Quantum Hall states at $\nu = \frac{2}{k+2}$: Analysis of the particle-hole conjugates of the general level- k Read-Rezayi states

Waheb Bishara,¹ Gregory A. Fiete,¹ and Chetan Nayak^{2,3}

¹Department of Physics, California Institute of Technology, MC 114-36, Pasadena, California 91125, USA

²Microsoft Research, Station Q, CNSI Building, University of California, Santa Barbara, California 93106, USA

³Department of Physics, University of California, Santa Barbara, California 93106, USA

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We study the $\nu = \frac{2}{k+2}$ quantum Hall states which are particle-hole conjugates of the $\nu = \frac{2}{k+2}$ Read-Rezayi states. We find that equilibration between the different modes at the edge of such a state leads to an emergent $SU(2)_k$ algebra in the counter-propagating neutral sector. Heat flow along the edges of these states will be in the opposite direction of charge flow. In the $k=3$ case, which may be relevant to $\nu = 2 + \frac{2}{5}$, the thermal Hall conductance and the exponents associated with quasiparticle and electron tunneling distinguish this state from competing states such as the hierarchy and/or Jain state.

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The most robust state in the second Landau level (SLL) is the $\nu = 5/2$ state.¹⁻³ As a result of its even denominator, it cannot belong to the usual hierarchy and/or “composite fermion” sequence of Abelian states⁴⁻⁹ which seems to explain all of the observed states in the lowest Landau level (LLL). The leading candidate theories of the $\nu = 5/2$ state are non-Abelian: the Pfaffian state¹⁰⁻¹² and its particle-hole conjugate, the anti-Pfaffian state.^{13,14} Thus, one may wonder whether the other fractions observed in the second Landau level, such as $\nu = 7/3$, $12/5$, $8/3$, and $14/5$,^{2,15,16} are also non-Abelian even though they occur at odd-denominator filling fractions. The state at $\nu = 12/5 = 2 + \frac{2}{5}$ has been the subject of particular interest because its filling fraction is the particle-hole conjugate³¹ of that of the $k=3$ Read-Rezayi (RR) state.¹⁷ This is an exciting possibility because this state is capable of supporting universal topological quantum computation.^{18,19} Alternatively, a state at the lowest level of a non-Abelian hierarchy built on a $\nu = 5/2$ Pfaffian state also occurs at $\nu = 12/5$.²⁰ Finally, the $\nu = 12/5$ state may simply be the transposition to the second Landau level of the Abelian state which is believed to occur at $\nu = 2/5$.

In this paper, we analyze the particle-hole conjugates of the general level- k Read-Rezayi states, which we call the level- k RR states. These states possess multiple gapless modes of edge excitations, which are of particular interest for charge and heat transport. We formulate the low-energy effective field theories of the edges of the level- k RR states and show that an $SU(2)_k$ Kac-Moody symmetry emerges when the different edge modes equilibrate. One notable feature is that the thermal Hall conductance due to this state, $\kappa_{xy} = -(\frac{2k-2}{k+2})\pi^2 k_B^2 T / 3h$, is opposite in sign to the electrical Hall conductance, $\sigma_{xy} = \frac{2}{k+2} \frac{e^2}{h}$. We then focus on the $k=3$ RR state and compare it to other possible $\nu = 2 + 2/5$ states. We show that charge transport through a quantum point contact and thermal transport can distinguish this state from its competitors.

The action of the edge of the level- k RR state¹⁷ at filling fraction $\nu = \frac{k}{k+2}$ is composed of charged and neutral sectors. The charged sector is described by a chiral bosonic field propagating with velocity v_c . The neutral sector is a chiral Z_k

parafermionic theory²¹ propagating with velocity v_n . The Z_k parafermion theory is an $SU(2)_k/U(1)$ coset with central charge $c = \frac{2k-2}{k+2}$ which can be represented by an $SU(2)_k$ chiral Wess-Zumino-Witten (WZW) model in which the $U(1)$ subgroup has been gauged²² (note that the gauge field is not minimally coupled²³). Thus, we can write

$$S = \frac{1}{4\pi\nu} \int dx d\tau \partial_x \varphi \bar{\partial} \varphi + S_{WZW,k} + \frac{k}{4\pi} \int dx d\tau \text{tr}(A_x \bar{\partial} g g^{-1} - \bar{A} g^{-1} \partial_x g + A_x g \bar{A} g^{-1} - A_x \bar{A}), \quad (1)$$

