

The ZTF Source Classification Project: II. Periodicity and variability processing metrics

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ABSTRACT

The current generation of all-sky surveys is rapidly expanding our ability to study variable and transient sources. These surveys, with a variety of sensitivities, cadences, and fields of view, probe many ranges of timescale and magnitude. Data from the Zwicky Transient Facility (ZTF) yields an opportunity to find variables on timescales from minutes to months. In this paper, we present the codebase, `ztfperiodic`, and the computational metrics employed for the catalogue based on ZTF's Second Data Release. We describe the publicly available, graphical-process-unit optimized period-finding algorithms employed, and highlight the benefit of existing and future graphical-process-unit clusters. We show how generating metrics as input to catalogues of this scale is possible for future ZTF data releases. Further work will be needed for future data from the Vera C. Rubin Observatory's Legacy Survey of Space and Time.

Keywords: techniques: photometric, stars: statistics, methods: data analysis, catalogues, surveys

1. INTRODUCTION

The study of variable and transient sources is rapidly expanding based on the large data sets available from wide-field survey telescopes. Amongst others these include the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS; Morgan et al. 2012), the Asteroid Terrestrial-impact Last Alert System (ATLAS; Tonry et al. 2018), the Catalina Real-Time Transient Survey (CRTS; Drake et al. 2009, 2014), the All-Sky Automated Survey for SuperNovae (ASAS-SN; Shappee et al. 2014; Kochanek et al. 2017), the Zwicky Transient Facility (ZTF; Bellm et al. 2018; Graham et al. 2019; Dekany et al. 2020; Masci et al. 2018), the Visible and Infrared Survey Telescope for Astronomy (VISTA; Lopes et al. 2020) and in the near future, the Vera C. Rubin Observatory's Legacy Survey of Space and Time (LSST; Ivezić et al. 2019). Catalogs of variable sources

from these surveys are building upon the highly successful work identifying periodic variable stars in the Magellanic Clouds and Galactic center; we highlight catalogs such as those from MACHO (Alcock et al. 2000) and OGLE (Udalski 2003; Udalski et al. 2015) as examples. Coupled with Gaia's second data release (DR2; Gaia Collaboration 2018) and soon DR3, we are currently experiencing a dramatic expansion of our capabilities for doing time-domain astrophysics. The range of sensitivities and cadences change the parameter space of magnitude ranges and timescales probed, which can span from years down to minutes for many of these surveys.

While these surveys can be used to follow-up objects of interest, here, we are primarily interested in “untargeted” searches, i.e., searches without a priori knowledge about objects, for variable and periodic objects. These searches enable identification of populations of objects,

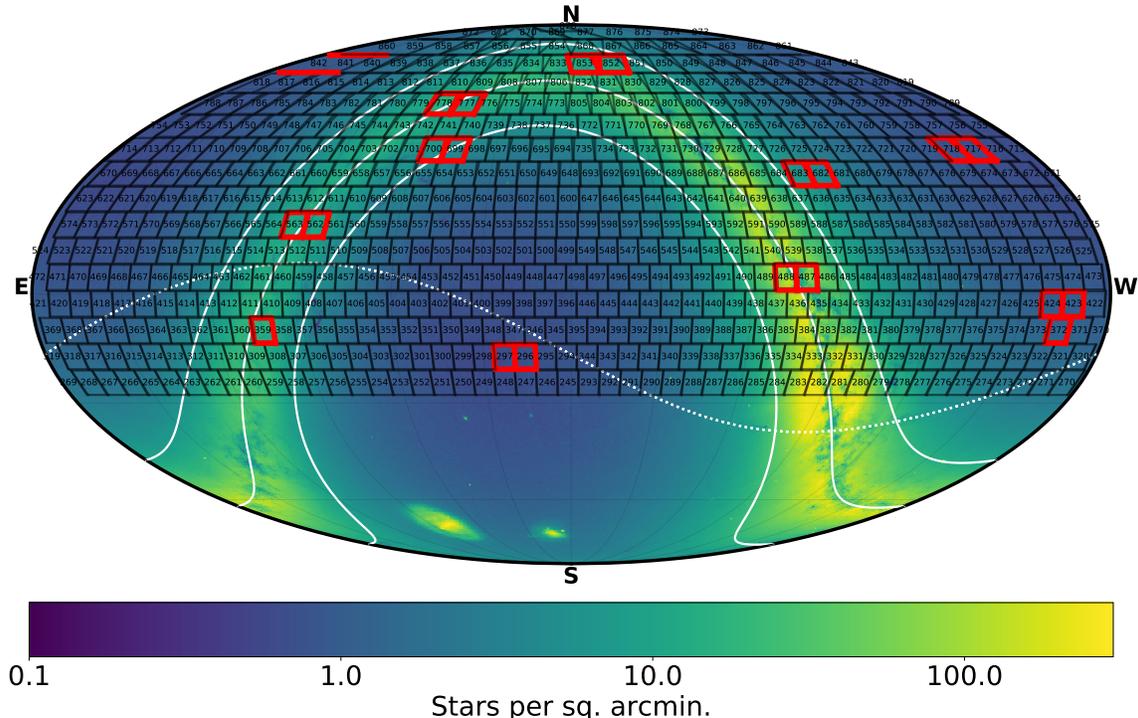


Figure 1. Mollweide projection in equatorial coordinates of the ZTF field coverage as a function of stellar density, using Gaia DR2 (Gaia Collaboration 2018). The black lines correspond to ZTF field boundaries, with the number contained corresponding to the field number. The solid white lines correspond to galactic latitude of -15 , 0 , and 15 , while the dashed line corresponds to the ecliptic. We highlight the 20 fields in the initial catalog release in red (van Roestel et al. in prep).

such as cataclysmic variables (Szkody et al. 2020), Be stars (Ngeow et al. 2019), Cepheids and RR Lyrae, useful for studying the processes and properties of galaxy formation (Saha 1984, 1985; Catelan 2009) and measuring the expansion rate of the Universe (Freedman et al. 2001). In addition, they are also sensitive to rare objects, such as short-period white dwarf binaries (Burdge et al. 2019a,b; Coughlin et al. 2020), large-amplitude, radial-mode hot subdwarf pulsators (Kupfer et al. 2019), and ultracompact hot subdwarf binaries (Kupfer et al. 2020).

To highlight one source class, we are discovering and characterizing the population of so-called ultra-compact binaries (UCBs), which have two stellar-mass compact objects with orbital periods $P_{\text{orb}} < 1$ hour (Burdge et al. 2019a,b; Coughlin et al. 2020). Many of the UCBs emit gravitational waves in the milliHertz regime with sufficient strain for the upcoming *Laser Interferometer Space Antenna* (*LISA*) to detect (Amaro-Seoane et al. 2017). These *LISA* “verification sources” will serve as crucial steady-state sources of gravitational waves that will not only verify that *LISA* is operating as expected (Kupfer et al. 2018), but also themselves serve as probes of binary stellar evolution (Nelemans & Tout 2005; Kremer et al. 2018; Banerjee 2018; Antonini et al. 2017), white

dwarf structure (Fuller & Lai 2011), Galactic structure (Breivik et al. 2020), accretion physics (Cannizzo & Nelemans 2015) and general relativity (Burdge et al. 2019a; Kupfer et al. 2019).

