Measurement of the $CP$-violating phase $\phi_s$ in the $B^0_s \rightarrow J/\psi \phi(1020) \rightarrow \mu^+\mu^-K^+K^-$ channel in proton-proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration

Abstract

The $CP$-violating weak phase $\phi_s$ and the decay width difference $\Delta\Gamma_s$ between the light and heavy $B^0_s$ mass eigenstates are measured with the CMS detector at the LHC in a sample of 48 500 reconstructed $B^0_s \rightarrow J/\psi \phi(1020) \rightarrow \mu^+\mu^-K^+K^-$ events. The measurement is based on a data sample corresponding to an integrated luminosity of 96.4 fb$^{-1}$, collected in proton-proton collisions at $\sqrt{s} = 13$ TeV in 2017–2018. To extract the values of $\phi_s$ and $\Delta\Gamma_s$, a time-dependent and flavor-tagged angular analysis of the $\mu^+\mu^-K^+K^-$ final state is performed. The analysis employs a dedicated tagging trigger and a novel opposite-side muon flavor tagger based on machine learning techniques. The measurement yields $\phi_s = -11 \pm 50$ (stat) $\pm 10$ (syst) mrad and $\Delta\Gamma_s = 0.114 \pm 0.014$ (stat) $\pm 0.007$ (syst) ps$^{-1}$, in agreement with the standard model predictions. When combined with the previous CMS measurement at $\sqrt{s} = 8$ TeV, the following values are obtained: $\phi_s = -21 \pm 44$ (stat) $\pm 10$ (syst) mrad, $\Delta\Gamma_s = 0.1032 \pm 0.0095$ (stat) $\pm 0.0048$ (syst) ps$^{-1}$, a significant improvement over the 8 TeV result.

1 Introduction

Precision tests of the standard model (SM) of particle physics have become increasingly important, since no direct evidence for new physics has been found so far at the CERN LHC. Decays of $B^0_s$ mesons present important opportunities to probe the consistency of the SM. In this Letter, a new measurement of the CP-violating weak phase $\phi_s$ and the decay width difference $\Delta \Gamma_s$ between the light ($B^0_s$) and heavy ($B^0_s^{*}$) $B^0_s$ meson mass eigenstates is presented. Charge-conjugate states are implied throughout, unless stated otherwise.

The weak phase $\phi_s$ arises from the interference between direct $B^0_s$ meson decays to a CP eigenstate of $c\bar{c}s\bar{s}$ and decays through mixing to the same final state. In the SM, $\phi_s$ is related to the elements of the Cabibbo–Kobayashi–Maskawa matrix via $\phi_s \simeq -2\beta_s = -2 \arg(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*)$, neglecting penguin diagram contributions, where $\beta_s$ is one of the angles of the unitary triangles. The current best determination of $-2\beta_s$ comes from a global fit to experimental data on b hadron and kaon decays. Assuming no physics beyond the SM (BSM) in the $B^0_s$ mixing and decays, a $-2\beta_s$ value of $-36.96^{+0.72}_{-0.84}$ mrad is determined [1]. New physics can modify this phase via the contribution of BSM particles to $B^0_s$ mixing [2,3]. Since the numerical value of $\phi_s$ in the SM is known very precisely, even a small deviation from this value would constitute evidence of BSM physics. The decay width difference between the $B^{0}_s$ and $B^{0}_s^{*}$ eigenstates, on the other hand, is predicted less precisely at $\Delta \Gamma_s = 0.091 \pm 0.013$ ps$^{-1}$ [4]. Its measurement provides an important test for theoretical predictions and can be used to further constrain new-physics effects [4].

The weak phase $\phi_s$ was first measured by the Fermilab Tevatron experiments [5–9], and then at the LHC by the ATLAS, CMS, and LHCb experiments [10–19], using $B^0_s \rightarrow J/\psi \phi (1020)$ (referred to as $B^0_s \rightarrow J/\psi \phi$ in what follows), $B^0_s \rightarrow J/\psi f_0(980)$, and $B^0_s \rightarrow J/\psi h^+h^-$ decays, where $h$ stands for a kaon or pion. Measurements of $\phi_s$ in $B^0_s$ decays to $\psi(2S)\phi(1020)$ and $D_s^+D_s^-$ were performed by the LHCb Collaboration [20,21].

In this Letter, CMS results on the $B^0_s \rightarrow J/\psi \phi$ decay to the $\mu^+\mu^-K^+K^-$ final state are presented, and possible additional contributions to this final state from the $B^0_s \rightarrow J/\psi f_0(980)$ and nonresonant $B^0_s \rightarrow J/\psi K^+K^-$ decays are taken into account by including a term for an additional $S$-wave amplitude in the decay model. Compared to our previous measurement [14] at $\sqrt{s} = 8$ TeV, we benefit from the increase in the center-of-mass energy from 8 to 13 TeV that nearly doubles the $B^0_s$ production cross section and a novel opposite-side (OS) muon flavor tagger. The new tagger employs machine learning techniques and achieves better discrimination power than previous methods. We also make use of a specialized trigger that requires an additional (third) muon, which can be used for flavor tagging, improving the tagging efficiency at the cost of a reduced number of signal events. As a result, the new measurement, while based on a similar number of $B^0_s$ candidates as the earlier one [14], allows us to double the precision in the determination of $\phi_s$, as well as measure some of the parameters that were constrained to their world-average values in our previous work [14]. At the same time, the precision on parameters that do not benefit from the tagging information, such as $\Delta \Gamma_s$, is comparable to that in the previous measurement.

Final states that are mixtures of CP eigenstates require an angular analysis to separate the CP-odd and CP-even components. A time-dependent angular analysis can be performed by measuring the decay angles of the final-state particles and the proper decay length of the reconstructed $B^0_s$ candidate, which is equal to the proper decay time $t$ multiplied by the speed of light, and referred to as $ct$ in what follows.

In this measurement, we use the transversity basis [22] defined by the three decay angles
\[ \Theta = (\theta_T, \psi_T, \varphi_T) \text{, as illustrated in Fig. 1.} \]

The angles \( \theta_T \) and \( \varphi_T \) are, respectively, the polar and azimuthal angles of the \( \mu^+ \) in the rest frame of the \( J/\psi \) meson, where the \( x \) axis is defined by the direction of the \( \phi \) meson momentum and the \( x-y \) plane is defined by the plane of the \( \phi \to K^+K^- \) decay. The helicity angle \( \psi_T \) is the angle of the \( K^+ \) meson momentum in the \( \phi \) meson rest frame with respect to the negative \( J/\psi \) meson momentum direction.

![Diagram](image)

**Figure 1:** Definition of the three angles \( \theta_T, \psi_T, \) and \( \varphi_T \) describing the topology of the \( B_s^0 \to J/\psi \phi \to \mu^+\mu^- K^+K^- \) decay.

The differential decay rate of \( B_s^0 \to J/\psi \phi \to \mu^+\mu^- K^+K^- \) is described by a function \( \mathcal{F}(\Theta,ct,\alpha) \), as in Ref. [23]:

\[
\frac{d^4\Gamma(B_s^0)}{d\Theta d(ct)} = \mathcal{F}(\Theta,ct,\alpha) \propto \sum_{i=1}^{10} O_i(ct,\alpha) g_i(\Theta),
\]

where \( O_i \) are time-dependent functions, \( g_i \) are angular functions, and \( \alpha \) is a set of physics parameters.

The functions \( O_i(ct,\alpha) \) are:

\[
O_i(ct,\alpha) = N_i e^{-\Gamma_{i} t} \left[ a_i \cosh \left( \frac{\Delta \Gamma_{i} t}{2} \right) + b_i \sinh \left( \frac{\Delta \Gamma_{i} t}{2} \right) + c_i \cos(\Delta m_{i} t) + d_i \sin(\Delta m_{i} t) \right],
\]

where \( \Delta m_{i} (\Delta \Gamma_{i}) \) is the absolute mass (decay width) difference between the \( B_{s}^{i} \) and \( B_{s}^{n} \) mass eigenstates, and \( \Gamma_{i} \) is the average decay width, defined as the arithmetic average of the \( B_{s}^{i} \) and \( B_{s}^{n} \) decay widths. The functions \( g_i(\Theta) \) and the parameters \( N_i, a_i, b_i, c_i, \) and \( d_i \) are defined in Table 1.

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The coefficients \( C, S, \) and \( D \) contain the information about \( CP \) violation, and are defined as:

\[
C = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}, \quad S = \frac{-2|\lambda| \sin \phi_s}{1 + |\lambda|^2}, \quad D = \frac{-2|\lambda| \cos \phi_s}{1 + |\lambda|^2},
\]

using the same sign convention as that in the LHCb measurement \[16\]. The amount of \( CP \) violation in the \( B_0^s \to B_0^s \) system is given by the complex parameter \( \lambda \), defined as \( \lambda = (q/p)(A_f/A_f) \), where \( A_f(A_f) \) is the decay amplitude of the \( B_0^s(\bar{B}_0^s) \) meson to the final state \( f \), and the parameters \( p \) and \( q \) relate the mass and flavor eigenstates through \( B_0^{s*} = p|B_0^0\rangle - q|\bar{B}_0^0\rangle \) and \( B_0^s = p|B_0^0\rangle + q|\bar{B}_0^0\rangle \) \[24\]. The parameters \( |A_\perp|^2, |A_0|^2, \) and \( |A_\parallel|^2 \) are the magnitudes of the perpendicular, longitudinal, and parallel transversity amplitudes of the \( B_0^s \to J/\psi \phi \) decay, respectively; \( |A_\bot|^2 \) is the magnitude of the S-wave amplitude from \( B_0^0 \to J/\psi f_0(980) \) and non-resonant \( B_0^0 \to J/\psi K^+K^- \) decays, and the parameters \( \delta_\perp, \delta_0, \delta_\parallel, \) and \( \delta_S \) are the respective strong phases.

Equation (1) represents the model for the \( B_0^0 \) meson decay, while the model for the \( \bar{B}_0^s \) meson decay is obtained by changing the sign of the \( c_i \) and \( d_i \) terms in Eq. (2).

## 2 The CMS detector

The central feature of the CMS apparatus is a superconducting solenoid of 6 m internal diameter, providing a magnetic field of 3.8 T. Within the solenoid volume are a silicon pixel and strip tracker, a lead tungstate crystal electromagnetic calorimeter, and a brass and scintillator hadron calorimeter, each composed of a barrel and two endcap sections. Forward calorimeters extend the pseudorapidity (\( \eta \)) coverage provided by the barrel and endcap detectors. Muons are detected in gas-ionization chambers embedded in the steel flux-return yoke outside the solenoid.

The silicon tracker measures charged particles within the range \( |\eta| < 2.5 \). During the LHC running period when the data used in this Letter were recorded, the silicon tracker consisted of 1856 silicon pixel and 15 148 silicon strip detector modules.

Muons are measured in the range \( |\eta| < 2.4 \), with detection planes made using three technologies: drift tubes, cathode strip chambers, and resistive plate chambers. The efficiency to reconstruct and identify muons is greater than 96%. Matching muons to tracks measured in the silicon tracker results in a relative transverse momentum (\( p_T \)) resolution, for muons with \( p_T \) up to 100 GeV, of 1% in the barrel and 3% in the endcaps \[25\].

Events of interest are selected using a two-tiered trigger system \[26\]. The first level (L1), composed of custom hardware processors, uses information from the calorimeters and muon detectors to select events at a rate of around 100 kHz within a fixed time interval of less than 4 \( \mu \)s. The second level, known as the high-level trigger (HLT), consists of a farm of processors running a version of the full event reconstruction software optimized for fast processing, and reduces the event rate to around 1 kHz before data storage.

