Lensing reconstruction of primordial cosmic microwave background polarization

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We discuss the possibility to directly reconstruct the cosmic microwave background (CMB) polarization field at the last scattering surface by accounting for modifications imposed by the gravitational lensing effect. The suggested method requires a tracer field of the large scale structure lensing potentials that deflected propagating CMB photons from the last scattering surface. This required information can come from a variety of observations on the large scale structure matter distribution, including the convergence reconstructed from lensing shear studies involving galaxy shapes. In the case of so-called curl, or *B* modes of CMB polarization, the reconstruction allows one to identify the distinct signature of inflationary gravitational waves.

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I. INTRODUCTION

The advent of high sensitivity cosmic microwave background (CMB) experiments will soon allow detailed observational studies on the CMB polarization field. In addition to confirmation of the basic picture of how anisotropies were generated during the recombination era [1], CMB polarization observations, at large angular scales, may allow measurement of the reionization redshift based on excess power created during the rescattering of the CMB quadrupole at the reionization surface [2]. An additional use of polarization observations is the potential detection of inflationary gravitational waves based on its contribution to the so-called curl or *B* modes of polarization [3].

In addition to primordial contributions involving density perturbations, which contribute solely to *E* modes, and tensor modes or gravitational waves, the polarization field one observes today also contains secondary contributions associated with large scale structure. This is similar to the well known secondary contributions that dominate the small scale temperature anisotropy signal [4]. In the case of *B* modes, the arcminute scale polarization is dominated by a fractional conversion of the dominant *E* mode contribution via the gravitational lensing angular deflection of CMB photons [5]. In Ref. [6], we discussed an approach to separate the gravitational-wave signature in *B* modes from the dominant, and contaminant, weak lensing contribution at arcminute scales. We also investigated the limits on gravitational wave *B* mode detection after a model-independent lensing subtraction. This calculation followed previous discussions in the literature on how CMB data can be used for the extraction of statistics involving the gravitational lensing effect, such as shear or deflection angles [7–9].

In general, the reconstruction of lensing deflections from CMB data requires *a priori* information on the primordial CMB contribution at the last scattering surface. This knowledge cannot be easily obtained from data alone since the CMB contribution measured today involves contributions resulting from secondary effects and any modifications during the transit to us from the last scattering surface. On the other hand, we can consider an alternative approach to an analogous problem involving a reconstruction of the primordial CMB contribution, at the last scattering surface, by accounting for modifications due to large scale structure. This is, effectively, an inverse problem from what was discussed in Refs. [8,9] and requires prior information related to the lensing effect on CMB, such as the mass distribution of large scale structure in which photons propagated. In the present study, we briefly discuss this possibility using a tracer field of the large scale structure potentials that deflected CMB photons via the gravitational lensing effect.

The discussion presented here follows the recent work of Ref. [9] where we discussed the lensing reconstruction from CMB data given information related to primordial CMB anisotropy and polarization fields. We refer the reader to [9] for the basic details of lensing and other important ingredients related to the calculation. To illustrate our results, we assume the currently favored cold dark matter model with a cosmological constant (ΛCDM) cosmology.

II. CALCULATION

The lensing effect on CMB data can be described simply as a transfer of power via changes to photon propagation directions on the sky. In Fourier space, we can consider the lensing modification following Ref. [10] and write

$$ {}_{\pm}Y(\mathbf{l}) = {}_{\pm}\tilde{Y}(\mathbf{l}) - \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} {}_{\pm}\tilde{Y}(\mathbf{l}_1) e^{\pm 2i(\varphi_{\mathbf{l}_1} - \varphi_{\mathbf{l}})} L(\mathbf{l},\mathbf{l}_1), \tag{1} $$

