The Politics of Asymmetric Extremism

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Abstract

In real-world policymaking, concrete and viable policy alternatives do not just appear out of thin air; they must be developed by someone with both the expertise and willingness to do so. We develop a model that explores the implications of strategic policy development by ideologically motivated actors, who craft competing high quality policies for a decisionmaker. We find that the process is characterized by unequal participation, inefficiently unpredictable and extreme outcomes, wasted effort, and an apparent bias toward extreme policies. When one proposer becomes asymmetrically extreme or capable they develop more extreme proposals, while their competitor moderates their proposals, increasingly declines to participate, and is harmed. Despite this, the decisionmaker benefits due to the increasing quality investments of the more extreme or capable proposer. The model thus provides rationale for why an ideologically extreme faction may come to dominate policymaking that is rooted in the nature of productive policy competition.
The reasonable man adapts himself to the world: the unreasonable one persists in trying to adapt the world to himself. Therefore all progress depends on the unreasonable man.

– George Bernard Shaw, *Man and Superman* (1903)

Ideological conflict is a key driver of competition among political elites (Converse (1964); Hinich and Munger (1994)); politically-active citizens, politicians, parties, and interest groups compete through elections and the policymaking process to have public policies enacted that reflect their ideological interests. Correspondingly, the political science literature has devoted a great deal of attention to studying these processes in order to better understand why some policies become law and not others. However, for the most part the literature has paid less attention to the process of developing the public policies themselves. Instead, the final adoption of a policy is typically thought of, or explicitly modeled, as some choice that is simply made from an available set of alternatives.

But in real world policymaking, policies that achieve a particular goal cannot just be freely chosen from the “theoretical infinity of them.” Rather, as Kingdon (1984) puts it in his sweeping work on the policy process, “before a subject can attain a solid position on a decision agenda, a viable alternative [must be] available for decisionmakers to consider” (p142, emphasis added). He recounts one presidential staffer’s perspective on the development of viable policies as follows (p132):

> Just attending to all the technical details of putting together a real proposal takes a lot of time. There’s tremendous detail in the work. It’s one thing to lay out a statement of principles or a general proposal, but its quite another thing to staff out all the technical work that is required to actually put a real detailed proposal together.

Who will “invest the resources – time, energy, reputation, and sometimes money” (Kingdon (1984), p122) to craft concrete, specific, and viable policy alternatives that reflect a particular goal or set of ideological interests? When and why do they do so? And how does the process of costly policy development itself, and the competition between participants in the process, influence what policies are ultimately adopted?

In this paper we analyze a parsimonious model of policy development in which policies must be designed by competing actors, at some cost to themselves, and with no guarantee that their policy will
actually be adopted. To do so we generalize the symmetric model of competitive policy development in Hirsch and Shotts (2015), which in turn builds on a burgeoning literature modeling policies as consisting of both an ideological component – over which participants in the policymaking process disagree – and a quality component – that all participants in the process value. Such models and related variants have been applied to a variety of settings in the political science literature, including policy development in legislatures (Hirsch and Shotts (2012); Hitt, Volden and Wiseman (2017)), expertise acquisition in bureaucracies (Ting (2011); Turner (2017)), judicial opinion writing (Lax and Cameron (2007)), lobbying and rulemaking (Hirsch and Shotts (2018)), federalism (Sasso (2019)), and industry (self) regulation (McCarty (2017)).

Because two proposers have the opportunity to craft policies in our model, the policymaking process takes the form of a contest between them (Tullock (1980)). Specifically, the proposers craft competing policy proposals that reflect their ideological interests, but then also make costly up-front investments in their proposal’s quality to improve its appeal to a decisionmaker. Quality is modelled as a policy-specific “public good” distinct from ideology that all participants in the process value, and is intended to capture the sorts of “criteria for survival” that Kingdon identifies for policy proposals; among these are technical and administrative feasibility, efficacy, equity, efficiency, and cost-effectiveness (Kingdon (1984), Ch. 6). But crucially, like the “policy entrepreneurs” in Kingdon’s theory, the proposers’ predominant motive for investing in quality is ideological – they seek to entice the decisionmaker to select their policy proposal over their competitor’s because “they want to promote their values, or affect the shape of public policy” (Kingdon (1984), p123). Thus, although ideology and quality are theoretically distinct in our framework, once the set of actually-available policies is considered they cannot be disentangled; in our model high quality policies must be crafted by somebody at cost to themselves, and the individuals and groups willing to do so are ideologically-motivated actors.¹

When crafting their respective policies, the proposers in our model face a natural tradeoff between crafting a more extreme proposal that better reflects their ideological interests – but then also must be

¹Worth noting is that the “policy entrepreneurs” in Kingdon’s theory engage in advocacy, while “specialists” who “see the world in similar ways” engage in policy development. However, the artificiality of this distinction is actually highlighted by Kingdon himself, who shortly after drawing it recounts conversations with a “knot of ideological liberals” who “have a view of the proper package of social insurance” and “work to fill in the gaps in that package, piece by piece” (p133). It is also directly critiqued by Sabatier (1991) – whose influential framework for understanding policy change is based on competing “advocacy coalitions” that integrate both types of actors.
higher quality to be appealing to the decisionmaker – vs. a more moderate proposal that requires fewer such investments to be competitive. The proposers also fear that should their policy fail to be adopted, they will instead have to live under a policy crafted by their competitor that is ideologically distant, and of insufficient quality to make up for it. How the proposers resolve these tensions determines their equilibrium behavior, and the effect of their attributes on the final policy outcome.

Our analysis of the model yields a large number of results; we therefore group them in three broad categories for the purposes of discussion. First, what are the basic properties and observable patterns of “policymaking as a contest”? Second, what are the consequences of asymmetric extremism between participants in the contest – that is, when one of the policy proposers has more extreme ideological preferences than the other? Finally, what are the consequences of asymmetric ability between participants in the contest – that is, when one of the policy proposers has greater resources for policy development, or is intrinsically more capable at doing so?

Policymaking as a Contest We uncover six key properties of policymaking as a contest.

The first three properties pertain to the proposers’ patterns of behavior. First, although the participants know everyone’s preferences and abilities, their behavior and final policy outcomes are characterized by unpredictability – when developing their policies, the proposers are always somewhat uncertain about what exactly their competitor will develop, whose proposal will prevail, and what the final policy outcome will be. This uncertainty is necessary to incentivize the proposers’ participation in policymaking – if a proposer were certain that their proposal would fail to be adopted they would decline to develop it, and were they certain it would be adopted they would scale back their quality investments. Second, participation in the policymaking process is generally asymmetric – one proposer always makes a new policy proposal with a strictly-positive quality investment (which we term being active), but the other sometimes declines to do so, expecting to be outmatched. These asymmetries are a natural consequence of asymmetries in the proposer’s ideological preferences and abilities at developing high quality policies. Third, policy proposals never converge to the decisionmaker’s ideal ideology. Instead, whenever a proposer develops a policy, its ideology diverges from the decisionmaker’s ideal in the direction of his own preferences. The reason is that the primary motive for developing a proposal is to induce a policy outcome closer to one’s ideal than what would otherwise prevail.

The next two properties pertain to the welfare of the participants. First, we find that competition
over policy benefits the decisionmaker, but is costly to the proposers. Specifically, the decisionmaker is always strictly better off under competition as compared to only receiving proposals from one of the two proposers. Surprisingly, this is true even if one of the proposers is perfectly aligned with her; the decisionmaker would never want to preclude participation by even a very ideologically-distant competitor. In contrast, a proposer is always strictly harmed by facing competition as compared to having “monopoly” proposal rights, even when his competitor makes substantial equilibrium investments in quality. Second, despite benefitting the decisionmaker, policymaking as a contest is very inefficient in several ways that would be avoided if the decisionmaker could commit ex-ante to whose policy she would choose, and under what conditions. For one, the expected ideological outcome is generically different from both the decisionmaker’s ideal, and the outcome that would maximize social welfare. In addition, the contest generates substantial uncertainty about the ideological outcome, and there is a high likelihood that the final outcome will be ideologically extreme in either direction; this harms all participants in the process due to risk aversion. Lastly, there is significant waste, as the benefits of any quality invested to improve the losing proposal are lost.

The final property pertains to the decisionmaker’s behavior. Surprisingly, her observable policy choices exhibit an endogenous bias toward extreme policies. Specifically, in equilibrium the decisionmaker is more likely to choose more extreme proposals from a given player than more moderate ones. In the special case of equally-capable proposers with equally extreme preferences, the decisionmaker will always choose the most extreme proposal presented (see also Hirsch and Shotts (2015)). This counterintuitive behavior results from the proposers’ strategic investments in quality. When a proposer chooses to make a more extreme proposal, he naturally anticipates that it will need to be greater quality to be competitive. However, in equilibrium he actually overinvests in the quality of extreme proposals, so that they are overall more appealing to the decisionmaker. The model thus provides a rationale for why extreme proposals may come to dominate the policy conversation – one that is rooted in the strategic incentives for productive policy development, rather than a biased or captured process that excludes moderate voices. An additional empirical implication is that when policy proposals are developed strategically, the underlying preferences of decisionmakers cannot be straightforwardly inferred from just the ideology of the policies that they choose.
Asymmetric Extremism  We next consider the implications of asymmetric extremism for policymaking as a contest. Asymmetries in extremism arise naturally at the level of national politics as the parties vacillate between extremism and moderation in response to the preferences of their base and internal organizational dynamics (Masket (2011)); indeed, many have recently argued that contemporary politics in the United States is characterized by asymmetric extremism (Grossmann and Hopkins (2016); McCarty (2015)). Asymmetric extremism also arises naturally within policymaking institutions including legislatures, bureaucracies, and courts as a result of electoral outcomes and political appointments (Lewis (2011)); for example, civil servants within a federal agency may consist of members of both parties, but the appointed political leadership will more closely reflect the President’s preferences and so be better aligned with one faction within the agency.

The politics of asymmetric extremism exhibit a number of surprising patterns. First, participation in policy development is unsurprisingly asymmetric. Intuition might suggest that it is the more moderate proposer who will be more likely to develop a proposal because he is better aligned with the decisionmaker. But in fact, it is the more extreme proposer who is more likely to develop a proposal – the reason is that he is more highly motivated to make costly investments in quality in order to move ideological outcomes in his direction. The extreme proposer unsurprisingly makes (first order stochastically) more extreme proposals than his moderate competitor, and they are also higher quality in order to “compensate” the decisionmaker for their greater extremism. Strikingly, however, an extremist invests so much more in quality by virtue of his greater motivation that his proposals are also better for the decisionmaker’s (first order stochastically) than the moderate’s; the extremist is thus strictly more likely to have his proposal implemented despite its greater extremism!

We next analyze what happens to the proposers’ behavior, policy outcomes, and the players’ welfare as one of the proposers becomes unilaterally more extreme. Greater intrinsic extremism causes a proposer to become more active in policymaking and to also develop more extreme proposals, but that are also higher quality and better overall for the decisionmaker. Interestingly, his competitor reacts by becoming increasingly inactive in policymaking – expecting to be outgunned – and increasingly moderate in his own proposals. Thus, observable moderation by a policy proposer may not actually reflect intrinsically moderate preferences, but instead the extremism of his opponent. As these effects occur the competitor becomes increasingly unhappy; he loses out to more and more ideologically
distant proposals that are of insufficient quality to make up for it. Surprisingly however, even as the extremist develops ever more extreme proposals that are increasingly uncontested, the decisionmaker becomes increasingly better off due to the extremist’s growing quality investments.

Our analysis of asymmetric extremism thus provides a coherent theoretical account for why an ideologically extreme faction may come to dominate the policy process despite their observably extreme behavior; an account that is rooted in the nature of productive policy competition rather than political dysfunction, bias, capture, or some other systemic failure. Further, the model illustrates how striking imbalances in political participation may arise naturally as a result of the differing motivations of the potential participants in the process, rather than exogenous and harmful constraints on participation or one side’s superior resources. In addition, it highlights how ideologically extreme preferences may have beneficial effects, as such preferences can motivate investments in “good policy” that are made to gain political support for ideologically-extreme proposals. Finally, it illustrates how centrist decisionmakers vs. more extreme participants may come to have highly polarized views about the desirability of what emerges from the policy process.

Asymmetric Ability We last analyze the politics of asymmetric ability. Asymmetries in policymaking talent between national parties may arise over time in particular issue areas or in general, as parties fluctuate in their success at cultivating a community of policy experts who share their ideological goals (Rich (2004)); again, some observers have argued that there is presently an asymmetry in expertise between the contemporary parties in the United States. More obviously, asymmetries in expertise and resources are a defining feature in policy domains heavily influenced by interest group politics (Berry and Wilcox (2015)) or subject to the formal rulemaking process (Yackee and Yackee (2006)); particularly when the primary axis of conflict is between poorly funded public interest groups – such as environmental organizations – and well-resourced business interests.