We are interested in computationally efficient algorithms for measuring the variability and periodicity of large suites of light curves for the purpose of creating catalogs of sources. These catalogs are useful for identifying classes of sources such as those mentioned above, as well as for mitigating the presence of known variable sources in searches for new transient objects, such as short γ -ray burst or gravitational-wave counterparts, e.g., Coughlin et al. (2019a,b). The size of all-sky survey data are large, enabling identification of a large number of sources. However, the computational cost scales linearly with the number of objects. In ZTF’s Second Data Release (DR2)¹, which we will employ here, there are 3,153,256,663 light curves, regardless of passband; for comparison, a cross-match to a Pan-STARRS catalog indicates that ~ 1.3 billion of these sources are unique. In the following, each individual light curve will be processed if they exceed 50 detections after surviving data quality and other cuts described below; we will focus

¹ <https://www.ztf.caltech.edu/page/dr2>

Field	RA [°]	Dec [°]	Gal. Long. [°]	Gal. Lat. [°]
296	15.79	-17.05	141.27	-79.14
487	281.19	4.55	36.55	3.03
682	266.86	33.35	58.61	26.73
851	351.43	69.35	115.73	7.93

Table 1. Locations of the fields (measured at their centers) highlighted in this paper.

our discussion in this paper on a small fraction of the fields, covering a range of stellar densities, consistent with those chosen for classification studies in the accompanying catalog paper (van Roestel et al. in prep). We note that this separately treats g -, r -, and i -band observations of the same light curves, and the set includes many objects with hundreds of detections in multiple passbands. In the following, we analyze single-band lightcurves, and so the same objects in different passbands are analyzed separately. We seek to choose variability metrics and periodicity algorithms appropriate for analyzing this data-set. We have taken inspiration from variability and periodicity codebases, such as FATS (Nun et al. 2015), *astrobase* (Bhatti et al. 2020) and *cesium* (Naul et al. 2016) in choosing the metrics, with a focus on using scalable graphical processing unit (GPU) periodicity algorithms to make the required large-scale processing tractable.

In this paper, we will describe the pipeline *ztfperiodic*², which we use to systematically identify variable and periodic objects in ZTF; we use a mix of metrics designed to efficiently identify and characterize variable objects as well as algorithms to phase-fold light-curves in all available passbands. We use a variety of available arrays of GPUs to period search all available photometry. The arrays of GPUs available are ideal for period finding large data sets and already have shown significant speed-ups relative to central processing units (CPUs).

2. OBSERVATIONAL DATA

We employ data predominantly from DR2 in this analysis, which covers public data between 2018-03-17 and 2019-06-30, and private data between 2018-03-17 to 2018-06-30; in our analysis, we also include private data up until 2019-06-30. ZTF’s Mid-Scale Innovations Program in Astronomical Sciences (MSIP) program covers the observable night sky from Palomar with a 3 night cadence, and a nightly cadence in both g and r filters in the Galactic plane (Bellm et al. 2019). The ZTF partnership is also conducting a moderate cadence survey

over a 3000 square degree field at high declination, visiting the entire field 6 times per night in g and r filters, resulting in more than 1000 epochs. Within this field, we expect to probe variables at the 20 mmag level for objects at 16th magnitude, and around 100 mmag at 20th magnitude (Masci et al. 2018). Also, ZTF conducts a high cadence survey in the Galactic plane, with galactic latitude $|b| < 14^\circ$ and galactic longitude $10^\circ < l < 230^\circ$, where the camera continuously sits on a field for several hours with a 40 s cadence. Overlaid by stellar densities, we show the ZTF primary field grid in Figure 1; we note there is also a secondary grid filling in gaps in the primary grid. In this Figure, we also include outlined in red the fields targeted in the forthcoming catalog paper (van Roestel et al. in prep).

The left panel of Figure 2 shows the probability density function of the number of detections for individual objects passing the data quality and time cuts for four example fields. The locations of these fields are given in Table 1, chosen to span a variety of galactic latitudes and longitudes and therefore stellar densities. This analysis removes any observation epochs indicating suspect photometry according to the *catflags* value in the light curve metadata³. It also removes any “high cadence” observations, defined as observations within 30 minutes of one another; for any series of observations with observation times less than 30 minutes apart, we keep only the first observation in the series. This is to support the periodicity analyses; an over-abundance of observations in very densely observed sets otherwise dominate the period finding statistics, as these observations will have the same weight as all temporally separated observations, but lack the sensitivity to periods longer than the single night in which they are taken. This short-coming will be a point of study moving forward. In addition to those observations removed, some subset of sources have fewer observations within the same field; these occur because faint stars have fewer detections because of lower detection efficiency. We also remove any stars within $13''$ of either an entry in the Yale Bright Star Catalog (Hoffleit & Jaschek 1991) and Gaia (Gaia Collaboration 2018) stars brighter than 13th magnitude; we arrived at this value through evaluating the measured variability of stars near cataloged objects, and $13''$ was a threshold beyond which the variability was consistent with background. This helps to remove bright blends from the variability selection process. On the right of Figure 2, we show the probability density function of the weighted

³ For details, see the DR2 Release Summary <https://www.ztf.caltech.edu/page/dr2>, specifically Section 9b.

² <https://github.com/mcoughlin/ztfperiodic>

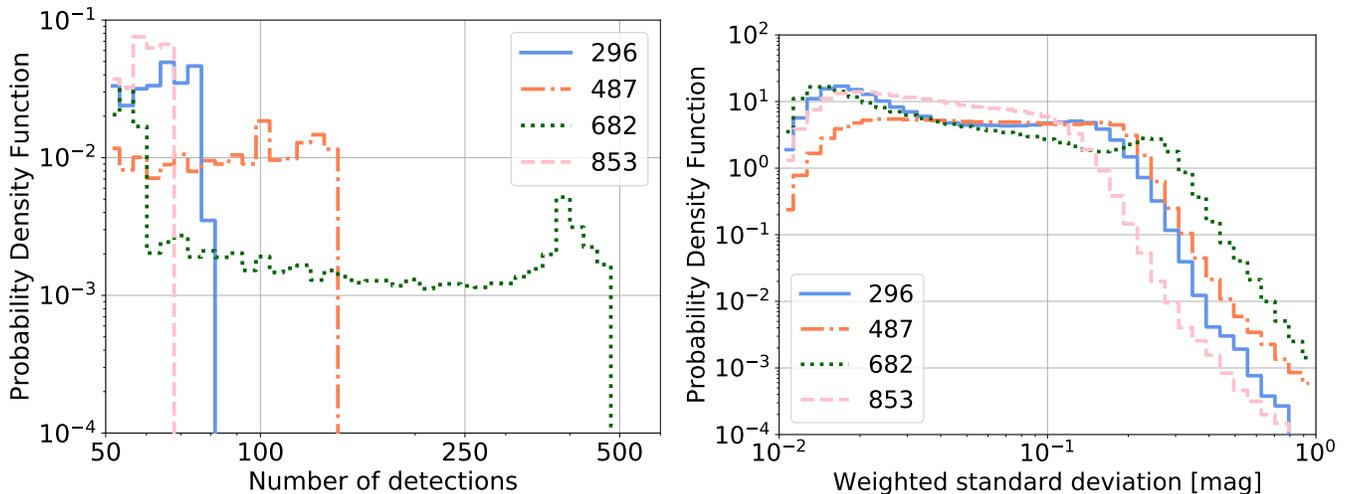


Figure 2. Left: Probability density function of the number of detections for individual objects passing the data quality and time cuts for four example fields. The field IDs are given in the legend, and their locations given in Table 1. Right: Probability density function of the weighted standard deviations for the light curves of individual objects.

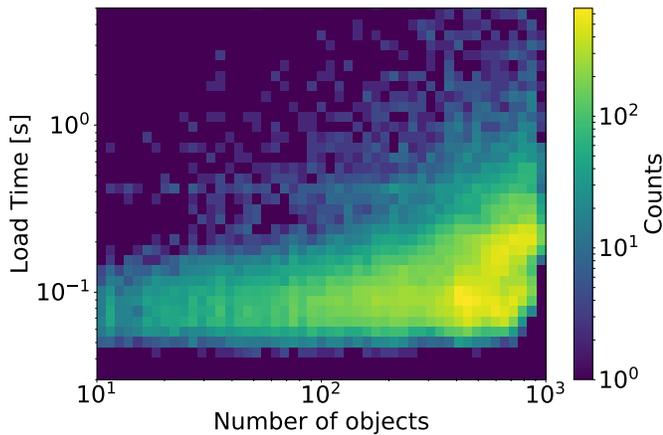


Figure 3. Two dimensional histogram of the read time from the ZTF light curve database “Kowalski” vs. the number of objects returned. The load time is mostly independent of the number of objects returned.

standard deviation for the light curves of individual objects in the example fields, showing consistency of this metric across fields.

