Supplemental Material (not copyedited or formatted) for: Michael Gibilisco. 2021. "Decentralization, Repression, and Gambling for Unity." The Journal of Politics 83(4). DOI: https://doi.org/10.1086/711626.

Online Appendix

A Continuation Values and Expected Utilities

Let $\bar{V}_i$ denote i's continuation value after a history in which the Periphery was won control of the territory. These values are independent of a strategy profile σ and take the form $\bar{V}_C = 0$ and $\bar{V}_P = \frac{\pi_P^P}{1-\delta}$.

Let $V_i^\sigma(g)$ denotes i's continuation value from beginning the game with grievance g when the Periphery has not won control of its territory and actors subsequently playing according to profile σ. In a similar vein, $U_C^\sigma(r; g)$ and $U_P^\sigma(m; g)$ denote the Center and Periphery's dynamic payoffs from choosing $r \in \{\emptyset, 0, 1\}$ and $m \in \{0, 1\}$ given grievance g when actors subsequently play according to profile σ. For the Center, $U_C^\sigma(r; g)$ takes the following form:

$$U_C^\sigma(r; g) = \begin{cases} 0 & \text{if } = \emptyset \\ \pi_C^C - \kappa_C + \delta V_C^\sigma(g+1) & \text{if } r = 1 \\ -\sigma_P(g)F(g)\psi + (1 - \sigma_P(g)F(g))\left(\pi_C^C + \delta V_C^\sigma(\max\{g-1, 0\})\right) & \text{if } r = 0. \end{cases}$$

For the Periphery, $U_P^\sigma(m;g)$ denotes the its dynamic payoff conditional on having reached its decision node, i.e., the Center chooses $r = 0$, in state g. Thus, $U_P^\sigma(m;g)$ takes the form

$$U_P^\sigma(m;g) = \begin{cases} -\kappa_P + F(g)\bar{V}_P + (1 - F(g))\left(\pi_P^C + \delta V_P^\sigma(\max\{g-1,0\})\right) & \text{if } m = 1 \\ \pi_P^C + \delta V_P^\sigma(\max\{g-1,0\}) & \text{if } m = 0. \end{cases} \tag{5}$$

With this notation in hand, the next definition states the equilibrium conditions.

Definition 1 *Strategy profile σ is an equilibrium if the following hold:*

$$\begin{aligned} \sigma_C(r;g) > 0 &\implies U_C^\sigma(r;g) \geq U_C^\sigma(r';g), \\ \sigma_P(g) > 0 &\implies U_P^\sigma(1;g) \geq U_P^\sigma(0;g), \text{ and} \\ \sigma_P(g) < 1 &\implies U_P^\sigma(0;g) \geq U_P^\sigma(1;g) \end{aligned}$$

for all grievance g and polices $r, r' \in \{\varnothing, 0, 1\}$.

Because the game is a dynamic game with a countable state space and a finite number of actions, an equilibrium from Definition 1 exists in mixed strategies. Notice that for some grievance g, the Center's continuation value, $V_C^\sigma(g)$, takes the form

$$V_C^\sigma(g) = \sum_{r\in\{\varnothing,0,1\}} \sigma(r;g) U_C^\sigma(r;g).$$

Thus, if σ is an equilibrium and $\sigma(r;g) > 0$ for some grievance g and action $r \in \{\varnothing, 0, 1\}$, then $V_C^\sigma(g) = U_C^\sigma(r;g)$ or else C has a deviation by playing some $r' \in \{\varnothing, 0, 1\}$.

B Proof of Proposition 1

Proposition 1 *If grievances are small, then the Periphery never mobilizes, the Center neither represses nor grants independence, and grievances dissipate on the equilibrium path. That is, $g \leq g^-$ implies $\sigma_P(g) = 0$ and $\sigma_C(0;g) = 1$ in every equilibrium σ.*

Proof. The proof that $g \leq g^-$ implies the Periphery does not mobilize with positive probability is covered in the main text. We prove that $g \leq g^-$ implies the Center does not repress or grant independence with positive probability. To see this, suppose $\sigma_C(r;g) > 0$ for some $g \leq g^-$, $r \neq 0$, and equilibrium σ. There are two cases.

Case 1: $r = 1$, repression. Then, C's expected utility is

$$\begin{aligned} U_C^\sigma(1; g) &= \pi_C - \kappa_C + \delta V_C^\sigma(g+1) \\ &\leq \pi_C - \kappa_C + \delta \frac{\pi_C^C}{1-\delta} \\ &< \frac{\pi_C^C}{1-\delta}. \end{aligned}$$

However, $\frac{\pi_C^C}{1-\delta}$ is C's continuation value if it takes action $r = 0$ in all future periods because grievances will never increase and P will never mobilize with positive probability along the subsequent path of play. Hence, taking action $r = 0$ in all future periods is a profitable deviation, a contradiction.

Case 2: $r = \emptyset$, independence. Then, C's expected utility is

$$U_C^\sigma(\emptyset; g) = 0 < \frac{\pi_C^C}{1-\delta}.$$

As in Case 1, this inequality implies taking action $r = 0$ in all future periods is a profitable deviation, a contradiction. □

C Properties of $\tilde{V}_C$

We first state and prove three Lemmas concerning properties of $\tilde{V}_C$.

Lemma 1 *1.* $\tilde{V}_C(g) > \frac{-F(g)\psi + (1-F(g))\pi_C^C}{1-(1-F(g))\delta}$ *for all g such that $F(g) > 0$.*

2. $\tilde{V}_C(g-1) > \tilde{V}_C(g)$ *for all $g > g^-$.*

3. If Assumption 1 holds, then $\lim_{g\to\infty} \tilde{V}_C(g) = \frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta}$.

Proof. To show (1), consider some g such that $F(g) > 0$ and $F(g') = 0$ for all $g' < g$. Such a g exists because $F(0) = 0$ and $\lim_{g\to\infty} F(g) = p > 0$. In addition, $F(g) < 1$ because there exists at least one g such that $F(g) \in (0, 1)$ by assumption. Then we have

$$\begin{aligned} \tilde{V}_C(g) &= -F(g)\psi + (1-F(g))\left(\pi_C^C + \delta \frac{\pi_C^C}{1-\delta}\right) \\ &= (1-(1-F(g))\delta)\frac{-F(g)\psi + (1-F(g))\pi_C^C}{1-(1-F(g))\delta} + (1-F(g))\delta \frac{\pi_C^C}{1-\delta} \\ &> \frac{-F(g)\psi + (1-F(g))\pi_C^C}{1-(1-F(g))\delta}, \end{aligned}$$

Supplemental Material (not copyedited or formatted) for: Michael Gibilisco. 2021. "Decentralization, Repression, and Gambling for Unity." The Journal of Politics 83(4). DOI: https://doi.org/10.1086/711626.

where the strict inequality follows because $F(g) \in (0, 1)$

For induction, consider some g such that $F(g) > 0$ and $F(g-1) > 0$, which implies $g - 1 > 0$. Suppose the inequality holds for all $g' < g$ such that $F(g') > 0$. Then we have

$$\begin{aligned}
\tilde{V}_C(g) &= -F(g)\psi + (1 - F(g))(\pi_C^C + \delta \tilde{V}_C(g-1)) \\
&> -F(g)\psi + (1 - F(g))\left(\pi_C^C + \delta \frac{-F(g-1)\psi + (1 - F(g-1))\pi_C^C}{1 - (1 - F(g-1))\delta}\right) \\
&\geq -F(g)\psi + (1 - F(g))\left(\pi_C^C + \delta \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta}\right) \\
&= \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta},
\end{aligned}$$

where the third line follows because the fraction $\frac{-F(g)\psi + (1-F(g))\pi_C^C}{1-(1-F(g))\delta}$ is decreasing in $F(g)$.