We find that the characteristic feature of competitive policymaking with asymmetric ability is that it observationally equivalent to policymaking with asymmetric extremism. That is, a proposer who is no more ideologically extreme than his competitor, but who is more capable at producing high quality policies, is also strictly more active in policymaking than his less expert competitor, and develops policies that are more ideologically extreme than his competitor but also higher quality and

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better for the decisionmaker. Similarly, as a proposer becomes increasingly expert, his own proposals become more extreme but also higher quality and better for the decisionmaker, and his competitor both moderates his proposals and becomes increasingly unlikely to make one. In the process, the competitor becomes increasingly worse off, and the decisionmaker becomes increasingly better off despite the increasing imbalance in political participation, and extremism of the expert’s proposals.

The model with asymmetric ability thus illustrates the difficulty of inferring the underlying preferences of participants in the policymaking process from their observable behavior. Specifically, competing policy developers may exhibit asymmetrically extreme behavior due to differences in their abilities at generating political support for their proposals, rather than an underlying difference in their intrinsic extremism. In addition, it starkly highlights how the ideology and quality of public policies are inextricably linked because of how ideological actors exploit quality to further their ideological aims. In the model, a proposer who becomes more expert is only getting better at producing a public good that benefits all players; this nevertheless has ideological and distributional consequences, benefitting himself (with more ideologically desirable outcomes) and the decisionmaker (with higher quality policies, even if more extreme) at the expense of an ideological competitor.

Related Literature

Classical Models of Policy Expertise We first relate to a large body of work studying the strategic acquisition of policy expertise based on the canonical model by Crawford and Sobel (1982) of “cheap talk” communication. In these works, a policy outcome $p$ result from the sum of a policy choice $y$ and an “unknown state of the world” $\omega$ distributed according to some distribution with mean 0 and variance $Var(\omega)$; this yields an expected utility of $-(x_i - y) - Var(\omega)$ when a player has a quadratic loss function over policy outcomes $p$ from an ideal point $x_i$. In this framework, an expert “develops” high quality policies by acquiring private knowledge of $\omega$. Then by making policy for, or recommending policies to, a decisionmaker using this knowledge of $\omega$, he can reduce the variance associated with final outcomes and benefit everyone – but at the cost of also inducing outcomes biased in favor of his own ideological interests.

This framework has been widely adopted in political science to study the institutional determinants of effective policymaking in a wide variety of institutional settings, particularly legislatures and
bureaucracies (see for example Gailmard and Patty (2012) for a review). The key strategic tension in such models is that privately-informed experts worry that their information will be used to implement policy outcomes that do not reflect their own ideological preferences. This causes them to take actions that strategically distort some of their policy-relevant information, which in turn reduces both the quality of policy outcomes (in a variance sense) and their initial willingness to acquire expertise (e.g. Gilligan and Krehbiel (1987)).

At the heart of the classical framework is a property that Callander (2008) critiques and terms *invertability*; if an individual learns how to accurately achieve a *particular* policy outcome (say a conservative one) then they must also know the state of the world \( \omega \), which in turn implies that they also know how to achieve *any* desired policy outcome (including a liberal one). In intuitive terms, the sort of expertise captured in these models is therefore “general” to all potential policies, rather than specific to one or another. A key consequence is that expertise is therefore also highly “expropriable,” in the sense that a decisionmaker can easily learn what an expert knows and apply that knowledge to achieve very different ends (Hirsch and Shotts (2012)), which in turn generates the fundamental strategic tension in these models.

**Policy Valence** Our model, in contrast, is part of a growing literature in political science that uses an alternative conceptualization of expertise. In it, individuals can make costly investments to *endow specific* policy proposals with quality that is valued by all players. This quality is sometimes termed *valence*, following the electoral literature that adopted this term to describe candidate traits that all voters value such as competence and charisma (Stokes (1963)). This conceptualization of expertise generates very different strategic incentives than models built on Crawford and Sobel (1982); rather than fear that their information will be expropriated, experts “attempt to exploit their monopoly power over investments to compel decision makers to accept policies that promote their interests” (Hirsch and Shotts (2015)), an effect that is akin to “real authority” in Aghion and Tirole (1997). Such models have now been used to analyze political decisionmaking in a wide variety of institutional settings, including interbranch bargaining (Londregan (2000)), bureaucratic expertise (Ting (2011); Turner (2017)), lobbying and influence (Hirsch and Shotts (2018)), policymaking in legislatures (Hitt, Volden and Wiseman (2017)), and judicial opinion writing (Lax and Cameron (2007)). Like us, the latter two works consider competition (the former briefly), but they assume that one proposer moves
first while then the second then directly responds.

A common interpretation of policy-specific quality in such models is that it represents a reduction in the variance associated with policy outcomes. These works “microfound” policy-specific quality by beginning with the familiar technology of Crawford and Sobel (1982), but then assuming that (i) outcomes are determined by policy-specific “states of the world” \( \omega_y \), and (ii) costly policy-specific effort by an expert can reduce the variance of a policy-specific state according to some function \( v_y(e) \). A policy’s “quality” is then the difference \( v_y(0) - v_y(e) \) between its variance with no effort and the reduced variance generated by the expert. Interpreting policy quality in this way is certainly plausible and has the virtue of embedding the models within a familiar framework. However, most valence models – including ours – are themselves agnostic as to whether the public good “produced” by an expert’s policy-specific investments represent uncertainty reduction, or something else entirely.

Also closely related is work by Callander (2008, 2011) on expertise in complex environments, which models the “unknown state” determining the relationship between policies and outcomes as an entire function that is the realized path of a Brownian motion. This approach has the property that learning how to implement a particular ideological outcome is helpful for implementing nearby outcomes, but not any outcome. It therefore effectively functions as an intermediate step between the “general” expertise of Crawford-Sobel models and the “policy-specific” expertise of valence models. Recent work on industry self-regulation (McCarty (2017)) also takes an intermediate approach.

**Political Contests** Separately, our model is related to a large literature studying political contests in a variety of forms. An important early contribution is Tullock (1980), who modelled competing interest groups as exerting costly and wasteful effort to secure “politicallly-contestable rents” that are equally valuable to all groups. Follow on work by Hillman and Riley (1989) relaxed this critical assumption by analyzing a contest in which (i) different interest groups may have asymmetric values for control of government policy, and (ii) control of policy always (rather than sometimes, as in a Tullock contest) goes to the interest group that puts in the most effort. This model is now more familiarly known as the all pay contest due to its close relationship to an all-pay auction, and was subsequently studied in greater depth and generality by economic theorists (e.g. Baye, Kovenock and de Vries (1996); Siegel (2009)). This model has been applied to several questions pertaining to the design of political institutions, including the effect of caps on political lobbying (Che and Gale
excluding some interest groups from political competition (Baye, Kovenock and Vries (1993)), and campaign spending limits (Meirowitz (2008)).

Our model is a successor to the asymmetric all pay contest; the quality investments made in a proposal are “all pay,” and the proposers may be highly asymmetric in both their preferences and abilities. However, there are several crucial differences that both complicate the analysis and generate surprising new insights. First, investing in quality is not wasteful, or a transfer to the decisionmaker – instead, it is a productive public good that benefits all players. Second, the proposers choose both a quality investment and an ideology to invest it into; as in other “multidimensional contests” they thus face a tradeoff between these two very different means of gaining support (Che and Gale (2003); Siegel (2009)). Finally and most importantly, the proposers are policy motivated rather than “rent seeking,” as befits political competition between ideologically motivated actors; they thus care only about which is policy implemented rather than whose policy is implemented.3 This crucial property generates the surprising result that asymmetric extremism and/or ability can benefit the decisionmaker despite the resulting imbalance in political participation.

In modeling a contest in which the players’ strategies have both an “all pay” component and an “ideological” or spatial component, our work also joins a large and diverse literature (Ashworth and Bueno de Mesquita (2009); Epstein and Nitzan (2004); Herrera, Levine and Martinelli (2008); Munster (2006); Serra (2010); Wiseman (2006); Zakharov (2009)). These models study either campaign spending in electoral contests or lobbying with wasteful expenditures, rather than productive policy development. These works also differ from each other and ours in terms their sequencing of the players’ moves (e.g. ideological component first and all-pay component second, all-pay first and ideology second, or one player first and then the other second), the players’ motivations (purely winning the contest, policy-motivated, or a mixture), and the players’ knowledge of the “decisionmaker’s” preferences (modelled abstractly with a contest success function (CSF) as in a Tullock contest, modelled explicitly with uncertainty, or known). Our model is unique in that all the proposers’ strategic choices – both ideological and all pay, and by both proposers – are made simultaneously. This is natural in fluid policymaking environments where there are no fixed rules constraining when and how potential proposers can become involved in policymaking. And although it appears to complicate the

3In the terminology of Baye, Kovenock and Vries (2012) the model features a rank order spillover.
equilibrium analysis by requiring the use of mixed strategies, it in fact dramatically simplifies it. In contrast to the aforementioned works, our model yields a unique equilibrium, with a succinct analytical characterization and easily analyzed comparative statics, across the entire parameter space.

The Model

Two proposers labelled −1 (left) and 1 (right) develop competing policies for consideration by a decisionmaker (DM), labelled player 0. A policy \((\gamma, q)\) consists of an ideology \(\gamma \in \mathbb{R}\) and a level of valence or quality \(q \in [0, \infty) = \mathbb{R}^+\). All three players \(N = \{-1, 0, 1\}\) are purely policy-motivated, in the sense that their final policy payoffs depend only on the ideology and quality of the policy implemented. The utility of player \(i \in N\) for a policy \((\gamma, q)\) is \(U_i(\gamma, q) = \lambda q - (\gamma - i \cdot x_i)^2\). The expression \(i \cdot x_i\) is player \(i\)’s ideological ideal point; the decisionmaker is located at 0, the left proposer is distance \(x_{-1}\) to her left, and the right proposer is distance \(x_1\) to her right. A proposer’s distance \(x_i\) from the decisionmaker reflects his ideological extremism. In contrast to ideology, a policy’s quality \(q\) is a public good that all players value equally at weight \(\lambda\); higher \(\lambda\) thus means that the players collectively care more about choosing a high quality policy, vs. one that reflects their ideological interests.

The game proceeds in two stages. In the first stage, the two proposers simultaneously select the ideology and quality their respective policy proposals \((\gamma_i, q_i)\). Endowing a policy with quality \(q_i\) costs its proposer \(c_i(q_i) = a_i q_i\). This is sunk regardless of which policy is ultimately chosen, and reflects the initial time and energy needed to improve a policy’s quality. The parameter \(a_i\) is player \(i\)’s marginal cost of endowing his policy with additional quality, and reflects his ability to make productive policy improvements that benefit all players. We also let \(\alpha_i = a_i / \lambda\) denote the ratio of a proposer’s marginal cost of quality to its marginal benefit; this quantity captures the net cost of investing in quality to proposer \(i\) once its intrinsic value is taken into consideration. For simplicity we assume this is \(> 1\) for both proposers, implying that neither would invest in quality for its intrinsic value alone.

In the second stage the DM chooses a single policy to be implemented as the final policy outcome. This may be one of the two policies developed by the proposers, or any other policy from an exogenous set of outside options \(O\) which contains all 0-quality policies \((\gamma, 0)\) (including the DM’s ideal point). This captures the idea that the DM has the power to choose policy, but not the capacity or skill to develop new policies with quality above some initial baseline.
Preliminary Analysis

The Monopoly Problem It is illuminating to first review equilibrium when there is a “monopoly” proposer who is the only player with the ability to endow policies with quality; without loss of generality suppose this is the right proposer ($i = 1$).

Because a proposer does not value quality enough to invest in it for its own sake, his predominant motive to develop a high quality policy is to influence the decisionmaker’s policy choice. Whether and how he does so depends crucially on the decisionmaker’s “outside option” – that is, the best policy ($\gamma_0, q_0$) she could implement for herself if she declines to take up the proposer’s policy. The monopoly problem is depicted in Figure 1, with ideology on the x-axis and quality on the y-axis. A monopolist’s calculus over whether to develop a new policy is described by the following inequality:

$$\arg\max_{q_1 - \gamma_1 \geq q_0 - \gamma_0} \left\{ \left( \lambda q_1 - (\gamma_1 - x_1)^2 \right) - a_1 q_1 \right\} \geq \lambda q_0 - (\gamma_0 - x_1)^2$$  (1)

The left hand side of the equation is the proposer’s maximum utility from developing some policy, and it illustrates how the decisionmaker’s outside option ($\gamma_0, q_0$) functions as a constraint on policy development. Specifically, to be adopted the proposer’s policy must be at least as good for the decisionmaker as her outside option, i.e., $\lambda q_1 - \gamma_1^2 \geq \lambda q_0 - \gamma_0^2$; in Figure 1 the monopolist’s policy must be above the indifference curve running through ($\gamma_0, q_0$). This illustrates a key motive for investing in quality; by doing so, a proposer can make more ideologically-extreme policies palatable to the decisionmaker. The right hand side is the proposer’s utility from doing nothing and living under the decisionmaker’s outside option, and it illustrates how the decisionmaker’s outside option functions as a motive for policy development. Specifically, the greater is the distance between the proposer’s ideal and the decisionmaker’s outside option, the more appealing is policy development ceteris paribus. In Figure 1, any potential outside option on the indifference curve running through ($\gamma_0, q_0$) is equally difficult to “beat” – but policies further to the left are more desirable to beat.