To query the photometry, we are using “Kowalski”⁴, an efficient non-relational database that uses MongoDB to efficiently store and access both ZTF alert/light curve data and external catalogs (including 230M+ alerts and 3.1B+ light curves) (Duev et al. 2019). We show the read time from the ZTF light curve database “Kowalski” vs. the number of objects returned on the left of Figure 3. We note that the typical 100-200 ms includes the typical 54 ms latency between California (where Kowal-

ski resides) and Minnesota (where the test was performed), including ~ 20 ms of light travel time. As can be seen, the analysis takes ~ 1 s per light curve across the range of light curves analyzed at a single time (100 to 1000 which is the memory limit for the GPUs).

The light curves are analyzed in groups of ~ 1000 to fill out the RAM available on most GPUs employed; when the jobs are being distributed, the total number of light curves in a given quadrant on a particular CCD within a field are queried. This is possible as ZTF has two grids on which all observations are performed, with a repeating pointing accuracy varying ~ 50 -200”; therefore, a particular object will generally only appear twice, once on each grid. For reference, there are 64 quadrants, 4 quadrants for each of the 16 CCDs. Based on this, the number of jobs required to analyze the total number of light curves, in chunks of 1000, are computed. To ensure the efficacy of the variability and periodicity metrics we discuss below, we place a threshold of at least 50 detections in a single band for an object to be analyzed. This requirement means fewer than 1000 light curves are sometimes analyzed because some will not meet the 50 observation limit required for analysis. Because `ztfperiodic` analyzes the data in chunks like these, it is simple to parallelize across GPUs, with different GPUs running on different sets of ~ 1000 light curves; an `hdf5` file is written at the end to disk containing the statistics for a particular job (see below), each chunk receiving a different name based on a simple convention (field, CCD, quadrant, job index).

3. VARIABLE SOURCE METRICS

Due to the significant amount of astronomical time series data provided by ZTF and other all-sky surveys, it

⁴ <https://github.com/dmitryduev/kowalski>

is important to have robust selection criteria and algorithms to find variable objects. In ground-based surveys like ZTF, light curves are irregularly sampled, have gaps, and can have large statistical errors, which means that these algorithms must be robust in order to efficiently find true signals. The catalog includes two main types of metrics: variability and periodicity. These metrics are typically useful for machine learning algorithms to group light curves into categories through feature extraction based on light curve data. The goal of these features is to encode numerical or categorical properties to characterize and distinguish the different variability classes, such that machine learning algorithms can distinguish between classes of light curves.

There are a variety of computationally cheap variability metrics we use, ranging from relatively basic statistical properties such as the mean or the standard deviation to more complex metrics such as Stetson indices (Stetson 1996). As shown by Pashchenko et al. (2017), many of the commonly used features are strongly correlated. We therefore chose the set of features suggested by Pashchenko et al. (2017), with the addition of robust measurements of the amplitude. We also performed a Fourier-decomposition to characterize the shape of the folded light curve better. We detail the choices we have made below, which include the number of measurements, weighted mean and median magnitudes (RMS and percentile-based), kurtosis, skewness, variance, chi-square, Fourier indices, amongst many others. The statistics simply require three vectors for each light curve, encoding the time, magnitude, and magnitude error.

We provide a summary of the metrics employed in Table 2. While the accompanying catalog paper (van Roestel et al. in prep) will discuss their use extensively, to demonstrate their efficacy, Figure 4 shows the probability density for a subset of the features in the analysis for a “variable” and “non-variable” set of objects. We define “variable” objects as those that statistically change from observation to observation inconsistent with their assigned error bars; “non-variable” objects do not display these traits at the signal-to-noise reached by ZTF. A number show clear differentiation between the two sets, indicating their suitability as metrics for this purpose. We note that while some metrics have similar marginalized distributions, this marginalized format masks potential higher order correlations between parameters, and therefore we choose to use all metrics in our classifier.

4. PERIOD FINDING

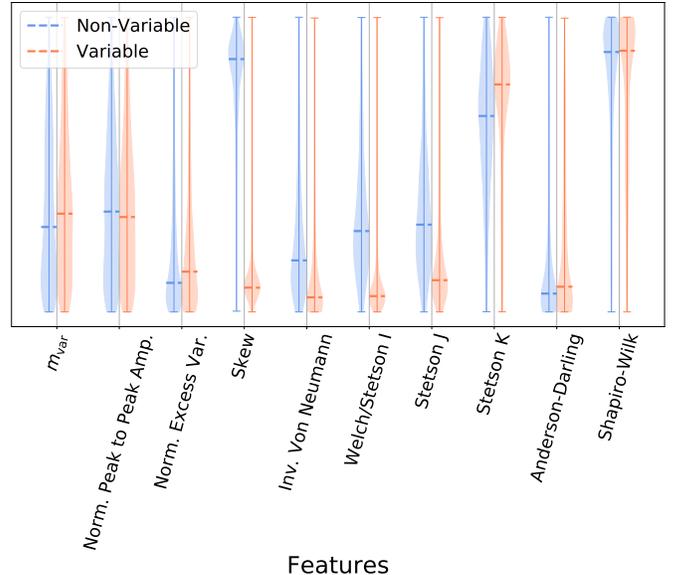


Figure 4. Probability density for a subset of the features in the analysis for a “variable” and “non-variable” set of objects (van Roestel et al. in prep), as assessed by a machine learning algorithm XGBoost (Chen & Guestrin 2016). The dashed lines correspond to the 50th percentile for the features. For simplicity, we have normalized all features such that they appear on the same plot, with values increasing in the upwards direction on the otherwise arbitrary y-axis.

We use period-finding algorithms to estimate periods for all objects; phase-folded light-curves, such as those in Figure 10, are used when scanning and classifying sources, typically for those with significant periodicities. There are a variety of period finding algorithms in the literature (see Graham et al. 2013b for a comparison and review), including those based on least-squares fitting to a set of basis functions (Zechmeister, M. & Kürster, M. 2009; Mortier, A. et al. 2015; Mortier, A. & Collier Cameron, A. 2017). By far the dominant source of computational burden is in the period finding. The algorithm we employ is hierarchical, with two period finding algorithms supplying candidate frequencies to a third.

The first algorithm is a conditional entropy (CE; Graham et al. 2013a) algorithm. CE is based on an information theoretic approach; broadly, information theory-based approaches improve on other techniques by capturing higher order statistical moments in the data, which are able to better model the underlying process and are more robust to noise and outliers. Graphically, CE envisions a folded light curve on a unit square, with the expectation that the true period will result in the most ordered arrangement of data points in the region. More specifically, the algorithm minimizes the entropy associated with the system relative to the phase, which