To show (2), note that it must hold when $g = g^- + 1$, because $\psi > 0$ and $F(g) > 0$ as $g > g^-$. Now consider some $g > g^- + 1$. For induction, suppose $\tilde{V}_C(g' - 1) > \tilde{V}_C(g')$ for all g' such that $g^- < g' < g$. Then

$$\begin{aligned}
\tilde{V}_C(g) &= -F(g)\psi + (1 - F(g))(\pi_C^C + \delta \tilde{V}_C(g-1)) \\
&\leq -F(g-1)\psi + (1 - F(g-1))(\pi_C^C + \delta \tilde{V}_C(g-1)) \\
&< -F(g-1)\psi + (1 - F(g-1))(\pi_C^C + \delta \tilde{V}_C(g-2)) \\
&= \tilde{V}_C(g-1),
\end{aligned}$$

where the second line follows because

$$\tilde{V}_C(g) > \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta} \geq -\psi$$

and $F(g)$ is increasing in g.

To prove (3), consider a sequence $\{g_n\}_{n=1}^{\infty}$ such that $\lim_{n\to\infty} g_n = \infty$ and $g_n < g_{n+1}$. Then the sequence $\left\{\tilde{V}_C(g_n)\right\}_{n=1}^{\infty}$ is weakly decreasing due to above arguments. In addition, $\left\{\tilde{V}_C(g_n)\right\}_{n=1}^{\infty}$ is bounded below because C's payoffs are finite and C discounts with rate $\delta < 1$. Thus, $\left\{\tilde{V}_C(g_n)\right\}_{n=1}^{\infty}$ has a limit, call it L. If the Periphery does value independence, the we have

$$\begin{aligned}
L &= \lim_{n\to\infty} \tilde{V}_C(g_n) \\
&= \lim_{n\to\infty} F(g_n)(-\psi) + \lim_{n\to\infty}(1 - F(g_n))(\pi_C^C + \delta \tilde{V}_C(g_n - 1)) \\
&= -p\psi + (1 - p)\left(\pi_C^C + \delta L\right),
\end{aligned}$$

which implies $L = \frac{-p\psi+(1-p)\pi_C^C}{1-(1-p)\delta}$. □

The next Lemma demonstrates that C's gambling for unity utility, $\tilde{V}_C$ is a lower bound on its equilibrium expected utility, V_C^σ.

Lemma 2 *For all grievances g, $V_C^\sigma(g) \geq \tilde{V}_C(g)$ in every equilibrium σ.*

Proof. To see this, suppose not. That is, suppose there exist grievance g and equilibrium σ such that $V_C^\sigma(g) < \tilde{V}_C(g)$. Then by the construction of $\tilde{V}_C$ and Proposition 1, $g > g^-$, or else $V_C^\sigma(g) = \frac{\pi_C^C}{1-\delta} = \tilde{V}_C$.

Next consider a deviation for C, labeled σ'_C, such that $\sigma'_C(0; g') = 1$ for all $g' \leq g$. I now demonstrate that $V_C^{\sigma'}(g) \geq \tilde{V}_C(g)$, where $\sigma' = (\sigma'_C, \sigma_P)$, which implies σ'_C is a profitable deviation because $\tilde{V}_C(g) > V_C^\sigma(g)$ by supposition.

The proof is by induction. The inequality, $V_C^{\sigma'}(g') \geq \tilde{V}_C(g')$, holds when $g' \leq g^-$ by the construction of $\tilde{V}_C$ and Proposition 1. Now consider some $g' > g^-$ and suppose $V_C^{\sigma'}(g'') \geq \tilde{V}_C(g'')$ for all $g'' < g'$. Then we have

$$\begin{aligned}
V_C^{\sigma'}(g') &= -\sigma_P(g')F(g')\psi + (1 - \sigma_P(g')F(g'))\left(\pi_C^C + \delta V_C^{\sigma'}(g' - 1)\right) \\
&\geq -\sigma_P(g')F(g')\psi + (1 - \sigma_P(g')F(g'))\left(\pi_C^C + \delta \tilde{V}_C(g' - 1)\right) \\
&\geq -F(g')\psi + (1 - F(g'))\left(\pi_C^C + \delta \tilde{V}_C(g' - 1)\right) \\
&= \tilde{V}_C(g').
\end{aligned}$$

Hence, $V_C^{\sigma'}(g) \geq \tilde{V}_C(g)$ as required. □

The final Lemma demonstrates that the cutpoint g^+ exists if and only if Assumptions 1 and 2 hold.

Lemma 3 *The cutpoint g^+ solving Equation (3) exists if and only if the Periphery values independence (Assumption 1) and secession is costly (Assumption 2).*

Proof. For necessity, suppose Assumptions 1 and 2 hold. Then Lemma 1 and Assumption 1 imply that $\tilde{V}_C(g)$ is weakly decreasing in g and converges to

$$\lim_{g\to\infty} \tilde{V}_C(g) = \frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta}.$$

Because $\tilde{V}_C(g) = \frac{\pi_C^C}{1-\delta} > 0$ for all $g \leq g^-$ and $\tilde{V}_C(g)$ is strictly decreasing in g when $g > g^-$, we require

$$\frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta} < \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}. \tag{6}$$

We now demonstrate that the inequality in Equation (6) holds when $\pi_C^C > \kappa_C$, the proof when $\pi_C^C < \kappa_C$ is identical. Suppose $\pi_C^C - \kappa_C > 0$. Then Equation (6) reduces to

$$\frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta} < \frac{\pi_C^C - \kappa_C}{1-\delta},$$

which is equivalent to

$$\psi > \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)}.$$

Because $\pi_C^C - \kappa_C > 0$, Assumption 2 reduces to

$$\psi > \min\left\{\frac{\pi_C^C(1-p)}{p}, \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)}\right\} = \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)}.$$

Thus, the inequality in Equation (6) holds, and therefore g^+ exists.

For sufficiency, suppose Assumption 1 does not hold, then $\kappa_P \geq F(g)\frac{\pi_P^P - \pi_P^C}{1-\delta}$ for all grievances g. Thus, $\tilde{V}_C(g) = \frac{\pi_C^C}{1-\delta} > \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}$ for all grievances g. Now suppose Assumption 1 holds but not Assumption 2. Then Lemma 1 implies that, for all g

$$\tilde{V}_C(g) \geq \frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta} \geq \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}. \qquad \square$$

D Preliminary Results

In this section, we state and prove two technical results that are essential to characterize equilibria in the remainder of the paper.

Lemma 4 *If $\sigma_C(1; g) > 0$ and $\sigma_C(0; g+1) = 1$ for some grievance g, then $\sigma_P(g+1) < 1$ in every equilibrium σ.*

Supplemental Material (not copyedited or formatted) for: Michael Gibilisco. 2021. "Decentralization, Repression, and Gambling for Unity." The Journal of Politics 83(4). DOI: https://doi.org/10.1086/711626.