where ${}_{\pm}Y(\mathbf{l}) = E(\mathbf{l}) \pm iB(\mathbf{l})$ follows the *E*- and *B*-mode decomposition introduced in Ref. [3]. Note that we denote the primordial contribution at the last scattering surface with a $\tilde{Y}$, while the polarization field observed today, after gravitational lensing, is denoted with Y. The lensing effect on the polarization field in Eq. (1) is captured by

$$ L(\mathbf{l}_1,\mathbf{l}_1{}') = \phi(\mathbf{l}_1 - \mathbf{l}_1{}')(\mathbf{l}_1 - \mathbf{l}_1{}')\cdot \mathbf{l}_1{}' + \frac{1}{2}\int \frac{d^2\mathbf{l}_1''}{(2\pi)^2}\phi(\mathbf{l}_1'') \times \phi^*(\mathbf{l}_1'' + \mathbf{l}_1{}' - \mathbf{l}_1)(\mathbf{l}_1''\cdot \mathbf{l}_1{}')(\mathbf{l}_1'' + \mathbf{l}_1{}' - \mathbf{l}_1)\cdot \mathbf{l}_1{}', $$

to the second order in the projected lensing potential ϕ given by [9,10]

$$\phi(\hat{\mathbf{m}}) = -2\int_0^{r_0} dr \frac{d_A(r_0-r)}{d_A(r)d_A(r_0)}\Phi(r,\hat{\mathbf{m}}r). \tag{2}$$

Here, r is the conformal time or comoving radial distance, d_A is the analogous comoving angular diameter distance, and Φ is the gravitational potential. The latter is related to fluctuations in the density field via the Poisson equation. The deflection angle associated with lensing is the gradient of the projected potential, $\alpha(\hat{\mathbf{n}}) = \nabla\phi(\hat{\mathbf{n}})$. Using deflections to the first order, we write

$$\begin{aligned} E(\mathbf{l}) = \tilde{E}(\mathbf{l}) - \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}[\tilde{E}(\mathbf{l}_1)\cos 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}}) \\ -\tilde{B}(\mathbf{l}_1)\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}})]\phi(\mathbf{l}-\mathbf{l}_1)(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}_1, \\ B(\mathbf{l}) = \tilde{B}(\mathbf{l}) - \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}[\tilde{E}(\mathbf{l}_1)\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}}) \\ +\tilde{B}(\mathbf{l}_1)\cos 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}})]\phi(\mathbf{l}-\mathbf{l}_1)(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}_1. \end{aligned} \tag{3}$$

In order to extract the primordial contribution, unaffected by lensing, consider the combination of E and B fields with a tracer field of the deflecting potential, which we label here as X. This tracer map has the property that it does not correlate strongly with either $\tilde{E}$ or $\tilde{B}$, the quantities at the last scattering. Thus the only correlation expected is the one resulting due to lensing, and to a lesser extent from other secondary contributions to polarization. Since the latter contributions involving scattering effects associated with large scale structure are significantly smaller [11], we ignore them in this discussion. In fact, as discussed in Ref. [12], the secondary polarization contributions resulting from effects due to clusters do not constitute a background for detection of either the gravitational lensing signal or the primordial polarization. If they did matter for some reason, most secondary scattering effects associated with polarization can be removed from data based on their spectral dependences. This is similar to the approach suggested in Ref. [13] to remove the Sunyaev-Zel'dovich effect (SZ; [14]) from dominant temperature anisotropies.

We consider a map created by taking the product of the polarization field and the tracer field, e.g., EX. In Fourier space, we can write this product as a convolution of the Fourier moments of the two fields:

$$\begin{aligned} (EX)(\mathbf{l}) = \frac{1}{2}\int \frac{d^2\mathbf{l}_1}{(2\pi)^2}[E(\mathbf{l}_1)X(\mathbf{l}-\mathbf{l}_1) \\ +X(\mathbf{l}_1)E(\mathbf{l}-\mathbf{l}_1)]W(\mathbf{l},\mathbf{l}_1), \end{aligned} \tag{4}$$

