1. Vessel Heating Setup

Experiments with heavier multicomponent fuels require heating of the entire experimental setup. A multi-zone heating system is installed, with a total available power of 10.8 kW, which is subdivided into six zones that can be separately powered and controlled, each with a power of 1.8 kW per zone. Each zone has a number of flexible silicone-rubber heaters from Omega and a type E thermocouple for temperature monitoring. The heaters are arranged into zones such that there is a compromise between an even power split between zones and keeping all heaters in a zone relatively close to one another to enable accurate temperature monitoring by thermocouple. Closed-loop control of temperature is achieved with Watlow 935A PID controllers. This ensures the temperature is relatively uniform across the vessel (within approximately ± 5 K). This, combined with the mixing of the gas inside the vessel before the experiments, ensures a uniform temperature in the flammable mixture at the start of the experiment.

Thermal insulation blankets were installed around the vessel to minimize heat loss. The heating system was able to raise the temperature of the experimental setup to over 400 K. The full details of the heating setup can be found in Jones [1].
2. Cylinder Heating Temperature History

Figure 2 shows an example of a typical temperature history during heating of a cylinder. The surface temperature of the cylinder is monitored by a thermocouple (placed above the point where ignition occurs so that the presence of the thermocouple does not disturb or alter the thermal boundary layer in that region) and by a two-color pyrometer. The surface temperature of the cylinder is fed back to a LabView script that provides closed-loop control on the surface temperature. When the temperature of the cylinder is within five percent of the set steady-state temperature in LabView, the control loop kicks in and drives the temperature to the set value. Note that the pyrometer reading is noisy at the start of the heating process due to a small signal-to-noise ratio from minimal light emission at the two pyrometer-measured wavelengths when the surface temperature is low.

3. Interferometer Validation

The interferometer is validated by investigating the temperature fields it captures from heating a well-characterized surface (Autolite glowplug from Boettcher [2], Melguizo-Gavilanes et al. [3]). The glowplug is placed in the vessel in the interferometer field of view (Fig. 3). The vessel is filled with nitrogen and the glowplug is heated while its temperature is monitored by a type K thermocouple and by pyrometry. Interferometer images are captured and post processed as described by Coronel et al. [4]. The temperature fields from interferometry are compared to a simulation from Melguizo-Gavilanes et al. [3], shown in Fig. 4. The interferometer temperature fields compare well with previous numerical results, with less than 10% error everywhere except right along the centerline above the glowplug as shown in Figures 5 and 6. A narrow region of higher error occurs along the centerline due to the nature of Nestor-Olsen algorithm, which converts line of sight integrated quantities into radial quantities by working from the outside of the image in towards the centerline. This causes a buildup of errors along the centerline of the processed image. This validation demonstrates the accuracy of the interferometer in extracting quantitative gas temperature fields surrounding heated surfaces. Details of the optical engineering and design can be found in Jones [1].
4. Natural Convection Boundary Layer Model

A similarity solution is used to model the steady boundary layer created by the natural convection flow induced by the hot cylinder. The goal of this part of the study was to explore the features of the steady boundary layer prior to ignition so the gas is modeled as inert air and no chemical reactions are considered. The temperature difference between the wall and the ambient gas is large enough that gas properties are considered a function of temperature. In order to take advantage of prior work on the similarity solution approach, a Cartesian geometry is considered, i.e., the hot surface is approximated as a vertical plate. This is a reasonable assumption as long as the boundary layer is sufficiently thin compared to the cylinder radius [5]. This is validated by comparison of interferometry results and computed boundary layer temperature profiles as discussed in Section 3 of the article. The following section describes the similarity solution used to develop a prediction for boundary layers in a natural convection flow with variable properties.

The analysis of the boundary layer is based on the low-speed, variable-density, two-dimensional steady boundary layer equations (1)-(3). Following the analysis of Sparrow and Gregg [6], we make the assumptions of constant pressure, variable density flow, an ideal gas, and neglect pressure work and viscous dissipation.

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0, \tag{1}
\]

\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = g (\rho_\infty - \rho) + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right), \tag{2}
\]

\[
\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right). \tag{3}
\]

The approach by Sparrow and Gregg [6] uses a stream function, \( \psi \), that accounted for the variable density character of the flow. The equation for conservation of mass is automatically satisfied by using \( \psi \), and the momentum and energy equations replace \( u \) and \( v \) with derivatives of \( \psi \)

\[
u = \frac{\rho_r}{\rho} \frac{\partial \psi}{\partial y}, \quad v = -\frac{\rho_r}{\rho} \frac{\partial \psi}{\partial x}, \tag{4}
\]
This yields a set of coupled, two-dimensional partial differential equations for $\psi$ and $T$ as functions of $x$ and $y$. Instead, we opt to transform the momentum and energy equations into ordinary differential equations through a transformation to a single independent similarity variable $\eta$.