where $\bar{\partial} \equiv i\partial_\tau + v_c \partial_x$ and $\bar{A} \equiv A_\tau - i v_n A_x$. The neutral sector is the sum of the second and third terms which we will call $S_{Z_k} = \int \mathcal{L}_{Z_k}$. The second term, the WZW action, is given by

$$S_{WZW,k} = \frac{k}{16\pi} \int d\tau dx \text{tr}(\partial_x g^{-1} \bar{\partial} g) - i \frac{k}{24\pi} \int dx d\tau dr \epsilon^{\mu\nu\lambda} \text{tr}(\partial_\mu g g^{-1} \partial_\nu g g^{-1} \partial_\lambda g g^{-1}). \quad (2)$$

The field g takes values in $SU(2)$. The second integral is over any three-dimensional manifold M which is bounded by the two-dimensional space-time of the edge ∂M . The value of this integral depends only on the values of the field g at the boundary ∂M . As a result of the gauging (1), the primary fields $\Phi_{j,m}$ of this model are essentially the spin- j primary fields of the WZW model (2) dressed by charge- m Wilson lines of the $U(1)$ gauge field; as a result of the latter, they are invariant under the $U(1)$ subset of the gauge group. The $k(k+1)/2$ primary fields $\Phi_{j,m}$ are, consequently, indexed by half-integers j, m satisfying $0 \leq j \leq k/2$, $m \in (-j, -j+1, \dots, j)$ with the identifications $(j, m) \equiv (\frac{k}{2} - j, m + \frac{k}{2})$, $(j, m) \equiv (j, m+k)$. The field $\Phi_{j,m}$ has dimension $\Delta_{j,m} = \frac{j(j+1)}{k+2} - \frac{m^2}{k}$. Of particular importance is the parafermion field $\psi_1 \equiv \Phi_{\frac{k}{2}, -\frac{k}{2}+1}$ of dimension $\Delta = 1 - \frac{1}{k}$. For $k=1$, the

theory is trivial; the $k=1$ RR state is simply the $\nu=1/3$ Laughlin state which has no neutral sector. The $k=2$ RR state is the Pfaffian state; in the special case $k=2$, the $SU(2)_2/U(1)$ coset can be alternately represented as a Majorana fermion. The three primary fields are then $\Phi_{0,0}=1$, $\Phi_{1/2,1/2}=\sigma$, $\Phi_{1,0}=\psi$.

In the RR state, the electron creation operator is a charge-1 fermionic operator, $\Psi_e^\dagger = \psi_1 e^{i\frac{k+2}{k}\phi}$, where ψ_1 is the $\mathbb{Z}_k$ parafermion field described above (simply the Majorana fermion in the $k=2$ case). With the bosonic field ϕ normalized as in Eq. (1), the scaling dimension of $e^{i\alpha\phi}$ is $\nu\frac{\alpha^2}{2}$. Consequently, the electron operator has scaling dimension $\frac{3}{2}$. The neutral sector does not enter the charge current, $J = \frac{1}{2\pi}\partial\varphi$, so the level- k RR state has a quantized Hall conductance $\sigma_{xy} = \frac{k}{k+2} \frac{e^2}{h}$. If this fractional quantum Hall state occurs in the second Landau level and the lowest Landau level (of both spins) is filled and inert, then $\sigma_{xy} = (2 + \frac{k}{k+2}) \frac{e^2}{h}$. The energy momentum tensor is the sum of the two energy momentum tensors, $T = T_c + T_{Z_k}$. Consequently, the thermal Hall conductivity is proportional to the sum of the two central charges:²⁴ $\kappa_{xy} = \frac{3k}{k+2} \frac{\pi^2 k_B^2}{3h} T$. If this fractional quantum Hall state occurs in the second Landau level, then $\kappa_{xy} = (2 + \frac{3k}{k+2}) \frac{\pi^2 k_B^2}{3h} T$.

