9.

Analysis and Testing of the FBA-11 Force Accelerometer

W. J. Rihn, M.EERI, W. F. Miller, C. B. Crouse, M.EERI, E. L. Holbrook, and B. Hushmand, M.EERI

The FBA-11 is a feedback-controlled accelerometer widely used to measure and record accelerations arising from earthquakes. It has found application both for structural response and for ground motion studies. The design intent of the FBA-11 was to provide electronic control of the natural frequency, damping, and output voltage. Included in this paper are (1) a circuit analysis yielding the complete closed-loop transfer function, and (2) the corroborative test results from shake table evaluations. The transfer function can be used to correct recorded accelerations for instrument response.

INTRODUCTION

The FBA-11 is a spring-mass device with variable capacitance transduction. The mass is a coil of wire which moves in a magnetic field. When acceleration is applied, a voltage is produced which is proportional to the displacement of the mass. This voltage is filtered and equalized, and a current is generated for feedback to the coil-mass. This feedback stiffens the system, greatly increasing the natural frequency. The feedback also provides the system damping.

The general characteristics of an earlier generation of feedback-controlled accelerometer (FBA-1) have been described by Rihn [3], and the FBA-1 was analyzed by Amini and Trifunac [1]. Crouse and Hushmand [2] used the results of the work presented here to correct the data from vibration experiments for the frequency-response characteristics of FBA-11s.

(WJR, ELH) Kinemetrics Systems, 222 Vista Ave., Pasadena, CA, 91107
(WFM) Seismological Laboratory, CIT, Pasadena, CA, 91125
(CBC, BH) Earth Technology Corp., 100 West Broadway, Long Beach, CA 90802
(CBC present address: Dames and Moore, 500 Market Place Tower, 2025 First Avenue, Seattle, WA 98121)

CIRCUIT ANALYSIS

The design of the FBA is shown schematically in Figure 1.

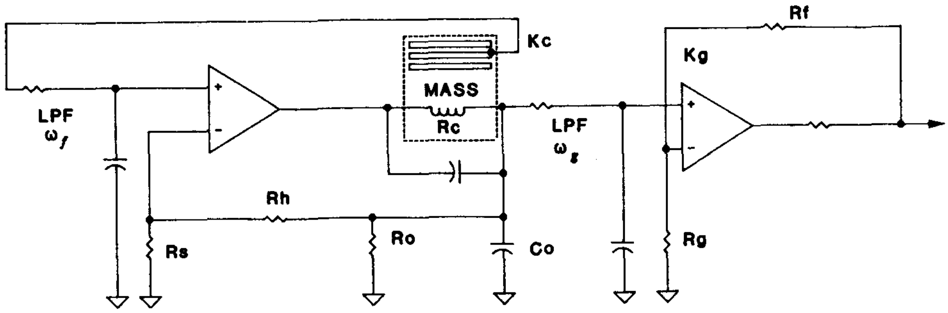


Figure 1

Before defining all of the terms and symbols, let us first examine the op amp circuit which provides the heart of the feedback:

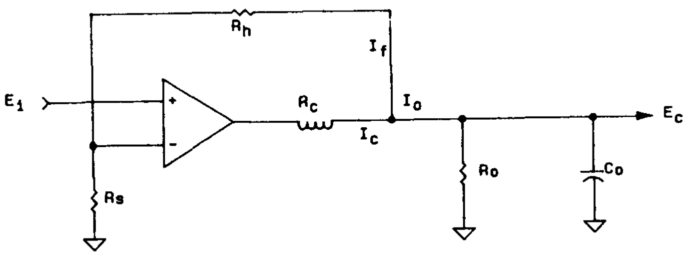


Figure 2

We wish to determine the current, I_c, through the feedback coil, R_c, as a function of the applied voltage, E_i. To facilitate the analysis, we assume an "ideal" op amp. The ideal op amp has: (a) infinite gain and (b) zero potential difference between input pins.

