LOW-FREQUENCY EARTHQUAKES IN THE MEXICAN SWEET SPOT

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1. SUPPLEMENTARY MATERIALS

1.1. Network Waveform Correlation Search Detection Threshold. A majority of previous studies that employed a network waveform correlation search used a detection threshold of eight times the median absolute deviation (MAD) (e.g. Bostock et al., 2012; Shelly et al., 2007). We, however, used a slightly lower threshold of five times the RMS of the daily CC sum (∼8% smaller than a threshold of eight times the MAD for a gaussian distribution) to gather a larger amount of potential detections to run through our robustness analysis (detailed in the following section; additionally). See Supplementary Table 1 for a comparison study of the number of detected LFEs with different detection thresholds.

1.2. Multi-band Correlation Coefficient Sum. Due to the limited bandwidth (1 - 2 Hz) used when searching for Mexican LFEs, there is a large risk of cycle skipping when performing the network waveform correlation search as shown in Supplementary Figure 1a. We overcame this obstacle by performing multiple correlation coefficient (CC) sums with increasingly higher frequency bands and then summing their results together; we defined this as the multi-band CC sum. The higher frequencies increase the dynamic range of the correlation coefficients, highlighting which detections are more (or less) ambiguous. We quantified the ambiguity of our detections by calculating a lobe ratio at every potential detection time. The lobe ratio is defined as the mean value of the two neighboring side lobes divided by the main lobe; an unambiguous detection therefore has a small lobe ratio. The three frequency bands we used to perform this analysis are 1 - 2, 1.5 - 4, and 2 - 6 Hz. The first band was chosen to include the highest SNR signal, while the second two bands were chosen to reduce the dependence on the nearly monochromatic main signal and to include the more discriminating higher frequencies.

The distribution of the multi-band lobe ratios is less uniform than the single-band lobe ratios, highlighting some detections as extremely ambiguous and others less so (Supplementary Figure 1). Going forward, we chose to only use detections that had a lobe ratio less than 0.8, attempting to reduce the chance of analyzing cycle-skipped detections; the qualifying detections are considered robust. Other lobe ratio cutoff values were evaluated, but the quality of our LFE family stacks degraded quickly with a cutoff value lower than 0.8.
1.3. Source Location Misfit Function. The misfit function we used to locate the 15 LFE families is defined here:

\[
\chi(\phi, \theta, z) = \sum_{\tau=1}^{T} \left( \sum_{i=1}^{I} \frac{\left| \delta_{\tau,m}^{i}(\phi, \theta, z) - \delta_{\tau,o}^{i} \right|^{2}}{w_{\tau,i}} \right),
\]

where \((\phi, \theta, z)\) are the coordinates of the theoretical source in latitude, longitude, and depth, \(i\) is the \(i^{th}\) (of \(I\) total) trace, \(\delta_{\tau,m}^{i}\) is the theoretical moveout (or delays) of the \(\tau^{th}\) time delay vector (of 3 total in this study: P-wave, S-wave, and S−P), \(\delta_{\tau,o}^{i}\) is the observed moveout (or delays) of the \(\tau^{th}\) time delay vector, and \(w_{\tau,i}\) is the weighting factor of the \(\tau^{th}\) time delay vector on the \(i^{th}\) trace based on the pick confidence with a weight of 1 indicating a confident pick and 2 indicating a less confident pick.

1.4. Focal Mechanism Coherent Energy. The maximum of the coherent energy between a synthetic seismogram and a stacked LFE family waveform is defined here:

\[
E(\Theta, \Phi, \Lambda) = \sum_{p=1}^{P} \sum_{i=1}^{I} \max \left[ \frac{CC\left(S_{p,m}^{i}(\Theta, \Phi, \Lambda, t), S_{p,o}^{i}(t)\right)}{N_{p}} \right],
\]

where \(E\) is the amount of in-phase energy, \(\Theta\) is the strike, \(\Phi\) is the dip, \(\Lambda\) is the rake, \(CC\) is the cross-correlation function, \(i\) is the \(i^{th}\) (of \(I\) total) trace, \(S_{p,m}^{i}\) is the synthetic seismogram of the \(p^{th}\) phase arrival (of \(P\) total), \(S_{p,o}^{i}\) is the observed stacked seismogram of the \(p^{th}\) phase arrival, \(t\) is the time, \(w_{p}\) is the weight attributed to the \(p^{th}\) phase arrival with a weight of 2 for P-waves and a weight of 1 for S-waves, and \(N_{p}\) is the number of compared traces of the \(p^{th}\) phase. An example energy distribution for a LFE family is shown in Supplementary Figure 2a.