where W is a Fourier-space based filter that we will design to maximize the reconstruction of the primordial polarization information. Similarly, we can also construct another quadratic statistic involving B and X fields. Using Eq. (3) in Eq. (4), we can simplify to obtain

$$\begin{aligned} (EX)(\mathbf{l}) = \frac{1}{2}\tilde{E}(\mathbf{l})\int \frac{d^2\mathbf{l}_1}{(2\pi)^2}W(\mathbf{l},\mathbf{l}_1)[C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}\cos 2 \\ \times(\varphi_{\mathbf{l}}-\varphi_{\mathbf{l}_1}) + C^{X\phi}_{l_1}\mathbf{l}_1\cdot\mathbf{l}\cos 2(\varphi_{\mathbf{l}}-\varphi_{|\mathbf{l}-\mathbf{l}_1|})] \\ -\frac{1}{2}\tilde{B}(\mathbf{l})\int \frac{d^2\mathbf{l}_1}{(2\pi)^2}W(\mathbf{l},\mathbf{l}_1)[C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}\sin 2 \\ \times(\varphi_{\mathbf{l}}-\varphi_{\mathbf{l}_1}) + C^{X\phi}_{l_1}\mathbf{l}_1\cdot\mathbf{l}\sin 2(\varphi_{\mathbf{l}}-\varphi_{|\mathbf{l}-\mathbf{l}_1|})], \end{aligned} \tag{5}$$

while analogous contribution to the quadratic statistic involving B and X fields, $(BX)(\mathbf{l})$, is

$$\begin{aligned} (BX)(\mathbf{l}) = \frac{1}{2}\tilde{E}(\mathbf{l})\int \frac{d^2\mathbf{l}_1}{(2\pi)^2}W(\mathbf{l},\mathbf{l}_1)[C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}\sin 2 \\ \times(\varphi_{\mathbf{l}}-\varphi_{\mathbf{l}_1}) + C^{X\phi}_{l_1}\mathbf{l}_1\cdot\mathbf{l}\sin 2(\varphi_{\mathbf{l}}-\varphi_{|\mathbf{l}-\mathbf{l}_1|})] \\ +\frac{1}{2}\tilde{B}(\mathbf{l})\int \frac{d^2\mathbf{l}_1}{(2\pi)^2}W(\mathbf{l},\mathbf{l}_1)[C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}\cos 2 \\ \times(\varphi_{\mathbf{l}}-\varphi_{\mathbf{l}_1}) + C^{X\phi}_{l_1}\mathbf{l}_1\cdot\mathbf{l}\cos 2(\varphi_{\mathbf{l}}-\varphi_{|\mathbf{l}-\mathbf{l}_1|})]. \end{aligned} \tag{6}$$

As written, these quadratic combinations are proportional to the primordial polarization contributions at the last scattering surface, $\tilde{E}(\mathbf{l})$ and $\tilde{B}(\mathbf{l})$. This is essentially the basis for the suggested reconstruction. In [8,9], the suggested quadratic statistics involve combinations of EE and EB such that these lead to terms which are proportional to ϕ times integrals involving $C_l^{\tilde{E}\tilde{E}}$ and $C_l^{\tilde{B}\tilde{B}}$. Thus, with prior knowledge on primordial polarization power spectra, it was possible to reconstruct lensing deflection potentials. Our approach, though similar, involves the inverse problem of constructing $\tilde{E}$ and $\tilde{B}$ given information related to ϕ.