Then:

$$I_c = I_o + I_f \tag{1}$$

$$E_i = I_f R_s \tag{2}$$

$$I_f (R_h + R_s) = I_o Z_o \tag{3}$$

where Z_o is the parallel combination of R_o and C_o.

Combining (2) and (3), we have:

$$\frac{E_i}{R_s}(R_h+R_s) = I_o Z_o \tag{4}$$

Substituting (2) and (4) into (1):

$$I_c = \frac{E_i}{R_s}\frac{(R_h+R_s)}{Z_o} + \frac{E_i}{R_s}$$

$$= E_i\left(\frac{R_h+R_s+Z_o}{R_s Z_o}\right) \tag{5}$$

It is readily shown that

$$Z_o = \frac{R_o \omega_o}{s+\omega_o}\ , \tag{6}$$

where s is the Laplace operator

and $\omega_o = \dfrac{1}{R_o C_o}$.

Then

$$I_c = E_i \frac{(R_h+R_s)\ (s+\omega_o)\ R_o\omega_o}{R_s R_o \omega_o}$$

$$= E_i \frac{s(R_h+R_s) + \omega_o(R_h+R_s+R_o)}{R_s R_o \omega_o}$$

$$= E_i \frac{R_h+R_s}{R_s R_o}\left(\frac{s + \omega_o \dfrac{(R_h+R_s+R_o)}{(R_h+R_s)}}{\omega_o}\right)$$

or in simplified form,

$$I_c = E_i K_f \left(\frac{s+\omega_p}{\omega_o} \right) \tag{7}$$

where

$$K_f = \frac{R_h+R_s}{R_s R_o}$$

and

$$\omega_p = \frac{1}{R_o C_o} \frac{(R_h+R_s+R_o)}{(R_h+R_s)} \,.$$

The bracketed portion of the expression for I_c is called a "one-zero equalizer".

Next, let us proceed to the analysis of the overall system. The FBA schematic (shown in Figure 1) is illustrated in block diagram and transfer function forms in Figure 3.

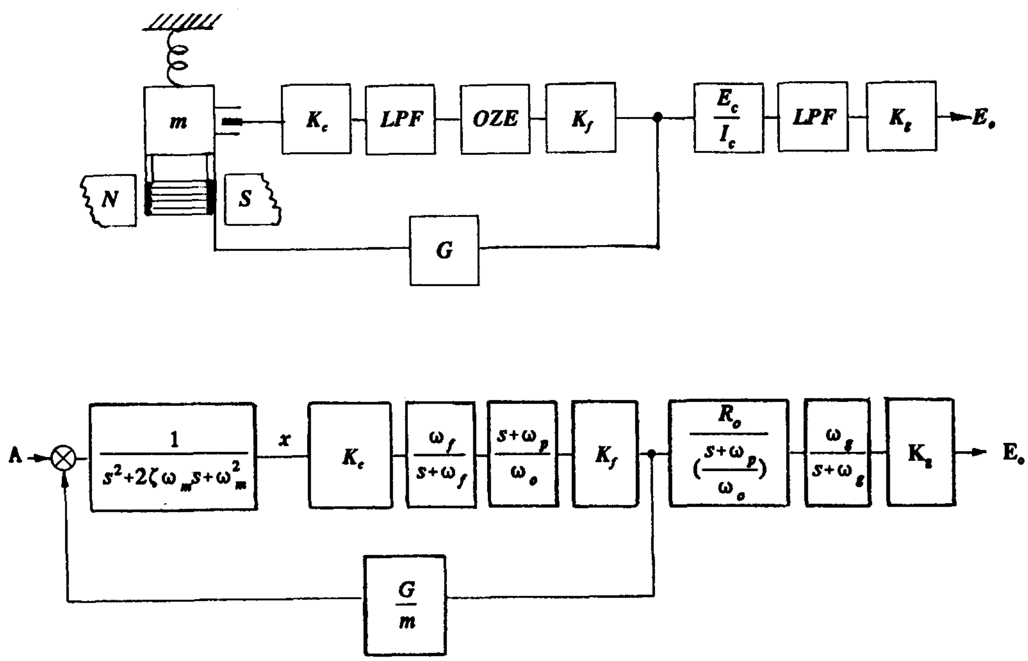


Figure 3

The terms used in the figures are defined as follows:

N & S represent the permanent magnet field

m is the mass of the accelerometer

G is the motor constant of the feedback coil in the magnetic field

LPF means a low pass filter

OZE is the "one-zero equalizer"

ζ is the intrinsic damping (without feedback)

ω_m is the natural frequency of the spring-mass system (without feedback)

K_c is the voltage to displacement constant of the capacitive transduction element

ω_f is the corner frequency of the low pass filter inside the feedback loop

K_f is the voltage to feedback current conversion

ω_g is the corner frequency of the post amplifier

K_g is the post amplifier gain

The feedback portion has a closed loop transfer function in the form of:

$$\frac{E_c}{A} = \frac{F}{1+CF}.$$

Thus, from figure 3, we write:

$$\frac{E_o}{A} = \frac{K_c\omega_f \frac{s+\omega_p}{\omega_o}K_f}{(s^2+2\zeta\omega_m s+\omega_m^2)(s+\omega_f)+\frac{G}{m}K_c\omega_f\frac{s+\omega_p}{\omega_o}K_f}\,\frac{R_o}{\frac{s+\omega_p}{\omega_o}}\,\frac{\omega_g}{s+\omega_g}K_g \tag{8}$$

which is the complete transfer function for the force balance accelerometer. A good approximation can be derived by making the following assumptions:

a) ω_f is much larger than the frequencies of interest

b) ζ is much smaller than the damping controlled by feedback

c) ω_m is much smaller than the natural frequency controlled by feedback

d) $\omega_p = \omega_o$, that is R_o is much smaller than $R_h + R_s$.

The resulting approximation is:

$$\frac{E_o}{A} = \frac{K_cK_f}{s^2+\frac{G}{m}\frac{K_cK_f}{\omega_o}s+\frac{G}{m}K_cK_f}R_o\frac{\omega_g}{s+\omega_g}K_g. \tag{9}$$

By comparison with the solution of an ordinary second-order differential equation, the closed loop natural frequency, ω_n, and damping, h, are:

$$\omega_n^2 = \frac{G}{m}K_cK_f$$

and

$$2h\omega_n = \frac{G}{m}\frac{K_cK_f}{\omega_o};\ h = \frac{\omega_n}{2\omega_o}$$

The output voltage at zero frequency is determined by letting s equal zero; then:

$$\frac{E_o}{A} = \frac{R_oK_g}{\frac{G}{m}}$$

The validity of these approximations can be verified using typical values:

$$G = 22\ N/A$$

$$m = .0041\ kg$$

$$K_c = 10{,}000\ V/m$$

$$K_f = \frac{R_h + R_s}{R_s R_o} = \frac{5.3k + 10k}{10k \times .80k} = 1.91 \times 10^{-3}$$

$$\omega_o = \frac{1}{R_o C_o} = \frac{1}{.80k \times 5.6 \times 10^{-6}} = 223\ radian/sec$$

$$K_g = 1 + \frac{R_f}{R_g} = 1 + \frac{7.50k}{10k} = 1.75$$

Therefore,

$$\omega_n^2 = \frac{22}{.0041}(10{,}000)(1.91 \times 10^{-3})$$

$$\omega_n = 320\ radian/sec;\ f_n = \frac{\omega_n}{2\pi} = 51\,Hz$$

$$h = \frac{320}{2 \times 223} = .72\ critical$$

and

$$\frac{E_o}{A} = \frac{(.80k)(1.75)}{22/.0041}\, 9.8 \frac{m}{sec^2}/g$$

$$= 2.55\ V/g$$

The amplitude and phase response as a function of frequency can be determined from equation 8. The amplitude will be flat to f_n, then fall as the square of frequency to f_g, after which it falls as the frequency cubed up to f_f. Above f_f, the amplitude decreases with the fourth power of frequency.