Due to the first-order nature of our velocity model, we had to account for any slight inaccuracies related to the phase and the travel time by searching for maxima in the cross-correlation function between \(-0.2\) s and \(0.2\) s lagtime and not only at zero lagtime. We do not consider this "slack" in the coherent energy determination to generate any error as the signal’s dominant frequency lies between 1 and 2 Hz so \(0.2\) s is at most only \(2/5\) of a period; it is not possible that a completely out-of-phase synthetic would be able to be "slide" over half a period, becoming completely in-phase.

We determine the stacked normalized coherent energy distributions in the following manner:

\[
E(\theta, \phi, \lambda) = \sum_{f=1}^{F} \frac{E^{f}(\theta, \phi, \lambda)}{\max [E^{f}(\theta, \phi, \lambda)]},
\]

where \(f\) is the \(f^{th}\) (of \(F\) total) LFE family. The resulting distribution is shown in Supplementary Figure 2b.

**REFERENCES**


SUPPLEMENTARY FIGURE 1. Multi-band correlation coefficient sum (CC sum) analysis of the LFE detection shown in Figure 2. The single-frequency band CC sum is shown in black while the multi-band CC sum is shown in red. **a)** A zoom of the CC sum spike from Figure 2b is seen to be ambiguous with two potential detection times (at 5.51175 and 5.5119 hours). The multi-band CC sum analysis discriminates against the first detection and favors the second. **b)** The distribution of lobe ratios (mean value of the side lobes divided by the main lobe) for the two different analyses shows that the multi-band analysis permits one to more easily determine which detections are ambiguous and therefore not to be used to calculate the LFE family’s stack.
**Supplementary Figure 2.** Coherent energy distribution with varying focal mechanisms for low-frequency earthquake (LFE) family #3 chosen on March 20th, 2005 and stacked coherent energy distribution for all 15 LFE families. Multiple maxima in the two distributions illustrate the inherent ambiguity of a shallow-dipping thrust mechanism. b) Each of the drawn focal mechanism beach balls are linked to both their corresponding cell in the distribution and the cell that represents the same mechanism but with a strike and rake rotated 180°. The maxima of the stacked energy distribution coincide with the maxima of the representative LFE family normalized energy distribution in a, indicating that a large majority of the LFE families have similar shallow-thrusting mechanisms that reflect the general subduction regime of the Mexican subduction zone.
**Supplementary Figure 3.** Vertical and horizontal distributions of the source location misfit distribution of LFE family #1 chosen on March 20th, 2005. The value of the darkest blue is 5 times larger than the minimum misfit, represented by the darkest red. The black box indicates the NVT source region determined by Husker et al. (2012) by inverting for NVT energy profiles with respect to the MASE network; the light blue shaded box indicates the Mexican Sweet Spot. The 10 broadband MASE stations used in this study are indicated by the black inverted triangles. The geometry of the top of the subducting Cocos slab is shown by the black contours and their associated depths in a and by the solid black line in b (Kim et al., 2010). a) Horizontal profile of the misfit distribution at the depth of the most likely source location projected on to the surface. b) Vertical profile of the misfit distribution projected on to the black line shown in a.
**Supplementary Figure 4.** Vertical and horizontal distributions of the source location misfit distribution of LFE family #2 chosen on November 2\textsuperscript{nd}, 2005. The description of this figure is the same as Supplementary Figure 3.