Index	Statistic	Calculation
1	N	Number of observations passing cuts
2	m_{median}	Median magnitude
3	m_{mean}	Weighted mean
4	m_{var}	Weighted variance
5	χ^2	$\frac{1}{N-1} \sum_i \frac{(m_{\text{median}} - m_i)^2}{\sigma_i^2}$
6	RoMS	$\frac{1}{N-1} \sum_i \frac{ m_{\text{median}} - m_i }{\sigma_i}$
7	Median absolute deviation	median ($ m - m_{\text{median}} $)
8	Normalized Peak to Peak Amplitude	$\frac{\max(m-\sigma) - \min(m+\sigma)}{\max(m-\sigma) + \min(m+\sigma)}$
9	Normalized Excess Variance	$\frac{1}{Nm_{\text{mean}}^2} \sum_i (m_{\text{mean}} - m_i)^2 - \sigma_i^2$
10-14	Ranges	Inner 50%, 60%, 70%, 80%, and 90% Range
15	Skew	$\frac{N}{(N-1)(N-2)} \sum_i \frac{(m_{\text{mean}} - m_i)^3}{\sigma_i^3}$
16	Kurtosis	$\frac{N(N+1)}{(N-1)(N-2)(N-3)} \sum_i \frac{(m_{\text{mean}} - m_i)^4}{\sigma_i^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$
17	Inverse Von Neumann Statistic	$\eta = \left(\frac{1}{\sum_i (\frac{1}{\Delta t_i})^2 m_{\text{var}}} \right) \sum_i \left(\frac{\Delta m_i}{\Delta t_i} \right)^2$ where $\Delta t_i = t_{i+1} - t_i$ and $\Delta m_i = m_{i+1} - m_i$
18	Welch/Stetson I	$\frac{N}{N-1} \sum_i \left(\frac{m_i - m_{\text{mean}}}{\sigma_i} \right) \left(\frac{m_{N-i} - m_{\text{mean}}}{\sigma_{N-i}} \right)$
19	Stetson J	$\sqrt{\frac{N}{N-1}} \sum_i \text{sgn}((m_i - m_{\text{mean}})(m_{N-i} - m_{\text{mean}})) \sqrt{\left \left(\frac{m_i - m_{\text{mean}}}{\sigma_i} \right) \left(\frac{m_{N-i} - m_{\text{mean}}}{\sigma_{N-i}} \right) \right }$
20	Stetson K	$\sqrt{\frac{1}{N}} \sum_i \left \left(\frac{m_i - m_{\text{mean}}}{\sigma_i} \right) \left(\frac{m_{N-i} - m_{\text{mean}}}{\sigma_{N-i}} \right) \right / \sqrt{\sum_i \left(\frac{m_i - m_{\text{mean}}}{\sigma_i} \right) \left(\frac{m_{N-i} - m_{\text{mean}}}{\sigma_{N-i}} \right)^2}$
21	Anderson-Darling test	Stephens (1974)
22	Shapiro-Wilk test	Shapiro & Wilk (1965)
23-35	Fourier Decomposition	$y = c_1 + c_2 \times t + \sum_{i=1}^5 (a_i \cos(\frac{2\pi t i}{P}) + b_i \sin(\frac{2\pi t i}{P}))$
36	Bayesian Information Criterion	χ^2 likelihood keeping different number of Fourier harmonics
37	Relative χ^2	$y = c_1 + c_2 \times t$ to full Fourier Decomposition

Table 2. Statistics calculated based on the light curves and period finding. N is the number of observations, m_i is the i th observation magnitude, and σ_i is the i th observation magnitude error. The variables c_1 , c_2 and the a_i 's and b_i 's are constants to be fit for in statistics 23-35 and 37. In addition to these statistics, we save the best fit period based on the hierarchical analysis as well as its significance.

naturally accounts for the non-trivial phase space coverage of real data. The CE version we use is `gce` (Katz et al. 2020)⁵. It is implemented on Graphics Processing Units (GPU) in CUDA (Nickolls et al. 2008) and wrapped to Python using Cython (Behnel et al. 2011) and a special CUDA wrapping code (McGibbon & Zhao 2019).

The software `gce` is a Python code that prepares light curves and their magnitude information for input into the Cython-wrapped CUDA program. Therefore, the user interface is entirely Python-based. The period range searched varies between 30 min and half of the baseline, T ; 30 min was chosen as a lower bound for computational considerations. We use a frequency step df

of $\frac{1}{3 \times T}$, oversampled by a factor of 3 in order to account for the irregular sampling. This oversampling term was measured empirically by trying the frequency grid on a variety of test cases, and shown to be the minimum factor that gave the correct period for a small sample of eclipsing binaries and RR Lyrae as determined by a higher resolution analysis. To evaluate the efficacy of this choice, Figure 6 shows a two dimensional histogram of the relative difference between an analysis of variance computation with an oversampling factor of 3, as used in the main analysis, and an oversampling factor of 10. We plot this relative difference as a function of the highest significance period identified by the oversampling factor of 10 analysis. We plotted any relative difference values below that of 10^{-3} in the bottom row of the histogram, indicating agreement at the 0.1% level. The main feature in this histogram, beyond the build-up of support of

⁵ <https://github.com/mikekatz04/gce>

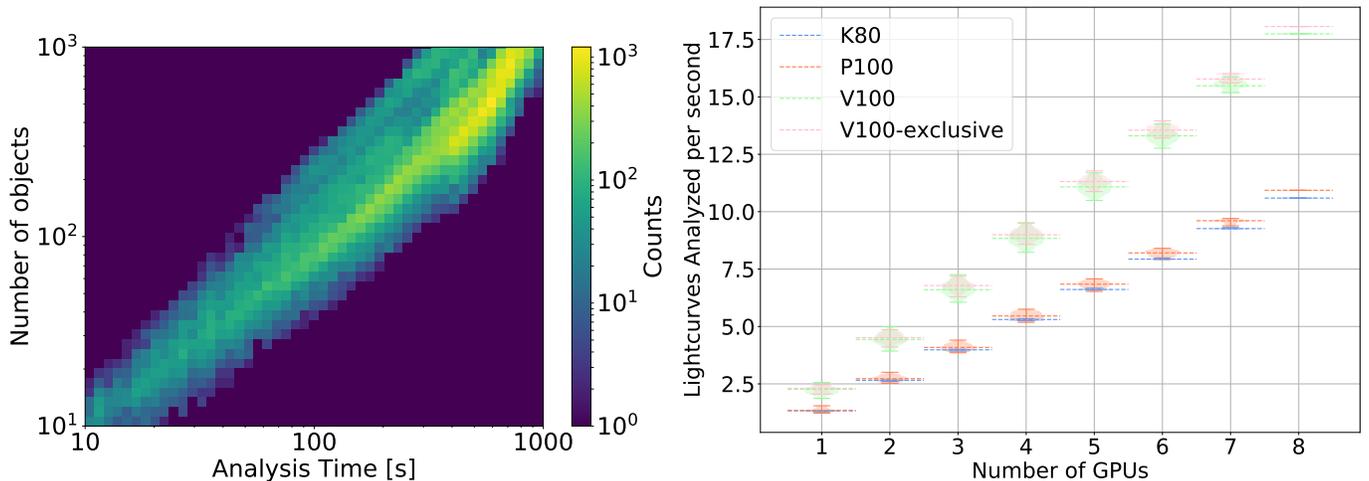


Figure 5. Left: Two dimensional histogram of the number of objects analyzed vs. analysis time for the ZTF light curves. Right: Scaling data based on GPU resources for the algorithm discussed in the text. We note that the V100 and V100-exclusive analysis (where jobs are restricted to run on their own on the GPU) are closely overlapping.

equal periods at the bottom, is the approach of a curve to a relative difference of 1.0, indicating those sources that disagree by a factor of 2 are modulated by the effect of diurnal sampling. There is a less dense horizontal line at those sources at half of the frequency. This analysis indicates that a more refined period grid, perhaps as a secondary step in the analysis, may be useful in the future. We note that phasing agreement to better than the 0.1% level is already achieved, and accuracies at this level are required to, for example, detect small but detectable period changes present in a variety of astrophysical processes.

We note that this baseline will change for each chunk analyzed; the frequency array is then simply $f = f_{min} + df \times [0..\text{ceil}((f_{max} - f_{min})/df)]$. We use 20 phase bins and 10 magnitude bins in the conditional entropy calculation, which amounts to the size of the phase-folded, two dimensional histogram. We note that conditional entropy, as a 2D binning algorithm, does not currently have weights implemented. In principle, this could be done similarly to the case of Lomb-Scargle below, although careful consideration should be taken for how to handle, for example, eclipsing systems where down-weighting fainter points with correspondingly larger error bars could be detrimental to identifying eclipses.