A more detailed description of the CMS detector, together with a definition of the coordinate system used and the relevant kinematic variables, can be found in Ref. \[27\].

## 3 Event selection and simulated samples

The analysis is performed using data collected in proton-proton (\( pp \)) collisions at \( \sqrt{s} = 13 \) TeV during 2017–2018, corresponding to an integrated luminosity of 96.4 fb\(^{-1}\). A trigger optimized
for the detection of b hadrons decaying to $J/\psi$ mesons, along with an additional muon potentially usable for flavor tagging, is used to collect the data sample for the analysis. At L1, the trigger requires three muons, with the minimum $p_T$ requirement on the highest $p_T$ (leading, $\mu_1$) and second-highest $p_T$ (subleading, $\mu_2$) muons of $p_T > 5$ and 3 GeV, respectively, and the dimuon invariant mass $m_{\mu_1\mu_2} < 9$ GeV. There is no $p_T$ requirement on the third muon at L1. At the HLT, the three muons are required to be within the CMS geometrical acceptance $|\eta| < 2.5$; two of these muons must be oppositely charged, each have $p_T > 3.5$ GeV, form a $J/\psi$ candidate with an invariant mass in the range $2.95$–$3.25$ GeV, and have a probability to originate from a common vertex larger than 0.5%. The third muon is required to have $p_T > 2$ GeV and can be used to infer the flavor of the $B_s^0$ meson at production (i.e., its particle/antiparticle state), exploiting semileptonic $b \to \mu^- + X$ decays, as discussed further in Section [4].

Additional selection criteria are applied to events passing the HLT requirements. The numerical values of the selection cuts have been optimized with the help of the TMVA package [28,29], using a genetic algorithm, to maximize the signal purity. First, $J/\psi$ meson candidates are constructed using pairs of opposite-sign muons with $p_T > 3.5$ GeV and $|\eta| < 2.4$, and compatible with originating from a common vertex, obtained from a Kalman fit [30]. Candidates are accepted only if their invariant mass is within 150 MeV of the world-average $J/\psi$ meson mass [31]. Next, pairs of opposite-sign tracks satisfying the high-purity requirement [32] with $p_T > 1.2$ GeV and $|\eta| < 2.5$, not associated with the muons that form the $J/\psi$ candidate, are used to form $\phi$ candidates. The $\phi$ candidates are selected if the track pair has an invariant mass, assuming the kaon mass for both particles, within 10 MeV of the world-average $\phi$ meson mass [31]. Finally, the $J/\psi$ and $\phi$ candidates are combined to form $B_s^0$ candidates: a common vertex ("$B_s^0$ vertex") is obtained from a fit with the four tracks, two for muons and two for kaons. The invariant mass of the $B_s^0$ candidate is obtained from a kinematic fit, where the invariant mass of the two muons is constrained to the world-average $J/\psi$ meson mass [31]. The mass of the $\phi$ candidate is not constrained since its natural width exceeds the mass resolution.

Due to the high instantaneous luminosity of proton-proton collisions at the LHC, several primary vertices (PVs) are reconstructed in each event. The vertex that minimizes the angle between the $B_s^0$ candidate momentum vector and the line connecting this vertex with the $B_s^0$ decay vertex is chosen as the production vertex and is used to determine the characteristics of the $B_s^0$ candidate, such as proper decay length. We used simulations to study if the PV selection procedure introduces any bias in the measurement. It was found that in about 97% of the events, the selected PV is also the closest one to the point of origin of the $B_s^0$ meson. The impact of choosing a different vertex in the remaining cases on the final results is found to be negligible with respect to the total systematic uncertainties discussed in Section [6]. The proper decay length is measured as $ct = cm^{PDG}_{B_s^0} L_{xy}/p_T$, where $m^{PDG}_{B_s^0}$ is the world-average $B_s^0$ mass [31] and $L_{xy}$ is the reconstructed transverse decay length, which is defined as the distance in the transverse plane from the production vertex to the $B_s^0$ vertex. Additional selection criteria are applied to $B_s^0$ candidates, requiring $p_T > 11$ GeV, the four-track vertex fit $\chi^2$ probability $> 2\%$, an invariant mass in the 5.24–5.49 GeV range, and a proper decay length $ct > 70 \mu$m, with an uncertainty $\sigma_{ct} < 50 \mu$m. The proper decay length uncertainty is obtained by propagating the uncertainties in the decay distance and the $p_T$ of the $B_s^0$ candidate to $ct$. In about 2% of the events more than one $B_s^0$ candidate is selected. In these cases, the candidate with the highest vertex fit probability is chosen. The impact of this choice on the measurement has been evaluated by redoing the analysis using the candidate with the lowest vertex fit probability. No sizable bias has been observed with respect to the total systematic uncertainties discussed in Section [6]. A total of 65 500 $B_s^0 \to J/\psi \phi$ candidates are selected.
Simulated event samples are used to measure the selection efficiency and the flavor tagging performance. These samples are produced using the Pythia 8.230 Monte Carlo (MC) event generator [33] with the underlying event tune CP5 [34] and the parton distribution function set NNPDF3.1 [35]. The b hadron decays are modeled with the EVTGEN 1.6.0 package [36]. Final-state photon radiation is accounted for in the EVTGEN simulation with PHOTOS 215.5 [37, 38]. The response of the CMS detector is simulated using the GEANT4 package [39]. The effect of multiple collisions in the same or neighboring bunch crossings (pileup) is accounted for by overlaying simulated minimum bias events on the hard-scattering process. Simulated samples are then reconstructed using the same software as for collision data.

The simulation is validated via comparison with background-subtracted data in a number of control distributions. The B^0_s candidate invariant mass distribution after the signal selection is shown in Fig. 2, whereas the proper decay length and its uncertainty distributions are shown in Fig. 3.

![Figure 2: The invariant mass distribution of the B^0_s \rightarrow J/\psi \phi \rightarrow \mu^+ \mu^- K^+ K^- candidates in data. The vertical bars on the points represent the statistical uncertainties. The solid line represents a projection of the fit to data (as discussed in Section 5, solid markers), the dashed line corresponds to the signal, the dotted line to the combinatorial background, and the long-dashed line to the peaking background from B^0 \rightarrow J/\psi K^*(892)^0 \rightarrow \mu^+ \mu^- K^- \pi^-, as obtained from the fit. The distribution of the differences between the data and the fit, divided by the combined uncertainty in the data and the best fit function for each bin (pulls) is displayed in the lower panel.](image-url)

4 Flavor tagging

The flavor of the B^0_s candidate at production is determined with an OS flavor tagging algorithm. The OS approach is based on the fact that b quarks are predominantly produced in b\overline{b} pairs, and therefore one can infer the initial B^0_s meson flavor by determining the flavor of the other (“OS”) b quark in the event.

In this analysis, the flavor of the OS b hadron is deduced by exploiting the semileptonic b \rightarrow \mu^- + X decay, where the muon sign \( \xi \) is used as the tagging variable (\( \xi = -1 \) for B^0_s). This technique works on a probabilistic basis. If no OS muon is found, the event is considered as untagged (\( \xi = 0 \)). The tagging efficiency \( \epsilon_{tag} \) is defined as the fraction of candidate events that
Figure 3: The ct distribution (left) and its uncertainty (right) for the $B_0^s \to J/\psi \phi \to \mu^+ \mu^- K^+ K^-$ candidates in data. The notations are as in Fig. 2.

are tagged. When a muon is found, the tag is defined to be correct (“right tag”) if the flavor predicted using the muon sign and the actual $B_0^s$ meson flavor at production coincide. The correlation between the muon sign and the signal $B_0^s$ meson flavor is diluted by wrong tags (mistags) originating from cascade $b \to c \to \mu^+ + X$ decays, oscillation of the OS $B^0$ or $B_0^s$ meson, and muons originating from other sources, such as $J/\psi$ meson and charged pion and kaon decays. The mistag fraction $\omega_{\text{tag}}$ is defined as the ratio between the number of wrongly tagged events and the total number of tagged events. It is used to compute the dilution $D = 1 - 2\omega_{\text{tag}}$, which is a measure of the performance degradation due to mistagged events. The tagging power $P_{\text{tag}} \equiv \epsilon_{\text{tag}} D^2$ is the effective tagging efficiency, which takes into account the dilution and is used as a figure of merit in maximizing the algorithm performance.

To maximize the sensitivity of this measurement, we have developed a novel OS muon tagger taking advantage of machine learning techniques. The use of deep neural networks (DNNs) in the new tagger leads to lowering of the mistag probability $\omega_{\text{tag}}$ and reducing of the related systematic uncertainties. The use of a dedicated trigger, which requires an OS muon, dramatically increases the fraction of tagged candidates compared to our earlier measurement [14]. Taken together, these two improvements increase the muon tagging performance by $\approx 20\%$ compared to that in Ref. [14].

For each event, we search for a candidate OS muon consistent with originating from the same production vertex as the signal $B_0^s$ meson. This tagging muon is required to have $p_T > 2$ GeV, $|\eta| < 2.4$, the longitudinal impact parameter with respect to the production vertex $IP_z < 1.0$ cm, and the distance from the $B_0^s$ candidate momenta in the $(\eta, \phi)$ plane $\Delta R_{\mu, \phi} > 0.4$. Tracks that belong to the reconstructed $B_0^s \to J/\psi \phi \to \mu^+ \mu^- K^+ K^-$ decay are explicitly excluded from consideration. In order to reduce the contamination from light-flavor hadrons misreconstructed as tagging muons, a discriminator based on a DNN was developed using the KERAS library [40] within the TMVA toolkit. This discriminator, called the “DNN against light hadrons” in the following, uses 25 input features related to the muon kinematics and reconstruction quality, and is trained with $3.5 \times 10^6$ simulated muon candidates of which $2.5 \times 10^5$ are misreconstructed hadrons. The following DNN hyperparameters are optimized through a grid scan to maximize the discrimination power: number of layers, number of neurons for each layer, and the dropout probability. No signs of overtraining are observed at the chosen hyperparameters configura-
tion when comparing the output distributions from the testing and training samples. Tagging muons are required to pass a working point of the DNN output that has an efficiency of $\approx 98\%$ for genuine muons and $\approx 33\%$ for misreconstructed light-flavor hadrons, when evaluated using muon candidates reconstructed with the CMS particle-flow (PF) algorithm [41]. In $\approx 3\%$ of the events where more than one tagging muon candidate passes all the above selections, only the highest $p_T$ one is kept.

Another DNN is used to further discriminate the right- and wrong-tag muons, as well as to predict the mistag probability on a per-event basis. This DNN, referred to as the muon tagger DNN, has been developed using the KERAS library within the TMVA toolkit, based on simulated $B^0 \rightarrow J/\psi \phi \rightarrow \mu^+\mu^- K^+K^-$ events, and calibrated with self-tagging $B^\pm \rightarrow J/\psi K^\pm$ MC and data samples, as described below.