The phase response is the sum of the phase lags dictated by f_n, f_g, and f_f. The first of these, f_n, causes a linear phase lag to 90° at f_n, then approaches a 180°asymptote at 2 f_n. The latter two, f_g and f_f, each cause a linear phase lag to 45° at f_g (and f_f), then approach a 90° asymptote at 2 f_g (and f_f). Thus the maximum phase lag approaches 360° as the frequency becomes infinite.

TEST RESULTS

Frequency response tests were conducted on two FBA-11 accelerometers. One of the FBA-11 units had a factory-determined natural frequency of 55 Hz and damping of .66 critical; the other unit had a natural frequency of 100 Hz and damping of .65 critical. The shake table used for the tests was an Acoustic Power Systems' Electro-Seis Model 113. The table was instrumented with an Endevco Model 2272 piezoelectric accelerometer which has a natural frequency of 15 kHz. The ratio of outputs of the FBA-11 and the 2272 was measured using an HP 3582A spectrum analyzer. Data was recorded for both the amplitude ratio and phase lag. For each of the two FBA-11s, the tests were conducted first with the shake table motion horizontal. Then the accelerometer was re-oriented to be vertical, the mass recentered, and the shaking test repeated for vertical motion.

The results are plotted on Figures 4 through 7. The experimental data is indicated by open circles. The continuous lines were calculated from poles and zeros. In addition, dotted line asymptotes have been added. For the amplitude plots, the asymptotic corners are at:

f_n = 55 Hz (Figures 4 and 5)
or f_n = 100 Hz (Figures 6 and 7)
f_g = 160 Hz
f_f = 320 Hz

For the phase plots, asymptotes have been drawn to points which are the sum of the phase lags at 2 f_n, and at 2 f_g, and then points toward 360° at 2 f_f.

CONCLUSIONS

The experimental results are very close to the theoretical predictions. For the horizontal shake table results, the amplitudes are somewhat below the theoretical curves at the highest frequencies. For the vertical shake table results, the amplitudes of the

100 Hz FBA-11 are slightly above the theoretical curve for frequencies greater than f_n, while for the 55 Hz FBA-11, the amplitudes precisely match the theory. These slight deviations of experimental results from theoretical curves are undoubtedly due to the difficulties of controlling the shake table.

ACKNOWLEDGEMENTS

The FBA was designed by E.L. Holbrook of Kinemetrics Systems. The circuit analysis was carried out by W. F. Miller of the Caltech Seismological Laboratory. The shake table tests were conducted by E. L. Holbrook of Kinemetrics Systems and C. B. Crouse and B. Hushmand of Earth Technology, Inc. Preparation of the original manuscript, using the scientific word processor, Chi Writer, by J. A. Viengkhou and of the final manuscript in Word Perfect 5.1 by M. J. Donaldson, is gratefully acknowledged. Partial support for the shake table tests was provided by the National Science foundation under Grant ECE-8419431.

REFERENCES

1. Amini, A. and Trifunac, M. (1983) Analysis of a Feedback Transducer [FBA-1]. Department of Civil Engineering, Report 83-03, University of Southern California, Los Angeles, California.

2. Crouse, C. B. and Hushmand, B. (1989). Soil-structure interaction at CDMG and USGS accelerograph stations, *Bull.Seism.Soc.Am.*,79, 1-14.

3. Rihn, W. J. (1973). Magnetic tape recording and processing of strong-motion data, *Fifth World Conf. Earth. Engin.*, Rome, 129.