The second algorithm is a Lomb-Scargle (LS, Lomb 1976; Scargle 1982) implementation; these algorithms are similar to Fourier Transforms, but for irregularly sampled data. In this version, it decomposes the time series into the frequency domain using a linear combination of sine waves $y = a \cos \omega t + b \sin \omega t$. If we define T as the period with the angular frequency $\omega = 2\pi T$,

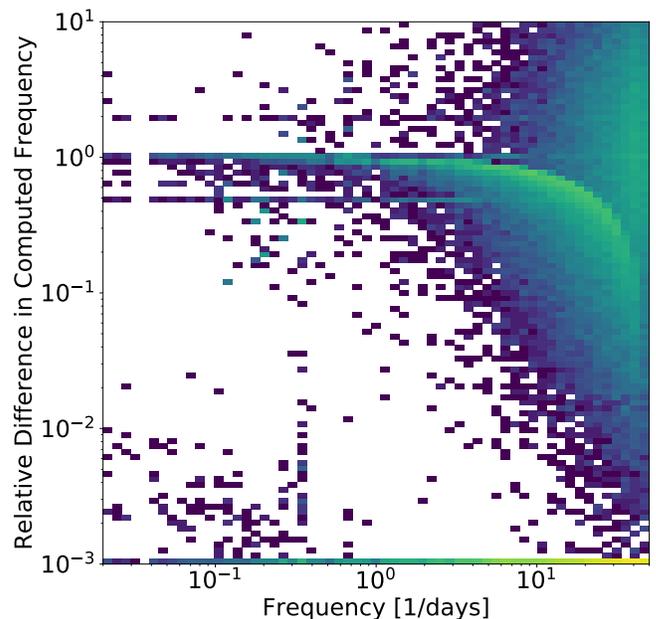


Figure 6. Two dimensional histogram of the relative difference between an analysis of variance computation with oversampling factors of 3, as used in the main analysis, and an oversampling factor of 10 vs. highest significance period identified by the oversampling of 10 analysis.

the periodogram is defined as:

$$P(\omega) = \frac{1}{2\sigma^2} \left\{ \frac{\left[\sum_{n=1}^N w_n (m_n - \bar{m}) \cos [\omega(t_n - \tau)] \right]^2}{\sum_{n=1}^N \cos^2 [\omega(t_n - \tau)]} + \frac{\left[\sum_{n=1}^N w_n (m_n - \bar{m}) \sin [\omega(t_n - \tau)] \right]^2}{\sum_{n=1}^N \sin^2 [\omega(t_n - \tau)]} \right\},$$

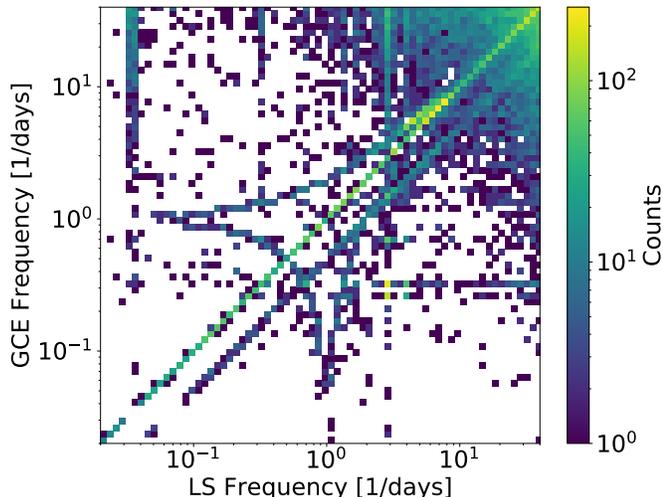


Figure 7. Two dimensional histogram comparing LS (x-axis) and GCE (y-axis).

where

$$w_n = \frac{\frac{1}{\sigma_n^2}}{\sum_{n=1}^N \frac{1}{\sigma_i^2}}; \tau = \tan(2\omega\tau) = \frac{\sum_{n=1}^N \sin(2\omega t_n)}{\sum_{n=1}^N \cos(2\omega t_n)}. \quad (1)$$

The algorithm we use is also implemented on GPUs ⁶.

Figure 7 shows a two dimensional histogram comparing LS (x-axis) and GCE (y-axis). For many objects, the peak frequencies identified are in agreement, with a subset identified as differing by half or twice the period. When this occurs, LS preferentially finds the shorter period, which occurs in a number of common scenarios. This includes when analyzing eclipsing binaries whose primary and secondary eclipses do not differ greatly in depth; in this case, LS tends to find a period equal to half the true value, while CE will find the correct value. For a further subset, the peak frequencies identified are different, requiring, in principle, a tie-breaker algorithm to determine the “best” choice. A further, interesting feature in these histograms are curved “lines,” symmetrical about the diagonal, that have vertical and horizontal asymptotes at periods corresponding to one day, which arise from the true period “beating” with a one day period arising from diurnal aliasing.

Figure 8 shows example periodic variables identified by GCE and LS as high significance, while the other period finding algorithm finds marginal significance; the GCE example has both eclipsing and underlying modulating behavior, more difficult for LS to recover, while the LS example identifies low-amplitude modulation in otherwise noisy data, difficult for GCE to recover within

its 2-D histograms. To address this issue, we take the top 50 frequencies identified by each of the algorithms, with no use of harmonic summing or otherwise period combining techniques, and run each through a CPU-based multi-harmonic analysis of variance (AOV, Schwarzenberg-Czerny 1998) code in the 200 frequency bins, covering a frequency range between peak frequency $- 100 \times \text{df}$ to peak frequency $+ 100 \times \text{df}$ ⁷. To determine the best period, we evaluate their ad-hoc “significance” using the statistic array s , which is the same length as the frequency array:

$$\text{significance} = (\max(s) - \text{mean}(s)) / \text{std}(s). \quad (2)$$

We note that s , which is parameterized by the period array, corresponds to, for example, the entropy in the phased 2D histograms for conditional entropy, or an estimate of the Fourier power for LS. While other examples of “significance” are also possible, such as approximate significance estimates for LS (Baluev 2008), we find this is sufficient for simply rank-ordering the frequencies across all algorithms and make simple comparisons between objects. We also point out that this technique throws away information from sub-dominant periods in a light curve, and so for objects where multiple periods are important, this technique will be sub-optimal. We can evaluate the threshold for significance appropriate for evaluating a confident periodic source; in Figure 9, we plot the significance vs. period for the “periodic” set of objects (van Roestel et al. in prep). For comparison, we show marginalized one-dimensional histograms for both the periodic and non-periodic sets of objects, which show distinct differences in both. The non-periodic set show a distinct peak at low frequencies and around the lunar cycle, while the periodic set peaks distinctly in the range 0.1–1 days, due to the high rate of RR Lyrae and Delta Scuti variables in this set. For scanning purposes, we can also compare the distribution of significances for these sets; for example, the 2nd percentile for periodic objects is ~ 14.4 , which corresponds to the 95th percentile of non-periodic objects. This would mean that a significance threshold of 14.4 would yield a false dismissal probability of 2% while having a false alarm probability of 5%. For further comparison, the relative rate of periodic to non-periodic sources is $\sim 0.13\%$.