The input features of the muon tagger DNN are of two kinds: muon variables and cone variables. The muon variables are the muon $p_T$, $\eta$, transverse and longitudinal impact parameters with respect to the production vertex, along with their uncertainties, the distance $\Delta R_{\eta,\phi}$ to the signal $B^0$ candidate, and the discriminant of the DNN against light hadrons. The cone variables are related to the activity in a cone of radius $\Delta R_{\eta,\phi} = 0.4$ around the muon momentum direction and include the relative PF isolation [41], the scalar $p_T$ sum of all additional tracks within the cone, the sum of their charges weighted by the track $p_T$, the muon relative momentum and $\Delta R_{\eta,\phi}$ with respect to the vector sum of the momenta of all additional tracks within the cone, and the ratio of the energy of the muon to the total energy of all additional tracks within the cone (assuming the pion mass for each track). The muon tagger DNN is trained on $2.8 \times 10^5$ simulated $B^0 \rightarrow J/\psi \phi$ events, of which about 85 000 have a wrong tag. Its structure is optimized similarly to that for the DNN against light hadrons. The optimal DNN has three dense layers of 200 neurons, each with a rectified linear unit activation function. A dropout layer with a dropout probability of $40\%$ is placed after each dense layer. The cross-entropy loss function and the Adam optimizer [42] are used. The DNN is constructed in such a way that its output score $d$ is equal to the probability of tagging the event correctly. Therefore, the per-event mistag probability is simply $\omega_{\text{evt}} = 1 - d$.

The output $d$ of the tagger is calibrated using a self-tagging data sample of $B^\pm \rightarrow J/\psi K^\pm \rightarrow \mu^+\mu^- K^\pm$ events, where the charge of the kaon corresponds to the charge and flavor of the $B^\pm$ meson. The same trigger and $J/\psi$ candidate reconstruction requirements as for the $B^0$ signal sample are applied. A charged particle with $p_T > 1.6$ GeV, assumed to be a kaon, is combined in a kinematic fit with the dimuon pair to form the $B^\pm$ candidate. The calibration is performed separately for the 2017 and 2018 data samples, by comparing the measured mistag fraction ($\omega_{\text{meas}}$) with the $\omega_{\text{evt}}$ predicted by the muon tagger DNN. The $B^\pm$ events are divided into 100 bins in $\omega_{\text{evt}}$ and the right- and wrong-tag events are separately counted in each bin to extract the corresponding $\omega_{\text{meas}}$ value. The $B^\pm$ signal in each bin is discriminated from the background via a binned likelihood fit to the $J/\psi K^\pm$ invariant mass distribution in the 5.10–5.65 GeV range.

The calibration results for the 2017 and 2018 $B^\pm$ data are shown in Fig. 4. The data points are fitted with a linear function $a + b\omega_{\text{evt}}$. The calibration parameters returned by the fit for the 2017 (2018) data samples are $a = -0.0010 \pm 0.0040$, $b = 1.012 \pm 0.013$ ($a = 0.0031 \pm 0.0031$, $b = 1.011 \pm 0.010$), statistically compatible with a unit slope and zero offset.

The calibration of the DNN output is also verified with a procedure similar to that described above using an independent sample of simulated $B^0_s \rightarrow J/\psi \phi$ and $B^\pm \rightarrow J/\psi K^\pm$ events. The reconstructed $B^0_s$ and $B^\pm$ mesons are matched to the generated ones in order to find their true flavor at production. In general, the measured mistag probability is predicted very accurately by $\omega_{\text{evt}}$ over the entire measured range for all the examined samples and processes, with more
Figure 4: Results of the calibration of the per-event mistag probability $\omega_{\text{evt}}$ based on $B^\pm \rightarrow J/\psi K^\pm \rightarrow \mu^+\mu^- K^\pm$ decays from the 2017 (left) and 2018 (right) data samples. The vertical bars represent the statistical uncertainties. The solid line shows a linear fit to data (solid markers). The pull distributions between the data and the fit function in each bin are shown in the lower panels.

than 90% of the tagged events falling in the $\omega_{\text{evt}} = 0.1–0.5$ range in all cases. Residual differences are well approximated by linear functions with slopes close to unity and offsets consistent with zero. The $\chi^2$ per degree of freedom values for all fits are below 2. We conclude that the value of $\omega_{\text{evt}}$ returned by the tagging DNN is a good approximation of the true mistag probability in data, with minor residual differences taken into account with calibration functions.

The calibrated flavor tagger performance, evaluated using $B^\pm \rightarrow J/\psi K^\pm$ events in data, is shown in Table 2. A tagging efficiency of $\approx 50\%$ and a tagging power of $\approx 10\%$ are achieved in both the 2017 and 2018 data samples. The efficiency is much higher than the semileptonic $b$ hadron branching fraction due to the requirement of an additional OS muon at the HLT, as described in Section 3.

Possible differences in the mistag probability calibration between the $B^0_s$ and $B^\pm$ samples, as well as the statistical uncertainties in the calibration parameters and possible variations from linearity of the calibration function, are considered as systematic uncertainties and described in Section 6.

Table 2: Calibrated opposite-side muon tagger performance evaluated using $B^\pm \rightarrow J/\psi K^\pm$ events in the 2017 and 2018 data samples. The uncertainties shown are statistical only.

<table>
<thead>
<tr>
<th>Data sample</th>
<th>$\varepsilon_{\text{tag}}$ (%)</th>
<th>$\omega_{\text{tag}}$ (%)</th>
<th>$P_{\text{tag}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2017</td>
<td>$45.7 \pm 0.1$</td>
<td>$27.1 \pm 0.1$</td>
<td>$9.6 \pm 0.1$</td>
</tr>
<tr>
<td>2018</td>
<td>$50.9 \pm 0.1$</td>
<td>$27.3 \pm 0.1$</td>
<td>$10.5 \pm 0.1$</td>
</tr>
</tbody>
</table>

5 Maximum-likelihood fit

An unbinned multidimensional extended maximum-likelihood fit is performed on the combined data samples using 8 observables as input: the $B^0_s$ candidate invariant mass $m_{B^0_s}$, the three decay angles $\Theta$ of the reconstructed $B^0_s$ candidate, the flavor tag decision $\xi$, the mistag fraction $\omega_{\text{evt}}$, the proper decay length of the $B^0_s$ candidate $ct$, and its uncertainty $\sigma_{ct}$.
From the multidimensional fit, the physics parameters of interest \( \phi, \Delta \Gamma_s, \Gamma_s, \Delta m_s, |\lambda| \), the squares of amplitudes \( |A_\perp|^2, |A_0|^2, |A_s|^2 \), and the strong phases \( \delta_0, \delta_\perp, \), and \( \delta_{s,1} \) are determined, where \( \delta_{s,1} \) is defined as the difference \( \delta_s - \delta_\perp \). The \( B_0^0 \rightarrow J/\psi \phi \) amplitudes are normalized to unity by constraining \( |A_\perp|^2 \) to \( 1 - |A_\perp|^2 - |A_0|^2 \). The fit model is validated with simulated pseudo-experiments and with simulated samples with different input parameter sets.

The likelihood function is composed of the probability density functions (pdfs) describing the signal and background components. The likelihood fit algorithm is implemented using the ROOFIT package \cite{roofit,roofitRef}. The signal and background pdfs are formed as the product of functions that model the invariant mass distribution and the time-dependent decay rates of the reconstructed candidates. In addition, the signal pdf includes the efficiency functions. The event pdf \( P \) is defined as:

\[
P = \frac{N_{\text{sig}}}{N_{\text{tot}}} P_{\text{sig}} + \frac{N_{\text{bkg}}}{N_{\text{tot}}} P_{\text{bkg}},
\]

where

\[
P_{\text{sig}} = \varepsilon(\mu) \varepsilon(\Theta) \left[ \tilde{F}(\Theta, \mu, \alpha) \otimes G(\mu, \sigma_\mu) \right] P_{\text{sig}}(m_{B_0^0}) P_{\text{sig}}(\sigma_{ct}) P_{\text{sig}}(\xi)
\]

and

\[
P_{\text{bkg}} = P_{\text{bkg}}(\cos \theta_\perp, \phi_\perp) P_{\text{bkg}}(\cos \psi) P_{\text{bkg}}(\mu) P_{\text{bkg}}(\sigma_{ct}) P_{\text{bkg}}(\xi).
\]

The corresponding negative log likelihood is:

\[
- \ln \mathcal{L} = - \sum_{i=0}^{N_{\text{evt}}} \ln P_i + N_{\text{tot}} - N_{\text{evt}} \ln N_{\text{tot}}.
\]

Here, \( P_{\text{sig}} \) and \( P_{\text{bkg}} \) are the pdfs that describe the \( B_0^0 \rightarrow J/\psi \phi \rightarrow \mu^+\mu^- K^+K^- \) signal and background contributions, respectively. The yields of signal and background events are \( N_{\text{sig}} \) and \( N_{\text{bkg}} \), respectively, \( N_{\text{tot}} \) is their sum, and \( N_{\text{evt}} \) = 65,500 is the number of candidates selected in data. The pdf \( \tilde{F}(\Theta, \mu, \alpha) \) is the differential decay rate function \( F(\Theta, \mu, \alpha) \) defined in Eq. (1), modified to include the flavor information \( \xi \) and the dilution term \( (1 - 2\omega_{\text{evt}}) \), which are applied as multiplicative factors to each of the \( c_i \) and \( d_i \) terms in Eq. (2). In the \( F \) expression, the value of \( \delta_0 \) is set to zero, following a general convention \cite{6,8}, and the value of \( \Delta \Gamma_s \) is constrained to be positive, based on the LHCb measurement \cite{44}. All the parameters of the pdfs are allowed to float in the final fit, unless explicitly stated otherwise.

The functions \( \varepsilon(\mu) \) and \( \varepsilon(\Theta) \) model the dependence of the signal reconstruction efficiency on the proper decay length and the three angles of the transversity basis, respectively. The proper decay length efficiency is parameterized with a fourth-order Chebyshev polynomial multiplied by an exponential function with a negative slope, while the angular efficiency is parameterized with spherical harmonics and Legendre polynomials up to order six. Both parameterizations are obtained from fits to the respective efficiency histograms in \( B_0^0 \rightarrow J/\psi \phi \) simulated events, and are fixed in the fit to data.

The term \( G(\mu, \sigma_{ct}) \) is a Gaussian resolution function, which makes use of the per-event decay length uncertainty \( \sigma_{ct} \), scaled by a correction factor \( \kappa \) introduced to account for the residual effects when the decay length uncertainty is used to model the \( \mu \) resolution. The value of \( \kappa \) is estimated using simulated samples and is equal to \( \approx 1.2 \) for both the 2017 and 2018 data samples.

The signal mass pdf \( P_{\text{sig}}(m_{B_0^0}) \) is a Johnson’s \( S_U \) distribution \cite{35}, while the decay length uncertainty pdf \( P_{\text{sig}}(\sigma_{ct}) \) is described by the sum of two Gamma distributions. These pdfs best model each individual variable in one-dimensional fits to simulated samples.
The background pdf contains two terms to model both the combinatorial background and the peaking background, dominated by $B^0 \rightarrow J/\psi K^*(892)^0 \rightarrow \mu^+ \mu^- K^+ \pi^-$, where the pion is assumed to be a kaon candidate. The background from $\Lambda_b^0 \rightarrow J/\psi p K^- \rightarrow \mu^+ \mu^- p K^-$, where the proton is assumed to be a kaon candidate, is estimated using simulated events to have a negligible effect on the fit results compared to the systematic uncertainties discussed in Section 6. The background invariant mass pdf $P_{\text{bkg}}(m_{B^0})$ is described by an exponential function for the combinatorial background and a Johnson’s SU distribution for the peaking background. The background decay length pdf $P_{\text{bkg}}(c_t)$ is described by the sum of two exponential distributions for the combinatorial background, while a single exponential distribution is used for the peaking background. The angular parts of the background pdfs $P_{\text{bkg}}(\cos \theta_T, \phi_T)$ are described analytically by a series of Legendre polynomials for $\cos \theta_T$ and $\cos \psi_T$, and sinusoidal functions for $\phi_T$. For the $\cos \theta_T$ and $\phi_T$ variables, a two-dimensional pdf is used to take into account a possible correlation between the two. The background decay length uncertainty pdf $P_{\text{bkg}}(\sigma_{c_t})$ is described by a sum of two Gamma distributions for the combinatorial background, while the peaking background is fixed to that for the signal.