Proof. Suppose not. Then there exists a g such that $\sigma_C(1; g) > 1$, $\sigma_C(0; g+1) = 1$ and $\sigma_P(g+1) = 1$ in equilibrium σ. We can write $V_C^\sigma(g+1)$ as

$$\begin{aligned} V_C^\sigma(g+1) &= -F(g+1)\psi - (1 - F(g+1))\left(\pi_C^C + \delta V_C^\sigma(g)\right) \\ &= -F(g+1)\psi - (1 - F(g+1))\left(\pi_C^C + \delta U_C^\sigma(1; g)\right) \\ &= -F(g+1)\psi - (1 - F(g+1))\left(\pi_C^C + \delta\left(\pi_C^C - \kappa_C + \delta V_C^\sigma(g+1)\right)\right). \end{aligned}$$

Solving reveals that

$$V_C^\sigma(g+1) = \frac{(1 - F(g+1))(\pi(1+\delta) - \delta\kappa_C) - F(g+1)\psi}{1 - (1 - F(g+1))\delta^2}.$$

By Lemma 2, $V_C^\sigma(g+1) \geq \tilde{V}_C(g+1)$. By Lemma 1.1,

$$\tilde{V}_C(g) > \frac{(1 - F(g+1))\pi_C^C - F(g+1)\psi}{1 - (1 - F(g+1)\delta)}.$$

Stringing these two inequalities together,

$$V_C^\sigma(g+1) > \frac{(1 - F(g+1))\pi_C^C - F(g+1)\psi}{1 - (1 - F(g+1)\delta)}.$$

Substituting the closed form solution for $V_C^\sigma(g+1)$ into the inequality above and solving for κ_C reveals that

$$\kappa_C < \frac{F(g+1)(\pi_C^C + \psi(1-\delta))}{1 - (1 - F(g+1))\delta}.$$

To derive a contradiction, consider a deviation in which C plays $r = 1$ with probability 1 in all future periods beginning at grievance $g+1$. This is a profitable deviation if and only if

$$V_C^\sigma(g+1) < \frac{\pi_C^C - \kappa_C}{1-\delta} \iff \kappa_C < \frac{F(g+1)(\pi_C^C + \psi(1-\delta))}{1 - (1 - F(g+1))\delta}.$$

However, $\kappa_C < \frac{F(g+1)(\pi_C^C + \psi(1-\delta))}{1-(1-F(g+1))\delta}$ as shown above. Hence, C can profitably deviate by repressing in all future periods. □

Lemma 5 *Consider some $g > g^-$ and equilibrium σ. If (a) $\sigma_C(0; g-1) = 1$ or $\sigma_C(0; g) = 1$ and (b) $\sigma_C(\emptyset; g') = 0$ for all $g' < g$, then $\sigma_P(g) = 1$.*

Proof. Suppose not. That is, consider some equilibrium σ and grievance $g > g^-$ such that

(a) $\sigma_C(0; g-1) = 1$ or $\sigma_C(0; g) = 1$,

(b) $\sigma_C(\emptyset; g') = 0$ for all $g' < g$, and

(c) $\sigma_P(g) < 1$.

Because σ is an equilibrium, we require $U_P^\sigma(0; g) \geq U_P^\sigma(1; g)$ to rule out profitable deviations, which is equivalent to

$$\kappa_P \geq F(g)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g-1)\right].$$

Because $\sigma_C(0; g-1) = 1$ or $\sigma_C(0; g) = 1$, the path of play will never reach a grievance larger than g. Because $\sigma_C(\emptyset; g') = 0$ for all $g' \leq g$, the Center will never grant independence along the subsequent path of play. Recall that when the C represses, P stage payoff is π_P^C, which is its payoff if it chooses not to mobilize, and even if C does repress with positive probability at some $g' < g$, the subsequent path of play will still never reach a grievance larger than g. Then $g > g^-$ implies $V_P^\sigma(g-1)$ is bounded above by

$$\frac{F(g)\bar{V}_P + (1 - F(g))\pi_P^C - \kappa_P}{1 - (1 - F(g))\delta},$$

which is P's payoff if its grievance never depreciates along the path of play, C never represses, and P always mobilizes. Combining these two inequalities, we require

$$\begin{aligned}\kappa_P &\geq F(g)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g-1)\right] \\ &\geq F(g)\left[\bar{V}_P - \pi_P^C - \delta\frac{F(g)\bar{V}_P + (1 - F(g))\pi_P^C - \kappa_P}{1 - (1 - F(g))\delta}\right].\end{aligned}$$

Solving for κ_P implies

$$\kappa_P \geq F(g)\frac{\pi_P^P - \pi_P^C}{1 - \delta},$$

that is, $g \leq g^-$. But this contradicts the assumption $g > g^-$. □

E Proof of Proposition 2

This section characterizes equilibrium behavior at moderate grievances.

We now prove that $g < g^+$ implies $\sigma_C(0; g) = 1$ in every equilibrium σ, that is, the Center neither represses nor grants independence with moderate grievances. The result requires preliminary lemmas. Notice that if either Assumption 1 or 2 does not hold, $\tilde{V}_C(g) > \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}$ for all g, and we can set $g^+ = \infty$ in the subsequent results.

Lemma 6 *If $g < g^+$, then $\sigma_C(\emptyset; g) = 0$ in every equilibrium σ.*

Proof. If not, then $V_C^\sigma(g) = U_C^\sigma(\emptyset; g) = 0$. If $g < g^+$, this contradicts Lemma 2 because $\tilde{V}_C(g) > 0 = V_C^\sigma(g)$. □

Lemma 7 *For all g, $\sigma(r; g) > 0$ imply $\sigma(\emptyset; g+1) = 0$ in every equilibrium σ.*

Proof. First, if $\kappa_C < \pi_C^C$, then C cannot grant independence with positive probability in any equilibrium. Doing so would result in a payoff of 0, but C could repress for all future periods, giving a payoff of $\frac{\pi_C^C - \kappa_C}{1-\delta} > 0$. Thus, consider the case where $\pi_C^C - \kappa_C < 0$. Suppose $\sigma_C(r; g) > 0$ for some g and $\sigma_C(\emptyset; g+1) > 0$. Then

$$\begin{aligned} V_C^\sigma(g) &= U_C^\sigma(r; g) \\ &= \pi_C^C - \kappa_C + \delta V_C^\sigma(g+1) \\ &= \pi_C^C - \kappa_C + \delta U_C^\sigma(\emptyset; g) \\ &= \pi_C^C - \kappa_C < 0, \end{aligned}$$

but this means C can profitably deviate at g by granting independence, i.e., σ is not an equilibrium. □

Lemma 8 *Fix an equilibrium σ. Then there does not exist a $g < g^+$ such that $\sigma_C(1; g') > 0$ for all $g' \geq g$.*

Proof. Suppose not and consider such a $g < g^+$ where $\sigma_C(1; g') > 0$ for all $g' \geq g$ in equilibrium σ. Then

$$V_C^\sigma(g) = U_C^\sigma(1; g) = \pi_C^C - \kappa_C + \delta V_C^\sigma(g+1).$$

Because $V_C(g') = U_C^\sigma(r; g')$ for all g' such that $\sigma_C(r; g') > 0$, similar substitutions imply $V_C^\sigma(g) = \frac{\pi_C^C - \kappa_C}{1-\delta}$. However, $g < g^+$ implies

$$\tilde{V}_C(g) > \frac{\pi_C^C - \kappa_C}{1-\delta} = V_C^\sigma(g),$$

by Equation(3). However, $\tilde{V}_C(g) > V_C^\sigma(g)$ contradicts Lemma 2. □

With these lemmas in hand, we now state the main result of the section.