In Eqs. (5) and (6), $C_l^{X\phi}$ is the cross correlation between tracer X field and the lensing potentials ϕ defined such that

$$\langle X(\mathbf{l})\phi(\mathbf{l}')\rangle = (2\pi)^2\delta_D(\mathbf{l}+\mathbf{l}')C_l^{X\phi}. \tag{7}$$

We can write this cross angular power spectrum as

$$C_l^{X\phi} = \frac{2}{\pi}\int k^2 dk P_{\delta\delta}(k) I_l^X(k) I_l^{\phi}(k), \tag{8}$$

where

$$I_l^X(k) = \int_0^{r_0} dr W^X(k,r) j_l(kr), \tag{9}$$

and similarly for $I_l^{\phi}(k)$. Here, $W^i(k,r)$ is the window function of the X or ϕ in radial coordinates, j_l's are the spherical Bessel functions and $P_{\delta\delta}(k)$ is the power spectrum of density fluctuations (see, Ref. [9] for details).

To extract power spectra associated with the primordial polarization, $C_l^{\tilde{E}\tilde{E}}$ and $C_l^{\tilde{B}\tilde{B}}$, we can either use power spectra constructed from correlating quadratic statistics themselves, e.g., $\langle (EX)(\mathbf{l})(EX)(\mathbf{l}')\rangle$, or the quadratic statistic correlated with the polarization field, e.g., $\langle (EX)(\mathbf{l})E(\mathbf{l}')\rangle$. The latter approach involves an extraction of the power spectrum from the bispectrum formed by the fields traced by the quadratic statistic and either E or B; this is similar to what was suggested in Refs. [15,16] to extract the lensing-SZ correlation from a squared temperature-temperature power spectrum. The method based on $\langle (EX)(\mathbf{l})(EX)(\mathbf{l}')\rangle$ extracts the power spectrum information via the trispectrum formed by lensing and tracer field correlation associated with quadratic statistics. This is analogous to techniques in Refs. [8,9] to extract the lensing information using CMB data. Here, for simplicity, we will use the approach based on the bispectrum; the approach based on the trispectrum can be followed by modifying the discussion in Ref. [9] for the present calculation.

We first consider the power spectrum formed by $\langle (EX)(\mathbf{l})E(\mathbf{l}')\rangle$. Following our standard definition, we define the associated power spectrum in this case as

$$\langle (EX)(\mathbf{l})E(\mathbf{l}')\rangle = (2\pi)^2 \delta_D(\mathbf{l}+\mathbf{l}') C_l^{EX-E}. \tag{10}$$

Similarly, we define the power spectra involved with other combinations of E and B with (EX) and (BX).

We can write the angular power spectrum associated with the $\langle (EX)(\mathbf{l})E(\mathbf{l}')\rangle$ correlation as

$$C_l^{EX-E} = \frac{1}{2} C_l^{\tilde{E}\tilde{E}} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} W(\mathbf{l},\mathbf{l}_1)[C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}\cos 2(\varphi_{\mathbf{l}}-\varphi_{\mathbf{l}_1}) + C^{X\phi}_{l_1}\mathbf{l}_1\cdot\mathbf{l}\cos 2(\varphi_{\mathbf{l}}-\varphi_{|\mathbf{l}-\mathbf{l}_1|})]. \tag{11}$$

Note that there is no contribution coming from the $\tilde{B}$ term associated with $(EX)(\mathbf{l})$ since there is no correlation between $\tilde{E}(\mathbf{l}')$ and $\tilde{B}(\mathbf{l})$ due to parity considerations. Also, there is no resulting contribution associated with the first order lensing term of E as lensing deflections, ϕ, should not correlate with the primordial polarization field. Similarly, the contribution to $\langle (EX)(\mathbf{l})B(\mathbf{l}')\rangle$ results in a power spectrum given by

$$C_l^{EX-B} = -\frac{1}{2} C_l^{\tilde{B}\tilde{B}} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} W(\mathbf{l},\mathbf{l}_1)[C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}\sin 2(\varphi_{\mathbf{l}}-\varphi_{\mathbf{l}_1}) + C^{X\phi}_{l_1}\mathbf{l}_1\cdot\mathbf{l}\sin 2(\varphi_{\mathbf{l}}-\varphi_{|\mathbf{l}-\mathbf{l}_1|})]. \tag{12}$$