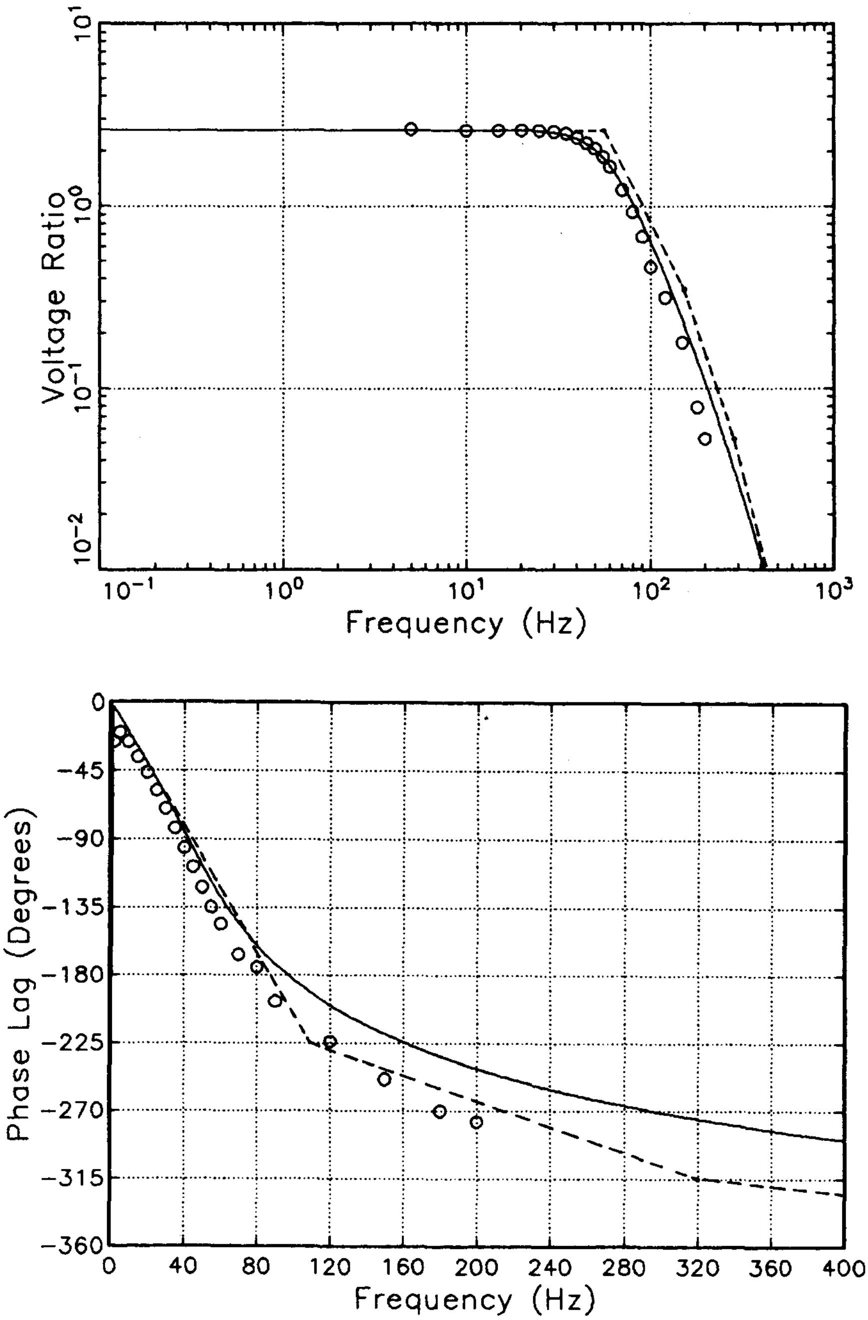


FIGURE 4

FBA-11, SERIAL 22388, Fn=55Hz

HORIZONTAL VIBRATION