To solve this problem and also aid in the subsequent analysis, it is helpful to re-express policies ($\gamma, q$) in terms of their ideology $\gamma$ and the utility they give the decisionmaker – we henceforth call this quantity the score of a policy, and denote it as $s$. Observing that $s = \lambda q - \gamma^2$ implies a quality level
of $q = \frac{s + \gamma^2}{\lambda}$ and substituting into the monopolist’s problem yields:

$$\arg\max_{s_1 \geq s_0} \left\{ \begin{array}{l}
- (\alpha_1 - 1) s_1 + 2\gamma_1 x_1 - \alpha_1\gamma_1^2 \\
\text{score effect} + \text{ideology effect}
\end{array} \right\} \geq s_0 + 2\gamma_0 x_1, \tag{2}$$

where $s_1$ and $s_0$ denote the score of the proposer’s policy and the decisionmaker’s outside option, respectively. It is straightforward to see from Equation 2 that should the proposer choose to develop a policy, the optimal one will be no better for the decisionmaker than her outside option ($s_1^* = s_0$), and thus be on the indifference curve running through $(\gamma_0, q_0)$. The reason is that there is a net cost to investing in quality absent ideological influence. The optimal ideology is then chosen by trading off the ideological benefit $2\gamma_1 x_1$ of a more extreme policy against the cost $\alpha_1\gamma_1^2$ of “compensating” the decisionmaker for it with additional quality, which yields an optimal ideology of $\gamma_1^* = \frac{\alpha_1}{\alpha_1}$. Thus, should the proposer choose to develop a proposal, the decisionmaker will not benefit even if it is very high quality; the proposer will force the decisionmaker to also accept an extreme ideology in exchange.

Lastly, substituting the optimal proposal $\left( s_1 = s_0, \gamma_1 = \frac{\alpha_1}{\alpha_1} \right)$ into the proposer’s problem and sim-
plifying yields that the proposer will choose to develop this policy vs. simply do nothing i.f.f.:

\[
\left(\frac{x_1}{\alpha_1}\right)^2 - 2 \left(\frac{x_1}{\alpha_1}\right) \gamma_0 \geq s_0
\]

This illustrates three crucial properties: (i) a higher ratio $\frac{x_1}{\alpha_1}$ of ideological extremism to ability motivates policy development, (ii) the ideological extremism $\gamma_0$ of the decisionmaker’s outside option in direction opposite the proposer also motivates policy development, and (iii) the decisionmaker’s happiness with her outside option $s_0$ disincentivizes policy development.

**The Competitive Problem** When there is a competing proposer, the decisionmaker’s best outside option $(s_0, \gamma_0)$ may no longer be some exogenous policy, but instead a proposal $(s_{-1}, \gamma_{-1})$ strategically developed by his competitor. Thus, the more ideologically distant he expects this competitor’s policy to be the more willing he is to develop his own.

The setup of the competitive problem is depicted in Figure 2. A proposer’s choice of policy is effectively a two-dimensional “bid” $(s_i, \gamma_i)$ consisting of the proposal’s appeal $s_i$ to the decisionmaker and its ideology $\gamma_i$, which together imply a level of quality $q_i = \frac{s_i + \gamma_i^2}{2}$ that the proposal must have. After seeing up to two proposals decisionmaker will then choose the one with the highest score (that is, on the highest indifference curve in Figure 2) provided that it exceeds the score 0 of her best outside option $(0, 0)$ to both proposers (i.e., that it is located in the shaded region).

![Figure 2: The Competitive Problem](image-url)
Developing a bid’s quality is the “all pay” component, since the investment must be made before the decisionmaker chooses. However, in contrast to a standard all pay contest, all the players will benefit from that quality should the proposal be implemented. A bid’s ideology determines both the value of “winning” to its proposer and the cost of “losing” to its competitor. To see the manifestation of a proposer’s tradeoff between generating political support through quality investments vs. ideological concessions, observe that the required quality of a proposal \( q_i = \frac{s_i + \gamma^2}{\lambda} \) is strictly increasing in its ideological extremism \( |\gamma_i| \) – i.e., holding a bid’s appeal to the decisionmaker fixed, greater costly investments in quality are necessary to support a more ideologically extreme bid.

Recall that in the monopoly problem, a proposer chooses to either develop no policy, or develop one no better than the decisionmaker’s outside option. Applying this insight to the competitive model straightforwardly yields that, as in the all pay auction, there is no pure strategy equilibrium. Were the proposers to expect a pair of concrete proposals from each other \((s_1, \gamma_1)\) and \((s_{-1}, \gamma_{-1})\) (as depicted in Figure 2), then the anticipated proposal from the competitor would function like the outside option in the monopoly problem. The proposers would then have to be developing proposals with exactly the same score; otherwise, one proposer would be proposing something strictly better than the decisionmaker’s best outside option (their competitor’s proposal) (in Figure 2 the left proposer is targeting a strictly higher score, which cannot be optimal). But if the proposals were tied in score, then at least one proposer would have a strict incentive to break the tie by either developing a proposal with a slightly higher score, or dropping out of policy development.\(^4\)

**Equilibrium**

The model features a unique equilibrium that is in asymmetric mixed strategies; the proposers randomize both over whether they develop a new policy proposal, as well as the exact ideology and quality of their policy when they do develop one. The unique equilibrium strategies are as follows; the derivation and all subsequent proofs are contained in an Online Appendix. All proofs are analytical.

\(^4\)Proving this property is rather more involved than the standard all pay contest because payoffs from “winning” and “losing” depend on the proposals made.
Proposition 1. For each proposer \( i \in \{-1, 1\} \), define the strictly decreasing function

\[
\epsilon_i(p) = \int_p^1 \frac{x_i}{\alpha_i - q} dq = x_i \log \left( \frac{\alpha_i - p}{\alpha_i - 1} \right).
\]

Let \( p_i(e) = \alpha_i - (\alpha_i - 1) e^{\frac{x_i}{\alpha_i}} \) denote the inverse of \( \epsilon_i(p) \), and let \( k \) denote the proposer with the smallest value of \( \epsilon_i(0) \).

- The probability that proposer \( i \) chooses an ideology closer to the DM than distance \( y \) is

\[
F^Y_i(y) = p_{-i} \left( \epsilon_i \left( \frac{y}{x_i/\alpha_i} \right) \right) = \alpha_{-i} - (\alpha_{-i} - 1) \left( \frac{x_i - y}{x_i - x_i/\alpha_i} \right)^{\frac{x_i}{x_i-1}}
\]

- When developing a policy at distance \( y \) from the DM, proposer \( i \) targets ideology \( \gamma_i(y) = iy \) and invests quality \( q^Y_i(y) = \frac{y^2 + s^Y_i(y)}{\lambda} \), where

\[
s^Y_i(y) = 2 \int_{\epsilon_i(y/x_i/\alpha_i)}^{\epsilon_i(0)} \left( \sum_{j \in \{-1, 1\}} \frac{x_j}{\alpha_j} p_j(e) \right) de
\]

Although the equilibrium strategies are straightforward to express and compute, they are somewhat difficult to interpret from the equations alone. We therefore describe the structure of equilibrium, as well as six key properties, with the aid of an example.

Figure 3 depicts the equilibrium strategies for a configuration in which the proposers are equally skilled at producing quality, but the right proposer has more extreme preferences than the left one \( (x_1 > x_{-1}) \). The left panel depicts the ideology and quality of the proposals that the left (purple) and right (blue) proposers randomize over. The decisionmaker’s indifference curves are in gray. The right panel depicts the probability distributions (PDFs) describing left (purple) and right’s (blue) ideological policy proposals. Specifically, when a proposer chooses to develop a new policy, that policy’s ideology is continuously distributed over an interval with the depicted density. In the example, the left proposer also sometimes chooses to develop no new policy. This choice is depicted in the left panel by the purple dot at the origin (the DM’s ideal point with zero quality); the probability this occurs is depicted in the right panel by the height of the thick purple segment along the y-axis. Finally, the density describing
the ideology of the final policy implemented by the DM is depicted by the gray dashed lines.\textsuperscript{5}

Properties of Equilibrium

1. Uncertainty  Although the game is of complete and perfect information, in the unique equilibrium each proposer is uncertain about exactly which ideology their competitor will propose, how much they will invest in its quality, and whose policy proposal will be implemented. This uncertainty remains regardless of how asymmetric the proposers are – there is always some chance that the winning policy comes from either player. This fundamental unpredictability arises from the need for both proposers to remain competitive in the policy process when each can ensure victory over a competing proposal by investing sufficiently in quality and/or making enough ideological concessions.

2. Asymmetric Participation  Although either proposer’s policy may eventually be chosen, the unique equilibrium generically exhibits asymmetric participation in the policy process. Specifically, one of the two proposers (in the example the right proposer) is always active, in the sense of developing a new policy with strictly positive quality and an ideology distinct from the DM’s ideal. The DM will thus always receive at least one new proposal that was not already available. The other proposer, however (in the example the left proposer, and generically proposer $k$ defined in Prop. 1) is only sometimes active; with strictly positive probability he instead develops nothing and acquiesces to

\textsuperscript{5}The distributions over both the ideology of the right player’s proposals, as well as the ideology of the final proposal, are atomless. Consequently, the area under the blue line, and the sum of the areas under the gray lines, are equal to one.
whatever his competitor develops. This asymmetry in political participation arises from differences in the proposers’ ideological motivation and/or ability to craft high quality policies.

3. Ideological Divergence The unique equilibrium exhibits strong ideological divergence in the policies developed. Specifically, whenever a proposer chooses to invest in developing a new policy, the policy’s ideology differs from the DM’s ideal. (In Figure 3 all positive-quality policy proposals have divergent ideologies). Active participation in the policy process is thus always accompanied by attempts to extract “ideological rents” with a policy closer to one’s ideal than the DM preference, which is natural given the proposers’ ideological motivations for investing in quality.

4. Beneficial and Costly Competition In equilibrium, new policies always diverge ideologically from the DM’s ideal; implementing them thus entails an ideological cost. It is therefore not obvious whether, and when, the DM actually benefits from quality investments. Indeed, when there is only a single monopoly proposer the DM does not benefit, as a “monopolist” extract all the benefits of quality in the form of ideological rents. Competition, however, strictly benefits the DM in a strong sense – with certainty the DM will receive at least one new policy that is strictly better than what she could achieve on her own. This holds regardless of the proposers’ characteristics, and even when one proposer is very unlikely to participate in the policy process. The property can be seen in Figure 3 by observing that all positive-quality policy proposals are strictly above the DM’s indifference curve through the origin, combined with the fact that one proposer always develops a new policy.

It is also not obvious whether the proposers benefit from or are harmed by competition – a competitor develops ideologically-unappealing policies, but gains support for them by making productive investments in its quality that benefit everyone. Despite this, competition turns out to strictly harm the proposers even as it benefits the DM – each proposer would instead strictly prefer to have monopoly proposal rights. In equilibrium, the proposers invest in enough quality to overcompensate the DM for her ideological losses, but not enough to compensate the more-distant competitor.

5. Inefficiency Although competition strongly benefits the decisionmaker, outcomes are also very inefficient in several ways. First, the average ideological location of the final policy generically differs from the DM’s ideal, as well as the ideology that would maximize the players’ joint utility. Second, the ideological location of the final policy is highly uncertain ex-ante, which harms all three players
due to their risk aversion. (Its distribution is depicted by the dashed grey lines in the right panel of Figure 3 – any particular location has zero probability). Finally, because the proposers must make quality investments before they know which policy will ultimately be implemented, all of the effort invested in the losing policy is wasted. These inefficiencies arise naturally from the fact that policy is developed and chosen via a “contest” rather than an orderly process designed by the decisionmaker, with ex-ante commitments to whose policy she will choose and under what conditions.

6. Endogenous Extremism  In equilibrium, the proposers naturally invest more in quality when developing more extreme policies in order to remain competitive \( q_i'(y) > 0 \). More surprising, however, is that the proposers invest so much more in extreme policies that these policies are actually more appealing to the DM overall and more likely to be chosen. (In the left panel of Figure 3 the proposers’ quality functions are steeper than the DM’s indifference curves). Thus, a surprising implication of developing policy via a “contest” is that the process will appear to be biased towards extreme policies. A further consequence is that the ideology of the final policy (distributed according to the gray dashed line in the right panel of Figure 3) always diverges from the DM’s ideal, and is even more extreme than the players’ initial policy proposals.

The reason for this counterintuitive effect is as follows. To gain the DM’s support, the proposers trade off making costly quality investments against making ideological concessions. When a proposer aims to craft a policy that is more appealing to the DM, ideological concessions become a costlier way to gain her support because the policy is more likely to actually be implemented. Reversing the statement, a policy proposal that makes fewer ideological concessions (i.e. that is more ideologically extreme) must also be more appealing to the DM, and so be more likely to be implemented.

Having characterized equilibrium and discussed a variety of general properties, we now examine the pattern of policy competition and comparative statics in three special cases.