The tag pdfs are defined as $P(\xi) = 1 - \varepsilon_{\text{tag}}$ for the untagged events ($\xi = 0$) and $P(\xi) = \varepsilon_{\text{tag}}(1 \pm A_{\text{tag}})/2$ for the tagged ones ($\xi = \pm 1$), where $\varepsilon_{\text{tag}}$ is the tagging efficiency and $A_{\text{tag}}$ is the tagging asymmetry, defined as the difference between the numbers of positively and negatively tagged events ($\xi = \pm 1$) divided by the total number. The measured tagging asymmetry is found to be compatible with zero.

The correlation between the different fit components has been studied in both data and simulations, and found to be negligible.

The peaking background part of $P_{\text{bkg}}$ is determined using simulated samples, while the initial combinatorial background part is found from a fit to the $B^0_s$ invariant mass sidebands 5.24–5.28 GeV and 5.45–5.49 GeV in data, and then left free to float in the final fit, starting from this initial pdf. The signal and background components of the decay length uncertainty pdf are fixed to the ones obtained from a two-dimensional fit together with the invariant mass pdf. The 2017 and 2018 data samples are fitted simultaneously. The joint likelihood function of the simultaneous fit shares the decay rate model, the invariant mass pdfs, the peaking background model, and the lifetime and angular components of the combinatorial background model between the two samples. The number of signal and background events are measured separately in each data sample, as is the tagging efficiency. The efficiency functions, $P(\sigma_{c_t})$ pdfs, tag pdfs, and $\kappa$ factors are also specific to each data sample.

6 Systematic uncertainties and results

The results of the fit with their statistical and systematic uncertainties are given in Table 3, whereas the statistical correlations between the measured parameters are reported in Appendix A. Statistical uncertainties are obtained from the increase in $-\log \mathcal{L}$ by 0.5, whereas systematic uncertainties are described below and summarized in Table 4. The measured number of $B^0_s \rightarrow J/\psi \phi \rightarrow \mu^+ \mu^- K^+ K^-$ signal events from the fit is $48,500 \pm 250$. The distributions of the input observables and the corresponding fit projections are shown in Figs. 2, 3, and 5.

Several sources of systematic uncertainties in the physics parameters are studied by testing the various assumptions made in the fit model and those associated with the fitting procedure.

Model bias: Possible biases in the fitting procedure are evaluated by generating 1000 pseudo-experiments, each statistically equivalent to the data samples, from the fitted model in data...
Table 3: Results of the fit to data. Statistical uncertainties are obtained from the increase in $-\log \mathcal{L}$ by 0.5, whereas systematic uncertainties are described below and summarized in Table 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit value</th>
<th>Stat. uncer.</th>
<th>Syst. uncer.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_s$ [mrad]</td>
<td>$-11$</td>
<td>$\pm 50$</td>
<td>$\pm 10$</td>
</tr>
<tr>
<td>$\Delta \Gamma_s$ [ps$^{-1}$]</td>
<td>$0.114$</td>
<td>$\pm 0.014$</td>
<td>$\pm 0.007$</td>
</tr>
<tr>
<td>$\Delta m_s$ [hs$^{-1}$]</td>
<td>$17.51$</td>
<td>$\pm 0.10$</td>
<td>$\pm 0.03$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$0.972$</td>
<td>$\pm 0.026$</td>
<td>$\pm 0.008$</td>
</tr>
<tr>
<td>$\Gamma_s$ [ps$^{-1}$]</td>
<td>$0.6531$</td>
<td>$\pm 0.0042$</td>
<td>$\pm 0.0026$</td>
</tr>
<tr>
<td>$</td>
<td>A_0</td>
<td>^2$</td>
<td>$0.5350$</td>
</tr>
<tr>
<td>$</td>
<td>A_{\perp}</td>
<td>^2$</td>
<td>$0.2337$</td>
</tr>
<tr>
<td>$</td>
<td>A_S</td>
<td>^2$</td>
<td>$0.022$</td>
</tr>
<tr>
<td>$\delta_{\parallel}$ [rad]</td>
<td>$3.18$</td>
<td>$\pm 0.12$</td>
<td>$\pm 0.03$</td>
</tr>
<tr>
<td>$\delta_{\perp}$ [rad]</td>
<td>$2.77$</td>
<td>$\pm 0.16$</td>
<td>$\pm 0.05$</td>
</tr>
<tr>
<td>$\delta_{S_{\perp}}$ [rad]</td>
<td>$0.221$</td>
<td>$\pm 0.083$</td>
<td>$\pm 0.048$</td>
</tr>
</tbody>
</table>

Figure 5: The angular distributions $\cos \theta_T$ (left), $\cos \psi_T$ (middle), and $\phi_T$ (right) for the $B_s$ candidates and the projections from the fit. The notations are as in Fig. 4.

(referred to as “nominal-model pseudo-experiments” in what follows). Each of them is fitted with the nominal model, and the pull distributions (i.e., the difference divided by the combined uncertainty) between the parameters obtained from the fit and their input values are produced. Each pull distribution is fitted with a Gaussian function, and the estimated central value is taken as the corresponding systematic uncertainty, if different from zero by more than its error. To avoid double-counting this uncertainty, whenever pseudo-experiments are used to evaluate other systematic uncertainties, the model bias is always subtracted. In these cases, the corresponding pull distributions are compared to those obtained with the nominal-model pseudo-experiments. If the mean of the pull distribution differs from the mean of the nominal-model distribution by more than their combined RMS, the difference is taken as the corresponding systematic uncertainty.

Model assumptions: The assumptions made in defining the likelihood functions are tested by generating pseudo-experiments with different hypotheses and fitting the samples with the nominal model. The following assumptions are tested: signal and background invariant mass models, background proper decay length model, and background angular model. Pull distributions with respect to the input values are used to evaluate the systematic uncertainty, as described in the “model bias” paragraph.

Angular efficiency: The systematic uncertainty related to the limited MC event count used to estimate the angular efficiency function is evaluated by regenerating the efficiency histograms
Table 4: Summary of the systematic uncertainties. The dashes (—) mean that the corresponding uncertainty is not applicable. The total systematic uncertainty is obtained as the quadratic sum of the individual contributions.

| Source                        | \(\phi_t\) | \(\Delta t_s\) | \(\Delta m_s\) | \(|A|\)  | \(\Gamma_s\) | \(|A_0|\) \(^2\) | \(|A_{\parallel}|\) \(^2\) | \(|A_{\perp}|\) \(^2\) | \(\delta_0\) | \(\delta_1\) | \(\delta_2\) |
|-------------------------------|------------|----------------|----------------|-------|-----------|----------------|----------------|----------------|-----------|-----------|-----------|
| Statistical uncertainty       | 50         | 0.014          | 0.10           | 0.026 | 0.0042    | 0.0047         | 0.0063         | 0.0077         | 0.12      | 0.16      | 0.083     |
| Model bias                    | 7.9        | 0.0019         | —              | 0.0035 | 0.0005    | 0.0002         | 0.0012         | 0.0011         | 0.020     | 0.016     | 0.006     |
| Model assumptions             | —          | —              | —              | 0.0046 | 0.0003    | —              | 0.0013         | 0.0011         | 0.017     | 0.019     | 0.011     |
| Angular efficiency            | 3.8        | 0.0006         | 0.007          | 0.0057 | 0.0002    | 0.0008         | 0.0010         | 0.0012         | 0.006     | 0.015     | 0.015     |
| Proper decay length efficiency | 0.3        | 0.0062         | 0.001          | 0.0002 | 0.0022    | 0.0014         | 0.0023         | 0.0041         | 0.001     | 0.002     | 0.002     |
| Proper decay length resolution | 3.5        | 0.0009         | 0.021          | 0.0015 | 0.0006    | 0.0007         | 0.0009         | 0.0007         | 0.006     | 0.025     | 0.022     |
| Data/simulation difference    | 0.6        | 0.0008         | 0.004          | 0.0003 | 0.0003    | 0.0044         | 0.0029         | 0.007          | 0.007     | 0.007     | 0.028     |
| Flavor tagging                | 0.5        | <10\(^{-4}\)  | 0.006          | 0.0002 | <10\(^{-4}\) | 0.0003 <10\(^{-4}\) | 0.0007 <10\(^{-4}\) | 0.0007         | 0.001     | 0.007     | 0.001     |
| Sig./bkg. \(\omega_{ct}\) difference | 3.0        | —              | —              | 0.0005 | —         | 0.0008 —       | —              | —              | 0.006     | 0.006     | 0.006     |
| Peaking background            | 0.3        | 0.0008         | 0.011          | <10\(^{-4}\) | 0.0002 | 0.0005         | 0.0002         | 0.0005         | 0.003     | 0.007     | 0.011     |
| \(S-P\) wave interference     | —          | 0.0010         | 0.019          | —       | 0.0005    | 0.0005         | —              | 0.013          | —         | 0.019     | 0.019     |
| \(P(\sigma_J)\) uncertainty   | <10\(^{-1}\) | 0.0019         | 0.028          | 0.0004 | 0.0008    | 0.0006         | 0.0008         | 0.0001         | 0.001     | 0.002     | 0.005     |
| Total systematic uncertainty  | 10.0       | 0.0070         | 0.032          | 0.0083 | 0.0026    | 0.0049         | 0.0045         | 0.016          | 0.028     | 0.045     | 0.048     |

1000 times using the reference one, with the fit repeated after reestimating the efficiency. The root mean square (RMS) of the obtained physics parameter distributions is taken as the systematic uncertainty.

**Proper decay length efficiency:** The proper decay length efficiency is first validated by fitting the \(B^{\pm}\) proper decay length distribution in the control \(B^{\pm} \rightarrow J/\psi K^{\pm}\) channel, using several different data-taking periods. Each fit is performed applying the efficiency function evaluated using simulated \(B^{\pm} \rightarrow J/\psi K^{\pm}\) samples with the same procedure used for the \(B^{0} \rightarrow J/\psi \phi\) analysis. We consider eight different data-taking periods, each with the number of \(B^{\pm}\) candidates comparable with the number of signal candidates in the \(B^{0}\) sample used in the analysis. We also consider the 2017 and 2018 data-taking periods as two additional large control data sets. The results are in good agreement with the world-average \(B^{\pm}\) meson lifetime \[^{[3]}\], with differences no larger than 1.5 standard deviations, showing no bias or instabilities during the data taking. Having verified that the efficiency parameterization does not introduce any noticeable bias, we evaluate the related systematic uncertainty by varying the parameters of the proper decay length efficiency function within their statistical uncertainties. The RMS of the distribution of each extracted physics parameter of interest with respect to the nominal fit value is taken as the corresponding systematic uncertainty. We assign a systematic uncertainty to the efficiency model by repeating the fit using the efficiency histogram instead of a smooth efficiency function, and taking the difference from the nominal result as the uncertainty.