There were two main advantages to this otherwise complicated method. The first is that there are known advantages to both CE and LS (Graham et al. 2013a), with one particularly successful for generic light curve

⁶ <https://github.com/johnh2o2/cuvarbase>

⁷ <https://github.com/joshuazd/lssfds>

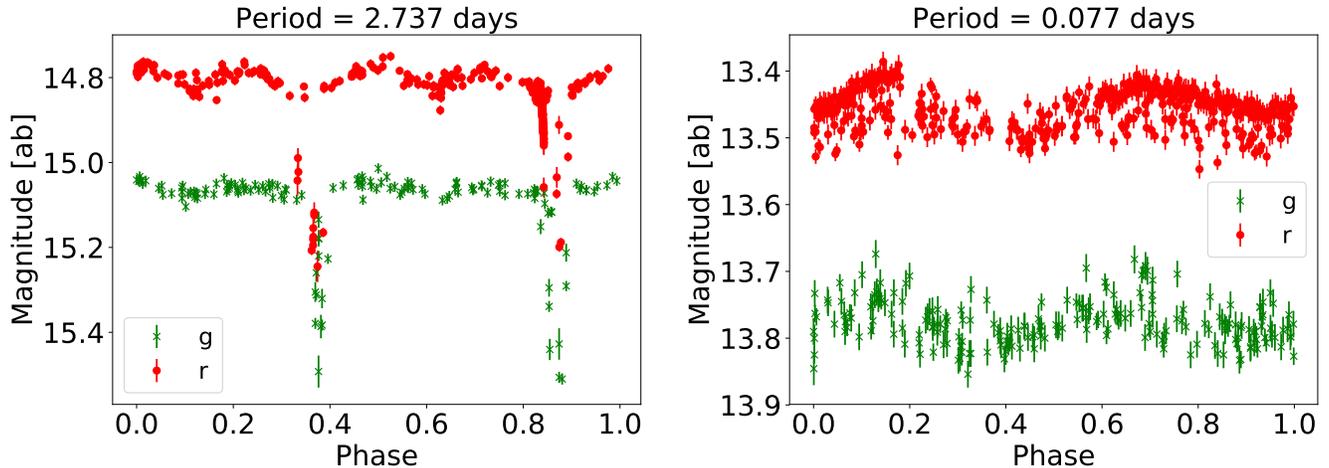


Figure 8. Example periodic variables (their estimated periods are given in the titles), identified by GCE (left) and LS (right) as high significance, while the other period finding algorithm finds marginal significance.

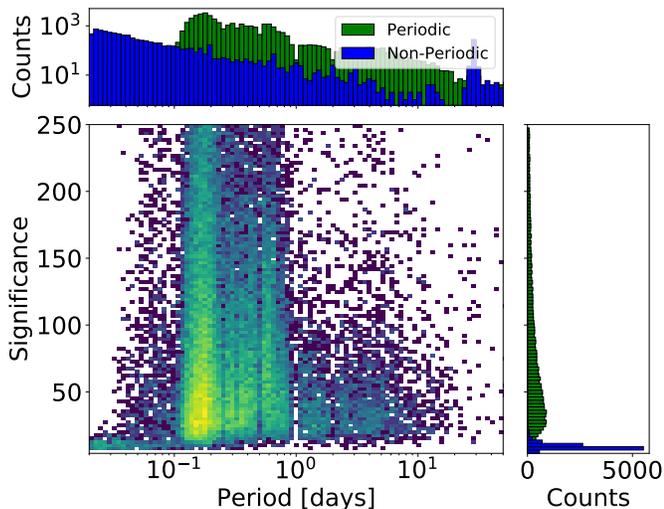


Figure 9. Two dimensional histograms of the significance vs. period for the “periodic” set of objects (van Roestel et al. in prep). The one dimensional histograms are marginalized versions of the two dimensional histogram (green). For comparison, we include a “non-periodic” set of objects in blue (van Roestel et al. in prep).

shapes and the second for low-amplitude variables, as demonstrated in Figure 8. To provide a single set of metrics based on these two algorithms, we needed a way to “choose” between CE and LS, from which came the choice of AOV, which has some of the benefits of both (generic light curve shapes with the potential for multiple harmonics). These motivates the second point, which is that the CPU-based version of AOV across the whole frequency grid would be computationally intractable, and so some of the benefits of the most suitable period finding algorithm are achieved despite its computational expense. The GPU implementations of

CE and LS were essential here, as CPU implementations of CE, AOV, and LS take ~ 10 s per light curve each, two orders of magnitude slower than their GPU counterparts. We highlight a few example periodic objects coming out of this analysis in Figure 10.

We show the analysis speed as a function of number of objects on the left of Figure 5. As can be seen, the analysis takes ~ 1 second per light curve across the range of light curves analyzed at a single time (~ 100 to ~ 1000 samples). At this point, we are dominated by the CPU cost of the AOV analysis, having put all other expensive computations on the GPU. Design of a GPU-based AOV analysis is ongoing, as well as breaking out the AOV process into separate processes (perhaps for objects that meet specific variability requirements). We now perform scaling tests for the numerical setup described above. The right of Figure 5 shows the scaling test results on 10 GPUs running concurrently. These tests have been performed at a number of facilities: (i) the Minnesota Supercomputing Institute, with both K40 (11 GB of RAM) and V100 GPUs (16 GB of RAM), (ii) XSEDE’s SDSC COMET cluster (Townsend et al. 2014) with K80 (12 GB of RAM) and P100 GPUs (16 GB of RAM), and (iii) NERSC’s CORI cluster with V100’s (16 GB of RAM). As expected, the number of analyzed light curves is linear with the number of GPUs allocated.

Figure 11 shows the two dimensional histogram of the period significance based on the analysis of variance computation vs. highest significance period identified for field 296 (top left), 487 (top right), 682 (bottom left) and field 853 (bottom right). We can identify a number of cases where a non-astrophysical signal is found due to the sampling pattern. It is clear based on the histograms that fractions and multiples of a sidereal day, as well as