The Politics of Symmetric Competition

We first discuss the politics of symmetric competition by reviewing results from Hirsch and Shotts (2015) – what is the nature of policy competition when the competing proposers are both equally extreme and capable of producing high quality policies?
always the decisionmaker’s ideal. Instead, both players’ proposals diverge ideologically from the decisionmaker’s ideal. When the proposers are symmetric ($x = x = x$ and $\alpha = \alpha = \alpha$).\footnote{Strategies in Proposition 1 simplify to $F^Y(y) = \frac{y}{y}$ and $q^Y(y) = y^2 + 4\frac{y^2}{x} \int_0^{\frac{y^2}{x}} \frac{x}{x-y} dp = y^2 + 4x \left( x \log \left( \frac{y}{x-y} \right) - y \right)$.}

- both proposers are always active and make symmetrically distributed proposals
- the decisionmaker always chooses the most extreme proposal presented
- as the proposers become more extreme (higher $x$) or skilled (lower $\alpha$), proposals become first-order stochastically more extreme, but also higher quality and better for the decisionmaker

Equilibrium strategies under symmetric competition are depicted in Figure 4. Since the proposers are exactly balanced in their extremism and ability, they naturally both always participate in the policy process, and develop proposals whose distributions are mirror images around the decisionmaker. Somewhat surprisingly, however, symmetry does not cause proposals to “converge” to the decisionmaker’s ideal. Instead, both players’ proposals diverge ideologically from the decisionmaker’s ideal with probability 1 (they are in fact just as likely to make extreme proposals as moderate ones). Most strikingly, the decisionmaker’s endogenous bias toward extreme proposals manifests as her always choosing the most extreme proposal.

Symmetric competition thus exhibits balanced and productive engagement in the policy process by two ideologically opposed sides, who each try to exploit quality to realize ideological gains. This benefits the decisionmaker despite observably extreme proposals and outcomes, and harms the proposers
who make costly investments without moving the average outcome away from the decisionmaker’s ideal. The comparative statics effect of making both proposers more ideologically extreme (higher $x$) or more capable (lower $\alpha$) reflect the benefits of productive ideological competition. First, proposals become first-order stochastically more extreme and higher quality; with greater intrinsic extremism the proposers become more willing to invest in quality to realize ideological gains, and with greater ability they become more capable of doing so. Second, despite the greater extremism of proposals and outcomes, the additional quality is sufficient to over compensate the decisionmaker, so that she is overall better off (not just in expectation, but also first-order stochastically).

The Politics of Asymmetric Extremism

We now turn to the politics of asymmetric extremism; what happens when the proposers are equally capable ($\alpha_1 = \alpha_{-1}$), but one is more ideologically extreme than the other ($x_i \neq x_{-i}$)? We begin by examining the pattern of policy competition. (For expositional clarity we call the more extreme player “the extremist” and the more moderate player “the moderate.”)

**Proposition 3.** If the proposers are equally skilled but asymmetrically extreme ($x_i > x_{-i}, \alpha_1 = \alpha_{-1}$),

- the extremist always develops a new policy, while the moderate only sometimes does
- the extremist’s proposals are first-order stochastically more extreme than the moderate’s proposals, but also first-order stochastically higher quality and better for the decisionmaker
- the extremist’s proposals are strictly more likely to be chosen

Recall that equilibrium strategies with asymmetric extremism are depicted in Figure 3.

As Proposition 3 illustrates, the politics of asymmetric extremism are quite surprising. First, asymmetric extremism results in an imbalance in political participation – the extremist always develops a new proposal with positive quality, but his moderate competitor (who is better represented by the decisionmaker) sometimes strategically disengages. The extremist also makes first-order stochastically more extreme proposals than his moderate competitor. Surprisingly, however, those proposals actually *fare better* in the policy process because they are also first-order stochastically higher quality – so
much so that they are also first-order stochastically better for the decisionmaker *despite* their greater extremism. Thus, far from being a hindrance, extreme preferences – in the sense of an ideal policy more distant from the decisionmaker – are an asset in the contest over policy. What explains an ideological extremist’s dominance in the policy process despite his observably extreme proposals? Simply put, it is because the ideological extremist is *more motivated* – motivated to craft proposals that are sufficiently appealing to prevent his opponent’s policy from being chosen, and *also* motivated to invest enough in their quality to make them appealing to the decisionmaker despite their extremism.

We continue our examination of asymmetric extremism by considering what happens to the proposers’ behavior when one becomes unilaterally more extreme.

**Proposition 4.** If proposer $i$ becomes more ideologically extreme (higher $x_i$), his own strategy and his opponent’s strategy are affected in the following ways:

*(Own strategy)*

- if he previously did not always make a proposal, he becomes strictly more likely to do so
- his proposals become first-order stochastically more extreme
- his proposals also become first-order stochastically higher quality, better for the decisionmaker, and more likely to be chosen

*(Opponent’s strategy)*

- if he did not always make a proposal, he becomes strictly less likely to do so
- his proposals become first-order stochastically more moderate
- there is no first-order stochastic change in his proposals’ quality or competitiveness

Although the above comparative statics apply to any configuration of preferences and costs, they are easiest to discuss in the special case of only asymmetric extremism ($\alpha_1 = \alpha_{-1}$).

The consequences of player $i$ becoming intrinsically more extreme on his own behavior are quite natural. If player $i$ begins as the moderate and becomes somewhat more extreme, then he becomes
strictly more likely to participate in the policy process; greater balance in extremism thus generates
greater balance in participation. Second, regardless of whether player $i$ is the moderate or the ex-
tremist, his proposals become more extreme but also higher quality and better for the decisionmaker,
which reflects both his greater motivation to both win, and to win with more extreme proposals.

More subtly, player $i$ becoming more extreme also directly influences his opponent’s behavior.
If player $i$ is the extremist and becomes even more extreme, then his opponent becomes even less
likely to participate, understanding that he is likely to be outgunned on quality investments. A
greater imbalance in ideological extremism thus generates a greater imbalance in political participation.
Moreover, when player $i$ becomes more extreme his opponent reacts by moderating the ideology of his
own proposals. Intuitively, when faced with a more ideologically-driven competitor who makes more
extreme but also more competitive proposals, a player’s best response is to try to defensively “block”
them with proposals that are better catered to the decisionmaker’s preferred ideology.

We last examine the effect of unilateral extremism on the players’ welfare; specifically, as a proposer
$i$ becomes more extreme, how does this affect the welfare of his opponent and of the decisionmaker?

We first consider the opponent’s welfare. By Proposition 4, proposer $i$’s proposals become more
extreme but also higher quality and better for the decisionmaker. Whether this shift helps or harms
the competitor thus depends on whether the additional quality is also sufficient to make these more
extreme proposals better for the competitor as well.

**Proposition 5.** If proposer $i$ becomes more ideologically extreme (higher $x_i$), the equilibrium utility
of his competitor $-i$ decreases

The welfare effect of greater unilateral extremism on an opponent is thus unambiguous – despite the
greater quality of the extremist’s proposals, the competitor becomes worse off. The reason is intuitive
– the extremist’s greater investments in quality are sufficient to “compensate” the decisionmaker for
the greater extremism of his proposals, but insufficient to compensate the more distant competitor.

We last consider the decisionmaker’s welfare. In the standard all-pay contest – in which two players
simply make costly up-front bids to win a fixed “prize” – asymmetries in the players’ valuations for
the prize always harm the decisionmaker (Hillman and Riley (1989)) – the reason is that they cause
the weaker player to participate less. In our contest over policy-development, however, the effect is far
from obvious. Similar to the standard all-pay contest, as the extremist becomes increasingly extreme, the moderate becomes increasingly unlikely to participate, which harms the decisionmaker. However, an increasingly extreme proposer still continues to develop proposals that are first-order stochastically better for the decisionmaker, which is a surprising positive. Whether the decisionmaker benefits or is harmed overall thus depends on whether improvement in the extremist’s proposals is enough to outweigh the decrease in competition from the moderate.

Proposition 6. Unilateral changes in extremism have the following effects on the decisionmaker.

• If the proposers begin symmetric \((x_i = x_{-i}, \alpha_i = \alpha_{-i})\) and proposer \(i\) becomes more ideologically extreme, the decisionmaker’s utility locally increases

• As a proposer becomes increasingly moderate \((x_i \to 0)\), both his probability of making a proposal and the decisionmaker’s utility approach 0

• As a proposer becomes increasingly extreme \((x_i \to \infty)\), the competitor’s probability of making a proposal approaches 0, but the decisionmaker’s utility approaches infinity

While it is difficult to characterize the precise impact of unilateral changes in extremism on the decisionmaker’s welfare at any particular set of parameters, the overall pattern is both simple and striking; the decisionmaker strongly benefits from unilateral extremism. First, if the proposers begin exactly symmetric and then one becomes unilaterally more extreme, the decisionmaker unambiguously benefits despite the resulting imbalance in participation. Second and more strikingly, as one proposer becomes increasingly moderate – with his ideal point approaching that of the decisionmaker – the decisionmaker becomes increasingly worse off; the moderate becomes increasingly unlikely to participate, and in the limit the extremist behaves as a monopolist, presenting a proposal that is no better for the decisionmaker than what she could achieve on her own. The decisionmaker is thus harmed by having a proposer share her ideological preferences because such a proposer loses their primary motivation to invest in quality.

Finally and most strikingly, as one proposer becomes very unilaterally extreme, his competitor is effectively driven out of the policymaking process entirely, and becomes increasingly worse off. Yet surprisingly, despite the absence of observable competition most of the time, the decisionmaker
becomes increasingly – and even unboundedly – better off! Unilateral extremism thus benefits the decisionmaker even as it results in highly asymmetric participation, a striking contrast from standard models of political contests (Hillman and Riley (1989)). This surprising effect arises precisely because the proposers care about policy outcomes even when they lose.

The Politics of Asymmetric Ability

We last examine the politics of asymmetric ability; what happens when the proposers are equally extreme \((x_i = x_{-i})\), but one is more capable of producing high quality policies \((\alpha_1 < \alpha_{-1})\)? For expositional clarity we call the more capable player “the expert” and the less capable player “the amateur” (but note that the “amateur” is still more expert than the decisionmaker, who cannot generate high quality proposals on her own). We first examine the pattern of competition.

**Proposition 7.** If the proposers are equally extreme but one is more skilled \((x_i = x_{-i}, \alpha_1 < \alpha_{-1})\), the pattern of competition is indistinguishable from when they are equally skilled but asymmetrically extreme \((x_i > x_{-i}, \alpha_1 = \alpha_{-1})\).

Equilibrium strategies with asymmetric ability are depicted in Figure 5. Surprisingly, asymmetric extremism and asymmetric ability turn out to be observationally equivalent in terms of the resulting pattern of behavior. In other words, an expert exploits his greater ability at developing high quality policies to make more competitive proposals that also better reflect his ideological preferences, while an amateur reacts by disengaging and also moderating his own proposals. The striking empirical implication is that observably extreme behavior by one political faction and moderation by the other may not actually reflect an asymmetry in their underlying extremism at all. Instead, it may reflect asymmetry in their ability at making the sort of “good policy” investments that are necessary to gain moderates’ support for ideologically-extreme proposals.

Unsurprisingly, the observational equivalence between asymmetric extremism and asymmetric ability extends to the comparative statics effects of making one proposer unilaterally more skilled.

**Proposition 8.** If proposer \(i\) becomes more skilled (lower \(\alpha_i\)) his own strategy and his opponent’s are affected in the same ways as when he becomes more ideologically extreme (higher \(x_i\)).
A proposer becoming unilaterally more skilled thus (i) increases his own political activity (if he was the amateur) and makes his proposals more extreme, higher quality, and better for the decisionmaker, and (ii) decreases his opponent’s political activity (if he was the amateur) and induces him to moderate his own proposals to stay competitive.

We next examine the effect of unilaterally greater ability on an opponent’s welfare.

**Proposition 9.** If proposer $i$ becomes more skilled, the equilibrium utility of his competitor decreases.

A proposer is thus unambiguously harmed when his competitor becomes more skilled. This is somewhat surprising, given that the skill in question is at making common value investments that benefit all players. However, in equilibrium these skills harm an ideological opponent because of the way they are strategically exploited to further the proposer’s ideological aims. A key implication is that “good policy” considerations cannot really considered separately from ideological ones even when they are, in theory, distinct. For example, a financial subsidy to one political faction – that they can only use to improve their policy proposals for everybody – will nevertheless have distributional effects, benefitting the subsidy recipient at the expense of his ideological opponents.

We conclude by examining how unilateral changes in ability effect the decisionmaker’s welfare.

**Proposition 10.** Unilateral changes in ability have the following effects on the decisionmaker.

- If the proposers begin symmetric ($x_i = x_{-i}, \alpha_i = \alpha_{-i}$) and proposer $i$ becomes more skilled, then
the decisionmaker’s utility locally increases.

- As a proposer becomes increasingly unskilled ($\alpha_i \to \infty$), both his probability of making a proposal and the decisionmaker’s utility approach 0.

- As a proposer becomes increasingly skilled ($\alpha_i \to 1$), the competitor’s probability of making a proposal approaches 0, but the decisionmaker’s utility approaches a strictly positive bound; this bound is strictly increasing in the inactive competitor’s extremism and ability.