**Proper decay length resolution:** A systematic uncertainty is assigned to the proper decay length resolution by varying the \(\kappa\) correction factor by \(\pm 10\%\), as estimated from a data-to-simulation comparison, repeating the fit, and taking the largest difference from the nominal result as the uncertainty. We also evaluate a systematic uncertainty related to the assumption that \(\kappa\) is independent of the proper decay length, by parametrizing \(\kappa\) as a function of \(ct\) using simulated samples. A systematic uncertainty is assigned with the same methodology used to evaluate the “model assumption” systematic uncertainties, using the \(\kappa(ct)\) parametrization as an alternative hypothesis.

**Data/simulation difference:** The efficiency parametrization is found to be very sensitive to the muon and kaon \(p_T\), and \(B^{0}\) meson rapidity distributions, hence a systematic uncertainty is assigned to cover the differences in each of these variables, between data and simulation. The effect is evaluated by reweighting the simulated distributions in each variable to agree with the data. The same weights are applied to the simulated samples used to estimate the efficiencies,
which are then recomputed. The fit is repeated in each case and the sum in quadrature of the differences from the nominal result is taken as the systematic uncertainty.

**Flavor tagging:** The uncertainties associated with the flavor tagging are propagated by varying the parameters of the mistag probability calibration curves within their statistical uncertainties. For each variation, new calibration curves are produced and the data are refitted. The RMS of each fitted parameter distribution is then taken as the corresponding systematic uncertainty. We also evaluate the effect of the assumption that the signal and calibration channels have the same mistag calibration. The difference between the $B_0^s$ and $B^\pm$ calibrations is evaluated using simulated samples and is taken as the systematic uncertainty. The effect of the calibration function shape is evaluated by repeating the fit using a third-order polynomial and taking the difference with respect to the nominal result as the systematic uncertainty. The combined contribution of the three sources to the total systematic uncertainty is negligible.

**Different \(\omega_{evt}\) distribution in signal and background:** A systematic uncertainty is assigned to the possible differences in the mistag probabilities between signal and background. The separate signal and background \(\omega_{evt}\) distributions in data are first measured by using the $B_0^s$ candidate invariant mass signal and sidebands regions. These distributions are separately modeled using the Kernel Density Estimation method [46, 47] and added to the fitting model. One thousand pseudo-experiments are generated and pull distributions with respect to the input values are used to evaluate the systematic uncertainty, as described in the “model bias” paragraph.

**Peaking background:** The systematic uncertainty related to the fixed yield of the peaking background component is evaluated by repeating the fit using a different yield obtained from a $B^0 \to J/\psi K^*(892)^0$ control sample in data. The difference with respect to the nominal result is taken as the systematic uncertainty. A systematic uncertainty is also assigned to the proper decay length modeling of the peaking background by forcing the lifetime to match the world-average value [31], repeating the fit, and taking the difference from the nominal result as the systematic uncertainty.

**S-P wave interference:** The fit model does not take into account the difference in the invariant mass dependence between the $P$-wave from the $B_0^s \to J/\psi \phi$ decay and the $S$-wave, which modifies their interference by a factor $k_{SP}$. The corresponding systematic uncertainty is estimated using pseudo-experiments. The $k_{SP}$ factor is computed by integrating the $P$- and $S$-wave interference term in the $\phi$ candidate mass range, assuming that the $P$-wave amplitude is described by a relativistic Breit–Wigner distribution and the $S$-wave amplitude by a constant, and found to be $k_{SP} = 0.54$. Different $S$-wave lineshapes are found to lead to very similar values of $k_{SP}$, with a variation no larger than \(\approx 2\%\). One thousand pseudo-experiments are generated applying $k_{SP} = 0.54$ to the $i = 8, 9, 10$ terms in Table 1 related to the $S$- and $P$-wave interference. Pull distributions with respect to the input values are used to evaluate the systematic uncertainty, as described in the “model bias” paragraph. The parameters $|A_S|^2$ and $\Delta m_s$ are the only ones whose total uncertainty is affected significantly by this approximation.

**$P(\sigma_{ct})$ uncertainty:** In the fit to data the proper decay length uncertainty pdf is fixed to the one obtained from a pre-fit, as described in Section 5. A systematic uncertainty is assigned by sampling this distribution 1000 times, using the parameter uncertainties obtained from the pre-fit. Each time the fit to data is repeated and the standard deviation of the obtained physics parameter distributions is taken as the systematic uncertainty.

A summary of the systematic uncertainties is given in Table 4. After adding the systematic uncertainties in quadrature, we measure the following values of the $CP$-violating phase and
the width difference between the two $B^0$ mass eigenstates:
\[
\phi_s = -11 \pm 50 \text{ (stat) } \pm 10 \text{ (syst) mrad},
\]
\[
\Delta \Gamma_s = 0.114 \pm 0.014 \text{ (stat) } \pm 0.007 \text{ (syst) ps}^{-1}.
\]

The $|\lambda|$ parameter is measured to be $|\lambda| = 0.972 \pm 0.026 \text{ (stat)} \pm 0.008 \text{ (syst)}$, consistent with no direct $CP$ violation ($|\lambda| = 1$). The average of the heavy and light $B^0_s$ mass eigenstate decay widths is determined to be $\Gamma_s = 0.6531 \pm 0.0042 \text{ (stat)} \pm 0.0026 \text{ (syst)} \text{ ps}^{-1}$, consistent with the world-average value $\Gamma_s = 0.6624 \pm 0.0018 \text{ ps}^{-1}$ [31]. The mass difference between the heavy and light $B^0_s$ meson mass eigenstates is measured to be $\Delta m_s = 17.51^{+0.10}_{-0.09} \text{ (stat)} \pm 0.03 \text{ (syst)} \hbar \text{ ps}^{-1}$, consistent with the theoretical prediction $\Delta m_s = 18.77 \pm 0.86 \hbar \text{ ps}^{-1}$ [4], and in slight tension with the world-average value $\Delta m_s = 17.757 \pm 0.021 \hbar \text{ ps}^{-1}$ [31]. The uncertainties in all these measured parameters are dominated by the statistical component. This analysis represents the first measurement by CMS of the mass difference $\Delta m_s$ between the heavy and light $B^0_s$ mass eigenstates and of the direct $CP$ observable $|\lambda|$.

7 Combination with 8 TeV results

The results presented in this Letter are in agreement with the earlier CMS result at a center-of-mass energy of 8 TeV [14]. As explained in Section 1, both measurements are performed with a similar number of events, with the one at $\sqrt{s} = 13\text{ TeV}$ having a higher tagging efficiency. This leads to an improvement in the uncertainty in quantities that require tagging, such as $\phi_s$, while but the uncertainties in those that do not use tagging, such as $\Delta \Gamma_s$, depend on the raw number of events and are not improved relative to the 8 TeV result. The two sets of results are combined using the BLUE method [48, 49] as implemented in the ROOT package [50–52] using the following physics parameters: $\phi_s, \Delta \Gamma_s, \Gamma_s, |A_0|^2, |A_\perp|^2, |A_S|^2, \delta_\parallel, \delta_\perp, \text{ and } \delta_{S\perp}$. The statistical correlations between the parameters obtained in each measurement are taken into account as well as the correlations of the systematic uncertainties discussed in Section 6. Different sources of systematic uncertainties are assumed to be uncorrelated. The systematic uncertainty correlation between the parameters of the 8 TeV result is assumed to be zero. This assumption has been found to not impact the results in a noticeable way. Since the muon tagging, the efficiency evaluation, and part of the fit model are different in the two measurements, the respective systematic uncertainties are treated as uncorrelated between the two sets of results. The combined results for the $CP$-violating phase and lifetime difference between the two mass eigenstates are:
\[
\phi_s = -21 \pm 44 \text{ (stat) } \pm 10 \text{ (syst) mrad},
\]
\[
\Delta \Gamma_s = 0.1032 \pm 0.0095 \text{ (stat) } \pm 0.0048 \text{ (syst) ps}^{-1},
\]
with a correlation between the two parameters of $+0.02$. The full combination results and the correlations between the various extracted parameters are reported in Appendix A.

The two-dimensional $\phi_s$ vs. $\Delta \Gamma_s$ likelihood contours at 68% confidence level (CL) for the individual and combined results, as well as the SM prediction, are shown in Fig. 6. The contours for the individual results are obtained with likelihood scans, which are used to obtain the combined contour. The contours only account for the statistical uncertainty and the correlation between the two scanned variables, while the results from the combination obtained using the BLUE method take into account the statistical and systematic correlations of a wider range of variables. The results are in agreement with each other and with the SM predictions.
The CP-violating phase $\phi_s$ and the decay width difference $\Delta\Gamma_s$ between the light and heavy $B^0_s$ meson mass eigenstates are measured using a total of 48,500 $B^0_s \rightarrow J/\psi \phi(1020) \rightarrow \mu^+\mu^-K^+\bar{K}^-$ signal events, collected by the CMS experiment at the LHC in proton-proton collisions at $\sqrt{s} = 13$ TeV, corresponding to an integrated luminosity of 96.4 fb$^{-1}$. Events are selected using a trigger that requires an additional muon, which can be exploited to infer the flavor of the $B^0_s$ meson at the time of production. A novel opposite-side muon tagger based on deep neural networks has been developed to maximize the sensitivity of the present analysis. A high tagging power of $\approx 10\%$ is achieved, aided by the requirement of an additional muon in the signal sample imposed at the trigger level.

The CP-violating phase is measured to be $\phi_s = -11 \pm 50$ (stat) $\pm 10$ (syst) mrad, consistent both with the SM prediction $\phi_s = -36.96^{+0.72}_{-0.84}$ mrad [1] and with the absence of CP violation in the mixing-decay interference. The decay width difference between the $B^0_s$ mass eigenstates is measured to be $\Delta\Gamma_s = 0.114 \pm 0.014$ (stat) $\pm 0.007$ (syst) ps$^{-1}$, consistent with the theoretical prediction $\Delta\Gamma_s = 0.091 \pm 0.013$ ps$^{-1}$ [4]. In addition, the CP-violating parameter $|\lambda|$ and the average lifetime of the heavy and light $B^0_s$ mass eigenstates, as well as their mass difference, have been measured. The uncertainties in all these measurements are dominated by the statistical components.