To find the edge structure of the level k anti-RR state (RR), we generalize the analysis done for $k=1$ in Ref. 25 and for $k=2$ in Refs. 13 and 14. Ignoring filled Landau levels (if any), we perform a particle-hole transformation of the partially filled Landau level (the second Landau level in the case of $\nu=12/5$). The edge between the level- k RR state ($\nu=1 - \frac{k}{k+2} = \frac{2}{k+2}$) and the vacuum ($\nu=0$) is mapped to the edge between the level- k RR state ($\nu=\frac{k}{k+2}$) and a $\nu=1$ state. Hence, the theory of this edge is described by a level- k RR edge theory and a counterpropagating bosonic charge mode which is the edge theory of the $\nu=1$ state. The low-energy effective Lagrangian is

$$\begin{aligned} \mathcal{L}_{\overline{\text{RR}}} = & \frac{1}{4\pi} \partial_x \phi_1 (i\partial_\tau + v_1 \partial_x) \phi_1 \\ & + \left(\frac{k+2}{k} \right) \frac{1}{4\pi} \partial_x \phi_2 (-i\partial_\tau + v_2 \partial_x) \phi_2 + \mathcal{L}_{Z_k} \\ & - \frac{2}{4\pi} v_{12} \partial_x \phi_1 \partial_x \phi_2 + \xi(x) \psi_1 e^{i[(k+2)/k]\phi_2} e^{-i\phi_1} + \text{H.c.}, \end{aligned} \quad (3)$$

where ϕ_1 is the $\nu=1$ edge charge mode and $v_{12} > 0$ is a repulsive density-density interaction along the edge. The final term is intermode electron tunneling which tunnels electrons from the outer $\nu=1$ edge to the inner edge with a random coefficient ξ which, for simplicity, we take to be of Gaussian white noise form, $\langle \xi(x) \xi^*(x') \rangle = W \delta(x-x')$. In the absence of intermode tunneling, this theory will not realize a universal value of the two-terminal conductance. The tunneling term allows the counterpropagating modes to equilibrate and achieve a universal two-terminal conductance, as is the case for the $\nu=2/3$ quantum Hall state.²⁵

For $v_{12}=0$, the intermode electron tunneling term is irrelevant, $dW/d\ell = -W$, as may be seen by using the replica trick

to integrate out ξ . However, for v_{12} sufficiently large, W becomes relevant. To see this, we introduce a set of fields defined by

$$\phi_\rho = \phi_1 - \phi_2, \quad \phi_\sigma = \phi_1 - \phi_2(k+2)/k, \quad (4)$$

corresponding to charged and neutral bosonic modes, respectively. In these variables, the Lagrangian takes the form $\mathcal{L}_{\overline{\text{RR}}} = \mathcal{L}_\rho + \mathcal{L}_\sigma + \mathcal{L}_{\text{tun}} + \mathcal{L}_{\rho\sigma}$, with

$$\begin{aligned} \mathcal{L}_\rho = & \frac{1}{4\pi} \left(\frac{k+2}{2} \right) \partial_x \phi_\rho (i\partial_\tau + v_\rho \partial_x) \phi_\rho, \\ \mathcal{L}_\sigma = & \frac{1}{4\pi} \frac{k}{2} \partial_x \phi_\sigma (-i\partial_\tau + v_\sigma \partial_x) \phi_\sigma + \mathcal{L}_{Z_k}(v_n), \\ \mathcal{L}_{\rho\sigma} = & 2v_{\rho\sigma} \partial_x \phi_\sigma \partial_x \phi_\rho, \end{aligned}$$

$$\mathcal{L}_{\text{tun}} = \xi(x) \psi_1 e^{i\phi_\sigma} + \xi^*(x) \psi_1^\dagger e^{-i\phi_\sigma}, \quad (5)$$

and $v_\sigma, v_\rho, v_{\rho\sigma}$ are functions of v_1, v_2 and v_{12} , e.g., $4\pi v_{\rho\sigma} = (k/2)^2 v_1 + (k+2/2)^2 v_2 - [k(k+2)/4 + (k/2)^2] v_{12}$. If $v_{\rho\sigma}=0$, then the electron tunneling operator has scaling dimension $[\psi_1 e^{i\phi_\sigma}] = 1$ and the intermode electron tunneling term is *relevant*: $dW/d\ell = W$.

We now show that when the disorder is a relevant perturbation, the edge theory flows to a new fixed point described by a freely propagating charged boson (responsible for the universal quantized Hall conductance) and a backward propagating neutral sector that possesses an $SU(2)$ symmetry. We will argue that due to the disordered tunneling the neutral modes will equilibrate and propagate at common average velocity $\bar{v}$ and show that the velocity mismatch and the mixing term $\mathcal{L}_{\rho\sigma}$ are irrelevant. An $SU(2)$ symmetry will thus emerge in the neutral sector. Note that for $k=2$ this reduces to the result obtained for the anti-Pfaffian.^{13,14} Let us write the neutral sector action $\mathcal{L}_\sigma$ as $\mathcal{L}_{SU(2)_k} + \mathcal{L}_{\delta v}$, with

$$\begin{aligned} \mathcal{L}_{SU(2)_k} = & \frac{1}{4\pi} \frac{k}{2} \partial_x \phi_\sigma (-i\partial_\tau + \bar{v} \partial_x) \phi_\sigma + \mathcal{L}_{Z_k}(\bar{v}), \\ \mathcal{L}_{\delta v} = & [\mathcal{L}_{Z_k}(v_n) - \mathcal{L}_{Z_k}(\bar{v})] + \frac{1}{4\pi} \frac{k}{2} (v_\sigma - \bar{v}) (\partial_x \phi_\sigma)^2. \end{aligned} \quad (6)$$