The overall pattern is that the decisionmaker benefits when a proposer becomes unilaterally more skilled, even though this skill is accompanied by more extreme proposals from the expert, and less participation from the amateur. First, if the proposers begin exactly symmetric and one becomes more skilled, then the decisionmaker benefits despite the resulting imbalance in participation. Thus, a situation of fully balanced competition is always strictly worse from the decisionmaker than some asymmetry. Second, if the amateur becomes increasingly incompetent, then outcomes again approach a situation of monopoly – in the limit the expert makes a proposal no better than what the decisionmaker can achieve on her own, and the amateur declines to participate.

Finally, if the expert becomes very skilled ($\alpha_i \to 1$), the effects resemble what happens when an extremist becomes very extreme ($x_i \to \infty$), but with some interesting differences. In both cases, the dominated player (in the former case amateur, and in the latter case the moderate) is driven out of policymaking, but the decisionmaker still concretely benefits from his potential participation. A key difference, however, is that when an extremist becomes very extreme the decisionmaker’s utility increases unboundedly, whereas when an expert becomes very skilled the decisionmaker’s utility only increases toward a strictly positive bound.\footnote{It is also true that if the proposers begin symmetric and then proposer $i$ becomes arbitrarily skilled, the decisionmaker will be better off in the limit of complete asymmetry than she was with full symmetry as long as $\alpha_{-i} \geq \alpha \approx 1.0435$.} Surprisingly, this bound depends on characteristics of the amateur. That is, even when the amateur is driven almost entirely out of the policy process, his personal characteristics exert outsized influence on the behavior of the expert. Specifically, if the amateur becomes more extreme or skilled (but still almost never participates because he is so outmatched by the expert), the expert still reacts by adjusting his proposals to be concretely better for the decisionmaker. Thus, in an environment of competitive policymaking, a lack of participation
cannot necessarily be interpreted as a lack of influence, or an indicator that formal constraints on participation would not be harmful.

**Discussion and Conclusion**

We have developed a model that explores the implications of strategic policy development by competing ideological actors. We find that the process exhibits several plausible patterns in light of real-world policymaking, including unequal participation, inefficiently unpredictable and extreme outcomes, wasted effort, and an apparent bias toward extreme policies. We explore the politics of asymmetric extremism and ability, and find that despite increasingly imbalanced proposals and outcomes, and even in an apparent absence of competition against an ideologically extreme faction, a moderate decisionmaker may nevertheless strongly benefit (although at the expense of the extremist’s ideological competitors). The model thus provides rationale for how ideological extremists may come to dominate policymaking and even benefit moderate decisionmakers; one rooted in the nature of productive policy competition rather than dysfunction, bias, capture, or some other systemic failure.

We model costly policy development as an individual or group making an ideology-specific investment in quality that is valued by all participants in policymaking, in contrast to a large literature that models policy expertise as the acquisition of private knowledge about an “unknown state of the world.” Our model is intentionally sparse and lacking in institutional detail in order to be applicable to the wide variety of settings in which competitive policy development occurs, including legislatures, bureaucracies, and courts. Broadening the interpretation of disagreement beyond “ideology” in a classic left-right sense, the model is applicable to any political environment where there are a mixture of competing and common interests, freedom among several individuals or groups to make proposals, and a decisionmaker who must make a single final choice.

One plausible application is thus decisionmaking in cabinets – when facing a specific crisis or immediate policy dilemma, a chief executive typically solicits proposals for how to proceed from his or her cabinet secretaries. These secretaries have a vested interest in good policy and the success of the administration, but also competing personal beliefs and/or interests for their respective departments (Kearns (2005)). An additional application is military decisionmaking. A nation may face an immediate security crises or seek to achieve a specific military goal, and the joint chairman of the
various service branches and/or the civilian leadership can solicit proposals from them. The branches
all value successful military outcomes, but also seek to further the resources and parochial interests
of their respective branches (Zimmerman et al. (2019)). A third is to policymaking in autocratic
one-party states. An important principle of decisionmaking in communist regimes is “democratic
centralism” – the idea that diverse interests within the party should be free to make proposals for how
to deal with a policy problem without fear of reprisal, but a central leader then has the authority to
implement a final policy decision that all party members must then obey (Angle (2005)).

Our analysis suggests two broad avenues for follow-on work.

The first avenue is to include additional elements that would allow the model to more closely
approximate real-world policymaking in a variety of settings. For example, what happens if there are
not just two but many potential participants in the policy development process, as in a legislature? It
is straightforward to show that when ability is common there is always an equilibrium in which only
the two most ideologically-extreme proposers are active. However, there many be other equilibria with
broader participation (see Baye, Kovenock and de Vries (1996)), and ability may be very unevenly
distributed among potential policy developers (see Hitt, Volden and Wiseman (2017)). Or, what
if the proposers can also engage in unproductive or even destructive activities in conjunction with
productive policy development order to enhance their policy’s prospects for adoption? For example,
they may try to bribe the decisionmaker with transfers, engage in unproductive advertising, lobbying,
or grassroots mobilization; or take aim at their opponent’s proposal, trying to harm its reputation
or even actively sabotage its functioning. Finally, what if the policy developers are not individuals
but teams – for example, aligned legislators and interest groups (Hall and Deardorff (2006)) – who
have common ideological interests, but must figure out how to distribute the costs of developing high
quality policies that achieve those interests among them?

The second avenue, following the classical literature on policy expertise, is to consider how political
institutions can be designed in order to encourage the development of high quality policies. For
example, what if the identities of the proposers can actually be chosen by the decisionmaker, as in
a legislature that collectively decides who will occupy its committee chairmanships? Relatedly, what
if there are already existing policy developers, but they can be subsidized with resources from the
decisionmaker? What if it is not the identities of the developers that can be chosen but that of the
decisionmaker, as in the appointment of an agency head who will consider policy proposals from career staff and/or outside interest groups? Finally, what if the decisionmaker is not a unitary actor but a collective choice body, as in a legislature? What sorts of collective choice rules will best encourage the development of high quality legislation?

References


The Politics of Asymmetric Extremism

June 21, 2019

Online Appendix

This Appendix is divided into two parts. Appendix A is a linear and self-contained treatment of the model with its own lemmas and propositions. Appendix B describes where to locate each result stated in the main text propositions in the general treatment in Appendix A.
A General Treatment

We begin with a slightly more general formulation of the model than stated in the main text. Two proposers labelled \(-1\) (left) and 1 (right) develop competing policies for consideration by a decision-maker (DM), labelled player 0. A policy \((\gamma, q)\) consists of an ideology \(\gamma \in \mathbb{R}\) and a level of quality \(q \in [0, \infty) = \mathbb{R}^+\). Utility over proposals takes the form

\[ U_i (\gamma, q) = \lambda q - (\gamma - X_i)^2, \]

where \(X_i\) is player \(i\)'s ideological ideal point, and \(\lambda\) is the weight all players place on quality. The proposers’ ideal points are on either side of the decisionmaker \((X_{-1} < X_D < X_1)\).

The game is as follows. First, the proposers simultaneously choose proposals \((\gamma_i, q_i)\). Making a proposal with quality \(q_i\) costs \(c_i (q_i) = a_i q_i\) up-front, where \(a_i > \lambda\). Second, the DM chooses one of the two proposals or something else from an exogenous set of outside options \(\mathcal{O}\), where \(\mathcal{O}\) may contain the DM's ideal point with no quality \((0, 0)\) and/or proposals that are strictly worse (and can be empty).

A.1 Preliminary Analysis

The game is a multidimensional contest in which the scoring rule applied to “bids” \((\gamma, q)\) is just the DM’s utility \(U_D (\gamma, q) = \lambda q - (X_D - \gamma)^2\). To facilitate the analysis we thus reparameterize proposals \((\gamma, q)\) to be expressed in terms of \((s, y)\), where \(y = \gamma - X_D\) is the (signed) distance of a proposal’s ideology from the DM’s ideal, and \(s = \lambda q - y^2\) is the DM’s utility for a proposal, or its score. The implied quality of a proposal \((s, y)\) is then \(q = \frac{s + y^2}{\lambda}\). Using this we re-express the proposers’ utility and cost functions in terms of \((s, y)\). Note that the decisionmaker’s ideal point with 0-quality has exactly 0 score, and is the most competitive “free” proposal to make.

Definition A.1.

1. Player \(i\)’s utility for proposal \((s, y)\) is

\[ V_i (s, y) = U_i \left( y + X_D, \frac{s + y^2}{\lambda} \right) = -x_i^2 + s + 2x_i y \]
where $x_i = X_i - X_D$ is the (signed) distance of $i$'s ideal from the DM.

2. Proposer $i$’s cost to make proposal $(s, y)$ is

$$c_i \left( \frac{s + y^2}{\lambda} \right) = \frac{a_i}{\lambda} (s + y^2) = \alpha_i (s + y^2)$$

where $\alpha_i = \frac{a_i}{\lambda}$ is $i$’s weighted marginal cost of generating quality.

Definition 1 reparameterizes proposals into score and ideological distance (henceforth just ideology) $(s, y)$, and the five primitives $(X_i, a_i, \lambda)$ into four parameters $(x_i, \alpha_i)$ describing the proposers' (signed) ideal ideological distance from the DM $x_i = X_i - X_D$ (henceforth just ideal ideology) and weighted marginal costs of generating quality $\alpha_i = a_i \lambda$ (henceforth just costs). Note that this notation differs slightly from the main text, where $x_i$ and $y$ denote the unsigned distance of a proposer’s ideal point and an ideological location from the DM.

### A.1.1 Necessary and Sufficient Equilibrium Conditions

In the reparameterized game, a proposer’s pure strategy $(s_i, y_i)$ is a two-dimensional element of $\mathbb{B} \equiv \{(s, y) \in \mathbb{R}^2 | s + y^2 \geq 0\}$. A mixed strategy $\sigma_i$ is a probability measure over the Borel subsets of $\mathbb{B}$, and let $F_i(s)$ denote the CDF over scores induced by $i$’s mixed strategy $\sigma_i$.\(^8\)

We now derive necessary and sufficient equilibrium conditions in a series of four lemmas. Let $\overline{\Pi}_i(s_i, y_i; \sigma_{-i})$ denote $i$’s expected utility for making proposal $(s_i, y_i)$ with $s_i \geq 0$ if a tie would be broken in her favor. Clearly this is $i$’s expected utility from making a proposal at any $s_i > 0$ where $-i$ has no atom, and $i$ can always achieve utility arbitrarily close to $\overline{\Pi}_i(s_i, y_i; \sigma_{-i})$ by making $\varepsilon$—higher score proposals. Now $\overline{\Pi}_i(s_i, y_i; \sigma_{-i}) =$

$$\overline{\Pi}_i(s_i, y_i; \sigma_{-i}) = -\alpha_i \left( s_i + y_i^2 \right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s_{-i} > s_i} V_i(s_{-i}, y_{-i}) d\sigma_{-i}. \quad (A.1)$$

The first term is the up-front cost of generating the proposal’s quality. The second term is the probability $i$’s proposal is selected, times her utility for it. The third term is $i$’s utility should she

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\(^8\)For technical convenience we restrict attention to strategies generating score CDFs that can be written as the sum of an absolutely continuous and a discrete distribution.
lose, which requires integrating over all her opponent’s proposals with higher score than \( s_i \). Taking the derivative with respect to \( y_i \) yields the first Lemma.

**Lemma A.1.** At any score \( s_i > 0 \) where \( F_{-i} (\cdot) \) has no atom, the proposal \((s_i, y^*_i (s_i))\), where \( y^*_i (s_i) = F_{-i} (s_i) \cdot \frac{x_i}{\alpha_i} \), is the strictly best score-\( s_i \) proposal.

**Proof:** Straightforward. QED

Lemma A.1 states that at almost every score \( s_i > 0 \), proposer \( i \)'s unique best combination of ideology and quality to generate that score is just a weighted average of the proposer’s and DM’s ideal ideologies \( \frac{x_i}{\alpha_i} \), multiplied by the probability \( F_{-i} (s_i) \) that \( i \)'s opponent makes a lower-score proposal. Note that \( i \)'s optimal ideology does not depend directly on her opponent \(-i \)'s ideologies, since a proposal’s ideology (holding score fixed) only matters conditional on winning. The optimal ideology also depends on the exact score \( s_i \) only indirectly through probability \( F_{-i} (s_i) \) the proposal wins the contest, since \( i \)'s utility conditional on winning is additively separable in score and ideology.

The second lemma establishes that at least one of the proposers is always active, in the sense of making a proposal with strictly positive score (all positive-score proposals are positive-quality, but the reverse is not necessarily true).