Since (BX) provides an additional statistic to correlated with polarization maps, we can also consider the cross correlation of $(BX)(\mathbf{l})$ with $E(\mathbf{l})$ and $B(\mathbf{l})$. We can write the two contributions as

$$C_l^{BX-E} = \frac{1}{2} C_l^{\tilde{E}\tilde{E}} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} W(\mathbf{l},\mathbf{l}_1)[C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}\sin 2(\varphi_{\mathbf{l}}-\varphi_{\mathbf{l}_1}) + C^{X\phi}_{l_1}\mathbf{l}_1\cdot\mathbf{l}\sin 2(\varphi_{\mathbf{l}}-\varphi_{|\mathbf{l}-\mathbf{l}_1|})], \tag{13}$$

and

$$C_l^{BX-B} = \frac{1}{2} C_l^{\tilde{B}\tilde{B}} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} W(\mathbf{l},\mathbf{l}_1)[C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}\cos 2(\varphi_{\mathbf{l}}-\varphi_{\mathbf{l}_1}) + C^{X\phi}_{l_1}\mathbf{l}_1\cdot\mathbf{l}\cos 2(\varphi_{\mathbf{l}}-\varphi_{|\mathbf{l}-\mathbf{l}_1|})], \tag{14}$$

respectively. As written, C_l^{EX-E} and C_l^{BX-E} are proportional to $C_l^{\tilde{E}\tilde{E}}$ while C_l^{EX-B} and C_l^{BX-B} are proportional to $C_l^{\tilde{B}\tilde{B}}$. This is the basis of the present approach which is leading to an extraction of these primordial polarization power spectra uncontaminated by the lensing contribution. With an appropriate normalization and a description for the filter, $W(\mathbf{l},\mathbf{l}_1)$, that optimizes the polarization extraction, one can effectively extract $C_l^{\tilde{B}\tilde{B}}$ and $C_l^{\tilde{E}\tilde{E}}$ from the higher order statistics involving polarization maps as observed today and a map of the large scale structure.

The signal-to-noise ratio for the extraction of the primordial power spectra of polarization, for example, in the case of $C_l^{\tilde{B}\tilde{B}}$ extracted using $\langle (BX)(\mathbf{l})B(\mathbf{l}')\rangle$, is

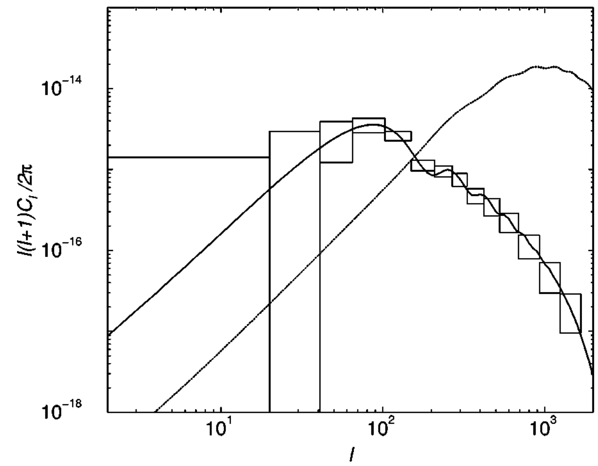
$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)^2 = \sum_l \frac{f_{\rm sky}(2l+1)[C_l^{\tilde{B}\tilde{B}}]^2}{[C_l^{\tilde{B}\tilde{B}}]^2 + 2\left[\int \frac{d^2\mathbf{l}_1}{(2\pi)^2} W(\mathbf{l},\mathbf{l}_1)\{C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}\cos 2(\varphi_{\mathbf{l}}-\varphi_{\mathbf{l}_1}) + C^{X\phi}_{l_1}\mathbf{l}_1\cdot\mathbf{l}\cos 2(\varphi_{\mathbf{l}}-\varphi_{|\mathbf{l}-\mathbf{l}_1|})\}\right]^{-2} N_l^{BX} C_l^{B,\mathrm{t}}}, \tag{15}$$