The Lagrangian $\mathcal{L}_{SU(2)_k}$ is, in fact, equivalent to (the opposite chirality version of) the chiral WZW action (2): the chiral boson ϕ_σ restores the $U(1)$ which was gauged out in Eq. (1). A simple way to see this is to note that the currents

$$J^+ = \sqrt{k} \psi_1 e^{i\phi_\sigma}, \quad J^- = \sqrt{k} \psi_1^\dagger e^{-i\phi_\sigma}, \quad J^z = \frac{k}{2} \partial_x \phi_\sigma \quad (7)$$

obey the same $SU(2)_k$ Kac-Moody commutation relations as the WZW currents

$$J^a = -\frac{ik}{2\pi} \text{tr}(T^a g^{-1} (i\partial_\tau - \bar{v} \partial_x) g), \quad (8)$$

where T^a , $a=x, y, z$ are $SU(2)$ generators and $J^\pm = J^a \pm iJ^y$.

We notice that the tunneling term $\mathcal{L}_{\text{tun}}$ can be written in terms of the currents,

$$\mathcal{L}_{\text{tun}} = \xi(x)J^+ + \xi^*(x)J^- \quad (9)$$

It is convenient to use the WZW representation since the tunneling term can be eliminated from the action by the gauge transformation $g \rightarrow gU$ with $U = P e^{\frac{i}{v} \int^x dx' \vec{\xi}(x') \cdot \vec{T}}$, where P denotes path ordering and $\vec{\xi}(x') = [2 \text{Re}(\xi(x')), -2 \text{Im}(\xi(x')), 0]$. Under this gauge transformation $\mathcal{L}_{\text{SU}(2)_k} \rightarrow \mathcal{L}_{\text{SU}(2)_k} - \vec{\xi} \cdot \vec{J}$, thus gauging away the tunneling term $\mathcal{L}_{\text{tun}}$.

We now turn to the effect of this gauge transformation on the velocity anisotropy terms. The velocity terms in $\mathcal{L}_\sigma$ can be written in the form

$$v_a \text{tr}(S_a \partial_x g^{-1} \partial_x g), \quad (10)$$

where S_a is a matrix satisfying $\text{tr}(S_a T_b T_c) = \delta_{ab} \delta_{ac}$ and v_a , $a=x, y, z$ can be expressed in terms of v_σ , v_n . Let us separate the traceless part M of the matrix $v_a S_a$: $M = v_a S_a - \text{tr}(v_a S_a) \times \mathbb{I}/3$. Then $\mathcal{L}_{\delta v}$ takes the form $\mathcal{L}_{\delta v} = \text{tr}(M \partial_x g^{-1} \partial_x g)$. Under the gauge transformation $g \rightarrow gU$, $\mathcal{L}_{\delta v} \rightarrow \text{tr}(M' \partial_x g^{-1} \partial_x g)$, where $M' = U M U^\dagger$ is random since the gauge transformation is a function of $\xi(x)$. The renormalization group flow of the mean square average of M' , $W_{M'}$, is $dW_{M'}/dl = (3 - 2\Delta)W_{M'}$,²⁶ where Δ is the scaling dimension of the term to which M' couples. In this case, M' couples to $\partial_x g^{-1} \partial_x g \propto J^2$ which has scaling dimension $\Delta = 2$ (i.e., M' is a velocity). Hence $W_{M'}$ and the velocity anisotropy are irrelevant. The part of the velocity term which is invariant under the gauge transformation is the average velocity $\bar{v} = \text{tr}(v_a S_a)/3$.

The mixing term, $\mathcal{L}_{\rho\sigma}$ is irrelevant. It can be written as $\mathcal{L}_{\rho\sigma} = 2v_{\rho\sigma}(\frac{2}{k}J^z)\partial_x \phi_\rho$; under the gauge transformation $g \rightarrow gU$ the current J^z is rotated with a random coefficient. Consequently, deviations from $v_{\rho\sigma} = 0$ are irrelevant, similar to the velocity anisotropy term above.