The results presented in this Letter are further combined with those obtained by CMS at $\sqrt{s} = 8$ TeV [14], yielding $\phi_s = -21 \pm 44$ (stat) $\pm 10$ (syst) mrad and $\Delta\Gamma_s = 0.1032 \pm 0.0095$ (stat) $\pm 0.0048$ (syst) ps$^{-1}$. These results are significantly more precise than those from the previous CMS measurement at 8 TeV, and can be used to further constrain possible new-physics effects in $B^0_s$ meson decay and mixing.
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A Supplemental material: correlation matrices and additional results

Table A.1: Statistical correlation matrix between the physics parameters as obtained from the ML fit to the 13 TeV data.

|       | $\phi_s$ | $\Delta\Gamma_s$ | $\Delta m_s$ | $|\lambda|$ | $\Gamma_s$ | $|A_0|^2$ | $|A_\perp|^2$ | $|A_S|^2$ | $\delta_{\parallel}$ | $\delta_{\perp}$ | $\delta_{S\perp}$ |
|-------|---------|-----------------|-------------|-------------|----------|--------|--------|--------|--------------|-----------|-----------|
| $\phi_s$ | +1.00   | -0.02           | -0.19       | +0.22       | 0.00     | -0.01  | +0.01  | -0.01  | -0.02       | -0.09     | +0.03     |
| $\Delta\Gamma_s$ | +1.00   | -0.02           | 0.00        | -0.48       | +0.63    | -0.71  | 0.00   | +0.01  | -0.01       | -0.04     |           |
| $\Delta m_s$ | +1.00   | -0.14           | +0.03       | -0.01       | +0.02    | +0.03  | +0.01  | +0.68  | -0.05       |           |           |
| $|\lambda|$ | 1.00    | -0.02           | 0.00        | -0.01       | -0.03    | -0.06  | 0.00   | +0.01  | -0.18       | +0.05     |           |
| $\Gamma_s$ | 1.00    | -0.31           | +0.42       | 0.15        | -0.02    | +0.02  | -0.05  |       |           |           |           |
| $|A_0|^2$ | +1.00   | -0.61           | +0.15       | -0.01       | -0.01    | -0.09  |       |       |           |           |           |
| $|A_\perp|^2$ | +1.00   | -0.11           | -0.06       | 0.00        | +0.06    |       |       |       |           |           |           |
| $|A_S|^2$ | +1.00   | -0.07           | +0.02       | -0.44       |       |       |       |       |           |           |           |
| $\delta_{\parallel}$ | 3.19    | ±0.12           | ±0.04       |           |       |       |       |       |           |           |           |
| $\delta_{\perp}$ | 2.78    | ±0.15           | ±0.06       |           |       |       |       |       |           |           |           |
| $\delta_{S\perp}$ | 0.238   | ±0.078          | ±0.046      |           |       |       |       |       |           |           |           |

Table A.2: Physics parameters values as obtained from the combination between the CMS 8 TeV and 13 TeV results.

<table>
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<tr>
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<tr>
<td>$\phi_s$ [mrad]</td>
<td>-21</td>
<td>±44</td>
<td>±10</td>
</tr>
<tr>
<td>$\Delta\Gamma_s$ [ps$^{-1}$]</td>
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<td>±0.0095</td>
<td>±0.0048</td>
</tr>
<tr>
<td>$\Gamma_s$ [ps$^{-1}$]</td>
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<tr>
<td>$\delta_{S\perp}$ [rad]</td>
<td>0.238</td>
<td>±0.078</td>
<td>±0.046</td>
</tr>
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</table>

Table A.3: Correlations between the physics parameters as obtained from the combination between the CMS 8 TeV and 13 TeV results. Correlations are both statistical and systematic.

|       | $\phi_s$ | $\Delta\Gamma_s$ | $\Gamma_s$ | $|A_0|^2$ | $|A_\perp|^2$ | $|A_S|^2$ | $\delta_{\parallel}$ | $\delta_{\perp}$ | $\delta_{S\perp}$ |
|-------|---------|-----------------|----------|--------|--------|--------|--------------|-----------|-----------|
| $\phi_s$ | +1.00   | -0.02           | -0.03    | +0.01  | -0.01  | +0.01  | -0.01       | -0.08     | +0.03     |
| $\Delta\Gamma_s$ | +1.00   | -0.45           | +0.43    | -0.57  | +0.01  | +0.01  | 0.00        | -0.01     | -0.01     |
| $\Gamma_s$ | +1.00   | -0.17           | +0.30    | +0.06  | -0.03  | 0.00   | -0.03       | 0.00      | -0.08     |
| $|A_0|^2$ | +1.00   | -0.56           | +0.25    | -0.03  | +0.01  | -0.18  |             |           |           |
| $|A_\perp|^2$ | +1.00   | -0.08           | -0.03    | +0.01  | +0.14  |       |             |           |           |
| $|A_S|^2$ | +1.00   | -0.02           | +0.02    | -0.20  |       |       |             |           |           |
| $\delta_{\parallel}$ | +1.00   | +0.26           | 0.00     |       |       |       |             |           |           |
| $\delta_{\perp}$ | +1.00   | -0.05           |           |       |       |       |             |           |           |
| $\delta_{S\perp}$ | +1.00   |                |           |       |       |       |             |           |           |
B  The CMS Collaboration

Yerevan Physics Institute, Yerevan, Armenia
A.M. Sirunyan†, A. Tumasyan

Institut für Hochenergiephysik, Wien, Austria

Institute for Nuclear Problems, Minsk, Belarus
V. Chekhovsky, A. Litomin, V. Makarenko, J. Suarez Gonzalez

Universiteit Antwerpen, Antwerpen, Belgium

Vrije Universiteit Brussel, Brussel, Belgium

Université Libre de Bruxelles, Bruxelles, Belgium

Ghent University, Ghent, Belgium

Université Catholique de Louvain, Louvain-la-Neuve, Belgium

Centro Brasileiro de Pesquisas Fisicas, Rio de Janeiro, Brazil
G.A. Alves, G. Correia Silva, C. Hensel, A. Moraes

Universidade do Estado do Rio de Janeiro, Rio de Janeiro, Brazil

Universidade Estadual Paulista a, Universidade Federal do ABC b, São Paulo, Brazil

Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria
A. Aleksandrov, G. Antchev, I. Atanasov, R. Hadjiiska, P. Iaydjiev, M. Misheva, M. Rodozov, M. Shopova, G. Sultanov
University of Sofia, Sofia, Bulgaria
M. Bonchev, A. Dimitrov, T. Ivanov, L. Litov, B. Pavlov, P. Petkov, A. Petrov

Beihang University, Beijing, China
W. Fang, Q. Guo, H. Wang, L. Yuan

Department of Physics, Tsinghua University, Beijing, China
M. Ahmad, Z. Hu, Y. Wang

Institute of High Energy Physics, Beijing, China

State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing, China

Sun Yat-Sen University, Guangzhou, China
Z. You

Institute of Modern Physics and Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) - Fudan University, Shanghai, China
X. Gao

Zhejiang University, Hangzhou, China
M. Xiao

Universidad de Los Andes, Bogota, Colombia
C. Avila, A. Cabrera, C. Florez, J. Fraga, A. Sarkar, M.A. Segura Delgado

Universidad de Antioquia, Medellin, Colombia
J. Jaramillo, J. Mejia Guisao, F. Ramirez, J.D. Ruiz Alvarez, C.A. Salazar Gonzalez, N. Vanegas Arbelaez

University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, Split, Croatia
D. Giljanovic, N. Godinovic, D. Lelas, I. Puljak, T. Sculac

University of Split, Faculty of Science, Split, Croatia
Z. Antunovic, M. Kovac

Institute Rudjer Boskovic, Zagreb, Croatia
V. Brigljevic, D. Ferenczek, D. Majumder, B. Mesic, M. Roguljic, A. Starodumov, T. Susa

University of Cyprus, Nicosia, Cyprus

Charles University, Prague, Czech Republic
M. Finger, A. Finger Jr., A. Kveton, J. Tomsa

Escuela Politecnica Nacional, Quito, Ecuador
E. Ayala

Universidad San Francisco de Quito, Quito, Ecuador
E. Carrera Jarrin
Academy of Scientific Research and Technology of the Arab Republic of Egypt, Egyptian Network of High Energy Physics, Cairo, Egypt
A.A. Abdelalim\textsuperscript{12,13}, S. Abu Zeid\textsuperscript{14}, S. Khalil\textsuperscript{13}

Center for High Energy Physics (CHEP-FU), Fayoum University, El-Fayoum, Egypt
A. Lotfy, M.A. Mahmoud

National Institute of Chemical Physics and Biophysics, Tallinn, Estonia
S. Bhowmik, A. Carvalho Antunes De Oliveira, R.K. Dewanjee, K. Ehataht, M. Kadastik, M. Raidal, C. Veelken

Department of Physics, University of Helsinki, Helsinki, Finland
P. Eerola, L. Forthomme, H. Kirschenmann, K. Osterberg, M. Voutilainen

Helsinki Institute of Physics, Helsinki, Finland
E. Brüken, F. Garcia, J. Havukainen, V. Karimäki, M.S. Kim, R. Kinnunen, T. Lampén, K. Lassila-Perini, S. Laurila, S. Lehti, T. Lindén, H. Siikonen, E. Tuominen, J. Tuominiemi

Lappeenranta University of Technology, Lappeenranta, Finland
P. Luukka, T. Tuuva

IRFU, CEA, Université Paris-Saclay, Gif-sur-Yvette, France

Laboratoire Leprince-Ringuet, CNRS/IN2P3, Ecole Polytechnique, Institut Polytechnique de Paris, Paris, France

Université de Strasbourg, CNRS, IPHC UMR 7178, Strasbourg, France

Université de Lyon, Université Claude Bernard Lyon 1, CNRS-IN2P3, Institut de Physique Nucléaire de Lyon, Villeurbanne, France

Georgian Technical University, Tbilisi, Georgia
T. Toriaishvili\textsuperscript{17}, Z. Tsmalaidze\textsuperscript{11}

RWTH Aachen University, I. Physikalisches Institut, Aachen, Germany

RWTH Aachen University, III. Physikalisches Institut A, Aachen, Germany
RWTH Aachen University, III. Physikalisches Institut B, Aachen, Germany

Deutsches Elektronen-Synchrotron, Hamburg, Germany

University of Hamburg, Hamburg, Germany

Karlsruher Institut fuer Technologie, Karlsruhe, Germany

Institute of Nuclear and Particle Physics (INPP), NCSR Demokritos, Aghia Paraskevi, Greece

National and Kapodistrian University of Athens, Athens, Greece

National Technical University of Athens, Athens, Greece
G. Bakas, K. Kousouris, I. Papakrivopoulos, G. Tsipolitis, A. Zacharopoulou

University of Ioánnina, Ioánnina, Greece
MTA-ELTE Lendület CMS Particle and Nuclear Physics Group, Eötvös Loránd University, Budapest, Hungary

Wigner Research Centre for Physics, Budapest, Hungary
G. Bencze, C. Hajdu, D. Horváth, F. Sikler, V. Veszpremi, G. Vesztergombi

Institute of Nuclear Research ATOMKI, Debrecen, Hungary
S. Czellar, J. Karancsi, J. Molnar, Z. Szillasi, D. Teyssier

Institute of Physics, University of Debrecen, Debrecen, Hungary
P. Raics, Z.L. Trocsanyi, B. Ujvari

Eszterhazy Karoly University, Karoly Robert Campus, Gyongyos, Hungary
T. Csorog, F. Nemes, T. Novak

Indian Institute of Science (IISc), Bangalore, India
S. Choudhury, J.R. Komaragiri, D. Kumar, L. Panwar, P.C. Tiwari

National Institute of Science Education and Research, HBNI, Bhubaneswar, India

Panjab University, Chandigarh, India

University of Delhi, Delhi, India
A. Ahmed, A. Bhardwaj, B.C. Choudhary, R.B. Garg, M. Gola, S. Keshri, A. Kumar, M. Naimuddin, P. Priyanka, K. Ranjan, A. Shah

Saha Institute of Nuclear Physics, HBNI, Kolkata, India

Indian Institute of Technology Madras, Madras, India

Bhabha Atomic Research Centre, Mumbai, India
D. Dutta, V. Jha, V. Kumar, D.K. Mishra, K. Naskar, P.K. Netrakanti, L.M. Pant, P. Shukla

Tata Institute of Fundamental Research-A, Mumbai, India
T. Aziz, M.A. Bhat, S. Dugad, R. Kumar Verma, U. Sarkar

Tata Institute of Fundamental Research-B, Mumbai, India

Indian Institute of Science Education and Research (IISER), Pune, India
S. Dube, B. Kansal, A. Kapoor, K. Kothekar, S. Pandey, A. Rane, A. Rastogi, S. Sharma