where $f_{\rm sky}$ is the fraction of sky covered and noise contributions to the map constructed by taking the product of B and X are given by the variance of $(BX)(\mathbf{l})$ and can be calculated by considering $\langle (BX)(\mathbf{l})(BX)(\mathbf{l}')\rangle$ to obtain

$$N_l^{BX} = \frac{1}{2}\int \frac{d^2\mathbf{l}_1}{(2\pi)^2} W^2(\mathbf{l},\mathbf{l}_1)[C^{B,\mathrm{t}}_{l_1} C^{X,\mathrm{t}}_{|\mathbf{l}-\mathbf{l}_1|} + C^{B,\mathrm{t}}_{|\mathbf{l}-\mathbf{l}_1|} C^{X,\mathrm{t}}_{l_1}]. \tag{16}$$

In the above, $C_l^{B,\mathrm{t}}$ and $C_l^{X,\mathrm{t}}$ are the total contributions to B mode and X fields, respectively. We can write these contributions as $C_l^{i,\mathrm{t}} = C_l^i + C_l^n + C_l^s$ where C_l^n is any noise contribution, for example, instrumental noise in the B map, and C_l^s is any secondary contribution, such as the dominant lensing contribution in the B map.

In order to maximize the signal-to-noise ratio for the extraction of primordial polarization power spectra, we follow the approach in Ref. [9] and consider a description for the filter. The maximum signal-to-noise ratio is obtained when



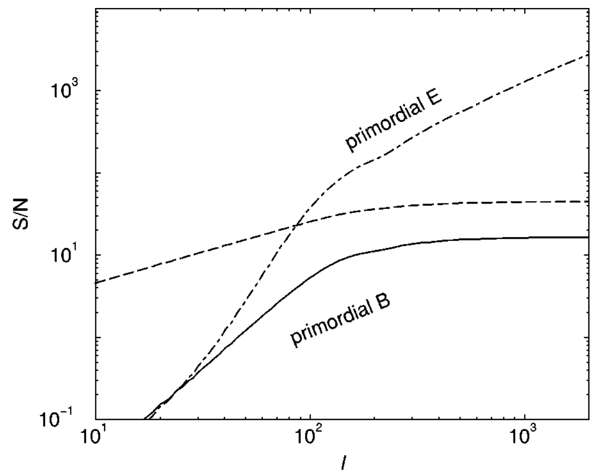


FIG. 1. Left: The CMB polarization B modes power spectra. We show the error bars associated with reconstruction of the primordial power spectrum using no-noise E and B maps and a tracer of the large scale structure involving lensing potentials. For comparison, the dotted line shows the contaminant contribution arising from the transfer of power from primordial E to B modes via lensing deflections. Right: The cumulative signal-to-noise ratios for the reconstruction of the primordial power spectra. The long-dashed line is the signal-to-noise ratio associated with the detection of a B mode power spectrum, under the assumption that contaminant lensing contribution can be treated as a source of noise.

$$W(\mathbf{l},\mathbf{l}_1)=\frac{[C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}\cos 2(\varphi_{\mathbf{l}}-\varphi_{\mathbf{l}_1})+C^{X\phi}_{l_1}\mathbf{l}_1\cdot\mathbf{l}\cos 2(\varphi_{\mathbf{l}}-\varphi_{|\mathbf{l}-\mathbf{l}_1|})]}{[C^{B,\mathrm{t}}_{l_1}C^{X,\mathrm{t}}_{|\mathbf{l}-\mathbf{l}_1|}+C^{B,\mathrm{t}}_{|\mathbf{l}-\mathbf{l}_1|}C^{X,\mathrm{t}}_{l_1}]}. \tag{17}$$

We can define similar filters for the extraction of $\tilde{E}$ from (BX) or (EX) estimators as well. In analogous to above, these filters involve the noise contributions in the (BX) and (EX) maps and the behavior of the function that integrates over the filter in each of the integrals in Eqs. (11)–(14). Since each estimate of $C_l^{\tilde{E}\tilde{E}}$ and $C_l^{\tilde{B}\tilde{B}}$ comes from two independent statistics involving (EX) and (BX), we add the individual signal-to-noise estimates of the two. This is equivalent to weighing individual noise contributions inversely.