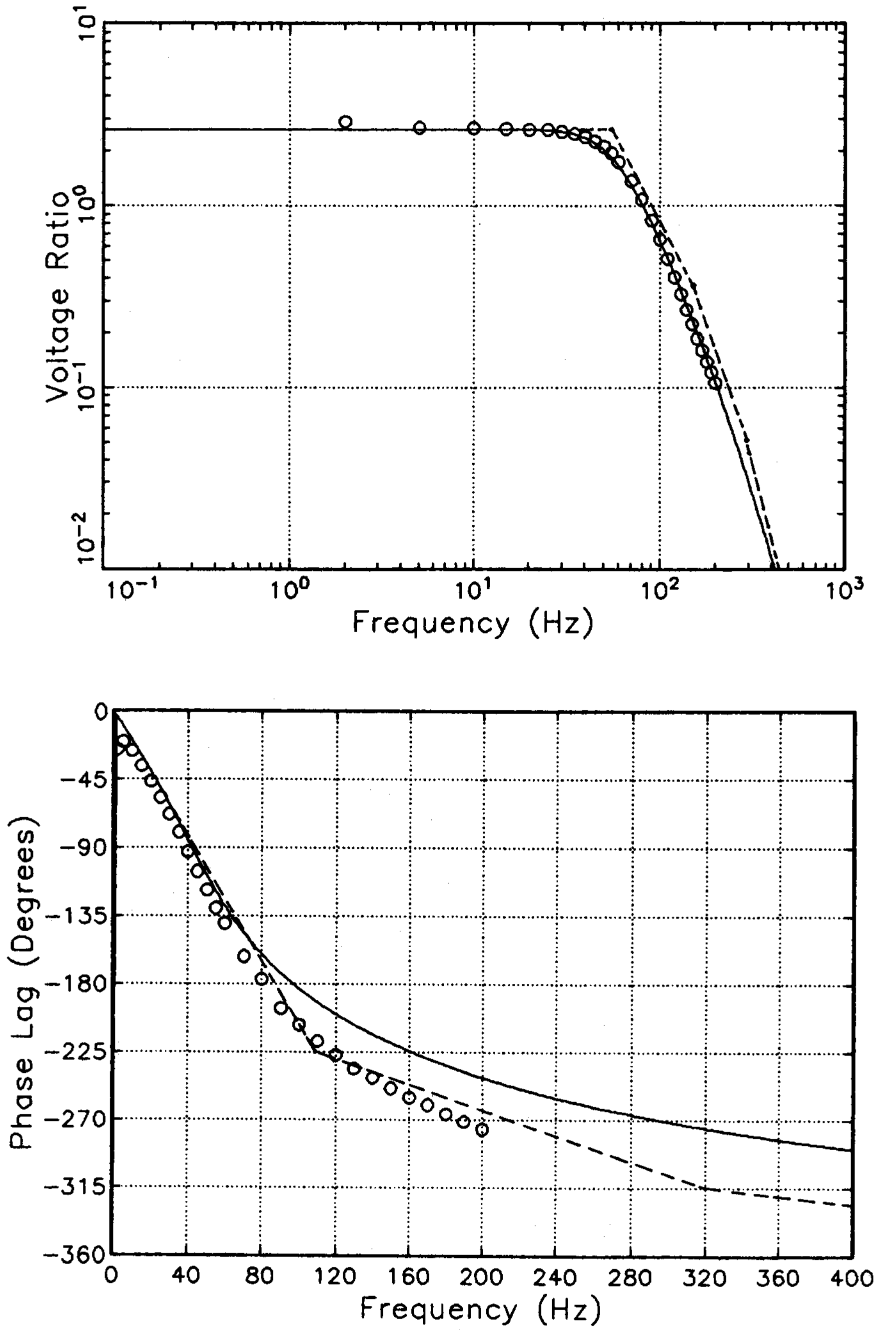


FIGURE 5

FBA-11, SERIAL 22388, Fn=55Hz