Proposition 2 *If grievances are moderate, then the Periphery always mobilizes, the Center neither represses nor grants independence, and grievances dissipate on the equilibrium path. That is, $g \in (g^-, g^+)$ implies $\sigma_P(g) = 1$ and $\sigma_C(0; g) = 1$ in every equilibrium σ.*

Proof. We first prove prove that $\sigma_C(0; g) = 1$ when $g \in (g^-, g^+)$ and σ is an equilibrium. Suppose not. By Lemma 6, $\sigma_C(1; g) > 0$. Furthermore, C represses with positive probability for at most some finite k periods by Lemma 8. That is, there exists a $\bar{g}$ such that $\sigma_C(1; g') > 0$ for $g' = g, ..., \bar{g}$ and $\sigma_C(1; \bar{g}+1) = 0$. By Lemma 7, this implies $\sigma_C(0; \bar{g}+1) = 1$. In addition, Proposition 1 and Lemma 7 imply $\sigma_C(\varnothing; g') = 0$ for all $g' < \bar{g}$. Thus, Lemma 5 and $\sigma_C(1; \bar{g}+1) = 0$ imply P mobilizes at $\bar{g}+1$ with probability 1. However, $\sigma_C(1; \bar{g}) > 0$, $\sigma_C(0; \bar{g}+1) = 1$, and $\sigma_P(\bar{g}+1) = 1$ contradict Lemma 4. To pin down P's strategy at $g \in (g^-, g^+)$, note that $\sigma_C(0; g') = 1$ for all $g' < g^+$. Then Lemma 5 implies $\sigma_P(g) = 1$. □

F Proof of Proposition 3

We now characterize equilibrium behavior at large grievances ($g \geq g^+$). We consider the generic case in which there does not exist $g \in \mathbb{N}_0$ such that $\tilde{V}_C(g) = \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}$, that is $\tilde{V}_C(g^+) < \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}$, where the inequality from Equation (3) holds strictly. If this held with equality, the Center would be indifferent leading to trivial indeterminacy. We consider high- and low-capacity regimes separately because the proof techniques vary dramatically between the two cases.

F.1 High repression capacity: $\kappa_C < \pi_C^C$

Lemma 9 *In high-capacity regimes, $\sigma_C(\varnothing; g) = 0$ for every grievance g and in every equilibrium σ.*

The proof is straightforward and omitted.

Lemma 10 *In high-capacity regimes, $\sigma_C(1; g^+) = 1$ and $\sigma_C(1; g) > 0$ for all $g > g^+$ in every equilibrium σ.*

Proof. The proof is by induction. First, we demonstrate that $\sigma_C(1; g^+) = 1$. To see this, suppose $\sigma_C(1; g^+) < 1$. Then Lemma 9 implies $\sigma_C(0; g^+) > 0$, in which case we have

$$U_C^\sigma(0; g^+) = \tilde{V}_C(g^+) < \frac{\pi_C^C - \kappa_C}{1-\delta}.$$

This means C can profitably deviate at grievance g^+ by repressing for an infinite number of periods, a contradiction.

For induction, consider some $g > g^+$ and assume $\sigma_C(1; g-1) > 0$. To derive a contradiction, assume $\sigma_C(1; g) = 0$. By Lemma 9, $\sigma_C(0; g) = 1$. Likewise, Lemma 9 guarantees C does not grant independence in any equilibrium, so Lemma 5 implies P mobilizes at g with probability 1. But then this contradicts Lemma 4. □

Lemma 11 *In high-capacity regimes, $g \geq g^+$ implies $V_C^\sigma(g) = \frac{\pi_C^C - \kappa_C}{1-\delta}$ in every equilibrium σ.*

Proof. If $g \geq g^+$, then Lemma 10 implies $\sigma_C(1; g') > 0$ for all $g' \geq g$. The remainder of the proof follows from an identical argument as the one in Lemma 8. □

Lemma 12 *In high-capacity regimes, $g > g^+$ and $\sigma_C(0; g) > 0$ imply $\sigma_P(g) < 1$ in every equilibrium σ.*

Proof. Suppose not. Then there exists $g > g^+$ such that $\sigma_C(0; g) > 0$ and $\sigma_P(g) = 1$. Because $g > g^+$, $g - 1 \geq g^+$. Likewise, $\sigma_C(1; g) > 0$ by Lemma 10, so it must be the case that $U_C^\sigma(0; g) = U_C^\sigma(1; g)$. Then we have

$$\begin{aligned} U_C^\sigma(0; g) = U_C^\sigma(1; g) &\iff -F(g)\psi + (1 - F(g))(\pi_C^C + \delta V_C^\sigma(g-1)) = \pi - \kappa_C + \delta V_C^\sigma(g+1) \\ &\iff -F(g)\psi + (1 - F(g))\left(\pi_C^C + \delta \frac{\pi - \kappa_C}{1-\delta}\right) = \frac{\pi - \kappa_C}{1-\delta} \\ &\iff \kappa_C = \frac{F(g)(\pi_C^C + (1-\delta)\psi)}{1(1 - F(g))\delta}, \end{aligned}$$

where we use Lemma 11 and $g - 1 \geq g^+$ to substitute for values $V_C^\sigma(g-1)$ and $V_C^\sigma(g+1)$.

Because σ is an equilibrium, we require $U_C^\sigma(1; g) = V_C^\sigma(g) \geq \tilde{V}_C(g)$, by Lemma 2. Then Lemma 1.1 implies

$$\begin{aligned} U_C^\sigma(1; g) > \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta} &\iff \frac{\pi_C^C - \kappa_C}{1-\delta} > \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta} \\ &\iff \kappa_C < \frac{F(g)(\pi_C^C + (1-\delta)\psi)}{1(1 - F(g))\delta}, \end{aligned}$$

which establishes the desired contradiction. □

Lemma 13 *In high-capacity regimes, there exists cutpoint $\bar{g} \in \mathbb{R}$ such that if $g > \bar{g}$, then $\sigma_P(g) = 1$ and $\sigma_C(1; g) = 1$ in every equilibrium σ.*

Proof. The proof is constructive. Define $\bar{g} \in \mathbb{N}_0$ to be a number that satisfies

$$g \geq \bar{g} \implies \kappa_P < F(g)\left[\bar{V}_P - \pi_P^C - \delta \frac{p\bar{V}_P + (1-p)\pi_P^C - \kappa_P}{1 - (1-p)\delta}\right].$$

Such a $\bar{g}$ exists because $F(g)\left[\bar{V}_P - \pi_P^C - \delta\frac{p\bar{V}_P+(1-p)\pi_P^C}{1-(1-p)\delta}\right]$ is positive and strictly increasing in g. Furthermore,

$$\lim_{g\to\infty} F(g)\left[\bar{V}_P - \pi_P^C - \delta\frac{p\bar{V}_P + (1-p)\pi_P^C}{1-(1-p)\delta}\right] = p\frac{\pi_P^P - \pi_P^C}{1-\delta},$$

and Assumption 1 implies

$$\kappa_P < p\frac{\pi_P^P - \pi_P^C}{1-\delta}.$$

We first show that $\sigma_P(g) = 1$ for $g \geq \bar{g}$. Suppose not; then there exists $g \geq \bar{g}$ such that $\sigma_P(g) < 1$. To rule out profitable deviations, we require $U_P^\sigma(0;g) \geq U_P^\sigma(1;g)$, which is equivalent to

$$\kappa_P \geq F(g)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g-1)\right].$$

