Supporting Information for "Resolving simulated sequences of earthquakes and fault interactions: Implications for physics-based seismic hazard assessment”

Valère Lambert¹ and Nadia Lapusta¹,²

¹Seismological Laboratory, California Institute of Technology, Pasadena, California, USA
²Department of Mechanical and Civil Engineering, California Institute of Technology, Pasadena, California, USA

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Text S1: Description of initial shear stress distributions for numerical simulations of long-term sequences of earthquakes and aseismic slip.

In our simulations of sequences of earthquakes and aseismic slip (SEAS), the distributions of shear stress and slip along the fault evolve depending upon the history of previous slip during periods of rapid seismic slip as well as slow aseismic slip and fault locking. We consider how the long-term evolution of fault slip differs among simulations using varying computational cell sizes and considerations of inertial effects, given the same initial conditions for shear stress, slip rate and the rate-and-state frictional state variable θ.

For all of our simulations, the velocity-strengthening (VS) portions of the fault are set to be initially creeping at steady state with the prescribed tectonic plate rate of $V_{ini} = V_{pl}$.
For points within the velocity-weakening (VW) segments of the fault, we first consider the initial shear stress distribution $S_1$, which favors the first rupture nucleating along the VW-VS boundary around $x = 33$ km and then jumping across the VS barrier to produce a two-segment rupture (e.g. Figure 2 of the main text):

$$
\tau_{\text{ini}VW}(x) = \begin{cases} 
\tau_{\text{ss}}(1 \text{ m/s}) + 3.5 \text{MPa} & \text{for } x \in [-33 \text{km}, -2 \text{km}) \\
\tau_{\text{ss}}(V_{pl}) + a_{VW} \ln \frac{0.1 \text{m/s}}{V_{pl}} - 1.5 \text{MPa} & \text{for } x \in [-2 \text{km}, -1 \text{km}) \\
\tau_{\text{ss}}(1 \text{ m/s}) + 5 \text{MPa} & \text{for } x \in (1 \text{km}, 27 \text{km}) \\
\tau_{\text{ss}}(V_{pl}) + a_{VW} \ln \frac{0.1 \text{m/s}}{V_{pl}} & \text{for } x \in [27 \text{km}, 33 \text{km}] 
\end{cases}
$$ (3)

In all of our simulations, points with the VW segments are initially locked with initial slip rate $V_{\text{ini}} = 10^{-10}$ m/s and the initial state variable $\theta$ chosen to be consistent with the corresponding initial shear stress and slip rate, given equation 2 in the main text.

In order to examine the convergence of long-term sequences of earthquakes with different initial conditions, we consider a second initial shear stress distribution $S_2$ (Figures 3A vs B in the main text), which favors the first rupture nucleating near the VS barrier around $x = 1$ km and propagating away from the barrier and spanning the entire right VW segment:

$$
\tau_{\text{ini}VW}(x) = \begin{cases} 
\tau_{\text{ss}}(1 \text{ m/s}) + 3.5 \text{MPa} & \text{for } x \in [-33 \text{km}, -1 \text{km}) \\
\tau_{\text{ss}}(V_{pl}) + a_{VW} \ln \frac{0.1 \text{m/s}}{V_{pl}} & \text{for } x \in (1 \text{km}, 7 \text{km}] \\
\tau_{\text{ss}}(1 \text{ m/s}) + 5 \text{MPa} & \text{for } x \in (7 \text{km}, 33 \text{km}] 
\end{cases}
$$ (4)
References

Figure S1. Inadequately-resolved simulations of fault model M1 exhibiting different simulated earthquake sequences and rates of two-segment ruptures. (A-B) History of cumulative slip over 4000 years in fully dynamic simulations of fault model M1 using oversized cells of (A) 500 m and (B) 1000 m, respectively. Contours of seismic slip are plotted every 0.5 s, with ruptures that jump across the VS barrier colored blue. (C) Spatial distribution of shear stress around the rupture front in a well-resolved simulation (Δx = 25 m, red) and the two simulations with oversized cells (Δx = 500 and 1000 m). As the cell size increases, the breakdown of shear stress at the rupture front is increasingly poorly resolved. (D-E) Frequency-magnitude histograms for events in (A-B), respectively. Simulations with oversized cells exhibit different long-term sequences of events compared to the well-resolved simulations (Figure 2 of main text), with increased production of small events and significantly different rates of two-segment ruptures.

