$$\Delta = f_1(L_{nh}, C_{nh}, R_{nh}, \omega) + jf_2(L_{nh}, C_{nh}, R_{nh}, \omega) = 0, \qquad (4.3)$$

$$\text{where } n = c, b, m$$

$$h = 1, 2, \ldots\ldots$$

and where f_1 and f_2 are real functions of real variables.

Equation (4.3) leads us to the following two equations, which must hold simultaneously for any values of the variables assigned:

$$\begin{aligned} f_1(L_{nh}, C_{nh}, R_{nh}, \omega) &= 0 \\ f_2(L_{nh}, C_{nh}, R_{nh}, \omega) &= 0 \end{aligned} \qquad (4.4)$$

It is thus seen that if we solve either equation for ω and substitute this value of ω in the other, both the frequency of oscillation and the relation between the circuit parameters which will cause the system to oscillate with this angular velocity may be obtained.

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[1] Miller, J. M., *Proc. I.R.E.*, **6**, No. 3, p. 141, 1918, and Chaffee, E. L., to be published.
[2] Chaffee, E. L., *Ibid.*, **15**, No. 3, p. 181, 1927.
[3] Hazeltine, L. A., *Ibid.*, **6**, No. 2, p. 79, 1918.
[4] Campbell, G. A., *Trans. A. I. E. E.*, **1911**.

ON THE SYMMETRY OF PROTONIC WAVE FUNCTIONS

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The following treatment of the symmetry properties of an H_2 molecule seem to indicate that protonic wave functions are symmetric and that, therefore, protons will satisfy the Einstein-Bose statistics.

Let p_1, e_1 be the proton and electron of a hydrogen atom a_1; p_2, e_2 those of an atom a_2. Let g_{11}, g_{12}, g_{13} be the coördinates of the center of gravity of a_1; g_{21}, g_{22}, g_{23} those of a_2. Let r_1, θ_1, ϕ_1; r_2, θ_2, ϕ_2 be, respectively, the polar coördinates of electrons e_1 and e_2, where the axis A of the polar coordinates is drawn from the center of gravity of a_1 to that of a_2 and the origin is at the center of gravity of the whole system.

The twelve coördinates g_{11}, g_{12}, g_{13}, g_{21}, g_{22}, g_{23}, r_1, θ_1, ϕ_1, r_2, θ_2, ϕ_2 completely specify the system of two protons and two electrons (except for spin coördinates). Let ψ be the wave function for the entire system.

ψ can be separated in the coördinates of the centers of gravity of the atoms and the electronic coördinates, that is

$$\psi = \psi_g \psi_e \tag{1}$$

where ψ_e is a function of r_1, θ_1, ϕ_1, r_2, θ_2, ϕ_2 and the electronic spin coordinates, while ψ_g is a function of the coördinates of the centers of gravity of the atoms and any other spin coördinates associated with them.

Considering the system as a symmetrical rotator capable of being in various rotational states, we can separate the rotational wave function ψ_R from ψ_g, that is,

$$\psi_g = \psi_R \psi_w. \tag{2}$$

ψ_R is considered as a function of ϑ and Φ, the polar angles of the axis A with respect to an axis B fixed in space. ψ_w will then involve all the coordinates, except ϑ and Φ, necessary to specify the positions of the centers of gravity of the atoms a_1 and a_2 and possible spin coördinates associated with them.

If p_1 is interchanged with p_2 and simultaneously e_1 with e_2, then this operation interchanges the centers of gravity of atoms a_1 and a_2. This has the effect of reversing the direction of the axis A so that ϑ becomes $\pi - \vartheta$ and Φ becomes $\Phi + \pi$. Then if p is the rotational quantum number, we have

$$\psi_{Rp}(p_1, e_1, p_2, e_2) = (-1)^p \psi_{Rp}(p_2, e_2, p_1, e_1) \tag{3}$$

as rotational wave functions for even p are unchanged by reversing the axis A while those for odd p are changed in sign by such a reversal. Reversing the direction of the axis A changes θ into $\pi - \theta$ and ϕ into $-\phi$. This operation leaves ψ_e unchanged when the molecule is in its lowest electronic state.

We know that the total wave function is antisymmetric in the electrons. If it is symmetric in the protons it will be changed in sign by reversing the axis A, while if it is antisymmetric in the protons it is unchanged by such a reversal. In the first case ψ_w will change its sign under the axis reversal when p is even while it will remain unchanged when p is odd and vice versa for the second case. This indicates that ψ_w does involve spin coördinates besides the vibrational wave function which latter is obviously unchanged by reversal of A. Hori's data on the band spectrum of hydrogen and Dennison's[1] theory of the rotational specific heat of hydrogen require that the relative weights of symmetric to antisymmetric rotational states should be as 1 to 3. Therefore, in case the total wave function is symmetric in the protons, the states with symmetric ψ_w should have three times the statistical weight of those with antisymmetric ψ_w and vice versa for the case when the total wave function is antisymmetric in the protons. The state of affairs is indicated in the following table.

POSSIBILITY NO.	I	II
Axial symmetry of ψ_w for odd rotational states	S	A
Symmetry of total wave function in protons	S	A
Axial symmetry of total wave functions	A	S
Axial symmetry of ψ_w for even rotational states	A	S

As a symmetric spin function has three times the statistical weight of an antisymmetrical spin function, it seems necessary to choose the first of the above alternatives requiring the total wave function to be symmetric in the protons. This result is independent of whether the protons themselves have or have not spins. The spins that give the correct relative weights to the rotational states are atomic spins pertaining to the elements of the rotator, the proton-electron pairs. Of course, it is possible that the atomic spins originate from protonic spins. If such is the case, then the coördinate function of the protons is at most antisymmetric in pairs of protons. This means physically that protons attract resonantly in groups and can repel dissonantly in pairs. If this is so it would help in the explanation of the formation of helium from protons and electrons in interstellar space as is assumed by Millikan and Cameron,[2] as the resonance forces acting between protons would tend to make their effective cross-sectional area for collision and capture large when the protons had the small velocities of the low temperatures of interstellar space.

The known information on the various isotopes of the elements show that there is a strong tendency toward what is apparently resonant pairing of nuclear electrons while no such tendency is shown for protons. This can be taken as additional evidence for resonant groupings of protons. The resonance forces between pairs of the relatively small numbers of nuclear electrons would not seem to be sufficient to stabilize the nucleus by acting against the powerful electrostatic repulsions of the protons.

If wave functions are symmetric in protons, then similar particles, atoms or molecules, containing an even number of electrons will satisfy the Einstein-Bose statistics, while those containing an odd number of electrons will satisfy the Fermi-Dirac statistics. Apparently almost all ordinary gases satisfy the Einstein-Bose statistics so that part of the van der Waal's forces in these gases will be attractive resonance forces. Probably all the van der Waal's forces for helium are of this character. According to Aston the principal isotope of xenon contains an odd number of electrons. This isotope should then show a greater vapor pressure at very low temperatures than the isotopes of even atomic weight. However, the unpaired electron is enclosed in a high potential energy wall so that equilibrium would be reached extremely slowly.

[1] *Proc. Roy. Soc.*, **1150**, 483–6, 1927.

[2] *Phys. Rev.*, **32**, 533–57, 1928.