**Lemma A.2.** In equilibrium \( F_k (0) > 0 \) for at most one \( k \in \{L, R\} \).

**Proof:** Suppose not, so \( F_i (0) > 0 \ \forall i \) in some equilibrium. Let \( U^*_i \) denote proposer \( i \)'s equilibrium utility, which can be achieved by mixing according to her strategy conditional on making score-\( s \leq 0 \) proposal. Let \( \bar{y}^0 \) denote the expected ideological outcome and \( \bar{s}^0 \) the expected score outcome conditional on both sides making score \( \leq 0 \) proposals. Since \( x_L < 0 < x_R \), we have \( V_k (\bar{s}^0, \bar{y}^0) \leq V_k (0, 0) \) for at least one \( k \), which implies \( k \) has a profitable deviation since \( U^*_k \leq \Pi_k (0, 0; \sigma_{-k}) < \Pi_k (0, y^*_k (0); \sigma_{-k}) \) (since \( F_k (0) > 0 \)). QED

The third Lemma establishes that in equilibrium there is 0 probability of a tie at a positive score.

**Lemma A.3.** In equilibrium there is 0-probability of a tie at scores \( s > 0 \).

**Proof:** Suppose not, so each proposer’s strategy generates an atom of size \( p^*_i > 0 \) at some \( s > 0 \). Proposer \( i \) achieves her equilibrium utility \( U^*_i \) by mixing according to her strategy conditional on a
score-$s$ proposal. Let $\bar{y}^s$ denote the expected ideological outcome conditional on both sides making score-$s$ proposals; then $V_k(s, \bar{y}^s) \leq V_k(s, 0)$ for at least one $k$, who has a profitable deviation. If $k$’s proposal at score $s$ is $(s, 0)$, then $U^*_k \leq \Pi_k(s, 0; \sigma_{-k}) < \Pi_k(s, y_k^*(s); \sigma_{-k})$ (since $F_{-k}(s) > 0$). If $k$ sometimes proposes something else, then $U^*_k < \left(1 - \frac{p_k}{F_{-k}(s)}\right) \Pi_k(s, E[y_k|s]; \sigma_{-k}) + \left(\frac{p_k}{F_{-k}(s)}\right) \Pi_k(s, 0; \sigma_{-k})$, which is $k$’s utility if she were to instead propose $(s, 0)$ with probability $\frac{p_k}{F_{-k}(s)}$, and the expected ideology $E[y_k|s]$ of her strategy at score $s$ with the remaining probability (and always win ties). QED

Lemmas A.1 – A.3 jointly imply that in equilibrium, proposer $i$ can compute her expected utility as if her opponent only makes proposals of the form $(s_{-i}, y^*_{-i}(s_{-i}))$. The utility from making any proposal $(s_i, y_i)$ with $s_i > 0$ where $-i$ has no atom (or a tie would be broken in $i$’s favor) is therefore

$$\Pi^*_i(s_i, y_i; F) = -\alpha_i(s_i + y_i^2) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s_i}^{\infty} V_i(s_{-i}, y^*_{-i}(s_{-i})) dF_{-i}. \tag{A.2}$$

Proposer $i$’s utility from making the best proposal with score $s_i$ is $\Pi^*_i(s_i, y^*_i(s_i); F)$, which we henceforth denote $\Pi^*_i(s_i; F)$.

Fourth and finally, we establish that equilibrium score CDFs must satisfy the following natural properties arising from the all pay component of the contest.

**Lemma A.4.** Support of the equilibrium score CDFs over $\mathbb{R}^+$ is common, convex, and includes 0.

**Proof:** We first argue $\hat{s} > 0$ in support of $F_i \rightarrow F_{-i}(s) < F_{-i}(\hat{s}) \forall s < \hat{s}$. Suppose not; so $\exists s < \hat{s}$ where $-i$ has no atom and $F_{-i}(s) = F_{-i}(\hat{s})$. Then $\Pi_i(\hat{s}, y_i; F) - \Pi_i(s, y_i; F) = -(\alpha_i - F_{-i}(\hat{s})) \cdot (\hat{s} - s) < 0$, implying $i$’s best score-$s$ proposal is strictly better than her best score-$\hat{s}$ proposal, a contradiction. We now argue this yields the desired properties. First, an $\hat{s} > 0$ in $i$’s support but not $-i$ implies $\exists \delta > 0$ s.t. $F_{-i}(s - \delta) = F_{-i}(s)$. Next, if the common support were not convex or did not include 0, then there would $\exists \hat{s} > 0$ in the common support s.t. neither proposer has support immediately below, so $F_i(s) < F_i(\hat{s}) \forall s < \hat{s}$ would imply both proposers have atoms at $\hat{s}$, a contradiction. QED

We conclude by combining the preceding lemmas to state a preliminary characterization of all equilibria in the form of necessary and sufficient conditions.
Proposition A.1. Necessary conditions for SPNE are as follows:

1. (Ideological Optimality) With probability 1, proposals are either
   (a) negative score \( s_i \leq 0 \) and 0-quality \( s_i + y_i^2 = 0 \)
   (b) positive score \( s_i > 0 \) with ideology \( y_i = y_i^* (s_i) = \left( \frac{\alpha_i s_i}{\alpha_i + \gamma_i} \right) F_{-i} (s_i) \).

2. (Score Optimality) The profile of score CDFs \((F_i, F_{-i})\) satisfy the following boundary conditions and differential equations.
   - (Boundary Conditions) \( F_k (0) > 0 \) for at most one proposer \( k \), and there \( \exists \bar{s} > 0 \) such that \( \lim_{s \to \bar{s}} \{ \bar{\Pi}_i (s; F) \} = 1 \) \( \forall i \).
   - (Differential Equations) For all \( i \) and \( s \in [0, \bar{s}] \),
     \[ \alpha_i - F_{-i} (s) = f_{-i} (s) \cdot 2x_i (y_i^* (s) - y_{-i}^* (s)) \]

The above and \( F_i (s) = 0 \) \( \forall i, s < 0 \) are sufficient for equilibrium.

Proof: (Score Optimality) A score \( \bar{s} > 0 \) in the common support implies \([0, \bar{s}]\) in the common support (by Lemma A.4) implying \( \lim_{s \to \bar{s}} \{ \bar{\Pi}_i (s; F) \} \geq U_i^* \). Equilibrium also requires \( \bar{\Pi}_i (s; F) \leq U_i^* \) \( \forall s \) so \( \bar{\Pi}_i (s; F) = U_i^* \) \( \forall s \in [0, \bar{s}] \), further implying the \( F \)'s are absolutely continuous over \((0, \infty)\) (given our initial assumptions), and therefore \( \frac{d}{ds} (\bar{\Pi}_i^* (s; F)) = 0 \) for almost all \( s \in [0, \bar{s}] \). This straightforwardly yields the differential equations for score optimality, with the boundary conditions implied by Lemma A.4. (Ideological Optimality) At most one proposer \( (k) \) makes \( \leq 0 \)-score proposals with positive probability, so \( F_{-k} (0) = 0 \). Such proposals lose for sure and never influence a tie, and therefore must be 0-quality with probability 1, yielding property (a). Atomless score CDFs \( \forall s > 0 \) implies \((s, y_i^* (s))\) is the strictly best score-\( s \) proposal (by Lemma A.1), yielding property (b). ( Sufficiency) Necessary conditions imply all \((s, y_i^* (s))\) with \( s \in (0, \bar{s}] \) yield a constant \( U_i^* \). \( F_{-k} (0) = 0 \) implies \( k \)'s strictly best score-0 proposal is \((0, y_k^* (0)) = (0, 0) \) and yields \( \bar{\Pi}_k (0; F) \), and \( F_k (s) = 0 \) for \( s < 0 \) implies \( k \) has a size \( F_k (0) \) atom here. Thus both proposers’ mixed strategies yield \( U_i^* \), and neither can profitably deviate to \( s \in (0, \bar{s}] \). To see neither can profitably deviate to \( s > \bar{s} \), observe \( \Pi_i^* (s; F) - \Pi_i^* (\bar{s}; F) = - (\alpha_i - 1) (s - \bar{s}) < 0 \). To see \( k \) cannot profitably deviate to \( s_k \leq 0 \), \( F_{-k} (0) = 0 \)
implies such proposals lose and never influence a tie, and so yield utility \( \leq U^*_k \). To see \(-k\) cannot profitably deviate to \( s_{-k} \leq 0 \), observe all such proposals result in either \((0, y_{-k})\) or \((0, 0)\) when \( s_k \leq 0 \) (since the DM’s other choices are \((0, 0)\) and \( \emptyset \)), and thus yield utility \( \leq \max \{ \Pi_{-k} (0, 0; F), \Pi_{-k} (0, y_{-k}; F) \} \) which is \( \leq U^*_{-k} \). QED

A.1.2 Preliminary Observations about Equilibria

Proposition A.1 implies that all equilibria have a simple form. At least one proposer (henceforth labelled \(-k\)) is always active – thus, competition not only strictly benefits the DM in expectation, but with probability 1. The other proposer (henceforth labelled \(k\)) may also always be active \((F_k (0) = 0)\), or be inactive with strictly positive probability \((F_k (0) > 0)\). Inactivity may manifest as proposing the DM’s ideal point with no quality \((0, 0)\), or as “position-taking” with more distant 0-quality proposals that always lose \((s_k < 0 \text{ and } s_k + y^2_k = 0)\). However, any equilibrium exhibiting the latter is payoff-equivalent to one exhibiting the former; we thus focus on the former for comparative statics.\(^9\) When either proposer \(i\) is active, she mixes smoothly over the ideologically-optimal proposals \((s, y^*_i (s)) = \left( s, \frac{x_i}{\alpha_i} F_{-i} (s) \right)\) with scores in a common mixing interval \([0, \bar{s}]\) according to the CDF \(F_i (s)\).\(^10\)

The differential equations characterizing the equilibrium score CDFs arise intuitively from the proposers’ indifference condition over \([0, \bar{s}]\). The left hand side is \(i’s\) net marginal cost of making a higher-score proposal, given a fixed probability \(F_{-i} (s)\) of winning the contest; the proposer pays marginal cost \( \alpha_i > 1 \) for sure, but with probability \(F_{-i} (s)\) her proposal is chosen and she enjoys a marginal benefit of 1 (because she values quality). The right hand side represents \(i’s\) marginal ideological benefit of increasing her score. Doing so increases by \(f_{-i} (s)\) the probability that her proposal wins, which changes the ideological outcome from her opponent’s optimal ideology \(y^*_{-i} (s)\) at score \(s\) to her own optimal ideology \(y^*_i (s)\).

\(^9\)Profiles with “position-taking” are equilibria if the position-taking does not invite a deviation by \(-k\) to negative scores; whether this is the case depends on \(k\)’s score-CDF below 0 and the DM’s outside options \(\emptyset\). When \((0, 0) \in \emptyset\) the necessary conditions are also sufficient.

\(^10\)Technically, the proposition does not state that the support interval is also bounded \((\bar{s} < \infty)\), but this is later shown indirectly through the analytical equilibrium derivation.
A.2 Activity, Strength, Dominance, and Ideology

We first derive properties of equilibrium that do not require a complete characterization of the equilibrium score CDFs. To do so we use the following simple result that describes the equilibrium relationship between the proposers’ score CDFs.

**Lemma A.5.** In any SPNE, \( \epsilon_i (F_{-i} (s)) = \epsilon_{-i} (F_i (s)) \) \( \forall s \geq 0 \), where

\[
\epsilon_i (p) = \int_p^1 \frac{|x_i|}{\alpha_i - q} dq = |x_i| \log \left( \frac{\alpha_i - p}{\alpha_i - 1} \right)
\]

**Proof:** Rearranging the differential equation in score optimality yields

\[
\frac{F_{-i} (s) - |x_i|}{\alpha_i - F_{-i} (s)} = \frac{F_i (s) - |x_{-i}|}{\alpha_i - F_i (s)} \forall s \in [0, \tilde{s}]
\]

\( \alpha_i = 0 \) \( \forall i \) and \( \epsilon_i (p) \) is strictly increasing in \( |x_i| \) and decreasing in \( \alpha_i \) \( \forall p \in [0, 1] \). QED

We refer to the property in Lemma A.5 as the *engagement equality*. To see why, observe that the decreasing function \( \epsilon_i (p) \) captures i’s relative willingness to deviate from a proposal winning with probability \( p \) to one that wins for sure (since the marginal ideological benefit of moving policy in her direction is \( |x_i| \), and the net marginal cost of increasing score on a proposal winning the contest with probability \( q \) is \( \alpha_i - q \)). We call this function i’s *engagement at probability p*. The engagement equality \( \epsilon_i (F_{-i} (s)) = \epsilon_{-i} (F_i (s)) \) states that at every score \( s \geq 0 \) both proposers must be equally engaged given the resulting probabilities of winning the contest, and therefore equally willing to deviate to the maximum score \( \tilde{s} \). It is easily verified that \( \epsilon_i (1) = 0 \) \( \forall i \) and \( \epsilon_i (p) \) is strictly increasing in \( |x_i| \) and decreasing in \( \alpha_i \) \( \forall p \in [0, 1] \).