Thus, we have found that at the fixed point where the edge modes equilibrate due to random electron tunneling, the edge theory of the anti-RR state is described by a single bosonic charge mode, $\mathcal{L}_\rho$, and an $\text{SU}(2)_k$ neutral sector, $\mathcal{L}_{\text{SU}(2)_k}$, moving in the opposite direction. The electron operator of the $\nu=1$ edge in the unequilibrated theory with $\xi=0$ in Eq. (3) is $e^{i\phi_1}$, which can be rewritten in the form $e^{i\phi_1} = e^{-i(k/2)\phi_\sigma} e^{i[(k+2)/2]\phi_\rho}$. As a result of equilibration, the dimension of this operator changes, from $\Delta_e = 1/2$ to $\Delta_e = (k+1)/2$. (The conformal spin, the difference between the right and left scaling dimensions, remains $1/2$, however.) Noting that $e^{i\phi_1}$ can be rewritten as $\chi_{j=k/2}^{m=-k/2} e^{i[(k+2)/2]\phi_\rho}$, we see that this operator is the lowest J^z eigenvalue, m , of a multiplet $\chi_{j=k/2}^m e^{i[(k+2)/2]\phi_\rho}$ with $m = -k/2, -k/2+1, \dots, k/2$. The other electron creation operators in this $\text{SU}(2)$ multiplet are obtained by acting multiple times on $e^{i\phi_1}$ with $J^\pm = \psi_1 e^{i\phi_\sigma}$; thus, they create an electron in the original $\nu=1$ edge and transfer multiple electrons from the RR edge to the $\nu=1$ edge. As a result of equilibration, all $k+1$ of these operators have the same scaling dimension. When electrons tunnel between two level- k RR droplets, the tunneling conductance $G \sim T^{2k}$ and, for finite $V > T$, $I_{\text{tun}} \sim V^{2k+1}$.

Quasiparticle operators can be obtained by the requirement that they are local with respect to these electron opera-

tors. The allowed quasiparticle operators (modulo the creation or annihilation of an electron) and their scaling dimensions are

$$\Phi_{\text{qp}}^{j,N} = \chi_j e^{i(j+N)\phi_\rho}. \quad (11)$$

The J^z eigenvalue is suppressed here; there is an $\text{SU}(2)$ multiplet of each of these operators all of which belong to the same quasiparticle species because they have the same topological properties. $\Phi_{\text{qp}}^{j,N}$ has right scaling dimension $(j+N)^2/(k+2)$ and left scaling dimension $j(j+1)/(k+2)$ and, therefore, total scaling dimension $[(j+N)^2 + j(j+1)]/(k+2)$ and topological spin $[(j+N)^2 - j(j+1)]/(k+2)$. For k even, $N=0, 1, \dots, \frac{k}{2}$. For k odd, $N=0, 1, \dots, \frac{k+1}{2}$ for integer j and $N=0, 1, \dots, \frac{k-1}{2}$ for half-integer j . Therefore, there are $(k+1)(k+2)/2$ different quasiparticle species. This is also the ground state degeneracy of the RR theory on the torus (which is 10 in the case of the $\nu=12/5$ state). The corresponding RR state has the same degeneracy. The minimal dimension of a quasiparticle operator is $[\Phi_{\text{qp}}^{1/2,0}] = [\Phi_{\text{qp}}^{0,1}] = \frac{1}{k+2}$. Consequently, when quasiparticles tunnel between the edges at a point contact, $R_{xx} \sim T^{-2k/(k+2)}$ and, at finite $V > T$, $I_{\text{tun}} \sim V^{(2-k)/(2+k)}$.

The thermal Hall conductivity of the anti-RR state is determined by the central charge of the edge theory.²⁴ Ignoring the filled Landau levels, the central charge of the bosonic charge sector is $c=1$ and the central charge of the $\text{SU}(2)_k$ theory is $\bar{c}=3k/(k+2)$. The thermal Hall conductivity of the anti-RR state is then

$$\kappa_{xy}^{\text{RR}} = \left(1 - \frac{3k}{k+2}\right) \frac{\pi^2 k_B^2}{3h} T. \quad (12)$$

