How Fast Can a Blob Go?

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1. Apparent Speed Limits

The spots of radio emission mapped with VLBI often appear to move across the sky with speeds $v \sim (1\text{–}10)h^{-1}c$. We learned at this workshop from Porcas (page 12) and Pearson, Readhead, and Barthel (page 94) that a large fraction, perhaps a majority, of radio sources with bright cores exhibit this superluminal motion. We also heard of two relatively poorly-studied sources (3C 454.3: Pauliny-Toth, page 55; CTA 102: Bååth, page 206) whose blobs might separate at $v \sim 15h^{-1}c$. Yet there do not appear to be any convincing claims of speeds in excess of $20h^{-1}c$. Here I discuss the significance of a characteristic transverse speed of VLBI knots $v \sim 10h^{-1}c$ and its relevance as a constraint on models of jet acceleration.

Is there really some characteristic maximum speed $\sim 10h^{-1}c$? One response would be that humanity has mapped a grand total of only 100–200 blobs with VLBI. If the blobs moved at random angles to the line of sight, then even if their three-dimensional pattern speeds approached c, the distribution of apparent speeds ($\mathcal{P}(> v/c) \lesssim 2(c/v)^2$) is such that we wouldn't *expect* to have seen any apparent speeds $> 20c$.

This is an unconvincing explanation for two (possibly unconvincing) reasons. First, the statistics of core-to-extended flux ratios (Hough and Readhead, page 114; Browne, page 129) suggest that in the sources with bright cores which are preferentially mapped, the emission from the moving patterns is at least roughly beamed in their directions of motion. If this beaming is over a fraction f of the sky and we map N blobs (with intrinsic pattern speed $\to c$) moving within the beaming cone, the apparent transverse speed of the fastest moving blob will typically be $\overline{v}_{\rm max} \sim c\sqrt{2N/f}$ and the probability that the actual maximum speed will be $v_{\rm max} < \overline{v}_{\rm max}$ is

$$\mathcal{P}(v_{\rm max}) = \left[1 - \frac{2}{f}\left(\frac{c}{v_{\rm max}}\right)^2\right]^N . \tag{1}$$

For likely values, $N \sim 100$, $f \lesssim 0.1$, this gives $\overline{v}_{\max} \gtrsim 45c$, and the observed upper limit is quite improbable if $H_0 = 100\,\mathrm{km\ s^{-1}Mpc^{-1}}$, but not significantly so if $H_0 = 50$!†

A second, more compelling argument is the absence of observational evidence for huge flares (e.g., rapid changes in intensity by more than a factor of 1000) or inferred brightness temperatures much in excess of 10^{15} K. Avoiding trouble with inverse Compton radiation does not require the synchrotron-emitting electrons (as opposed to the *pattern* constrained above) to have bulk Lorentz factors in excess of $5\,h^{-1}$. Even tiny amounts of jet material with a bulk Lorentz factor > 25 would cause easily detectable flares. On the other hand, there appears to be no observational objection to *smooth* jets with Lorentz factors as high as 50, provided their vectors of bulk velocity are spread over enough of the sky (10^{-1}–10^{-3}) to avoid embarrassing space densities of unbeamed counterparts. Although brightness temperatures $> 10^{15}$ K would then be physically possible, they need not occur if the brightness temperature in the comoving frame is typically $\ll 10^{12}$ K (there is no obvious reason why emitting plasma should always be near the inverse Compton limit in its rest frame, though an optically thick synchrotron source cannot have a rest-frame brightness temperature much less than $\sim 10^{10}$ K, whence the upper limit $\gamma \lesssim 50$).

2. The Compton Speed Limit

The particles in a relativistic jet see blue-shifted photons pouring in from the forward direction. The resulting radiation pressure on electrons in the jet will brake any jet whose initial Lorentz factor exceeds some critical value, converting the excess kinetic energy into a directed beam of scattered photons.

