Supplementary Material

Results from the Ice Thickness Models Intercomparison eXperiment phase 2 (ITMIX2)

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S1 “TamreBraun” model description

The model is based on mass conservation, requiring ice flux divergence to be matched by mass balance and rate of ice thickness change. In a glacier, the ice thickness change rate $\frac{\partial h}{\partial t}$ is determined by the divergence of ice flux $\nabla \cdot \vec{F}$ and the local mass balance rate $M$:

$$\frac{\partial h}{\partial t} = -\nabla \cdot \vec{F} + M.$$  \hspace{1cm} (1)

The ice flux is $\vec{F} = h\vec{u}$, where $h$ is the ice thickness and $\vec{u}$ is the vertically averaged horizontal ice velocity. That velocity consists of a deforming component and a sliding component:

$$\vec{u} = \vec{u}_d + \vec{u}_s.$$ \hspace{1cm} (2)

According to the Shallow Ice Approximation (SIA) \cite{?}, the deforming velocity is given by

$$\vec{u}_d = -f_d h^{n+1} |\nabla S|^{n-1} \nabla S$$ \hspace{1cm} (3)

and the sliding velocity by

$$\vec{u}_s = -f_s h^{n-1} |\nabla S|^{n-1} \nabla S,$$ \hspace{1cm} (4)

where $S$ is the surface elevation, $f_d$ and $f_s$ are two flow parameters (also referred to as “pre-factors”) \cite{?}, and $n$ is the exponent in Glen’s law \cite{?}.

The explicit form of $f_d$ is

$$f_d = 2A(g\rho)^\frac{3}{(n+2)}(n+2)^{-1},$$ \hspace{1cm} (5)

where $A$ and $n = 3$ are the flow rate factor and exponent in Glen’s law \cite{?}, respectively, $g$ is gravitational acceleration, and $\rho$ is ice density.

The used sliding law is inspired by Bindschadler (1983) \cite{?}, who proposes the form $u_s = \frac{k\tau^m}{N_e}$. $N_e = N - P$ is the effective normal pressure that a glacier exerts on its bed ($N$ being the ice overburden pressure and $P$ the basal water pressure), $\tau$ is the basal shear stress, and $k$ is a constant. Following \cite{?}, we set the exponent $m = n = 3$. We additionally assume that $N_e = aN$, where $a$ is between 0 and 1 (that is, $N_e$ is a fixed fraction of $N$). Since $N = \rho gh$, the explicit form of our sliding parameter becomes

$$f_s = \frac{k(\rho g)^\frac{n-1}{a}}{a}.$$ \hspace{1cm} (6)

Introducing the above into Equation 1, the ice thickness change rate becomes

$$\frac{\partial h}{\partial t} = \nabla \cdot [(f_d h^{n+2} + f_s h^n)|\nabla S|^{n-1} \nabla S] + M.$$ \hspace{1cm} (7)

To simplify this equation, we now make two substitutions: $d = f_d h^{n+2} + f_s h^n$ and $\vec{N} = |\nabla S|^{n-1} \nabla S$. This leads to

$$\frac{\partial h}{\partial t} = \nabla \cdot (d\vec{N}) + M$$ \hspace{1cm} \hspace{1cm} (8)

or, re-arranged:

$$\nabla \cdot (-d\vec{N}) = M - \frac{\partial h}{\partial t}.$$ \hspace{1cm} (9)

The model differs from past works in how we infer ice thickness from the surface shape and mass balance constraints. We integrate the derived equation over catchment area $A$ contributing to ice flow through some point $P$ on the glacier:

$$\int_A \nabla \cdot (-d\vec{N}) \, da = \int_A \left( M - \frac{\partial h}{\partial t} \right) \, da.$$ \hspace{1cm} (10)
This expression allows application of Gauss’s theorem to the left side of the equation, yielding

$$\oint_L -d\vec{N} \cdot \vec{n} dL = \int_A \left( M - \frac{\partial h}{\partial t} \right) da,$$

(11)

where $L$ is the boundary surrounding the catchment area and $\vec{n}$ is the outward-pointing normal vector of the boundary.

At the boundary of the catchment area, the direction of ice flow and thus the direction of $\vec{N}$ will always follow the boundary. Thus, $\vec{N}$ is perpendicular to $\vec{n}$ everywhere except at point $P$ itself (where the vectors point in exactly opposite directions), greatly simplifying the integral over the boundary:

$$\oint_L -d\vec{N} \cdot \vec{n} dL = d_P|\vec{N}|_P d\ell,$$

(12)

where $d\ell$ is the side length of the grid cell corresponding to point $P$ in the model. This allows us to calculate the value of ice thickness proxy $D$ at a point $P$ (i.e. $D_P$) directly from upstream corrected mass balance and local surface shape:

$$D_P = \frac{1}{|\vec{N}|_P d\ell} \int_A \left( M - \frac{\partial h}{\partial t} \right) da.$$

(13)

Given that $D = (\rho g)^n (f_d h^{n+2} + f_s h^n)$, we then solve this polynomial equation for $h$ at each point in the model area, keeping in mind that only positive real solutions reflect actual presence of ice.