Because the Center never grants independence in strong regimes, $V_P^\sigma(g-1)$ is bounded above by $\frac{p\bar{V}_P+(1-p)\pi_P^C-\kappa_P}{1-(1-p)\delta}$, which is the Periphery's dynamic payoff if it mobilizes in every period at maximum capacity, p. Combining these two inequalities gives us

$$\begin{aligned}\kappa_P &\geq F(g)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g-1)\right]\\ &\geq F(g)\left[\bar{V}_P - \pi_P^C - \delta\frac{p\bar{V}_P + (1-p)\pi_P^C - \kappa_P}{1-(1-p)\delta}\right],\end{aligned}$$

but this implies $g < \bar{g}$, which is contradiction. Thus, $\sigma_P(g) = 1$. Then Lemma 10 and the contrapositive of Lemma 12 imply $\sigma_C(1;g) = 1$, as required. □

Lemma 14 *In high-capacity regimes, if $g \geq g^+$, then $\sigma_P(g) = 1$ in every equilibrium σ.*

Proof. Suppose there exists $g \geq g^+$ such that $\sigma_P(g) < 1$. Lemma 13 implies that there exists grievance $g^\dagger \geq g$ such that $\sigma_P(g^\dagger) < 1$ and $\sigma_P(g') = \sigma_C(1;g') = 1$ for all $g' > g^\dagger$. To rule out profitable deviations, we require $U_P^\sigma(0;g^\dagger) \geq U_P^\sigma(1;g^\dagger)$. This implies

$$\kappa_P \geq F(g^\dagger)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g^\dagger - 1)\right].$$

Because P will never be able to mobilize at a larger grievance than $g^\dagger$ along the path of play and C never grants independence, $V_C^\sigma(g^\dagger - 1)$ is bounded above by

$$\frac{F(g^\dagger)\bar{V}_P + (1-F(g^\dagger))\pi_P^C - \kappa_P}{1-(1-F(g^\dagger)\delta)}.$$

Supplemental Material (not copyedited or formatted) for: Michael Gibilisco. 2021. "Decentralization, Repression, and Gambling for Unity." The Journal of Politics 83(4). DOI: https://doi.org/10.1086/711626.

Then we have

$$\begin{aligned}\kappa_P &\geq F(g^\dagger)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g^\dagger - 1)\right] \\ &\geq F(g^\dagger)\left[\bar{V}_P - \pi_P^C - \delta\frac{F(g^\dagger)\bar{V}_P + (1 - F(g^\dagger))\pi_P^C - \kappa_P}{1 - (1 - F(g^\dagger)\delta)}\right] \\ &= F(g^\dagger)\frac{\pi_P^P - \pi_P^C}{1 - \delta},\end{aligned}$$

which implies $g^\dagger \leq g^- \leq g^+$, a contradiction. □

We now prove Proposition 3.1, which characterizes equilibria in regimes with large grievances when $\pi_C^C > \kappa_C$.

Proof of Proposition 3.1. If $g \geq g^+$, then Lemma 14 implies $\sigma_P(g) = 1$. Because $g > g^+$ implies $\sigma_P(g) = 1$. Lemma 10 and the contrapositive of Lemma 12 imply $\sigma_C(1; g) = 1$, as required. □

F.2 Low repression capacity: $\kappa_C > \pi_C^C$

Lemma 15 *Fix an equilibrium σ. In low-capacity regimes, the there does not exist grievance g such that $\sigma_C(1; g') > 0$ for all $g' \geq g$.*

Proof. The result follows from the inequality $\pi_C^C - \kappa_C < 0$ and the argument proving Lemma 8. □

Lemma 16 *In low-capacity regimes, $\sigma_P(g^+) = 1$, $\sigma_C(0; g^+) = 0$, and $\sigma_C(\varnothing; g^+) > 0$ in ever equilibrium σ.*

Proof. First, P mobilizes at g^+ by Lemma 5 and Proposition 2.

Second, $\sigma_C(0; g^+) = 0$. If not, then with positive probability the Center chooses to enter the path of play into moderate grievance levels. That is, $V_C^\sigma(g^+) = U_C^\sigma(0; g^+) = \tilde{V}_C(g^+)$. But then $V_C^\sigma(g^+) < 0$ because the regime has low capacity, so C can profitably deviate by granting independence at g^+.

Third, $\sigma_C(1; g^+) < 1$. To see this, suppose not, i.e., suppose $\sigma_C(1; g^+) = 1$. By Lemma 15, there exists $g^\dagger \geq g^+$ such that $\sigma_C(1; g^\dagger + 1) = 0$ and $\sigma_C(1; g^\dagger) > 0$ for all $g' = g^+, ..., g^\dagger$. Then by Lemma 7, $\sigma_C(\varnothing; g') = 0$ for all $g' = g^+, .., g^\dagger + 1$. By Proposition 2, $\sigma_C(0; g') = 1$ for all $g' < g^+$. Then Lemma 5 implies $\sigma_P(g^\dagger + 1) = 1$. However, $\sigma_C(1; g^\dagger) > 0$, $\sigma_C(0; g^\dagger + 1) = 1$, and $\sigma_P(g^\dagger + 1) = 1$ contradict Lemma 4. Thus, $\sigma_C(1; g^+) < 1$, which implies $\sigma_C(\varnothing; g^+) > 0$ by the previous paragraph. □

Before proving the last technical lemma of this section, consider the following definitions. The set $\mathcal{G} \subseteq \mathbb{N}_0$ is *an absorbing set with respect to profile* σ if once the path of play enters grievance level g such that $g \in \mathcal{G}$, it never transitions to a grievance g' such that $g' \notin \mathcal{G}$ with positive probability. The set $\mathcal{G}$ is *an irreducible absorbing set with respect to* σ if $\mathcal{G}$ is an absorbing set with respect to σ and there does not exist a proper subset $\mathcal{G}' \subsetneq \mathcal{G}$ such that $\mathcal{G}'$ is an absorbing set with respect to σ.

Lemma 17 *Consider an equilibrium σ and some grievance $g \geq g^+$. Then the following hold:*

1. *beginning at grievance g, the path of play enters an irreducible absorbing set $\mathcal{G}$ with respect to σ,*
2. $\max \mathcal{G}$ *exists,*
3. $g^+ \leq \min \mathcal{G}$*, and*
4. *there exists $g' \in \mathcal{G}$ such that $\sigma_C(\varnothing; g) > 0$.*

Proof. To prove (1), consider $g \geq g^+$ and two cases. If $\sigma_C(1; g) = 0$, then the path of play enters the set $\{g^+, ..., g\}$, which is an absorbing set because $\sigma_C(0; g^+) = 0$ by Lemma 16. So the set $\{g^+, ..., g\}$ has a irreducible absorbing set, $\mathcal{G}$. If $\sigma_C(1; g) > 0$, then Lemma 15 implies there exists $g^\dagger \geq g$ such that $\sigma_C(1; g') > 0$ for all $g' = g, ..., g^\dagger$ and $\sigma_C(1; g^\dagger + 1) = 0$ from Lemma 7. Then the path of play enters the set $\{g^+, ..., g^\dagger + 1\}$, which is an absorbing set as well.

The proof of (2) and (3) follow immediately from the existence of $\mathcal{G}$ and Lemmas 15 and 16, respectively.

To prove (4), suppose not. Suppose $\sigma_C(\varnothing; g') = 0$ for all $g' \in \mathcal{G}$. I first claim that it must be the case that $\#\mathcal{G} > 1$. Suppose the contrary. Then $\mathcal{G} = \{g'\}$, and C cannot be repressing with positive probability at g, or else $\mathcal{G}$ is not absorbing. Also, if $\mathcal{G} = \{g'\}$ and $\sigma_C(0; g') > 0$, then $F(g) = 1$ and $\sigma_P(g) = 1$ or else the path of play would transition to $g - 1$ with positive probability. In this case, $U_C(0; g') = -\psi < 0$, but this means C has a profitable deviation by granting independence at g'. Thus, $\#\mathcal{G} > 2$ and as such $\max \mathcal{G} - 1 \in \mathcal{G}$.