Department of Physics, Isfahan University of Technology, Isfahan, Iran
H. Bakhshiansohi
The text is too large to convert to plain text.
INFN Sezione di Perugia $^a$, Università di Perugia $^b$, Perugia, Italy
M. Biasini$^{a,b}$, G.M. Bilei$^a$, D. Ciangottini$^{a,b}$, L. Fanò$^{a,b}$, P. Lariccia$^{a,b}$, G. Mantovani$^{a,b}$, V. Mariani$^{a,b}$, M. Menichelli$^a$, F. Moscatelli$^a$, A. Rossi$^{a,b}$, A. Santocchia$^{a,b}$, D. Spiga$^a$, T. Tedeschi$^{a,b}$

INFN Sezione di Pisa $^a$, Università di Pisa $^b$, Scuola Normale Superiore di Pisa $^c$, Pisa, Italy
K. Androsov$^a$, P. Azzurri$^c$, G. Bagliesi$^c$, V. Bertacchi$^{a,c}$, L. Bianchini$^a$, T. Boccali$^a$, R. Castaldi$^a$, M.A. Ciocci$^{a,b}$, R. Dell’Orso$^a$, M.R. Di Domenico$^{a,b}$, S. Donato$^a$, L. Giannini$^{a,b,c}$, A. Giassi$^a$, M.T. Grippo$^a$, F. Ligabue$^{a,b,c}$, E. Manca$^{a,b,c}$, G. Mandorli$^{a,b,c}$, A. Messineo$^{a,b}$, F. Palla$^a$, G. Ramirez-Sanchez$^{a,b,c}$, A. Rizzi$^{a,b}$, G. Rolandi$^{a,b,c}$, S. Roy Chowdhury$^{a,b,c}$, A. Scribano$^a$, N. Shafiei$^{a,b}$, P. Spagnolo$^a$, R. Tenchini$^a$, G. Tonelli$^a$, G. Bagliesi$^c$, A. Bazzocchi$^a$, A. Cappati$^a$, Sapienza Università di Roma $^a$, Rome, Italy
F. Cavallari$^a$, M. Cipriani$^{a,b}$, D. Del Re$^{a,b}$, E. Di Marco$^a$, M. Diemoz$^a$, E. Longo$^{a,b}$, P. Meridiani$^a$, G. Organtini$^{a,b}$, F. Pandolfi$^a$, R. Paramatti$^{a,b}$, C. Quaranta$^{a,b}$, S. Rahatlou$^{a,b}$, C. Rovelli$^a$, F. Santanastasio$^{a,b}$, L. Soffiti$^{a,b}$, R. Tramontano$^{a,b}$

INFN Sezione di Torino $^a$, Università di Torino $^b$, Torino, Italy, Università del Piemonte Orientale $^c$, Novara, Italy
N. Amapane$^{a,b}$, R. Arcidiacono$^{a,b,c}$, S. Argiro$^{a,b}$, M. Arneodo$^{a,b,c}$, N. Bartosik$^a$, R. Bellan$^{a,b}$, A. Bellora$^{a,b}$, C. Biino$^a$, A. Cappati$^{a,b}$, N. Cartiglia$^a$, S. Cometti$^a$, M. Costa$^{a,b}$, R. Covarelli$^{a,b}$, N. Demaria$^a$, B. Kiani$^{a,b}$, F. Legger$^a$, C. Mariotti$^a$, S. Maselli$^a$, E. Migliore$^{a,b}$, V. Monaco$^{a,b}$, E. Monteil$^{a,b}$, M. Monteno$^a$, M.M. Obertino$^{a,b}$, G. Ortona$^a$, L. Pacher$^{a,b}$, N. Pastrone$^a$, M. Pelliccioni$^a$, G.L. Pinna Angioni$^{a,b}$, M. Ruspini$^{a,b,c}$, R. Salvatico$^{a,b}$, F. Siviero$^{a,b}$, V. Sola$^a$, A. Solano$^{a,b}$, D. Soldi$^{a,b}$, A. Staiano$^a$, D. Trocino$^{a,b}$

INFN Sezione di Trieste $^a$, Università di Trieste $^b$, Trieste, Italy
S. Belforte$^a$, V. Candelise$^{a,b}$, M. Casarsa$^a$, F. Cossutti$^a$, A. Da Rold$^{a,b}$, G. Della Ricca$^{a,b}$, F. Vazzoler$^{a,b}$

Kyungpook National University, Daegu, Korea

Chonnam National University, Institute for Universe and Elementary Particles, Kwangju, Korea
H. Kim, D.H. Moon

Hanyang University, Seoul, Korea
B. Francois, T.J. Kim, J. Park

Korea University, Seoul, Korea
S. Cho, S. Choi, Y. Go, S. Ha, B. Hong, K. Lee, K.S. Lee, J. Lim, J. Park, S.K. Park, J. Yoo

Kyung Hee University, Department of Physics, Seoul, Republic of Korea
J. Goh, A. Gurtu

Sejong University, Seoul, Korea
H.S. Kim, Y. Kim

Seoul National University, Seoul, Korea
University of Seoul, Seoul, Korea

Yonsei University, Department of Physics, Seoul, Korea
H.D. Yoo

Sungkyunkwan University, Suwon, Korea

Riga Technical University, Riga, Latvia
V. Veckalns

Vilnius University, Vilnius, Lithuania
A. Juodagalvis, A. Rinkevicius, G. Tamulaitis

National Centre for Particle Physics, Universiti Malaya, Kuala Lumpur, Malaysia
W.A.T. Wan Abdullah, M.N. Yusli, Z. Zolkapli

Universidad de Sonora (UNISON), Hermosillo, Mexico
J.F. Benitez, A. Castaneda Hernandez, J.A. Murillo Quijada, L. Valencia Palomo

Centro de Investigacion y de Estudios Avanzados del IPN, Mexico City, Mexico
H. Castilla-Valdez, E. De La Cruz-Burelo, I. Heredia-De La Cruz, R. Lopez-Fernandez, A. Sanchez-Hernandez

Universidad Iberoamericana, Mexico City, Mexico
S. Carrillo Moreno, C. Oropeza Barrera, M. Ramirez-Garcia, F. Vazquez Valencia

Benemerita Universidad Autonomia de Puebla, Puebla, Mexico
J. Eysermans, I. Pedraza, H.A. Salazar Ibarguen, C. Uribe Estrada

Universidad Autónoma de San Luis Potosí, San Luis Potosí, Mexico
A. Morelos Pineda

University of Montenegro, Podgorica, Montenegro
J. Mijuskovic, N. Raicevic

University of Auckland, Auckland, New Zealand
D. Krofcheck

University of Canterbury, Christchurch, New Zealand
S. Bheesette, P.H. Butler

National Centre for Physics, Quaid-I-Azam University, Islamabad, Pakistan

AGH University of Science and Technology Faculty of Computer Science, Electronics and Telecommunications, Krakow, Poland
V. Avati, L. Grzanka, M. Malawski

National Centre for Nuclear Research, Swierk, Poland

Institute of Experimental Physics, Faculty of Physics, University of Warsaw, Warsaw, Poland
K. Bunkowski, A. Byszuk, K. Doroba, A. Kalinowski, M. Konecki, J. Krolikowski, M. Olszewski, M. Walczak
Laboratório de Instrumentação e Física Experimental de Partículas, Lisboa, Portugal

Joint Institute for Nuclear Research, Dubna, Russia

Petersburg Nuclear Physics Institute, Gatchina (St. Petersburg), Russia
G. Gavrilov, V. Golovtcov, Y. Ivanov, V. Kim, E. Kuznetsova, V. Murzin, V. Oreshkin, I. Smirnov, D. Soknov, V. Sulimov, L. Uvarov, S. Volkov, A. Vorobyev

Institute for Nuclear Research, Moscow, Russia

Institute for Theoretical and Experimental Physics named by A.I. Alikhanov of NRC ‘Kurchatov Institute’, Moscow, Russia
V. Epshteyn, V. Gavrilov, N. Lychkovskaya, A. Nikitenko, V. Popov, I. Pozdnyakov, G. Safronov, A. Spiridonov, A. Stepennov, M. Toms, E. Vlasov, A. Zhokin

Moscow Institute of Physics and Technology, Moscow, Russia
T. Aushev

National Research Nuclear University ‘Moscow Engineering Physics Institute’ (MEPhI), Moscow, Russia
O. Bychkova, R. Chistov, M. Danilov, D. Philippov, S. Polikarpov

P.N. Lebedev Physical Institute, Moscow, Russia
V. Andreev, M. Azarkin, I. Dremin, M. Kirakosyan, A. Terkulov

Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia
A. Belyaev, E. Boos, M. Dubinin, L. Dudko, A. Ershov, A. Gribushin, V. Klyukhin, O. Kodolova, I. Lokhtin, S. Obraztsov, S. Petrushanko, V. Savrin, A. Snigirev

Novosibirsk State University (NSU), Novosibirsk, Russia
V. Blinov, T. Dimova, L. Kardapoltsev, I. Ovtin, Y. Skovpen

Institute for High Energy Physics of National Research Centre ‘Kurchatov Institute’, Protvino, Russia

National Research Tomsk Polytechnic University, Tomsk, Russia
A. Babaev, A. Iuzhakov, V. Okhotnikov, L. Sukhikh

Tomsk State University, Tomsk, Russia
V. Borchsh, V. Ivanchenko, E. Tcherniaev

University of Belgrade: Faculty of Physics and VINCA Institute of Nuclear Sciences, Belgrade, Serbia
P. Adzic, P. Cirkovic, M. Dordevic, P. Milenovic, J. Milosevic
Centro de Investigaciones Energéticas Medioambientales y Tecnológicas (CIEMAT), Madrid, Spain

Universidad Autónoma de Madrid, Madrid, Spain
C. Albajar, J.F. de Trocóniz, R. Reyes-Almanza

Universidad de Oviedo, Instituto Universitario de Ciencias y Tecnologías Espaciales de Asturias (ICTEA), Oviedo, Spain
B. Alvarez Gonzalez, J. Cueva, C. Erice, J. Fernandez Menendez, S. Folgueras, I. Gonzalez-Caballero, E. Palencia Cortezon, C. Ramón Álvarez, V. Rodríguez Bouza, S. Sanchez Cruz

Instituto de Física de Cantabria (IFCA), CSIC-Universidad de Cantabria, Santander, Spain

University of Colombo, Colombo, Sri Lanka
MK Jayananda, B. Kailasapathy54, D.U.J. Sonnadara, DDC Wickramarathna

University of Ruhuna, Department of Physics, Matara, Sri Lanka
W.G.D. Dharmaratna, K. Liyanage, N. Perera, N. Wickramage

CERN, European Organization for Nuclear Research, Geneva, Switzerland

Paul Scherrer Institut, Villigen, Switzerland

ETH Zurich - Institute for Particle Physics and Astrophysics (IPA), Zurich, Switzerland

Universität Zürich, Zurich, Switzerland

National Central University, Chung-Li, Taiwan
C. Adloff, C.M. Kuo, W. Lin, A. Roy, T. Sarkar, S.S. Yu

National Taiwan University (NTU), Taipei, Taiwan

Chulalongkorn University, Faculty of Science, Department of Physics, Bangkok, Thailand
B. Asavapibhop, C. Asawatangtrakuldee, N. Srimanobhas

Çukurova University, Science Department, Science and Art Faculty, Adana, Turkey

Middle East Technical University, Physics Department, Ankara, Turkey
B. Isildak, G. Karapinar, K. Ocalan, M. Yalvac