The technique we use to evaluate the integral of mass balance over the catchment area of point $P$ is borrowed from geomorphology. In particular, it is a development of the FastScape flow routing algorithm [? ], which allows our model to be computationally very efficient. It finds all grid cells contributing to the flow through a point, and adds their area together to obtain the total catchment area.
Table S1: Source for the data used within ITMIX2. Abbreviations are as follows: OL = glacier outline, DEM = digital elevation model of the glacier surface, SMB = surface mass balance, Vel. = surface ice flow velocity, dh/dt = rate of ice thickness change, H = ice thickness measurements. Contact information for unpublished data are given in the footnotes. Where available, information on data accuracy is found within the provided references.

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(5) Russian Academy of Sciences, Institute of Geography, Moscow, Russia; contact person S. Kutuzov
(6) Zentralanstalt für Meteorologie und Geodynamik (ZAMG), Vienna, Austria; contact person D. Binder
(7) Norwegian Water Resources and Energy Directorate (NVE), Oslo, Norway; contact person L.M. Andreassen
(8) ASTER GDEM version 2; a product of NASA and METI
(9) Marius Schaefer, Universidad Austral de Chile, Institute of Physics and Mathematics, Valdivia, Chile
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Figure S1: Examples of the results provided by individual models. The glacier surface (solid blue line) is shown with the results of all experiments (grey lines). Experiments in which the given profile was available for calibration (dark grey lines) are distinguished from those in which the profile was not available (light grey). Direct ice thickness observations are given by the red dots. The selected profile refers to the compulsory test case “Unteraar”, and is the same as shown in the top row of Figure 2 of the main article. The profile’s location is given in the map displayed in the bottom right panel. The blue dot defines the profile’s left-hand side. The thick, dark grey vertical bar to the right of the first panel visualizes the mean glacier thickness. “AAD 1” (“AAD 0”) is the average absolute deviation between observed and modelled ice thickness when the given profile was (was not) available for calibration.
Figure S2: Same as Supplementary Figure S1, but for a longitudinal profile on “Unteraar”. The profile is the same as shown in the bottom row of Figure 2 of the main article.
Figure S3: Same as Supplementary Figure S1, for a south-north profile on the compulsory test case “Austfonna”.
Figure S4: Same as Supplementary Figure S1, for a west-east profile on the compulsory test case “Austfonna”.

AAD 0 = 125 m  AAD 1 = 109 m  
AAD 0 =  60 m  AAD 1 =  1 m  
AAD 0 =  50 m  AAD 1 =  6 m  
AAD 0 =  62 m  AAD 1 =  55 m  
AAD 0 =  62 m  AAD 1 =  55 m  
AAD 0 =  60 m  AAD 1 =  1 m  
AAD 0 =  50 m  AAD 1 =  6 m  
AAD 0 =  54 m  AAD 1 =  13 m  
AAD 0 =  40 m  AAD 1 =  10 m  
AAD 0 = 114 m  AAD 1 =  11 m  
AAD 0 =  65 m  AAD 1 =  59 m  
AAD 0 = 498 m  AAD 1 = 498 m  
AAD 0 =  44 m  AAD 1 =  38 m  
AAD 0 =  62 m  AAD 1 =  52 m  

Elevation (m a.s.l.)
Distance along profile (km)
Profile 19
30 km
Figure S5: Same as Supplementary Figure S1, for a cross-section of the compulsory test case “Synthetic 1”.

0.0 0.5 1.0 1.5 2.0 km
Distance along profile (km)

Elevation (m a.s.l.)

Profile 08

Brinkerhoff
AAD 0 = 7 m
AAD 1 = 7 m

Farinotti
AAD 0 = 21 m
AAD 1 = 0 m

Fuerst
AAD 0 = 35 m
AAD 1 = 9 m

Gantayat
AAD 0 = 31 m
AAD 1 = 31 m

GilletChaulet
AAD 0 = 8 m
AAD 1 = 2 m

Huss
AAD 0 = 37 m
AAD 1 = 17 m

Maurer
AAD 0 = 31 m
AAD 1 = 8 m

Morlighem
AAD 0 = 16 m
AAD 1 = 9 m

Rabatel
AAD 0 = 18 m
AAD 1 = 19 m

Ramsankaran
AAD 0 = 29 m
AAD 1 = 29 m

TamreBraun
AAD 0 = 22 m
AAD 1 = 23 m

VanPeltLeclercq
AAD 0 = 19 m
AAD 1 = 19 m

Werder
AAD 0 = 28 m
AAD 1 = 22 m

1.5 km

Figure S6: Same as Supplementary Figure S1, for a longitudinal profile of the compulsory test case “Synthetic 1.”
Figure S7: Same as Figure 6 in the main text but for the case in which only the three compulsory test cases are considered.
Figure S8: Equivalent of Figure 7 in the main text (identical for the “POOLED” panel) but for every single model. The blue solid (dotted) line is the fit through the median deviations of each model when all (only compulsory) test cases are considered. The dotted red line is identical for each panel, corresponds to the pooled fit, and is provided as a reference. The red dotted line in the “POOLED” panel only considers the compulsory test cases (defined in Table 1 of the main article). Note that the colour scale for the pooled result differs from the one in the other panels.
Figure S9: Same as Figure 8 in the main text but for the case in which only the three compulsory test cases are considered.