Second, because $\mathcal{G}$ is irreducible, $\sigma_C(1; \max \mathcal{G} - 1) > 0$, or else $\mathcal{G} \setminus \{\max \mathcal{G}\}$ would be absorbing as well. Furthermore, $\sigma_C(1; \max \mathcal{G}) = 0$ or else the path of play would transition with positive probability to $\max \mathcal{G} + 1$. Because $\sigma_C(1; \max \mathcal{G} - 1) > 0$ and $\sigma_C(1; \max \mathcal{G}) = 0$, Lemma 7 implies $\sigma_C(0; \max \mathcal{G}) = 1$ Because the path of play never leaves $\mathcal{G}$ nor transitions to grievance $g' > \max \mathcal{G}$ and C never grants independence along the path of play starting from $\max \mathcal{G}$, then $\sigma_P(\max \mathcal{G}) = 1$, which follows from an identical argument as the one in Lemma 5. However, this contradicts Lemma 4. □

The proof of Proposition 3.2 follows from Lemma 17.

G Proof of Proposition 4

First, the result in Proposition 4.1 follows immediately form Lemma 15. Second, the result in Proposition 4.2 is proved below in Lemma 18. Third, I construct an equilibrium that supports cycles of repression and mobilization, as described in Proposition 4.3, in Example 1. As part of this construction, I need a new result in Lemma 19.

Lemma 18 *If $\kappa_C > (1+\delta)\pi_C^C$, then the Center never represses in any equilibrium σ, i.e., $\sigma_C(1; g) = 0$ for every grievances g and every equilibrium σ.*

Proof. To derive a contradiction, suppose the contrary. That is, suppose $\kappa_C > (1+\delta)\pi_C^C$ and the Center represses in equilibrium σ. Thus, the regime is has low capacity, and there exist some g such that $\sigma_C(1; g) > 0$. By Lemma 15, there exists $g^\dagger \geq g$ such that $\sigma_C(1; g^\dagger + 1) = 0$ and $\sigma_C(1; g') > 0$ for all $g' = g, ..., g^\dagger$. Then by Lemma 7, $\sigma_C(\emptyset; g') = 0$ for all $g' = g+1, .., g^\dagger+1$. Hence, $\sigma_C(0; g^\dagger+1) = 1$. We can compute C's continuation value at $g^\dagger$ as

$$\begin{aligned}
V_C^\sigma(g^\dagger) &= \sigma_C(1; g^\dagger)U_C^\sigma(1; g^\dagger) + \sigma_C(0; g^\dagger)U_C^\sigma(0; g^\dagger) = U_C^\sigma(1; g^\dagger) \\
&= \pi_C^C - \kappa_C + \delta V_C^\sigma(g^\dagger + 1) \\
&= \pi_C^C - \kappa_C + \delta \Big[\sigma_P(g^\dagger + 1)\Big(-F(g^\dagger + 1)\psi + (1 - F(g^\dagger + 1))(\pi_C^C + \delta V_C^\sigma(g^\dagger)\Big) + \\
&\qquad (1 - \sigma_P(g^\dagger + 1))\Big(\pi_C^C + \delta V_C^\sigma(g^\dagger)\Big)\Big],
\end{aligned}$$

where the second equality follows because σ is an equilibrium and $\sigma_C(1; g^\dagger) > 0$. Solving for $V_C^\sigma(g^\dagger)$ reveals that

$$V_C^\sigma(g^\dagger) = \frac{\pi_C^C(1 + (1 - F(g^\dagger + 1)\sigma_P(g^\dagger + 1))\delta) - \kappa_C - F(g^\dagger + 1)\sigma_P(g^\dagger + 1)\delta\psi}{1 - (1 - \sigma_P(g^\dagger + 1)F(g^\dagger + 1))\delta^2},$$

which is decreasing in $\sigma_P(g^\dagger + 1)$. Because $\sigma_P(g^\dagger + 1) \geq 0$, then

$$V_C^\sigma(g^\dagger) \leq \frac{\pi_C^C(1 + \delta) - \kappa_C}{1 - \delta^2}$$

Thus, $\kappa_C > (1 + \delta)\pi_C^C$ implies $V_C^\sigma(g^\dagger) < 0$. But this implies C can profitably deviate at $g^\dagger$ by granting independence and guaranteeing itself a payoff of zero. □

Lemma 19 *In low-capacity regimes, if $F(g)(\pi_P^P - \pi_P^C) > \kappa_P$ and $g \geq g^+$, then $\sigma_P(g') = 1$ and $\sigma_C(1; g') = 0$ for all $g' \geq g$ in every equilibrium σ.*

Proof. By Equation (1), P mobilizes at g' if

$$\kappa_C < F(g')\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g'-1)\right].$$

An upper bound on $V_P^\sigma(g'-1)$ is $\frac{\pi_P^P}{1-\delta}$, which is the discounted sum of P's largest per-period payoff. Combining these two inequalities implies P mobilizes when $F(g')(\pi_P^P - \pi_P^C) > \kappa_P$, which holds because $F(g)(\pi_P^P - \pi_P^C) > \kappa_P$, and F is increasing.

Second, I claim that $\sigma_C(1; g') = 0$ for all $g' \geq g$. Suppose not. Then there exists a $g^\dagger$ such that $\sigma_C(1; g^\dagger) > 0$ and $\sigma_C(0; g^\dagger + 1) = 1$ by Lemmas 7 and 15. The previous paragraph demonstrates that P mobilizes with probability 1 with grievance $g^\dagger + 1$. But this contradicts Lemma 4. □

Example 1 *In this example, I assume $\pi_C^C = \pi_P^P = 1$, and $\pi_P^C = 0$. In addition, $\kappa_C = 1.2$ and $\kappa_P = .25$. This implies that the regime has low repression capacity. Finally, $\delta = .9$, $\psi = 6$, and F takes the form:*

$$F(g) = \begin{cases} 0 & \text{if } g = 0 \\ \frac{g}{700} + \frac{33}{175} & \text{if } g \geq 1 \text{ and } g \leq 8 \\ 1 & \text{otherwise.} \end{cases}$$

Thus, $g^- = 0$, and $g^+ = 7$, because $\tilde{V}_C(6) \approx .33$ and $\tilde{V}_C(7) \approx -.15$. By Proposition 2, the Periphery mobilizes with probability one for all $g \in \{1, 2, ..., 7\}$ and the Center neither represses nor grants independence for all $g \in \{0, 1, 2, ..., 6\}$. Note that $F(9)(\pi_P^P - \pi_P^C) > \kappa_P$, so Lemma 19 implies the Periphery mobilizes for all grievances $g \geq 9$ and the Center does not repress at grievance $g \geq 9$.

We specify remaining behavior as follows.