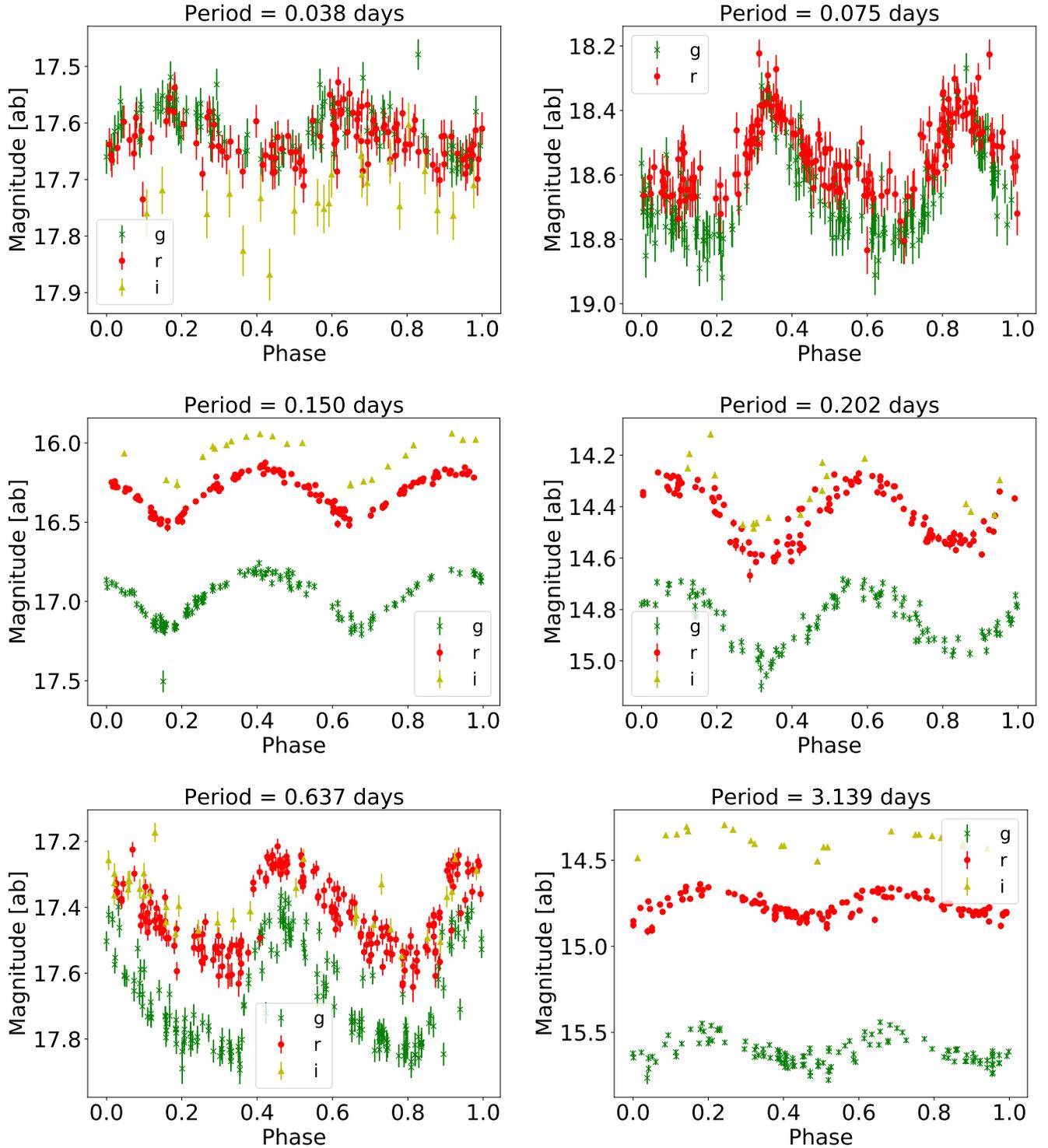


Figure 10. Example periodic variables (their computed periods are given in the titles) folded at twice their computed period, identified in the various bins we use for scanning. We fold at twice the computed period to evaluate the consistency of the reconstructed light curves across two orbits, useful for assessing the correctness of the period. These bins are chosen to simplify comparison of periodicity significances estimated between source variable on very different time scales: top left, 30–60 min (delta Scuti), top right, 1–2 hr (delta Scuti), middle left 2–4 hr (W UMa), middle right, 4–12 hr (W UMa), bottom left, 12–72 hr (RRab star) and bottom right, 3–10 days (Cepheid). For W UMa’s, the true period is two times longer than the computed period.

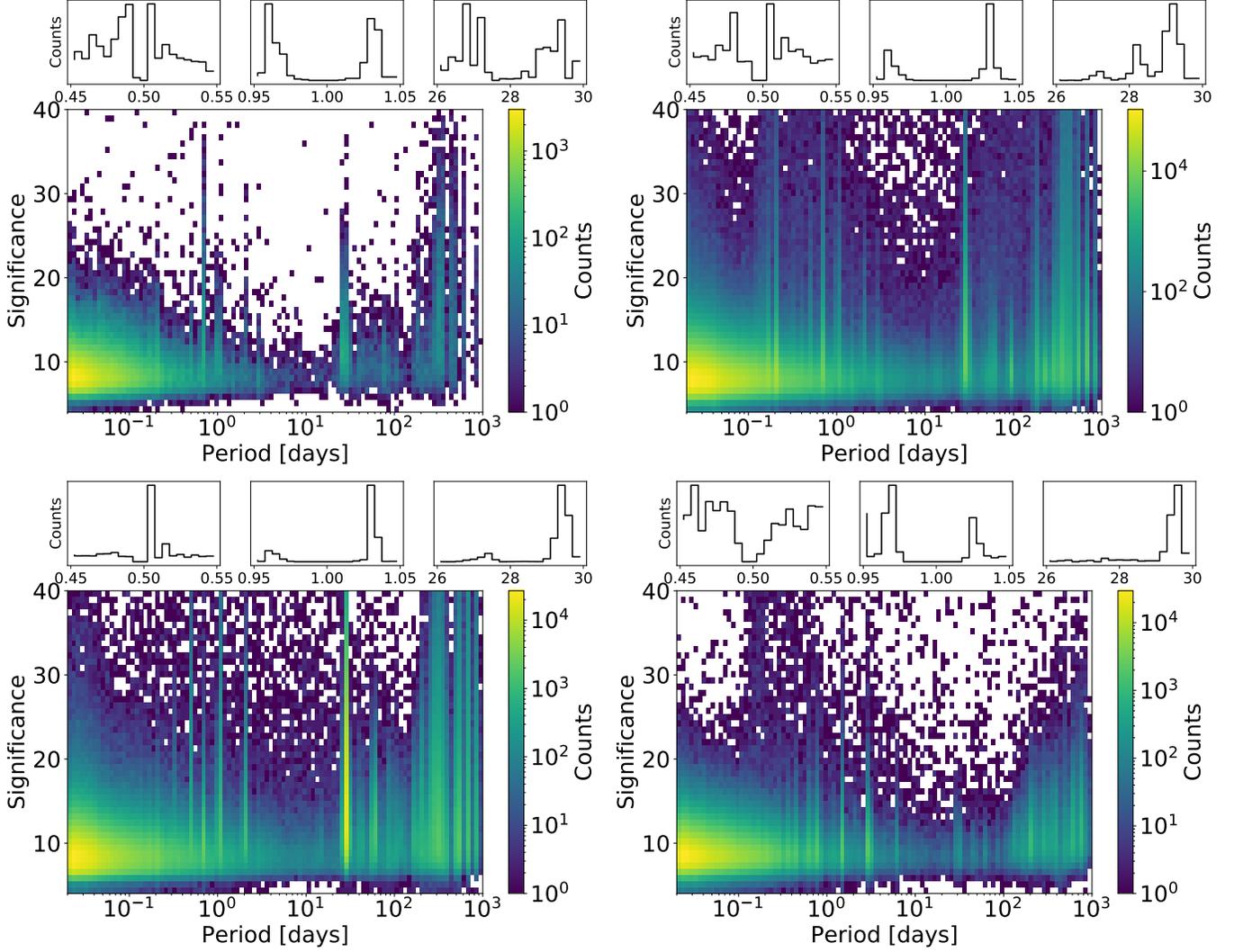


Figure 11. Two dimensional histograms of the period significance based on the analysis of variance computation vs. highest significance period identified for field 296 (top left), 487 (top right), 682 (bottom left) and field 853 (bottom right). The one dimensional histograms focus on periods of 0.5 days and 1 days, to evaluate the effect of diurnal sampling on aliasing, and 28 days, to evaluate the same for the lunar cycle.

near sidereal months with the lunar cycle, induce false period estimates. We remove at the period finding level bands around fractions and multiples of a sidereal day to help mitigate this. Specifically, we remove frequencies (in cycles/day) of 47.99–48.01, 46.99–47.01, 45.99–46.01, 3.95–4.05, 2.95–3.05, 1.95–2.05, 0.95–1.05, 0.48–0.52 and 0.03–0.04; this removes $\sim 1\%$ of the frequency range. We chose these frequencies based on an iterative procedure; we would evaluate a subset of objects, examine histograms of period excesses identified at these frequencies, remove them, and then reevaluate. This will be convolved with longer term trends, including seasonal and annual variations. We note that removal of these frequencies may have removed the true variables covering these period ranges. The marginalized histograms

in Figure 11 indicate that a small excess of sources at the edge of the removed frequency bands remains due to the diurnal and lunar aliases, showing that a broader removal could be useful.

The top left of Figure 12 shows a comparison between period finding analyses of field 700 using the setup used in this analysis and one where we do not remove these bands; in general, most of the objects are assigned the same period. In addition to some differences due to GPU errors (see below) and the differences that arise in the significance calculation where different frequencies reach the AOV stage, a significant fraction receive periods in narrow frequency bands that are otherwise cut, indicating the importance of these cuts. The top right of Figure 12 shows a comparison between period finding

analyses of field 700 using the setup used in this analysis and one where periods are searched down to 3 min; similar to before, in general, many of the objects are assigned the same period. Some objects are assigned a period that corresponds to the second harmonic of the original analysis. Also, some objects have very short periods assigned, as typically occurs for marginal periodic signals.