**Supplementary Figure 5.** Vertical and horizontal distributions of the source location misfit distribution of LFE family #3 chosen on March 20\textsuperscript{th}, 2005. The description of this figure is the same as Supplementary Figure 3.
Supplementary Figure 6. Vertical and horizontal distributions of the source location misfit distribution of LFE family #4 chosen on March 20<sup>th</sup>, 2005. The description of this figure is the same as Supplementary Figure 3.
Supplementary Figure 7. Vertical and horizontal distributions of the source location misfit distribution of LFE family #5 chosen on March 21st, 2005. The description of this figure is the same as Supplementary Figure 3.
**Supplementary Figure 8.** Vertical and horizontal distributions of the source location misfit distribution of LFE family #6 chosen on March 21st, 2005. The description of this figure is the same as Supplementary Figure 3.
**Supplementary Figure 9.** Vertical and horizontal distributions of the source location misfit distribution of LFE family #7 chosen on October 31st, 2005. The description of this figure is the same as Supplementary Figure 3.
SUPPLEMENTARY FIGURE 10. Vertical and horizontal distributions of the source location misfit distribution of LFE family #8 chosen on August 1st, 2005. The description of this figure is the same as Supplementary Figure 3.
SUPPLEMENTARY Figure 11. Vertical and horizontal distributions of the source location misfit distribution of LFE family #9 chosen on August 1st, 2005. The description of this figure is the same as Supplementary Figure 3.
Supplementary Figure 12. Vertical and horizontal distributions of the source location misfit distribution of LFE family #10 chosen on April 15th, 2006. The description of this figure is the same as Supplementary Figure 3.
Supplementary Figure 13. Vertical and horizontal distributions of the source location misfit distribution of LFE family #11 chosen on March 15th, 2006. The description of this figure is the same as Supplementary Figure 3.
Supplementary Figure 14. Vertical and horizontal distributions of the source location misfit distribution of LFE family #12 chosen on May 15th, 2006. The description of this figure is the same as Supplementary Figure 3.
Supplementary Figure 15. Vertical and horizontal distributions of the source location misfit distribution of LFE family #13 chosen on April 23rd, 2006. The description of this figure is the same as Supplementary Figure 3.
Supplementary Figure 16. Vertical and horizontal distributions of the source location misfit distribution of LFE family #14 chosen on July 16th, 2006. The description of this figure is the same as Supplementary Figure 3.
**Supplementary Figure 17.** Vertical and horizontal distributions of the source location misfit distribution of LFE family #15 chosen on July 6th, 2006. The description of this figure is the same as Supplementary Figure 3.
Table 1. Number of multiplets detected as a function of the detection threshold.  

<table>
<thead>
<tr>
<th>Date</th>
<th>Number of robustly detected multiplets</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>5 x RMS(^b)</td>
</tr>
<tr>
<td>March 20th, 2005</td>
<td>57</td>
</tr>
<tr>
<td>March 22th, 2005</td>
<td>70</td>
</tr>
<tr>
<td>November 1st, 2005</td>
<td>48</td>
</tr>
<tr>
<td>November 2nd, 2005</td>
<td>30</td>
</tr>
<tr>
<td>July 14th, 2006</td>
<td>29</td>
</tr>
<tr>
<td>July 16th, 2006</td>
<td>51</td>
</tr>
</tbody>
</table>

\(^a\)The LFE template used in this analysis was chosen on March 20\(^{th}\), 2005. Example days were chosen based on their relatively high LFE activity. \(^b\)Root-mean squared. \(^c\)Median absolute deviation.

Table 2. Number of multiplets detected as a function of the secondary low-frequency earthquake (LFE) template length based on the LFE family’s initial stack.  

<table>
<thead>
<tr>
<th>LFE template length</th>
<th>Number of detected multiplets</th>
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<tbody>
<tr>
<td>3.4</td>
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</tr>
<tr>
<td>5.4</td>
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<tr>
<td>6.4</td>
<td>163</td>
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<td>23.4</td>
<td>505</td>
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<tr>
<td>33.4</td>
<td>534</td>
</tr>
</tbody>
</table>

\(^a\)The LFE template used in this analysis was chosen on March 20\(^{th}\), 2005.