$$\eta = \frac{c}{x^\frac{1}{4}} \int_0^y \frac{\rho}{\rho_r} dy, \quad c = \left[ \frac{g(\rho_\infty - \rho_r)/\rho_r}{4\nu_r^2} \right]^\frac{1}{2},$$

where the reference condition $r$ is selected to be the value at the hot surface, denoted $w$ for wall. This transformation is motivated by the Howarth-Dorodnitsyn approach to compressible flow. This is analogous to the traditional similarity approach used for natural convection flows that are treated with the Boussinesq approach and constant fluid properties [7]. Transformed dependent variables are:

$$F(\eta) = \left( \frac{\psi}{x^\frac{1}{4}} \right) \left( \frac{1}{4\nu_r c} \right), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}.$$  

The perfect gas and constant pressure assumptions can be used to simplify the dimensionless buoyancy force:

$$\frac{(\rho_\infty/\rho) - 1}{(\rho_\infty/\rho_w) - 1} = \frac{T - T_\infty}{T_w - T_\infty} = \theta.$$  

The momentum and energy equations are transformed into ordinary differential equations where $F$ and $\theta$ are functions of $\eta$ alone:

$$\frac{\partial}{\partial \eta} \left[ \frac{\rho \mu}{\rho_r \mu_r} F'' \right] + 3F'F'' - 2(F')^2 + \theta = 0,$$

$$\frac{\partial}{\partial \eta} \left[ \frac{\rho \lambda}{\rho_r \lambda_r} \theta' \right] + 3Pr_r \left[ \frac{cp}{cp_r} \right] F\theta' = 0,$$

with isothermal boundary conditions at the boundaries and a no-slip condition at the hot surface:

$$F(0) = F'(0) = 0, \quad \theta(0) = 1, \quad F'(\infty) = \theta(\infty) = 0.$$  

Following the analysis laid out by Cairnie et al. [8], we assume the combination of fluid properties that appear as coefficients vary with temperature as follows:

$$M(\theta) = \frac{\rho \mu}{\rho_r \mu_r},$$
\[ \tilde{M}(\theta) = \frac{dM}{d\theta}, \quad (12) \]

\[ L(\theta) = \frac{1}{Pr_r} \left( \frac{c_{pr}}{c_p} \right) \frac{\rho \lambda}{\rho_r \lambda_r}, \quad (13) \]

\[ Y(\theta) = \frac{1}{Pr_r} \left( \frac{c_{pr}}{c_p} \right) \frac{d}{d\theta} \left( \frac{\rho \lambda}{\rho_r \lambda_r} \right). \quad (14) \]

The functions \( M, \tilde{M}, L, \) and \( Y \) were represented as fits to tabulated properties of high-temperature air. Using the defined property functions, the final form of the similarity version of the boundary layer equations is:

\[ MF'''' + \tilde{M}\theta'F'' + 3FF'' - 2(F')^2 + \theta = 0, \quad (15) \]

\[ L\theta'' + Y(\theta')^2 + 3F\theta' = 0, \quad (16) \]

The functions \( F \) and \( \theta \) are determined by numerical solution of the two-point boundary value problem (15) and (16) with boundary conditions (10) using MATLAB’s bvp4c solver. Our solution is validated against the results presented by Cairnie and Harrison [8] and the details of the validation are presented in [1]. The results for u-velocity, v-velocity, and temperature are shown in Fig. 7.

The numerical results in Figure 7 are for a 25.4-cm-tall plate. Results for a shorter plate can be obtained by truncating the numerical results at the desired total height of the plate. Velocity fields parallel (u-velocity) and perpendicular (v-velocity) to the wall and temperature fields are presented. The wall parallel velocity takes the expected structure for a natural convection flow: no-slip creates a velocity of zero next to the wall. Buoyancy forces increase the velocity as distance from the wall increases, reaching a maximum before velocity drops again to zero at the edge of the momentum layer to match ambient conditions. Similarly, wall perpendicular velocity appears as expected for a natural convection flow. The velocity is largest and negative at the leading edge of the cylinder as gas is entrained into the natural convection flow. The entrainment effect becomes less pronounced along the outer edge of the momentum layer as distance from the leading edge increases. As expected, the perpendicular velocity next to the wall is zero. Temperature results show the gas matches wall temperature immediately next to the surface, and then gas temperature drops off rapidly until ambient temperature
is approached at the edge of the thermal layer. Additionally, the width of
the momentum and thermal layers increase very quickly in approximately the
first centimeter along the plate, after which there is a much more gradual
increase in width as the distance from the leading edge increases.