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Figure S2. Final slip and evolution of slip for the first rupture in simulations of fault model M1 with different numerical resolution. (A) The final slip distribution for the first simulated rupture with the same initial conditions is practically the same for simulations using cell sizes of 12.5, 25, 125 and 250 m. (B) The evolution of slip is contoured every 0.5 s and comparable spacing between contours illustrates that the rupture speed is generally consistent for the first rupture in these well-resolved and marginally-resolved simulations. The evolution of slip and final slip are virtually identical for the two well-resolved simulations using 12.5 and 25 m cells and the average final slip for simulations with 12.5 and 250 m cell sizes differ by less than 0.8%.
Figure S3. Different long-term interaction of co-planar fault segments in simulations with different treatment of inertial effects. (A-C) History of cumulative slip over 4000 years in well-resolved (A) fully dynamic, (B) standard quasi-dynamic ($\beta = 1$) and (C) enhanced quasi-dynamic ($\beta = 3$) simulations of fault model M1 with initial conditions S1. Contours of seismic slip are plotted every 0.5 s with ruptures that jump across the VS barrier colored blue. The increased spacing between contours for the enhanced quasi-dynamic ruptures in (C) illustrate the higher effective rupture speeds that are more comparable to those of the fully dynamic ruptures in (A). Despite the higher rupture speeds and larger slip rates (Figure ??), the long-term slip behavior for both quasi-dynamic simulations is qualitatively comparable, with no ruptures jumping across the VS barrier.
**Figure S4.** Comparable scaling of average static stress drop versus moment with reasonable stress drop values between 1 to 10 MPa for simulated ruptures in the six sets of simulations shown in Figure 9 of the main text. The six models all use oversized cells of $\Delta x = 1000$ m and produce comparable earthquake frequency-magnitude statistics. Simulations using oversized cells produce small numerically-discrete ruptures consisting of only a few cells that fail to propagate due to the poorly resolved stress concentration of the shear stress at the (diffuse) rupture front. The small numerically-discrete ruptures produce variable amounts of slip, despite being restricted to the same rupture size of only 1 to several cells, leading to large, upward-sweeping trends in average stress drop with moment, which are purely numerical.
Figure S5. Normalized frequency-magnitude statistics over 20,000 years of SEAS simulations different fault models using oversized cells with comparable b-values (Figure 9 of main text). While the different models produce similar average static stress drops (Supplementary Figure S4) and similar cumulative frequency-magnitude statistics with b-values around 0.3, they do not have identical frequency-magnitude statistics and result in different rates of multi-segment ruptures.
Figure S6. Variability in specific frequency-magnitude statistics with finer magnitude binning (0.1) for models with similar b-values. (A-F) Cumulative frequency-magnitude histograms (Top) and frequency-magnitude histograms (Bottom) over 20,000 years in (A-C) fully dynamic and (D-F) quasi-dynamic SEAS simulations, as shown in Figures 8 and 9 of the main text.
Figure S7. Excellent resolution of local shear stress and slip rate for the first rupture of well-resolved fully dynamic simulations of model M5 shown in Figure 10. The evolution of local shear stress with slip at (A) $x = 20$ km and (B) $x = -20$ km is virtually identical. Evolution of (C-D) shear stress and (E-F) slip rate with time for the same points. The rupture nucleates near $x = 30$ km. Early in the rupture (A, C & E), the local behavior is comparable among the well-resolved simulations. Near the end of the first rupture (D & F), the simulations begin to deviate very slightly in their local behavior, consistent with the results of Day et al. (2005). While the simulated behavior in the first rupture is very similar, these small differences, resulting from different numerical approximations, compound over many sequences and eventually lead to diverging behavior, as seen in Figure 10.

