector field ξ^a is

$$(Q_\xi^8)_{ab} = \epsilon_{abcd}\left(4\xi^e A_e F_f{}^d F_g{}^c F^{gf}\right). \quad (95)$$

The constraints are given by

$$C_{bcda} = -2\epsilon_{ebcd}(E_8^g)^e{}_a - \epsilon_{ebcd}(E_8^A)^e A_a. \quad (96)$$

Finally, the above results can be summarized in the following compact form:

$$(\mathbf{Q}_\xi)_{c_3c_4} = \epsilon_{abc_3c_4}\left(M^{abc}\xi_c - E^{abcd}\nabla_{[c}\xi_{d]}\right), \quad (97)$$

where

$$M^{abc} \equiv -2\nabla_d E^{abcd} + E_F^{ab} A^c, \quad (98)$$

and

$$(\mathbf{C}^d)_{abc} = \epsilon_{eabc}(2E^{pqre}R_{pqr}{}^d + 4\nabla_f\nabla_h E^{efdh} + 2E_F^{eh}F^d{}_h - 2A^d\nabla_h E_F^{eh} - g^{ed}\mathbf{L}) \quad (99)$$

with

$$E^{abcd} \equiv \frac{\delta\mathbf{L}}{\delta R_{abcd}}, \quad E_F^{ab} \equiv \frac{\delta\mathbf{L}}{\delta F_{ab}}. \quad (100)$$

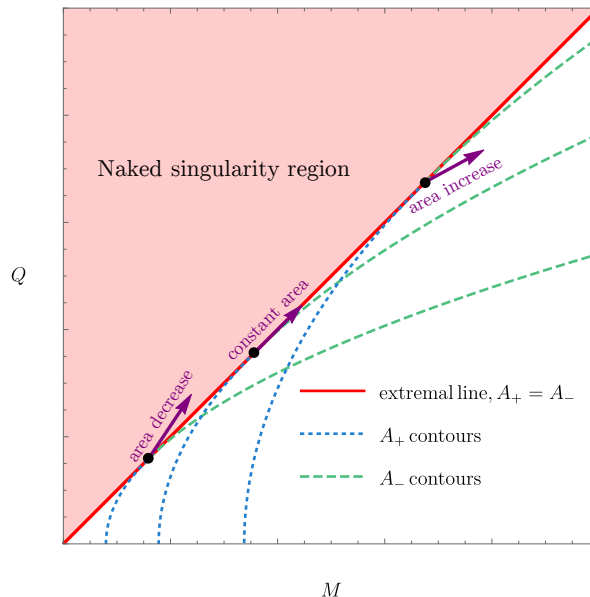


Figure 1: Extremality contour and constant area contours. Extremal black holes live on the red solid line which divides the whole parameter space into the naked singularity region and the non-extremal black hole region. The constant area contours are always tangent to the extremal line. A small perturbation around an extremal point then shifts the spacetime to one of the following: (i) a naked singularity when the horizon area is decreased ⁵; (ii) another extremal solution when the area is unchanged; and (iii) a nonextremal black hole when the area is increased.

D Proof that constant area direction is along the extremality curve

Suppose the radius, hence area A of the horizon is determined implicitly by the following equation

$$F(M, Q, A) = 0 \quad (101)$$

Extremality condition requires, in addition, that

$$\partial_A F(M, Q, A) = 0 \quad (102)$$

This is because the two roots of $1/g_{rr}$ coincide at this location.

⁵Of course the event horizons do not exist for naked singularities. In this sense we have the “horizon area” decreased to zero nonsmoothly.

Extremal black holes is a one-parameter family, with $Q_{\text{ext}}(M)$, $A_{\text{ext}}(M)$ determined jointly by Eqs. (101) and (102). In practice, when $Q < Q_{\text{ext}}(M)$, we will have contours of constant A (as shown in Fig. 1), determined by

$$\partial_M F dM + \partial_Q F dQ = 0 \quad (103)$$

or

$$(dQ/dM)_A = -\partial_M F / \partial_Q F \quad (104)$$

On the other hand, we can find out the direction of the extremality curve in the (M, Q, A) space. The tangent vector satisfies

$$\partial_M F \Delta M + \partial_Q F \Delta Q + \partial_A F \Delta A = 0 \quad (105)$$

However, because we have $\partial_A F$ on that curve, we have $\partial_A F = 0$ and also

$$(dQ/dM)_{\text{ext}} = -\partial_M F / \partial_Q F \quad (106)$$

This means, on the extremality contour, the direction at which area remains constant is the same as the contour itself. This does not mean that the contour all has the same area — instead, constant area contours reach the extremality contour in a tangential way, as shown in the figure.

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