In writing Eqs. (5) and (6), we have only used the expansion of ϕ in Eq. (1) to the first order. The next term involved, which is proportional to $\int d^2\mathbf{l}_1'\phi(\mathbf{l}_1')\phi^*(\mathbf{l}_1'+\mathbf{l}_1-\mathbf{l})f(\mathbf{l},\mathbf{l}_1,\mathbf{l}_1')W(\mathbf{l},\mathbf{l}_1)$—when combined with the tracer field X—leads to a term that is effectively the bispectrum formed by two ϕ terms and one X term. This higher order correction, say, is proportional to $\int d^2\mathbf{l}_1' B_{\phi\phi X}(\mathbf{l}_1',-\mathbf{l}_1'-\mathbf{l}_1+\mathbf{l},-\mathbf{l}+\mathbf{l}_1)f(\mathbf{l},\mathbf{l}_1,\mathbf{l}_1')W(\mathbf{l},\mathbf{l}_1)$ and is considerably smaller than the dominant term proportional to, say, $C^{X\phi}_{|\mathbf{l}-\mathbf{l}_1|}g(\mathbf{l},\mathbf{l}_1)W(\mathbf{l},\mathbf{l}_1)$ for two reasons. Though large scale structure at low redshifts is highly non-Gaussian, we can ignore higher order corrections to this calculation, such as the one above due to the bispectrum formed by ϕ and X, since lensing effect on CMB traces large scale structure at redshifts of 2 to 3 at scales where fluctuations are still linear. Thus, higher order correlations such as $\langle\phi\phi X\rangle$ are significantly smaller than the dominant term involving $\langle\phi X\rangle$.

Another important reason why such corrections are small arises from the product of the mode coupling term captured by $f(\mathbf{l}_1,\mathbf{l}_1',\mathbf{l}_1'')$, involving dot products between the three $\mathbf{l}$ vectors and the filter function, $W(\mathbf{l},\mathbf{l}_1)$. When integrated over $\mathbf{l}_1'$ and $\mathbf{l}_1$, the combination of $f(\mathbf{l}_1,\mathbf{l}_1',\mathbf{l}_1'')W(\mathbf{l},\mathbf{l}_1)$ leads to significant cancellations, since the associated dot products between length vectors lead to positive and negative oscillations which are summed over. As we find later, in the case of the dominant first order term of ϕ involving $C_l^{X\phi}$, the associated dot products between length vectors, when considering the combination of $g(\mathbf{l},\mathbf{l}_1)W(\mathbf{l},\mathbf{l}_1)$ following the above discussion, come in as squared quantities such that cancellations are avoided. Overall, the situation here is similar to the non-Gaussian noise contributions to the lensing reconstruction discussed in Ref. [9]; the additional corrections are suppressed under the dominant Gaussian behavior of the potential field that lenses CMB as well as the asymmetry of the filter function when compared to mode coupling involved in higher order correction terms.

III. DISCUSSION

We summarize our main results in Fig. 1 for the extraction of primordial polarization power spectra based on information related to lensing modification to CMB anisotropies. For the reconstruction, we assume a gravitational-wave contribution to B modes with a tensor-to-scalar ratio of 0.25 and assume no instrumental-noise contributions to polarization observations today. When calculating noise, we include additional secondary noise contribution from lensing to E and B maps. For simplicity, we also take a tracer field, X, that corresponds to lensing potentials themselves. Thus signal-to-noise ratios shown in Fig. 1 should be considered as optimistic values. Any noise contribution to polarization observations and a tracer field which is less correlated with ϕ will lead to a degradation in the signal-to-noise ratios of primordial polarization reconstruction from the ones shown here.