† The inequalities in the probability distributions in the preceding paragraphs are strictly valid only if it is possible to select blobs whose pattern velocities are randomly oriented. The scarcity of superluminal contractions and 180° misalignments with large-scale structure suggests that the line of sight must always be outside the cone defined by the direction of blob motion and the axis defined by larger-scale structure. Because of this and the general impression that high apparent speeds are "too common", there is interest in models which use beaming to inextricably link the direction of pattern motion to that of the line of sight. In the class of models considered by Scheuer (1984; see also Phinney 1985) the pattern moves at an angle $(1-1/n)$ times closer to the line of sight than the source axis. For randomly oriented source axes, this *increases* the probability of seeing high apparent speeds; however the maximum possible apparent speed is c times the Lorentz factor of the pattern, which is in turn somewhat *less* than the Lorentz factor of the front of particles. Thus, in these simple models, an observed upper limit to v/c constrains γ even more strongly than in simple "cannon-ball" models. It is, however, possible to construct contrived models (which do not show contractions from any viewing angle) in which the pattern speed greatly exceeds that of any physical particles.

The electrons, positrons, or electromagnetically coupled electron-proton pairs making up a (cold) jet in a radiation field of (frequency integrated) specific intensity $I(\theta, \phi)$ will gain or lose energy $p^0 = \gamma mc^2$ according to (Phinney 1982; O'Dell 1981)

$$c\beta \frac{dp^0}{dr} = \frac{dp^0}{dt} = -\sigma \left[\gamma^2 \int I(1-\beta\cos\theta)^2 \, d\Omega - \int I(1-\beta\cos\theta)\, d\Omega\right] \quad (2)$$

$$c\frac{dp^0}{dr} \simeq \sigma F \left[\frac{1}{4\gamma^2} - k_1\theta^2 - k_2\gamma^2\theta^4\right] - \frac{4}{3}\sigma\gamma^2 U_{\rm ISO} c\,, \quad (3)$$

where the second approximate equality holds if $\gamma \gg 1$ and the radiation field consists of a flux F confined to a range of angles $\theta \ll 1$ with respect to the direction of particle motion (e.g., at a distance r from a source of size R and isotropic luminosity L, $\theta \sim R/r$ and $F = \frac{L}{4\pi r^2}$), *plus* an isotropic component of energy density $U_{\rm ISO}$. The constants k_1 and k_2 depend on the details of the limb-darkening of the source, but are typically ~ 0.05. The first term on the right of (3) vanishes when $\gamma \sim 1.5/\theta$, i.e., when the particle velocity is such that there is no net momentum flux in the aberrated radiation field of its rest frame.

The radiation field in an active galactic nucleus is probably dominated by emission from some sort of accretion disk with luminosity L_d, inner radius $R_i \sim (2\text{–}10) r_g$ ($r_g \equiv GM_h/c^2$, M_h being the mass of the putative black hole), which emits from the annulus between R and $2R$ a luminosity $L(R) \sim L_d(R_i/R)$. As seen from a point a distance r out along the symmetry axis of the disk, the effective values of $F\theta^2$ and $F\theta^4$ are dominated by that part of the disk with $R \sim r$ ($\theta \sim 1$), and are $\sim L_d R_i/(4\pi r^3)$. In addition, when a broad-line region is present, we expect that a fraction $0.1\, f_{-1}$ of the total luminosity $10^{46}\, L_{46}$ erg s^{-1} will be reradiated roughly isotropically from a radius $R_{\rm BLR} \sim L_{46}^{1/2}$ pc (as lines—there may also be radiation Thomson scattered by the intercloud medium). It then follows from (3) that any jet emerging from the broad-line region will have had its Lorentz factor reduced to $\gamma < \gamma_{\rm BLR}$ (see also Rees 1984) where

$$\gamma_{\rm BLR} \equiv 10^3\, f_{-1}^{-1} L_{46}^{-1/2} \left(\frac{m}{m_e}\right) . \quad (4)$$

Jets are, however, probably produced far inside the broad-line region. Here drag due to photons from the accretion disk provides a much more severe constraint. For large γ, the term in k_2 in (3) dominates, and using the above estimate for the effective value of $F\theta^4$, we find from (3) that the jet will be rapidly decelerated (or accelerated!) to the equilibrium Lorentz factor if $\gamma > \gamma_d(r)$, where

$$\gamma_d(r) \equiv \frac{1}{k_2}\frac{1}{l_i}\left(\frac{r}{R}\right)^2 . \quad (5)$$