Thus, the conductivity due to the partially filled second Landau level is negative for all k . Focusing on the $\nu=2/5$ anti-RR state ($k=3$) its thermal Hall conductivity is $-\frac{4}{5}$ (in units of $\frac{\pi^2 k_B^2}{3h} T$), while the Abelian hierarchy state at $\nu=2/5$ has a positive thermal Hall conductance of $+2$, and the $\nu=2/5$ non-Abelian hierarchy state of Ref. 20, built on the $\nu=1/2$ Pfaffian state, would have a thermal Hall conductance of $+\frac{1}{2}$. We note that the construction of Ref. 20 can also produce a $\nu=2/5$ state built on the anti-Pfaffian state, with thermal Hall conductance $-\frac{3}{2}$. These thermal conductivities are achieved at length scales longer than the equilibration length of the edges. In the case of the $\nu=12/5$ state, the filled lower Landau level gives an additional contribution of $+2$, which would make all of the thermal conductivities positive, though differing in magnitude. Therefore, in order to distinguish the non-Abelian $\nu=12/5$ states from the Abelian one through the signs of their thermal Hall conductivities, it would be necessary to measure the thermal conductivity along an edge between $\nu=2$ and $\nu=2+\frac{2}{5}$, which would only have a contribution from the partially filled Landau level. On shorter length scales, the different modes on the edge do not equilibrate, in which case both the anti-RR state and the non-Abelian hierarchy state will have heat flow both upstream and downstream while the Abelian state will have purely chiral heat transport. In this case, the filled Landau levels simply give an additional contribution to the downstream heat transport.

The difference between the various proposed $\nu=12/5$ states would also be evident from the transport through a point contact. As a result of weak quasiparticle tunneling from one edge to the other, there is a nonzero longitudinal resistance $R_{xx} \sim T^{4\Delta_{qp}-2}$. At finite voltage $V > T$, we instead have $I_{\text{tun}} \sim V^{4\Delta_{qp}-1}$. In the Abelian hierarchy $\nu=2/5$ state, the most relevant tunneling operator is that of the charge $\frac{2}{5}e$ quasiparticle with $\Delta_{qp}=\frac{1}{5}$,^{9,27} leading to $R_{xx} \sim T^{-6/5}$. In the non-Abelian hierarchy state of Ref. 20, the most relevant tunneling operator is that of charge $\frac{1}{5}e$ quasiparticles with dimension $\Delta_{qp}=\frac{9}{80}$, leading to $R_{xx} \sim T^{-31/20}$. Its sister state, built on the anti-Pfaffian, rather than the Pfaffian, has $\Delta_{qp}=\frac{19}{80}$, hence $R_{xx} \sim T^{-21/20}$. Finally, in the $k=3$ RR state, the operator $\Phi_{1/5}=\Phi_{(\frac{1}{2})}e^{i\frac{1}{2}\phi_p}$ carries charge $\frac{1}{5}e$ and has scaling dimension $\Delta_{qp}=\frac{1}{5}$, while the operator $\Phi_{2/5}=e^{i\phi_p}$ carries charge $\frac{2}{5}e$ and has the same scaling dimension. Therefore, the longitudinal resistance in this theory will behave as $R_{xx} \sim T^{-6/5}$, precisely as in the Abelian hierarchy state. However, shot noise experiments²⁸⁻³⁰ can detect the charge of the tunneling quasiparticles. In the Abelian hierarchy state, the current is carried by charge $2e/5$ quasiparticles at the lowest temperatures, where the most relevant operator (in the renor-

malization group sense) will dominate. In the non-Abelian hierarchy state, charge $e/5$ quasiparticle tunneling is the most relevant operator. In the $k=3$ RR state, charge $e/5$ and charge $2e/5$ quasiparticle tunneling are equally relevant, but the bare tunneling matrix element for charge $e/5$ quasiparticles is presumably larger than for charge $2e/5$ quasiparticles [$\Gamma_{2/5} \sim (\Gamma_{1/5})^2$], so tunneling will be dominated by the former. In summary, we expect shot noise experiments in either of the non-Abelian states to result in a charge of $e/5$, as compared to charge $2e/5$ in the Abelian state. The two non-Abelian states can be distinguished from each other by the power laws with which R_{xx} depends on T or I_{tun} on V for $V > T$ in the limit of weak tunneling. In the opposite limit of strong tunneling, the droplet effectively breaks in two and all that remains is the weak tunneling of electrons between the two droplets. In this case, $G \sim T^{4\Delta_e-2}$; in both the Abelian and non-Abelian hierarchy states, $\Delta_e=3/2$ while in the $k=3$ RR state, $\Delta_e=2$.

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