1. *At grievance $g = 7$, the Periphery mobilizes with probability 1 and the Center mixes between repression and granting independence, $\sigma_C(1; 7) + \sigma_C(\emptyset; 7) = 1$*

2. *At grievance $g = 8$, the Center neither represses nor grants independence, i.e., $\sigma_C(0; 8) = 1$ and the Periphery mobilizes with probability $\sigma_P(8)$.*

We first characterize mixing probabilities, $\sigma_C(1; 7)$, $\sigma_C(\emptyset; 7)$, and $\sigma_P(8)$, such that the following hold:

$$\sigma_C(1;7) + \sigma_C(\emptyset;7) = 1$$
$$U_C^\sigma(1;7) = U_C^\sigma(\emptyset;7)$$
$$U_P^\sigma(1;8) = U_P^\sigma(0;8).$$

The first equation says the Center mixes between repression and granting independence at $g = 7 = g^+$. The second and third equations are C and P's indifference conditions, respectively. Because

$U_C^\sigma(\varnothing; 7) = 0$, C's indifference equations takes the form:

$$\pi_C^C - \kappa_C + \delta V_C^\sigma(8) = 0, \tag{7}$$

where

$$V_C^\sigma(8) = \sigma_P(8)\left[-F(8)\psi + (1 - F(8))\left(\pi_C^C + \delta V_C^\sigma(7)\right)\right] + (1 - \sigma_P(8))\left[\pi_C^C + \delta V_C^\sigma(7)\right].$$

In equilibrium, $V_C^\sigma(7) = U_C^\sigma(\varnothing; 7) = 0$. Thus, we have

$$V_C^\sigma(8) = \sigma_P(8)\left[-F(8)\psi + (1 - F(8))\pi_C^C\right] + (1 - \sigma_P(8))\pi_C^C.$$

Substituting the above equality into Equation (7), C's indifference condition takes the form:

$$\pi_C^C - \kappa_C + \delta\left(\sigma_P(8)\left[-F(8)\psi + (1 - F(8))\pi_C^C\right] + (1 - \sigma_P(8))\pi_C^C\right) = 0. \tag{8}$$

Next, consider P's indifference equation, $U_P^\sigma(1; 8) = U_P^\sigma(0; 8)$, which takes the form

$$-\kappa_P + F(8)\frac{\pi_P^P}{1 - \delta} + (1 - F(8))\left(\pi_P^C + \delta V_P^\sigma(7)\right) = \pi_P^C + \delta V_P^\sigma(7), \tag{9}$$

where

$$\begin{aligned} V_P^\sigma(7) &= \sigma_C(\varnothing; 7)\frac{\pi_P^P}{1 - \delta} + \sigma_C(1; 7)\left[\pi_P^C + \delta V_P^\sigma(8)\right] \\ &= \sigma_C(\varnothing; 7)\frac{\pi_P^P}{1 - \delta} + \sigma_C(1; 7)\left[\pi_P^C + \delta U_P^\sigma(0; 8)\right] \\ &= \sigma_C(\varnothing; 7)\frac{\pi_P^P}{1 - \delta} + \sigma_C(1; 7)\left[\pi_P^C + \delta\left(\pi_P^C + \delta V_P^\sigma(7)\right)\right] \end{aligned}$$

Here the second equality follows because $\sigma_C(0; 8) = 1$. Solving Equations (8) and (9) with the constraint $\sigma_C(\varnothing; 7) + \sigma_C(1; 7) = 1$ reveals that

$$\sigma_P(8) = \frac{(1 + \delta)\pi_C^C - \kappa_C}{(\pi_C^C + \psi)\delta F(8)} \approx .56$$

and

$$\sigma_C(1; 7) = \frac{\kappa_P - F(8)(\pi_P^P - \pi_P^C)}{\delta^2\kappa_P + \delta F(8)(\pi_P^P - \pi_P^C)} \approx .13.$$

Finally, we check profitable deviations. First, P's indifference condition precludes profitable deviations at $g = 8$. Second, C does not have a profitable deviation at $g = 7$ due to its indifference *equation and because $U_C^\sigma(0;7) = \tilde{V}_C(7) < 0$. Also, C has no profitable deviation at $g = 8$, because $V_C(8) > 0$. To see this, note that $U_C^\sigma(1;7) = \pi_C^C - \kappa_C + \delta V_C^\sigma(8) = 0$ by Equation (7), and $\pi_C^C - \kappa_C < 0$. If C deviates by granting independence at $g = 8$, then its payoff is zero. Likewise, if C deviates by repressing, its payoff is $\pi_C^C - \kappa_C + \delta V_C^\sigma(9)$, which reduces to $\pi_C^C - \kappa_C < 0$ because C is granting independence when $g = 9$. Lemma 19 implies that C cannot profitably deviate by using repression, at grievances $g \geq 9$. Thus, we only need to verify that C cannot profitably deviate by choosing to refrain from repression or granting independence, at grievances $g \geq 9$. Because the Periphery mobilizes at $g \geq 9$ and $F(g) = 1$, mobilization surely succeeds, implying $U_C^\sigma(0;g) = -\psi$ for all $g \geq 9$ which is strictly less than C's utility from following its equilibrium strategy of granting independence.*

H Exogenous Decentralization

In this section, I continue to analyze the numerical example in Figure 4 and prove Proposition 5.

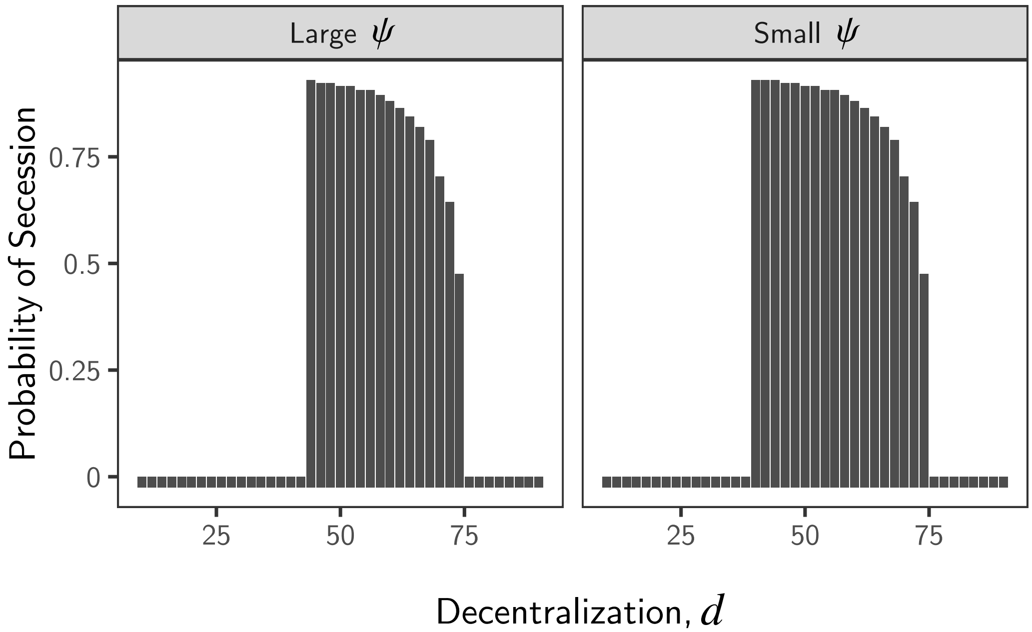
From, the example in Figure 4, I compute the probability that the country breaks apart due to secessionist mobilization —labeled probability of secession hereafter—as a function of decentralization. For a fixed d, three potential paths of play emerge at initial grievance g^1 in equilibrium. First, if $g^1 < g^+[d]$, the Center neither represses nor grants independence, and the probability secession is

$$\begin{cases} 0 & \text{if } g^1 \leq g^-[d] \\ 1 - \prod_{g' : g^\dagger[d] < g' \leq g^1} (1 - F(g')) & \text{otherwise.} \end{cases}$$

Second, if $g^1 \geq g^+[d]$ and the regime has high capacity ($\pi - d > \kappa_C$), then the Center represses in all future periods, and the probability of secession is zero. Third, if $g^1 \geq g^+[d]$ and the regime has low capacity ($\pi - d < \kappa_C$), the probability of secession is undefined. Although the Periphery will eventually gain control of its territory (Proposition 3.2), this may arise either from secessionist mobilization or Center-granted independence. This third case does not arise in the numerical example. As seen in Figure 4, if $g^1 \geq g^+[d]$ for some d, then the regime has high capacity.