Usefully, the engagement equality implies a simple functional relationship between the players’ score CDFs that must hold in equilibrium regardless of their exact values. Letting

\[
p_i (\epsilon) = \alpha_i - (\alpha_i - 1) e^{\frac{|x_i|}{\epsilon}}
\]

denote the inverse of \( \epsilon_i (p) \) (which is decreasing in \( p \), increasing in \( |x_i| \), and decreasing in \( \alpha_i \)) equilibrium requires that \( F_i (s) = p_{-i} (\epsilon_i (F_{-i} (s))) \) \( \forall s \in [0, \tilde{s}] \).
A.2.1 Activity and Strength

We first use the engagement equality to derive the identity of the sometimes-inactive proposer \( k \) and the probability \( F_k(0) \) that she is sometimes inactive, as well as perform comparative statics on \( F_k(0) \).

**Proposition A.2.** In equilibrium \( k \in \text{arg min}_i \{ \epsilon_i(0) \} \) and

\[
F_k(0) = p_{-k}(\epsilon_k(0)) = \alpha_{-k} - (\alpha_{-k} - 1) \left( \frac{\alpha_k}{\alpha_k - 1} \right)^{\frac{x_k}{x_{-k}}}. \]

The probability \( k \) is inactive \( F_k(0) \) is decreasing in her distance from the DM \( |x_k| \) and her opponent’s quality costs \( \alpha_{-k} \), and increasing in her opponent’s distance from the DM \( |x_{-k}| \) and her own quality costs \( \alpha_k \). In addition, \( \lim_{|x_k| \to 0} \{ F_k(0) \} = \lim_{|x_{-k}| \to \infty} \{ F_k(0) \} = \lim_{\alpha_k \to \infty} \{ F_k(0) \} = \lim_{\alpha_{-k} \to 1} \{ F_k(0) \} = 1 \).

**Proof:** Suppose \( \epsilon_k(0) < \epsilon_{-k}(0) \); then \( F_k(0) = 0 \) and the engagement equality would imply \( F_{-k}(0) < 0 \), a contradiction. Since \( F_i(0) = 0 \) for some \( i \) we must have \( F_{-k}(0) = 0 \) and \( F_k(0) = p_{-k}(\epsilon_k(0)) > 0 \). Comparative statics and limit statements follow from previous observations on \( \epsilon_i(\cdot) \) and \( p_i(\cdot) \). QED

The sometimes-inactive proposer is thus the one with the lowest engagement at probability 0 – that is, who is least willing to participate in the contest entirely.

We next use the engagement equality to derive the players’ probabilities of victory.

**Proposition A.3.** In equilibrium the probability proposer \( k \) loses the contest is

\[
\int_0^1 p_{-k}(\epsilon_k(p)) \, dp = \int_0^1 \left( \alpha_{-k} - (\alpha_{-k} - 1) \left( \frac{\alpha_k}{\alpha_k - 1} \right)^{\frac{x_k}{x_{-k}}} \right) \, dp
\]

which is decreasing in her distance from the DM \( |x_k| \) and her opponent’s quality costs \( \alpha_{-k} \), and increasing in her opponent’s distance from the DM \( |x_{-k}| \) and her own quality costs \( \alpha_k \).

**Proof:** The probability \( k \) loses the contest is \( \int_0^s f_{-k}(s) F_k(s) \, ds \); applying the engagement equality this is \( \int_0^s p_{-k}(\epsilon_k(F_{-k}(s))) f_{-k}(s) \, ds \), and applying a change of variables of \( F_{-k}(s) \) for \( p \) (recalling \( F_{-k}(0) = 0 \)) yields the result. QED
The probability $k$ loses thus obeys the same comparative statics as her probability of inactivity. Somewhat paradoxically, she becomes less likely to win when her preferences are closer to the DM or her opponent’s are more distant. More intuitively, she becomes more likely to win if she is more able or her opponent less able.

A.2.2 Dominance

In the standard asymmetric two-player all-pay contest there is always an unambiguously weaker player, who makes bids that are first-order stochastically worse for the DM. In the present contest, in contrast, there may be no unambiguously weaker player in this sense.

**Proposition A.4.** Proposer $i$ is dominated ($F_{-i} (s) < F_i (s)$ ∀$s ∈ (0, \bar{s})$) i.f.f. she is less engaged at every probability $p$ ($\epsilon_i (p) < \epsilon_{-i} (p)$ ∀$p ∈ (0, 1)$). Equivalently, she is dominated i.f.f. both $\int_0^1 \frac{|x_i|}{\alpha_{i-q}} dq ≤ \int_0^1 \frac{|x_{-i}|}{\alpha_{-i-q}} dq$ and $\frac{|x_i|}{\alpha_{i-1}} ≤ \frac{|x_{-i}|}{\alpha_{-i-1}}$, where the latter condition is stronger than the former i.f.f. $i$ has a cost advantage.

**Proof:** Lemma A.5 and the engagement function $\epsilon_i (p)$ strictly decreasing when $p ∈ [0, 1)$ immediately implies $\text{sign} (\epsilon_{-k} (F_{-k} (s)) − \epsilon_k (F_{-k} (s))) = \text{sign} (F_{k} (s) − F_{-k} (s)) ∀s ∈ (0, \bar{s})$, which straightforwardly yields the first statement. Now let $\delta (p) = \epsilon_{-k} (p) − \epsilon_k (p)$, so $\delta (0) ≥ 0 = \delta (1)$. We argue $\delta' (1) ≤ 0$ is necessary and sufficient. For necessity, $\delta' (1) > 0 = \delta (1) → \delta (p) < 0$ in a neighborhood below 1. For sufficiency, it is easily verified that $\delta' (p) = \frac{|x_k|}{\alpha_k-p} − \frac{|x_{-k}|}{\alpha_{-k}-p}$ crosses 0 at most once when the proposers are asymmetric; thus $\delta (0) ≥ 0 = \delta (1) ≥ \delta' (0)$ implies $\delta (p)$ strictly quasi-concave over $[0, 1]$ and $\delta (p) > \min \{\delta (0), \delta (1)\} ≥ 0$ for $p ∈ (0, 1)$.

We last argue $\delta (0) ≥ 0$ and $\alpha_k > \alpha_{-k} → \delta' (1) < 0$. Observe that $\alpha_k < \alpha_k$ and $\delta' (0) = \frac{x_k}{\alpha_k} − \frac{x_{-k}}{\alpha_{-k}} ≤ 0 → \delta' (1) = \frac{|x_k|}{\alpha_k} \left( \frac{1}{1-1/\alpha_k} \right) − \frac{|x_{-k}|}{\alpha_{-k}} \left( \frac{1}{1-1/\alpha_{-k}} \right) < 0$. If $\delta' (0) ≤ 0$ we are done; if $\delta' (0) > 0$ then $\delta' (1) ≥ 0 → \delta' (p) > 0 ∀p ∈ [0, 1) → \delta (1) > 0$, a contradiction. QED

Clearly, a proposer $k$ who is both less extreme ($|x_k| ≤ |x_{-k}|$) and less able ($\alpha_k ≥ \alpha_{-k}$) (with one strict) satisfies both conditions and is therefore dominated. However, when one proposer is more extreme while the other is more able, then lower engagement at probability 0 is necessary but not sufficient for the more able proposer to be dominated.

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A.2.3 Ideology

Lastly, the engagement equality directly yields simple expressions for the probability distribution over the ideology of each player’s proposals.

**Proposition A.5.** Let $G_i(y) = \Pr (|y_i| \leq |y|)$ denote the probability that $i$’s proposal is closer to the DM than $y$. Then

$$G_i(y) = p_{-i} \left( \epsilon_i \left( \frac{y}{x_i/\alpha_i} \right) \right) = \alpha_{-i} - (\alpha_{-i} - 1) \left( \frac{x_i - y}{x_i - x_i/\alpha_i} \right) \frac{|x_i|}{x_i},$$

which is first-order stochastically increasing in $i$’s extremism $|x_i|$, decreasing in her costs $\alpha_i$, decreasing in her opponent’s extremism $|x_{-i}|$, and increasing in her opponent’s costs $\alpha_i$.

**Proof:** Proposer $i$’s ideology at score $s$ is $y_i^*(s) = \frac{x_i}{\alpha_i} F_{-i}(s)$ (from ideological optimality), so $F_{-i}(s_i^*(y)) = \frac{y}{x_i/\alpha_i}$ where $s_i^*(y)$ is the inverse of $y_i^*(s)$. That is, the probability $-i$ makes a proposal with score $\leq s_i^*(y)$ is $\frac{y}{x_i/\alpha_i}$. Now the probability $G(y)$ that $i$ makes a proposal closer to the DM than $y$ is $F_i(s_i^*(y))$, which is $= p_{-i} (\epsilon_i (F_{-i}(s_i^*(y)))) = p_{-i} \left( \epsilon_i \left( \frac{y}{x_i/\alpha_i} \right) \right)$ from the engagement equality. Comparative statics are straightforward. QED

When a proposer $i$ becomes more ideologically extreme (higher $|x_i|$) or able (lower $\alpha_i$), she makes first-order stochastically more extreme proposals. Proposer $i$’s opponent $-i$ reacts to $i$ becoming more extreme or more able by moderating the ideological extremism of her own proposals.

A.3 Payoffs

We complete the analysis by calculating payoffs. This requires first characterizing score CDFs $F_i(s)$ satisfying Proposition A.1, which are shown constructively to be unique.

**Proposition A.6.** The unique score CDFs over $s \geq 0$ satisfying Proposition A.1 are $F_i(s) = p_{-i} (\epsilon(s)) \forall i$, where $\epsilon(s)$ is the inverse of

$$s(\epsilon) = 2 \int_{\epsilon}^{\epsilon_k(0)} \sum_j \frac{|x_j|}{\alpha_j} p_j(\hat{\epsilon}) d\hat{\epsilon}.$$
The inverse score CDFs are \( s_i(F_i) = s(\epsilon_{-i}(F_i)) \forall i \), and the score targeted at each ideology is 
\[ s'_i(y) = s \left( \epsilon_i \left( \frac{y}{\alpha_i} \right) \right). \]
The function \( s(\epsilon) \) is strictly increasing in \( x_i \) and strictly decreasing in \( \alpha_i \) \( \forall \epsilon \in [0, \epsilon_k(0)) \), and the maximum score is \( \bar{s} = s(0) \).

**Proof:** From the engagement equality \( \epsilon_i(F_{-i}(s)) = \int_{F_{-i}(s)}^{1} \frac{|x_i|}{\alpha_i - q} dq = \epsilon(s) \forall i, s \) for some \( \epsilon(s) \). We characterize the unique \( \epsilon(s) \) implying score CDFs \( F_i(s) = p_{-i}(\epsilon(s)) \) and optimal ideologies \( y_i(s) = \frac{x_i}{\alpha_i} p_i(\epsilon(s)) \) that satisfy score optimality. First observe that \( \epsilon'(s) = f_i(s) \epsilon'_{-i}(F_i(s)) = -\int_{\epsilon(s)}^{x_i(s)} \epsilon'(\bar{\epsilon}) d\bar{\epsilon} \). Next the differential equations may be rewritten as \( \frac{a_i - F_{-i}(s)}{F_{-i}(s) - |x_i|} = 2 \sum_j y_j(s) \). Substituting the preceding observations into both sides yields \( \frac{1}{\epsilon'(s)} = -2 \sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon(s)) \), and rewriting in terms of the inverse \( s(\epsilon) \) yields \( s'(\epsilon) = -2 \sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon) \).

Lastly \( \epsilon_k(F_{-k}(s)) = \epsilon(s) \) and \( F_{-k}(0) = 0 \) imply the boundary condition \( s(\epsilon_k(0)) = 0 \) so \( s(\epsilon) = \int_{\epsilon_k(0)}^{\epsilon_k(0)} s'(\hat{\epsilon}) d\hat{\epsilon} = 2 \int_{\epsilon_k(0)}^{\epsilon_k(0)} \sum_j \frac{|x_j|}{\alpha_j} p_j(\hat{\epsilon}) d\hat{\epsilon} \). Now \( s(\epsilon) \) is straightforwardly increasing in \( |x_i| \) and decreasing in \( \alpha_i \) given previous observations about \( p_j(\hat{\epsilon}) \). QED

The maximum score \( \bar{s} \) thus changes continuously with the parameters of both proposers, even when one is dominant. This contrasts with the standard 2-player all pay contest, where the mixing interval is unaffected by the parameters of the stronger player.

The preceding characterization transparently yields the following comparative statics.

**Corollary A.1.** Increasing a proposer’s extremism \( |x_i| \) or decreasing her costs \( \alpha_i \) first-order stochastically increases her own score CDF, but has ambiguous effects on her opponent’s score CDF.

To see that the effect of a proposer’s parameters on her opponent’s score CDF is necessarily ambiguous, suppose that the always-active proposer \(-k\) becomes even more extreme or able. Then her opponent \( k \) becomes less likely to be active, but also the range of scores \([0, \bar{s}]\) over which she mixes when she is active increases. She thus has a higher probability of making very high-score proposals, even while she is simultaneously less likely to enter the contest.

**A.3.1 Proposer Payoffs**

Using Proposition A.6, the proposers’ equilibrium payoffs are as follows.