Here l_i is the usual compactness parameter,

$$l_i = \frac{\sigma_T L_i}{4\pi m c^3 R_i} = 1836 \left(\frac{L_i}{L_{\rm Edd}}\right)\left(\frac{r_g}{R_i}\right)\left(\frac{m_e}{m}\right) . \qquad (6)$$

If a typical quasar has $L_i \sim L_{\rm Edd}$, $R_i \sim 5r_g$, while a typical radio galaxy has $L_i \sim 10^{-3} L_{\rm Edd}$, then the constraint (5) becomes $\gamma < 10(r/70r_g)^2$ for an e^+e^- jet in a quasar, and $\gamma < 10(r/2r_g)^2$ for an e–p jet in a quasar or an e^+e^- jet in a radio galaxy.

These already severe constraints would become even more severe in the presence of any of the likely complications to this simple picture – e.g., if a corona above the accretion disk scatters a significant fraction of the disk luminosity, if the outer parts of the disk absorb and reradiate photons from the inner parts, if the jet has a finite opening angle, or if the jet axis is misaligned with the axis of disk symmetry at large radii.

For e^+e^- jets born near R_i in Eddington-limited sources, the first, accelerating term on the right-hand side of (3) can be significant. The jet is then quickly brought to the equilibrium Lorentz factor, which increases as the jet moves outwards until radiation driving is no longer efficient. Solving (3), we find that the terminal Lorentz factor

$$\gamma_f \simeq \left(\frac{l_i}{16k_2}\right)^{1/7} \simeq l_i^{1/7} . \qquad (7)$$

For our Eddington-limited disk $\gamma_f \sim 2$ 3 for e^+e^-. Note that this is much less than the $\gamma_f \simeq l_i^{1/4} \simeq 5$ which we would have obtained if we had ignored the outer parts of the disk (the relevant radii here being $r \sim 7R_i \sim 35r_g$).

3. Discussion

Quasars very probably have both large compactness parameters l_i and jets with Lorentz factor $\gamma \sim 10$. Compton drag then imposes severe constraints on any model of jet acceleration (equation 5). The speed limit on electron-proton jets accelerated near the black hole in quasars is of the same order as that inferred from observation ($\gamma \sim 10$). There would be no such limit on electron-proton jets in radio galaxies, though electron-positron jets accelerated near the black hole would there also be limited to $\gamma \lesssim 10$.

Electron-positron jets in quasars cannot reach the desired γ until radii $\gtrsim 70r_g$, and must therefore be accelerated slowly and carefully. This seems very difficult to arrange in any local or purely hydrodynamical process unless the kinetic luminosity desired in the jet is a negligible fraction of the total source luminosity.

A slow acceleration can be arranged in magnetohydrodynamic wind models in which large-scale magnetic fields extract rotational energy from a black hole or accretion disk. Flow on rapidly diverging field lines (as might exist near the disk or black hole) is rapidly accelerated, and would be subject to Compton drag, lowering the efficiency with which the wind extracts power and producing a collimated beam of scattered ambient photons (whose polarization properties might constrain the structure of the jet: Begelman and Sikora 1987). But magnetic hoop stresses and the radiation pressure on the particles will tend to collimate the field lines, and on such lines the acceleration of particles (the centrifugal transfer of energy from electromagnetic Poynting flux to particle kinetic energy) is very slow. Even in a quasar an e^+e^- jet could thus escape the effects of Compton drag (detailed models have been constructed by Phinney, in preparation).

Such ideal, axisymmetric jets would not, however, have any internal dissipation and would be invisible until they ran into obstacles. Compton cooling in quasars prevents material not supported by rotation from existing within ~ 1 pc, so obstacles are not likely to be present inside 1 pc. Since blazars seem to have significantly beamed infrared flux (which can be understood as marginally self-absorbed synchrotron radiation from relativistic electrons at a few gravitational radii), it seems likely that internal dissipation driven by instabilities in the jet is an additional limiting factor.

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