VERTICAL VIBRATION

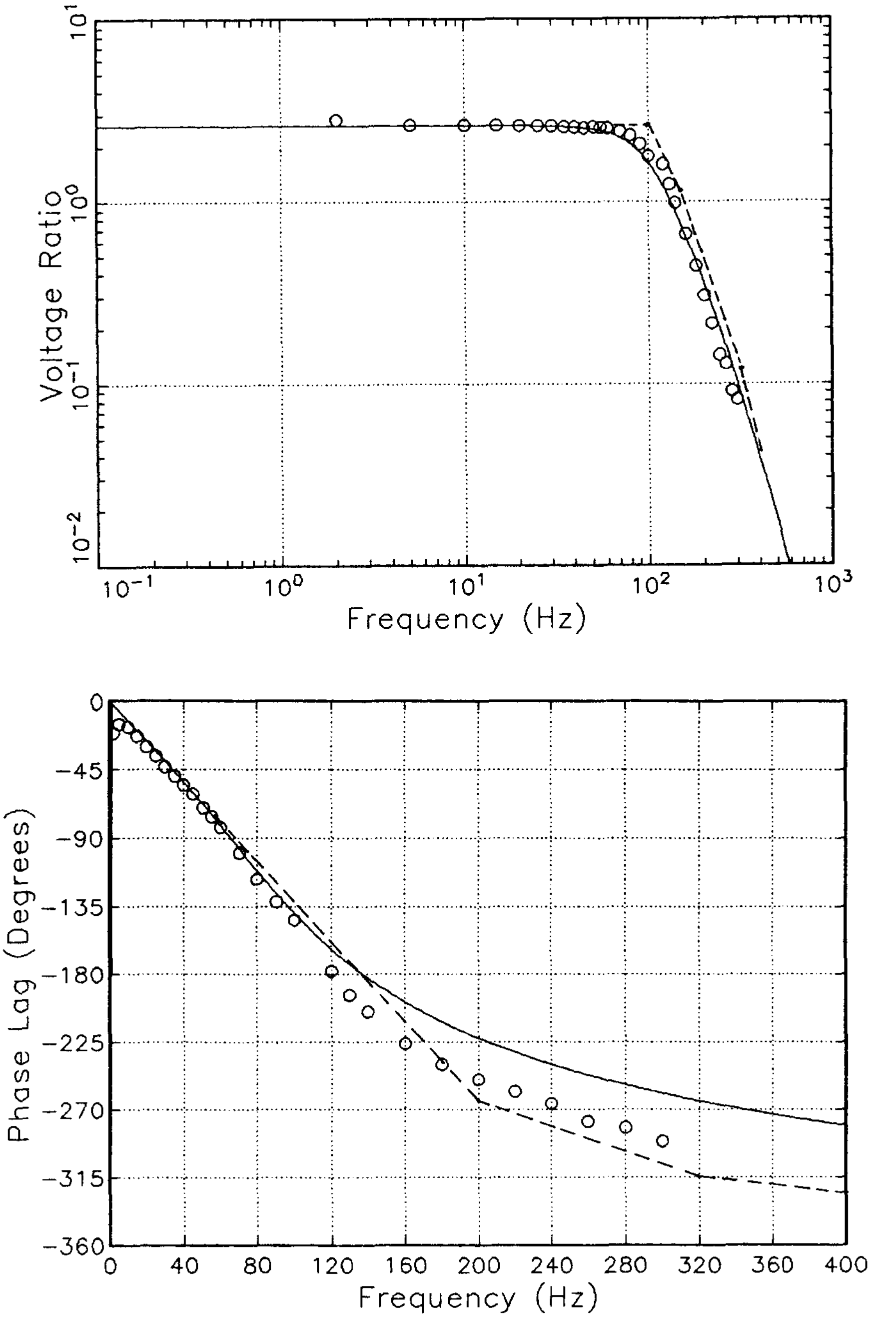
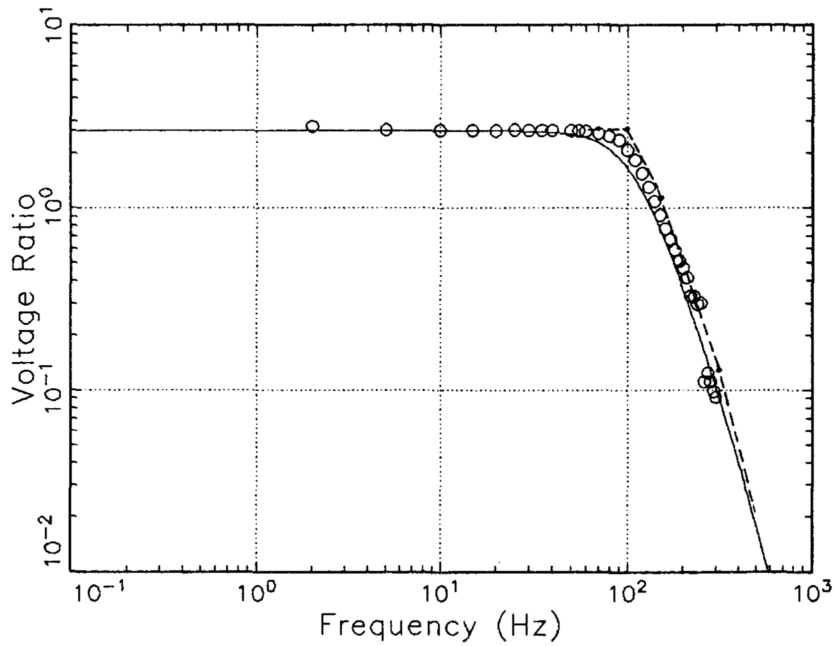