**Proposition A.7.** Proposer \( i \)'s equilibrium utility is \( \Pi_i^*(\bar{s}; F^*) = -\left( 1 - \frac{1}{\alpha_i} \right) x_i^2 - (\alpha_i - 1) \bar{s}, \) which is decreasing in her own costs \( \alpha_i \) as well as either players’ extremism \( |x_j| \forall j, \) and increasing in her opponent’s costs \( \alpha_{-i} \).
Proof: A proposer’s equilibrium utility is straightforward since \( (\bar{s}, y_i^*(\bar{s})) \) is in the support of their strategy and wins for sure. Comparative statics of a proposer’s \( i \)'s parameters on her opponent \(-i\)'s utility, as well as of \( x_i \) on her own utility, follow immediately from previously-shown statics on \( \bar{s} = s(\epsilon) \).

Taking the derivative with respect to \( \alpha_i \), substituting in 
\[
-\frac{p_i'(\epsilon) x_i}{\alpha_i - p_i(\epsilon)} = 1,
\]
performing a change of variables, and rearranging the expression yields 
\[
-1 \cdot 2 \int_0^{\epsilon_k(0)} \frac{|x_k|}{\alpha_k} \left( p_{-k}(\epsilon) - \left( \frac{\alpha_k}{\alpha_k - 1} \right)^{-1} p_{-k}(\epsilon_k(0)) \right) d\epsilon - \left( \frac{|x_i|}{\alpha_i} \right)^2 \left( 1 + 2 \int_{p_i(\epsilon_k(0))}^{1} \left( \frac{\alpha_i p}{\alpha_i - p} - 1 \right) dp \right).
\]
The first term is negative since \( p_{-k}(\epsilon) > p_{-k}(\epsilon_k(0)) \) for \( \epsilon < \epsilon_k(0) \) and \( \frac{1}{\alpha_k} < \int_0^{1} \frac{1}{\alpha_k - p} dp = \log \left( \frac{\alpha_k}{\alpha_k - 1} \right) \). The second term is also negative since 
\[
1 + 2 \int_{p_i(\epsilon_k(0))}^{1} \left( \frac{\alpha_i p}{\alpha_i - p} - 1 \right) dp = \int_{p_i(\epsilon_k(0))}^{1} (2p - 1) dp \geq 0.
\]
QED

A proposer’s equilibrium utility has two components. The first \(-\left( 1 - \frac{1}{\alpha_i} \right) x_i^2 \) is her utility if she could make proposals as a “monopolist” (and the DM’s outside option included \((0, 0)\)). The second \(- (\alpha_i - 1) \bar{s} \) is the cost generated by competition, which forces her to make proposals that leave the DM strictly better off than the best “free” proposal \((0, 0)\) in order to maintain influence. This competition cost is increasing in \( i \)'s marginal cost \( \alpha_i \) of generating quality (holding \( \bar{s} \) fixed) as well as the maximum score \( \bar{s} \), which in turn is increasing in both proposers’ ideological extremism and decreasing in their costs everywhere in the parameter space. A proposer is thus strictly harmed when her competitor becomes more extreme or able. This is distinct from all pay contests without spillovers (Siegel (2009)), where the equilibrium utility of the “sometimes inactive” player is pinned at her fixed value for losing.

A proposer also worse off when her own preference become more distant from the decisionmaker. Finally, a proposer is worse off when her costs of producing quality increase – even though there is a countervailing effect of reducing the intensity of competition (and indeed, the competition cost \((\alpha_i - 1)\bar{s} \) alone is not generically monotonic in \( \alpha_i \)).

A.3.2 DM Payoffs

Lastly, again using Proposition A.6 the DM’s equilibrium utility and the proposers’ average scores (which bound the DM’s utility from below) are as follows.
Proposition A.8. The DM’s equilibrium utility is \( U_{DM}^* = \int_{e_k(0)}^{e_k(0)} s(\epsilon) \cdot \frac{\partial}{\partial \epsilon} \left( \prod_j p_j(\epsilon) \right) \) de =

\[
2 \int_0^{e_k(0)} \left( 1 - \prod_j p_j(\epsilon) \right) \cdot \left( \sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon) \right) \) de
\]

Proposer i’s average score is \( E[s_i] = \int_{e_k(0)}^{e_k(0)} s(\epsilon) \cdot \frac{\partial}{\partial \epsilon} (p_{-i}(\epsilon)) \) de =

\[
2 \int_0^{e_k(0)} (1 - p_{-i}(\epsilon)) \cdot \left( \sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon) \right) \) de
\]

Proof: \( F_i(s) F_{-i}(s) \) is the CDF of \( \max\{s_i, s_{-i}\} \) so the DM’s utility is \( \int_0^s s \cdot \frac{\partial}{\partial s} \left( \prod_j F_j(s) \right) ds = \int_0^s s \cdot \frac{\partial}{\partial s} \left( \prod_j p_j(\epsilon(s)) \right) ds \). A change of variables from \( s \) to \( \epsilon \) yields the first expression and integration by parts and rearranging yields the second. A nearly identical series of steps yields i’s average score. QED

Direct comparative statics on the DM’s utility \( U_{DM}^* \) are difficult because changing a proposer’s parameters has mixed effects on her opponent’s score CDF. We thus consider two special cases; breaking symmetry, and the limiting cases of extreme imbalance. The effect of breaking symmetry is as follows.

Proposition A.9. When the proposers are symmetric \( |x_i| = |x_{-i} \) and \( \alpha_i = \alpha_{-i} \), the DM’s utility is locally increasing either’s extremism or ability.

Proof: First differentiating the DM’s utility \( U_{DM}^* \) with respect to \( |x_{-k}| \) and applying symmetry yields

\[
\frac{2}{\alpha} \int_0^{e_k(0)} \left( \left( 1 - 3p(\epsilon) \right)^2 \cdot \frac{\partial p(\epsilon)}{\partial \epsilon} + \left( 1 - (p(\epsilon))^2 \right) p(\epsilon) \right) \) de which is transparently \( \geq \frac{2}{\alpha} \int_0^{e_k(0)} \left( 1 - 3(p(\epsilon))^2 \right) \frac{\partial p(\epsilon)}{\partial \epsilon} \) de.
\]

Now substituting \( \frac{\partial p(\epsilon)}{\partial \epsilon} = -\log \left( \frac{\alpha - p(\epsilon)}{\alpha - 1} \right) p'(\epsilon) \) and a change of variables yields \( \frac{2}{\alpha} \int_0^1 \left( 1 - 3p^2 \right) \log \left( \frac{\alpha - p}{\alpha - 1} \right) dp = \frac{2}{\alpha} \int_0^1 \left( \frac{p - p^3}{\alpha - p} \right) dp > 0 \). Next differentiating \( U_{DM}^* \) w.r.t. \( \alpha_{-k} \) and applying symmetry yields

\[
2x \int_0^{e_k(0)} \left( 1 - (p(\epsilon))^2 \right) \frac{\partial}{\partial \alpha} \left( \frac{p(\epsilon)}{\alpha} \right) - \frac{2}{\alpha} \left( p(\epsilon) \right)^2 \frac{\partial p(\epsilon)}{\partial \alpha} \) de. Substituting \( \frac{\partial}{\partial \alpha} \left( \frac{p(\epsilon)}{\alpha} \right) = \frac{x}{(\alpha - 1)\alpha^2} p'(\epsilon), \frac{\partial p(\epsilon)}{\partial \alpha} = -\frac{1 - p(\epsilon)}{\alpha - 1}, -\frac{p'(\epsilon) x}{\alpha - p(\epsilon)} = 1 \), rearranging, and a change of variables yields \( \frac{2x^2}{(\alpha - 1)\alpha^2} \int_0^1 \left( 2p^2 \left( \frac{\alpha - p}{\alpha - x} \right) - (1 - p^2) \right) dp < 0 \). QED

The DM thus strictly benefits locally if symmetry between the players is broken by one becoming more
extreme or able – even though the other also becomes less active. The effect of extreme asymmetries is as follows.

**Proposition A.10.** The DM’s utility exhibits the following limiting behavior

\[ 0 = \lim_{\alpha_i \to \infty} U^*_{DM} = \lim_{x_i \to 0} U^*_{DM} < \lim_{x_i \to \infty} U^*_{DM} = \infty \]

and \( \lim_{\alpha_i \to 1} U^*_{DM} = 2x_k \int_0^1 \left( \frac{1-p}{\alpha_k-p} \right) \cdot \left( \frac{x_k}{\alpha_k} p + x-k \right) dp \), which is strictly increasing in \( x_k \) and strictly decreasing in \( \alpha_k \).

**Proof:** Observe that \( E [s-k] \leq U^*_{DM} \leq \bar{s} \). For the first two limiting statements it is easily verified that \( \bar{s} \to 0 \) as \( \alpha_k \to \infty \) or \( x_k \to 0 \). For the third limiting statement observe that \( E [s-k] \geq \frac{|x-k|}{\alpha_k} p-k (\epsilon_k (0)) \cdot 2 \int_0^{\epsilon_k (0)} (1-p_k (\epsilon)) d\epsilon \to \infty \) as \( |x-k| \to \infty \) since the first term \( \to \infty \) and the remaining terms are non-decreasing. For the fourth limiting statement, using the definition in Proposition A.8 and that \( \lim_{\alpha_k \to 1} \left\{ p-k (\epsilon) \right\} = 1 \forall \epsilon \in [0, \epsilon_k (0)] \) yields a limit of \( 2 \int_0^{\epsilon_k (0)} (1-p_k (\epsilon)) \cdot \left( \frac{x_k}{\alpha_k} p_k (\epsilon) + x-k \right) d\epsilon \). Observing that \( -\frac{p_k (\epsilon) x_k}{\alpha_k-p_k (\epsilon)} = 1 \), substituting into the expression, and applying a change of variables yields the expression, which straightforwardly obeys the stated comparative statics. QED

If an extreme imbalance is the result of one proposer’s incompetence or ideological moderation, the DM’s utility approaches 0, her utility if \( -i \) were a “monopolist” (and the DM’s outside options included \((0, 0)\)). (Proposer \(-i\)’s utility also approaches her utility if she were a monopolist). However, if extreme imbalance is the result of one proposer’s greater ability to produce quality (specifically, if her marginal cost of producing quality approaches its intrinsic value), then the DM’s utility is bounded away from 0. In this case the DM strictly benefits from the potential for competition, even though actual competition is almost never observed (since \( F_{-i} (0) = F_k (0) \) approaches 1). Finally, unilateral ideological extremism benefits the decisionmaker in a strong sense; the DM can achieve arbitrarily high utility with a proposer whose preferences are sufficiently distant from her own.
B  Guide to Main Text Propositions

In this Appendix we “prove” the main text propositions by describing where to locate each stated result in the preceding general treatment.

Proof of Proposition 1

The distribution over ideologies is provided in Proposition A.5. The score $s^Y_i(y) = s\left(\frac{y}{x_i/\alpha_i}\right)$ at each ideology is provided in Proposition A.6 (which also defines the function $s(\cdot)$), so the implied quality is $q^Y_i = y^2 + s\left(\frac{y}{x_i/\alpha_i}\right)$ as stated in the main text.

Proof of Proposition 2

This proposition restates a subset of the results provided in Hirsch and Shotts (2015) Propositions 1-3 and Corollary 1.

Proof of Proposition 3

The first statement follows from Proposition A.2 on activity.

The second statement on ideology is an implication of the ideology comparative statics stated in Proposition A.5, which state that as a proposer becomes unilaterally more extreme her ideologies become more extreme and her opponent’s ideologies simultaneously become more moderate. To see this compare strategies when the proposers have the same extremism as the moderate (and therefore develop symmetrically extreme proposals) to when one proposer becomes more extreme. The second statement on score follows from the necessary and sufficient conditions for score-dominance in Proposition A.4 – a proposer being more extreme and able with at least one strict is a sufficient condition for score dominance. Finally, the second statement on quality is an immediately implication of the extremist having more ideologically extreme but also higher score proposals (first order stochastically).

The third statement is an immediate implication of the second statement.

Proof of Proposition 4

The first bullet point under both “own strategy” and “opponent” strategy follow from Proposition A.2 on activity. The second bullet point under both “own strategy” and “opponent” strategy follow from Proposition A.5 on ideology. The third bullet point under “own strategy” is a joint implication.
of the ideology comparative statics in Proposition A.5 and the comparative statics on “own score” in Corollary A.1. The third bullet point under “opponent strategy” also follows from Corollary A.1 and the subsequent discussion.

**Proof of Proposition 5**
Follows immediately from Proposition A.7.

**Proof of Proposition 6**
The first bullet point is from Proposition A.9. The second and third bullet points are a joint implication of Propositions A.2 (on activity) and A.10 (on decisionmaker’s welfare).

**Proof of Proposition 7**
Follows from Propositions A.2, A.4, and A.5 according to an identical argument as in the proof of Proposition 3.

**Proof of Proposition 8**
Follows from Proposition A.2, A.5, and Corollary A.1 by an identical argument as in the proof of Proposition 4.

**Proof of Proposition 9**
Follows immediately from Proposition A.7.

**Proof of Proposition 10**
The first bullet point is from Proposition A.9. The second and third bullet points are a joint implication of Propositions A.2 (on activity) and A.10 (on decisionmaker’s welfare).