4.1. Streamlines

The similarity approach described previously can be used to model stream-
lines of a natural convection flow. The stream function $\psi$ is computed from:

$$\psi = 4\nu_w x^{3/4} \left( \frac{g(T_w - T_\infty)}{4\nu^2 T_w^2} \right)^{1/4} F(\eta).$$  \hspace{1cm} (17)

The similarity variable can be implicitly determined as a function $\eta(x, y)$ from
(5) once the function $\theta(\eta)$ has been calculated. The stream function, $\psi$, is
then normalized by the maximum value of the stream function in the domain:

$$\psi_n = \frac{\psi}{\psi_{max}}. \hspace{1cm} (18)$$

Streamline trajectories $x(y)$ are defined implicitly by solving (17) with a con-
stant value of $\psi$. A wide range of $\psi_n$ values were chosen to allow the selection
of streamlines close to the hot surface as well as streamlines that barely enter
the thermal layer. Nine values of the normalized stream function, $\psi_n$, were
used to define the streamlines shown in Fig. 8. Once streamlines are selected,
the $(x, y)$ position, the wall parallel and perpendicular velocities $u, v$, and the
temperature, $T$ at every point along each streamline are saved and used to
compute the temperature history.

The time history of the streamline is calculated by the following pro-
cedure: compute the arc length, $s$, along the streamline and integrate the
reciprocal of arc velocity with respect to arc length. The change in arc length
is defined as:

$$ds^2 = dx^2 + dy^2, \hspace{1cm} (19)$$

and we define $s = 0$ at the start of the streamline such that all subsequent
arc lengths can be found by:

$$s_i = s_{i-1} + \sqrt{dx_i^2 + dy_i^2}. \hspace{1cm} (20)$$
The total time to move along the streamline is given by:
\[ t_s = \int_0^s \frac{1}{\sqrt{u^2 + v^2}} ds. \] (21)

The velocities \((u, v)\) are calculated from the stream function. The temperature histories of gas elements moving along selected streamlines are shown in Fig. 9.

We observe that once streamlines enter the boundary layer, the wall normal distance remains relatively constant over the height of the plate. In particular, fluid elements on the streamlines entering the boundary layer closest to the bottom travel nearly parallel to the plate and move very slowly in the very low-speed portion of the momentum boundary layer adjacent to the wall. These fluid elements also reach high temperatures rapidly as shown in Figure 9. We anticipate on the basis of the temperature history that chemical reaction will occur rapidly on those streamlines closest to the surface. However, most of the energy released in the reaction process will be absorbed by the nearby wall if it is highly conductive, as is the case for our metal cylinder. So it is not possible for an ignition event to happen for those streamlines which have the longest residence time at high temperatures.

Instead, ignition will take place slightly further away on streamlines that are close but not too close to the wall. Although the gas will heat more gradually and not get as hot as the surface, it will not lose as much heat to the surface. Streamlines that are far from the surface never get sufficiently hot to react before the top of the plate is reached.

We conclude that there is an optimal streamline and distance from the plate for ignition to occur. The temperature history is sufficient to initiate reaction before the top of the plate is reached and energy released by chemical reaction on this optimal streamlines can create a sufficiently hot spot to ignite a propagating flame in the surrounding gas.

However, it is very challenging to identify the streamline or location in the boundary layer from this heuristic analysis. A detailed simulation accounting not only for heat transfer but also chemical reaction and species transport is required.
References


Figure 1: Zoned heating system for multicomponent fuel testing.
Figure 2: Temperature history of typical cylinder heating event. The dark red line represents the temperature from pyrometry and bright red line represents reconstructed temperature at the centerline as measured by the top thermocouple. The light and dark blue dashed lines represent the voltage readings of the two pyrometer photo detectors during the heating process.

Figure 3: Raw interferogram of glowplug.
Figure 4: Temperature fields from interferometry (left) compared with simulations from Melguizo-Gavilanes et al. [3] (right).
Figure 5: Thermal boundary layer profiles of experimental and numerical results compared for each of the four slices indicated in Figure 4. The labels on the abscissa are distance in pixels from the centerline (0.116 mm/px); each plot is labeled by the vertical distance above the top of the glow plug.
Figure 6: Percent error in thermal boundary layer between experimental and numerical results for each of the four slices as indicated in Figure 4. The labels on the abscissa are distance in pixels from the centerline (0.116 mm/px); each plot is labeled by the vertical distance above the top of the glow plug.
Figure 7: Results of similarity solution to thermal boundary layer on a vertical plate. Left: u-velocity field. Middle: v-velocity field. Right: temperature field. The wall temperature is 1100 K, the gas is dry air, and the plate is 25.4 cm long.
Figure 8: Normalized stream function, $\psi_n$, from similarity solutions. Selected streamlines for tracking highlighted with dashed gray lines.
Figure 9: Stream function temperature history from similarity solutions.