FIGURE 6

FBA-11, SERIAL 22389, Fn=100Hz HORIZONTAL VIBRATION



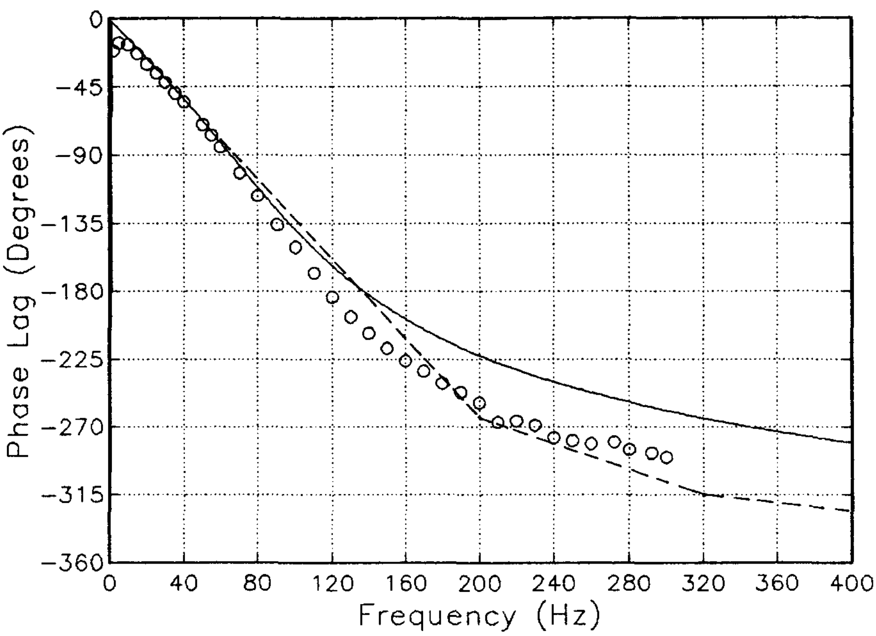


FIGURE 7

FBA-11, SERIAL 22389, Fn=100Hz

VERTICAL VIBRATION