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Figure S8. Differing frequency-magnitude histograms for 4000 years of sequences of earthquakes in fully dynamic simulations of fault model M5 with varying cell sizes. Simulations are performed using different computational cell sizes of (A) 25 m, (B) 12.5 m, (C) 6.25 m, (D) 125 m and (E) 500 m. Simulations exhibit differences in long-term behavior, even for well-resolved simulations (A-C) where the stress at the rupture front is spatially described by more than 3 cells. Simulations using oversized cells or with marginal resolution (D & E) produce more smaller events as small groups of cells nucleate into ruptures that fail to propagate due to the stress concentration at the rupture front being poorly resolved.
**Figure S9.** Evolution of local slip rate and shear stress with slip and time at two points at (left) $x = 20$ km and (right) $x = -20$ km during the first rupture of adequately-discretized fully dynamic simulations of fault model M5 with the effects of off-fault dissipation approximated using a velocity limit of $V_{\text{lim}} = 2$ m/s, as shown in Figure 17 of main text. The local behavior is nearly identical for all three simulations with different spatial discretization.
Figure S10. Simulations with diverging long-term sequences of earthquakes after small differences in a slow-slip transient. (A-B) Virtually indistinguishable spatial distribution of shear stress and slip after the 13th event in fully dynamic simulations of fault model M5 with the effects of off-fault dissipation approximated using a velocity limit of $V_{\text{lim}} = 2$ m/s, with cell sizes of $\Delta x = 6.25$ (red) and $\Delta x = 12.5$ (black dashed). (C) After a slow-slip transient following event 13, around $t = 210$ years, the timing of slip events begins to diverge. (D) The resolved shear stress changes due to the slow-slip transient within the nucleation region of event 14 mildly differs between the two simulations of different cell size, resulting in a slightly higher stress release for the simulation with cell size $\Delta x = 12.5$ m. (E-H) Following the slow-slip transient, there is a 3-year difference between the nucleation of event 14, leading to slightly higher prestress before the initiation of the rupture, and hence slightly different resulting slip distributions. These differences compound into greater discrepancies in shear stress and slip in subsequent ruptures (Figure 15).
Figure S11. Different sequences of earthquakes and rate of two-segment ruptures over 4000 years in quasi-dynamic simulations with different resolution of fault model M5. (Top) Slip history for simulations with varying spatial resolution showing different histories of events depending on the choice of cell size. Seismic slip is contoured every 0.5 s with ruptures jumping across the VS barrier colored blue. (Bottom) Frequency-magnitude statistics for the respective simulations of varying cell size shown above. Simulations using larger cell sizes produce a larger number of small events as small groups of cells nucleate but ruptures cannot propagate due to the inadequately resolved stress concentration at the rupture front. Even adequately-resolved simulations show different histories of events, including rates of ruptures jumping across the VS barrier.
Figure S12. Compounded effects of small numerical differences in well-resolved quasi-dynamic simulations result in diverging long-term earthquake sequences. Comparison of prestress before rupture (left) and resulting slip distributions (right) for several events over the first 2000 years of simulated time in two quasi-dynamic simulations of fault model M5 using cell sizes of 6.25 m (red) and 12.5 m (black). (A & B) The evolution of shear stress and accumulated slip during the first few sequences of events is practically identical, however small differences begin to appear due to different numerical approximations. (C) The small differences in shear stress accumulate over sequences of events, resulting in more noticeable variations in the amount of slip in larger events. (D) The accumulation of noticeable differences in shear stress, particularly in regions of rupture nucleation and near the VS barrier, leads to differing sequences of events, rupture sizes, and probabilities of rupture jumping across the VS barrier.
Figure S13. Comparison of local slip rate, shear stress and stress transfer with different treatment of inertial effects and considerations for plasticity. (A-B) Spatial distribution of (A) shear stress, (B) slip rate and (C) stress transfer along the fault during the first rupture with the same initial conditions in fully dynamic (black) and quasi-dynamic (red) simulations of fault model M5, as well as a fully dynamic simulation approximating the effects of off-fault plasticity with a slip velocity limit of 2 m/s. The stress transfer during fully dynamic ruptures is much more pronounced than quasi-dynamic ruptures, resulting in higher slip rates and more focused shear stresses at the rupture front. The approximation for off-fault plasticity limits the peak slip velocity and restricts the magnitude of the peak stress transfer along the fault. However, the stress transfer for the fully dynamic rupture including the plasticity approximation is still more pronounced than that of the quasi-dynamic rupture and remains more pronounced behind the rupture front.
Figure S14. Frequency-magnitude histograms for 4000 years of sequences of earthquakes in fully dynamic simulations of fault model M5 with the effects of off-fault dissipation approximated using a slip velocity limit of $V_{\text{lim}} = 2 \, \text{m/s}$. Simulations are performed using different computational cell sizes of (A) 25 m, (B) 12.5 m, (C) 6.25 m, (D) 125 m and (E) 500 m. The increased production of smaller events ($M_w \leq 4$) in simulations with large computations cells (D & E) is qualitatively similar to the fully dynamic simulations of fault model M5 with no velocity limit shown in Figure S4.