With $X=\phi$, the assumption here is that one can use a properly cleaned CMB temperature map to reconstruct lensing deflections following the approach in Refs. [8,9] and apply such a construction to polarization observations to reduce contamination from lensing to gravitational-wave detection. We include a noise contribution to the X field, when calculating $C_l^{X,\mathrm{t}}$, following the noise calculation for ϕ extracted from a no instrumental-noise CMB temperature map in Refs. [8,9]. Additional possibilities for X include a frequency cleaned SZ map, a map of the convergence from large scale structure lensing observations, and the galaxy distribution from surveys such as Sloan but imaged out to a higher redshift; since contributions to lensing in CMB comes at redshifts of a few and more, in general, a tracer field out to redshifts of $\sim 3-5$ will be required to appropriately apply this method.

As suggested, the primordial polarization construction is more important for B modes, given that they contain the distinct signature of gravitational waves, than E modes. For the extraction of the B-mode power spectrum, we estimate a cumulative signal-to-noise ratio of ~ 16. The cumulative signal-to-noise ratio for the detection of this contribution, under the assumption that the contaminant lensing contribution can be treated as a known source of noise, is of order ~ 40. The suggested reconstruction here leads to a roughly a factor of ~ 2.5 decrease in the associated signal-to-noise ratio. The reconstruction, however, has the advantage that one does not need to make any assumptions on the lensing contribution to the power spectrum of B modes. Relaxing our assumption of a tensor-to-scalar ratio of 0.25, we find that the suggested extraction can be used to separate primordial B modes with at least a signal-to-noise ratio of 1 when the tensor-to-scalar ratio is of order 0.01. In the case of Planck, one can use its temperature data to construct a ϕ estimator which can then be used to clean Planck's polarization maps. In this case, we determined a cumulative signal-to-noise ratio of less than 10^{-2} for the detection of primordial B modes. This low signal-to-noise ratio is dominated by a combination of the noise-contribution in the Planck data and the fact that Planck's poor resolution of the temperature map does not allow a reconstruction of the lensing signal down to arcminute scales required for an analysis such as the one suggested here. In an improved scenario, relative to Planck, we require temperature observations down to 1 to 2 arcminute scale, from Planck's 7–10 arc min, with at least a factor of 100 better noise in polarization maps.

Though we have not separated individual signal-to-noise ratio values, the reconstruction of B modes has less noise with the combination of the (BX) statistic correlated with a map of B modes instead of the (EX) correlated with B modes. This is because the dominant noise associated with E modes in (EX) limits the reconstruction of $\tilde{B}$ modes significantly. The signal-to-noise ratio values for the reconstruction of $\tilde{E}$ modes have similar values whether (EX) or (BX) is used. This reconstruction has a cumulative signal-to-noise ratio in excess of 10^3 both due to its high contribution and the fact that the $\tilde{E}$-mode power spectrum peaks at arcminute scales compared to the $\tilde{B}$ mode power spectrum which peaks at degree scales.

It is clear that the reconstruction analysis can be best done for the extraction of primordial $\tilde{E}$ modes. The low signal-to-noise ratio values for $\tilde{B}$ modes, however, should not discourage one from attempting to do an analysis like the one suggested here in upcoming CMB data. The ultimate, and challenging, detection of gravitational wave signature, which, in principle, gives a handle on the inflationary energy scale requires various statistical techniques so that this contribution can be identified confusion free. Since lensing contribution dominates the B-mode signal, we have exploited its use as a possible way to extract the primordial polarization field. Given the importance of detecting gravitational waves for future cosmological studies, statistics such as the one discussed are clearly warranted for further studies.

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