We also characterized GPU-based transient errors that affect a small fraction of objects when running on the HPC clusters used here (Tiwari et al. 2015). The bottom row of Figure 12 shows a comparison between two identical period finding analyses of field 700 using the setup used in this analysis and one where the GPU-based period finding is run 3 times; while two outliers remain, this is enough to clean up the distributions at the cost of computational efficiency. We expect that if there are algorithmic speed-ups enabling this to be performed feasibly over the entire data set, it will be useful to mitigate this issue.

5. CONCLUSION

In this paper, we introduce the variability metrics and the periodicity algorithms that are used to derive ZTF’s variable star catalog based on ZTF’s DR2. In addition to the variability metrics suggested by Pashchenko et al. (2017), we included robust measurements of the light curve amplitude and a Fourier-decomposition to characterize the shape of the folded light curves. We designed a hierarchical period-finding scheme that utilized multiple period-finding algorithms in order to be as sensitive as possible across the period parameter space of amplitude and period. This analysis provides the input statistical features that will provide training sets for the machine learning based catalogs, presented in a partner publication (van Roestel et al. in prep). This publication will present classifiers and associated metrics that evaluate the variability, periodicity, completeness, purity and classifications for these objects. To create the catalog, `ztfperiodic` has been applied to a variety of large-scale computing systems and NVIDIA-based GPUs. Setting up and executing the proposed analysis required access to GPU arrays of this size, and the computational needs will only grow in future releases from ZTF and larger data sets from future surveys.

Going forward, the priority will be to analyze the data from the ZTF’s Third Data Release (June 24, 2020)⁸. Many of the choices made in this paper should be revisited for this process. These include:

- Restricting to one point within 30 minutes (to remove high cadence data)
- Restricting to periods greater than 30 minutes (to make computing tractable)
- Restricting to a minimum of 50 points per light curve (for metric efficacy)
- Period range exclusions (to remove effects from cadence aliasing)
- Period finding choices, including the choice of the oversampling factor

Outside of more epochs yielding more light curves passing the 50 point cut, we expect to revisit the choices made in the period finding. Ongoing work includes devising faster algorithms to improve the scaling of the processing. Other options include combining photometric points from other surveys.

We also are exploring the benefits of “clipping” light curves for outlier removal, either using a comparison with a given σ difference or percentile based cuts. In general, we continue to improve the algorithms looking ahead, given that computational burdens are growing significant. For LSST in particular, the number of variable sources expected is significantly higher, because of both its increased depth and ~ 5 mmag precision, given that the fraction of variables increases as a power law with increasing photometric precision (Huber et al. 2006). Translation of the work described here to LSST is obviously of great interest, as LSST will overlap with the faint end of ZTF, and have a limiting magnitude far beyond what ZTF can reach, meaning that it can access a far larger volume of sources. However, translating the work described here to LSST will be accompanied by several challenges. Obviously the cadence will be substantially lower due to LSST’s smaller field of view relative to ZTF, which will impact primarily the ability to recover periods at the short end. Perhaps the biggest challenge is that LSST will not acquire sufficient samples to detect periodic behavior in many sources until several years into the survey, which will increase the baseline of the sampling relative to that of ZTF; this in turn means that a larger frequency grid must be searched to recover equivalent objects. Additionally, because LSST will contain far more sources than ZTF due to its depth, the computational cost will be significantly higher, as it is proportional to the total number of sources. This further motivates continued speed-ups of the algorithms presented here.

⁸ <https://www.ztf.caltech.edu/page/dr3>

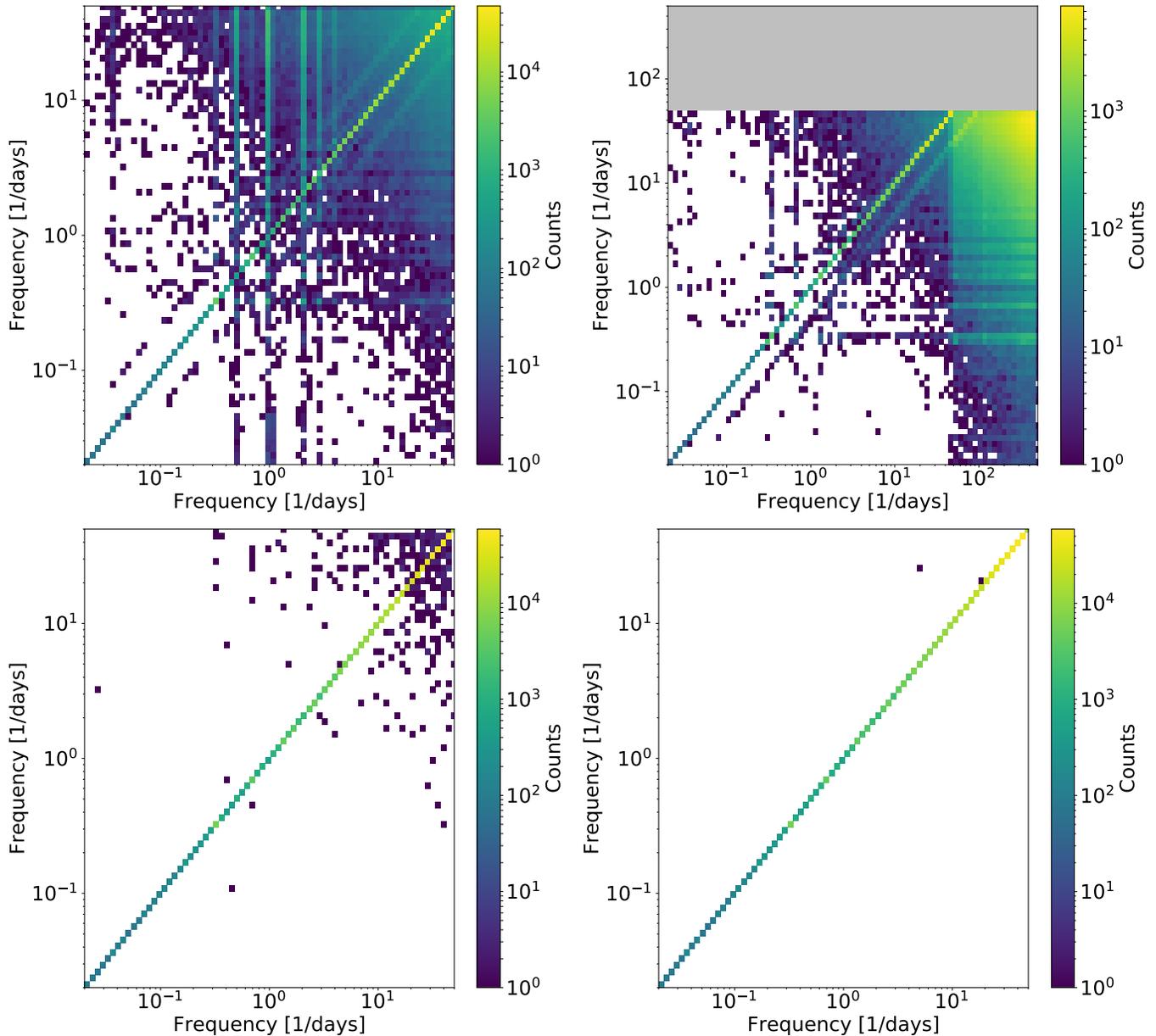


Figure 12. On the top left is a comparison between two period finding analyses of field 700 comparing one without the aliasing-dominated frequency bands removed (x-axis) and the typical setup (y-axis). On the top right is the same where the x-axis version has a period range searched down to 3 min. The gray region corresponds to a period region not available to that run. On the bottom row, we compare runs where we use the typical setup (left) and one where the GPU-based period finding is run 3 times (right).

The data underlying this article are derived from public code found here:

<https://github.com/mcoughlin/ztfperiodic>. DR2, including how to access the data, can be found at <https://www.ztf.caltech.edu/page/dr2>.

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⁹ <http://www.msi.umn.edu>

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