Bogazici University, Istanbul, Turkey
I.O. Atakisi, E. Gülmiz, M. Kaya, O. Kaya, Ö. Özçelik, S. Tekten, E.A. Yetkin

Istanbul Technical University, Istanbul, Turkey
A. Cakir, K. Cankocak, Y. Komurcu, S. Sen

Istanbul University, Istanbul, Turkey
F. Aydogmus Sen, S. Cerci, B. Kaynak, S. Ozkorucuklu, D. Sunar Cerci

Institute for Scintillation Materials of National Academy of Science of Ukraine, Kharkov, Ukraine
B. Grynyov

National Scientific Center, Kharkov Institute of Physics and Technology, Kharkov, Ukraine
L. Levchuk

University of Bristol, Bristol, United Kingdom

Rutherford Appleton Laboratory, Didcot, United Kingdom
Imperial College, London, United Kingdom

Brunel University, Uxbridge, United Kingdom
J.E. Cole, P.R. Hobson, A. Khan, P. Kyberd, C.K. Mackay, I.D. Reid, L. Teodorescu, S. Zahid

Baylor University, Waco, USA

Catholic University of America, Washington, DC, USA
R. Bartek, A. Domínguez, R. Uniyal, A.M. Vargas Hernandez

The University of Alabama, Tuscaloosa, USA
A. Buccilli, O. Charaf, S.I. Cooper, S.V. Gleyzer, C. Henderson, P. Rumerio, C. West

Boston University, Boston, USA

Brown University, Providence, USA

University of California, Davis, Davis, USA

University of California, Los Angeles, USA

University of California, Riverside, Riverside, USA

University of California, San Diego, La Jolla, USA

University of California, Santa Barbara - Department of Physics, Santa Barbara, USA

California Institute of Technology, Pasadena, USA
Carnegie Mellon University, Pittsburgh, USA

University of Colorado Boulder, Boulder, USA

Cornell University, Ithaca, USA

Fermi National Accelerator Laboratory, Batavia, USA

University of Florida, Gainesville, USA

Florida International University, Miami, USA
Y.R. Joshi

Florida State University, Tallahassee, USA

Florida Institute of Technology, Melbourne, USA

University of Illinois at Chicago (UIC), Chicago, USA

The University of Iowa, Iowa City, USA

Johns Hopkins University, Baltimore, USA

The University of Kansas, Lawrence, USA

Kansas State University, Manhattan, USA

Lawrence Livermore National Laboratory, Livermore, USA
F. Rebassoo, D. Wright

University of Maryland, College Park, USA

Massachusetts Institute of Technology, Cambridge, USA

University of Minnesota, Minneapolis, USA

University of Mississippi, Oxford, USA
J.G. Acosta, S. Oliveros

University of Nebraska-Lincoln, Lincoln, USA
K. Bloom, S. Chauhan, D.R. Claes, C. Fangmeier, L. Finco, F. Golf, J.R. González Fernández, I. Kravchenko, J.E. Siado, G.R. Snow†, B. Stieger, W. Tabb

State University of New York at Buffalo, Buffalo, USA

Northeastern University, Boston, USA

Northwestern University, Evanston, USA

University of Notre Dame, Notre Dame, USA
The Ohio State University, Columbus, USA
J. Alimena, B. Bylsma, B. Cardwell, L.S. Durkin, B. Francis, C. Hill, W. Ji, A. Lefeld, B.L. Winer, B.R. Yates

Princeton University, Princeton, USA

University of Puerto Rico, Mayaguez, USA
S. Malik, S. Norberg

Purdue University, West Lafayette, USA

Purdue University Northwest, Hammond, USA
T. Cheng, J. Dolen, N. Parashar, M. Stojanovic

Rice University, Houston, USA

University of Rochester, Rochester, USA

Rutgers, The State University of New Jersey, Piscataway, USA

University of Tennessee, Knoxville, USA
H. Acharya, A.G. Delannoy, S. Spanier

Texas A&M University, College Station, USA

Texas Tech University, Lubbock, USA

Vanderbilt University, Nashville, USA

University of Virginia, Charlottesville, USA
L. Ang, M.W. Arenton, B. Cox, G. Cummings, J. Hakala, R. Hirosky, M. Joyce, A. Ledovskoy, C. Neu, B. Tannenwald, Y. Wang, E. Wolfe, F. Xia

Wayne State University, Detroit, USA
P.E. Karchin, N. Poudyal, J. Sturdy, P. Thapa

University of Wisconsin - Madison, Madison, WI, USA
K. Black, T. Bose, J. Buchanan, C. Caillol, S. Dasu, I. De Bruyn, L. Dodd, C. Galloni,

†: Deceased
1: Also at Vienna University of Technology, Vienna, Austria
2: Also at Department of Basic and Applied Sciences, Faculty of Engineering, Arab Academy for Science, Technology and Maritime Transport, Alexandria, Egypt
3: Also at Université Libre de Bruxelles, Bruxelles, Belgium
4: Also at IRFU, CEA, Université Paris-Saclay, Gif-sur-Yvette, France
5: Also at Universidade Estadual de Campinas, Campinas, Brazil
6: Also at Federal University of Rio Grande do Sul, Porto Alegre, Brazil
7: Also at UFMS, Nova Andradina, Brazil
8: Also at Universidade Federal de Pelotas, Pelotas, Brazil
9: Also at University of Chinese Academy of Sciences, Beijing, China
10: Also at Institute for Theoretical and Experimental Physics named by A.I. Alikhanov of NRC ‘Kurchatov Institute’, Moscow, Russia
11: Also at Joint Institute for Nuclear Research, Dubna, Russia
12: Also at Helwan University, Cairo, Egypt
13: Now at Zewail City of Science and Technology, Zewail, Egypt
14: Also at Ain Shams University, Cairo, Egypt
15: Also at Purdue University, West Lafayette, USA
16: Also at Université de Haute Alsace, Mulhouse, France
17: Also at Tbilisi State University, Tbilisi, Georgia
18: Also at Erzincan Binali Yildirim University, Erzincan, Turkey
19: Also at CERN, European Organization for Nuclear Research, Geneva, Switzerland
20: Also at RWTH Aachen University, III. Physikalisches Institut A, Aachen, Germany
21: Also at University of Hamburg, Hamburg, Germany
22: Also at Department of Physics, Isfahan University of Technology, Isfahan, Iran, Isfahan, Iran
23: Also at Brandenburg University of Technology, Cottbus, Germany
24: Also at Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia
25: Also at Institute of Physics, University of Debrecen, Debrecen, Hungary, Debrecen, Hungary
26: Also at Physics Department, Faculty of Science, Assiut University, Assiut, Egypt
27: Also at MTA-ELTE Lendület CMS Particle and Nuclear Physics Group, Eötvös Loránd University, Budapest, Hungary, Budapest, Hungary
28: Also at Institute of Nuclear Research ATOMKI, Debrecen, Hungary
29: Also at IIT Bhubaneswar, Bhubaneswar, India, Bhubaneswar, India
30: Also at Institute of Physics, Bhubaneswar, India
31: Also at G.H.G. Khalsa College, Punjab, India
32: Also at Shoolini University, Solan, India
33: Also at University of Hyderabad, Hyderabad, India
34: Also at University of Visva-Bharati, Santiniketan, India
35: Also at Indian Institute of Technology (IIT), Mumbai, India
36: Also at Deutsches Elektronen-Synchrotron, Hamburg, Germany
37: Also at Department of Physics, University of Science and Technology of Mazandaran, Behshahr, Iran
38: Now at INFN Sezione di Bari a, Università di Bari b, Politecnico di Bari c, Bari, Italy
39: Also at Italian National Agency for New Technologies, Energy and Sustainable Economic Development, Bologna, Italy
40: Also at Centro Siciliano di Fisica Nucleare e di Struttura Della Materia, Catania, Italy
41: Also at Riga Technical University, Riga, Latvia, Riga, Latvia
42: Also at Consejo Nacional de Ciencia y Tecnología, Mexico City, Mexico
43: Also at Warsaw University of Technology, Institute of Electronic Systems, Warsaw, Poland
44: Also at Institute for Nuclear Research, Moscow, Russia
45: Now at National Research Nuclear University ‘Moscow Engineering Physics Institute’ (MEPhI), Moscow, Russia
46: Also at St. Petersburg State Polytechnical University, St. Petersburg, Russia
47: Also at University of Florida, Gainesville, USA
48: Also at Imperial College, London, United Kingdom
49: Also at P.N. Lebedev Physical Institute, Moscow, Russia
50: Also at California Institute of Technology, Pasadena, USA
51: Also at Budker Institute of Nuclear Physics, Novosibirsk, Russia
52: Also at Faculty of Physics, University of Belgrade, Belgrade, Serbia
53: Also at Università degli Studi di Siena, Siena, Italy
54: Also at Trincomalee Campus, Eastern University, Sri Lanka, Nilaveli, Sri Lanka
55: Also at INFN Sezione di Pavia a, Università di Pavia b, Pavia, Italy, Pavia, Italy
56: Also at National and Kapodistrian University of Athens, Athens, Greece
57: Also at Universität Zürich, Zurich, Switzerland
58: Also at Stefan Meyer Institute for Subatomic Physics, Vienna, Austria, Vienna, Austria
59: Also at Laboratoire d’Annecy-le-Vieux de Physique des Particules, IN2P3-CNRS, Annecy-le-Vieux, France
60: Also at Şırnak University, Sirnak, Turkey
61: Also at Department of Physics, Tsinghua University, Beijing, China, Beijing, China
62: Also at Near East University, Research Center of Experimental Health Science, Nicosia, Turkey
63: Also at Beykent University, Istanbul, Turkey, Istanbul, Turkey
64: Also at Istanbul Aydin University, Application and Research Center for Advanced Studies (App. & Res. Cent. for Advanced Studies), Istanbul, Turkey
65: Also at Mersin University, Mersin, Turkey
66: Also at Piri Reis University, Istanbul, Turkey
67: Also at Adiyaman University, Adiyaman, Turkey
68: Also at Ozyegin University, Istanbul, Turkey
69: Also at Izmir Institute of Technology, Izmir, Turkey
70: Also at Necmettin Erbakan University, Konya, Turkey
71: Also at Bozok Universitetesi Rektörlüğü, Yozgat, Turkey
72: Also at Marmara University, Istanbul, Turkey
73: Also at Milli Savunma University, Istanbul, Turkey
74: Also at Kafkas University, Kars, Turkey
75: Also at Istanbul Bilgi University, Istanbul, Turkey
76: Also at Hacettepe University, Ankara, Turkey
77: Also at Vrije Universiteit Brussel, Brussel, Belgium
78: Also at School of Physics and Astronomy, University of Southampton, Southampton, United Kingdom
79: Also at IPPP Durham University, Durham, United Kingdom
80: Also at Monash University, Faculty of Science, Clayton, Australia
81: Also at Bethel University, St. Paul, Minneapolis, USA, St. Paul, USA
82: Also at Karamanoğlu Mehmetbey University, Karaman, Turkey
83: Also at Bingol University, Bingol, Turkey
84: Also at Georgian Technical University, Tbilisi, Georgia
85: Also at Sinop University, Sinop, Turkey
86: Also at Mimar Sinan University, Istanbul, Istanbul, Turkey
87: Also at Nanjing Normal University Department of Physics, Nanjing, China
88: Also at Texas A&M University at Qatar, Doha, Qatar
89: Also at Kyungpook National University, Daegu, Korea, Daegu, Korea