Figure 6 graphs the probability of secession decentralization varies. When d is small, $g^1 > g^+[d]$ so the high-capacity regime represses and the probability of secession is zero. When d is large, $g^1 < g^-$, so grievances are small and a lasting peace emerges. When d is moderate, then the Center gambles for unity and secession occurs with positive probability. When ψ is large (left panel), all decentralization levels below $d = 44$ result in long-term repression and a zero probability of secession. When ψ is small (right panel), all decentralization levels below $d = 38$ result in long-term repression and a zero probability of secession.

Figure 6: Decentralization and comparative statics



Notes: The panels graph the probability of secession (vertical axis) for a fixed decentrlaization level (horizontal axis) with a large cost of secession $\psi = \frac{3\pi}{2}$ (left) and a small cost $\psi = \frac{\pi}{2}$ (right). The remaining parameters take on the following valules: $\pi = 100$, $\kappa_C = 50$, $\kappa_P = 50$, $\delta = 0.95$, and and $F(g) = 1 - \left(0.01g + 0.001g^2 - 1\right)^{-1}$.

Proposition 5 *Assume the regime has a high capacity for repression ($\kappa_C < \pi$) and initial grievances are large ($g^1 \geq g^+[0]$). There exist cutpoints $\underline{d}$ and $\overline{d}$ such that $0 \leq \underline{d} < \overline{d} < 1$ and secession occurs with positive probability on the equilibrium path only if decentralization is moderate, i.e., $\underline{d} < d < \overline{d}$.*

Proof. Set $\underline{d} = 0$. The regime has high repression capacity by assumption, and $g^1 \geq g^+[\underline{d}]$ implies that C represses with probability one in all future periods when the game begins at grievance g^1. As such the probability of secession is zero.

In addition, we can set $\bar{d}$ as follows

$$\bar{d} = \hat{d}(g^1) + \epsilon$$

where $\hat{d}$ is defined in Equation (4) above and $\epsilon \in \mathbb{R}$ is such that $0 < \epsilon < \max\{\frac{(1-\delta)\kappa_P}{F(g^1)}, 1\}$. Note that the fraction $\frac{(1-\delta)\kappa_P}{F(g^1)}$ is well defined because $F(g^1) \neq 0$. If $F(g^1) = 0$ then $g^1 \leq g^-[0] < g^+[0]$, a contradiction.

It suffices to show that $g^1 \leq g^-\left[\bar{d}\right]$ because this inequality implies that g^1 is small at decentralization level $\bar{d}$ and g^- is strictly increasing in d. As such, g^1 is small at decentralization levels $d > \bar{d}$. In addition, when $g^1 \leq g^-$ no mobilization occurs along the path of play by Proposition 1.

When $\pi_P^C = d$ and $\pi_P^P = \pi$, then we can write $g^-[d]$ as

$$g^-[d] = \max\left\{g \in \mathbb{N}_0 \;\middle|\; \kappa_P > F(g)\frac{\pi - d}{1-\delta}\right\}.$$

Thus, $g^1 \leq g^-\left[\bar{d}\right]$, as required. □

I Proof of Proposition 6

We first prove Proposition 6.1 and then present two numerical examples that establish Propositions 6.2 and 6.3.

Proof of 6.1. Consider equilibrium (d^*, σ). We first prove that $d^* \leq \min\{\hat{d}(g^1), \kappa_C\}$. First, $d^* \leq \kappa_C$. To see this, note that $V_C^\sigma(g; d^*) \leq \frac{\pi - d^*}{1-\delta}$. Thus, if C chooses $d^* > \kappa_C$, then $V_C^\sigma(g; d^*) < \frac{\pi-\kappa_C}{1-\delta}$, which means C can profitably deviate by choosing $d^* = 0$ and repressing in all future periods.

Second, $d^* \leq \hat{d}(g^1)$. When C chooses $d^* > \hat{d}(g^1)$, then $g^1 \leq g^-[d^*]$, which implies that $V_C^\sigma(g^1; d^*) = \frac{\pi-d^*}{1-\delta}$, which is strictly decreasing in d^*. So C has a profitable deviation by choosing decentralization $d = d^* - \epsilon$ for $\epsilon > 0$ but close to zero. This establishes the desired result.

Finally, we prove that if $\kappa_C < \max\left\{\frac{\pi}{2}, \pi - \hat{d}(g^1)\right\}$ and $d^* > 0$, then $g^1 < g^+[d^*]$, i.e., C never represses nor grants independence along the subsequent path of play. To do this suppose not and consider two relevant cases.

Case 1: $\pi - d^* - \kappa_C > 0$. Then $V_C^\sigma(g^1; d^*) = \frac{\pi-d^*-\kappa_C}{1-\delta}$, and C can profitably deviate by choosing $d^* = 0$ and repressing in all future periods.

Case 2: $\pi - d^* - \kappa_C \leq 0$. If $\kappa_C < \frac{\pi}{2}$, then

$$d^* \geq \pi - \kappa_C > \pi - \frac{\pi}{2} > \kappa_C,$$

which contradicts the upper bound described above. If $\kappa_C < \pi - \hat{d}(g^1)$, then we have

$$\begin{aligned} d^* &\geq \pi - \kappa_C \\ &> \pi - \left(\pi - \hat{d}(g^1)\right) \\ &= \pi - \left(\frac{(1-\delta)\kappa_P}{1-\delta}\right) \\ &= \hat{d}(g^1), \end{aligned}$$

which contradicts the upper bound described above. □

The next example illustrates that the Center decentralizes in equilibrium (d^*, σ) and the subsequent interaction entails gambling for unity.

Example 2 *For the exogenous parameters, we consider* $\pi = 100$, $\psi = 100$, $\kappa_C = 40$, $\kappa_P = 95$ *and* $\delta = 0.9$. *In addition, F takes the form:*

$$F(g) = \begin{cases} 0 & \text{if } g = 0 \\ \frac{1}{10} & \text{if } g \in \{1, \dots, 100\} \\ \frac{3}{10} & \text{if } g = 101 \\ 1 & \text{if } g \geq 102. \end{cases}$$

and initial grievances are $g^1 = 101$.

Note that $\kappa_C < \frac{\pi}{2}$, *so Proposition 6.1 implies that if C decentralizes in an equilibrium* (d^*, σ), *then it chooses to neither repress nor grant independence in all future periods, in which case, C's expected utility is* $\tilde{V}_C(g^1; d^*)$. *Thus, if C chooses* $d^* > 0$, *it will choose a* d^* *that solves*

$$F(g')\frac{\pi - d^*}{1 - \delta} - \kappa_P = 0$$

for some $g' > g^-[0]$ *and* $g' \leq g^1$. *In words, if C decentralizes, it will choose a decentralization level that makes the Periphery (at some grievance level* g^1*) indifferent between mobilizing and not along the subsequent path of play. If not, C can profitably deviate by offering slightly less decentralization without changing the Periphery's strategy in states* $g \leq g^1$.

Given this discussion and the construction of F, there are three possible decentralization levels to consider: $\{0, \hat{d}(1), \hat{d}(101)\}$. *Note that* $\hat{d}(101) = \frac{205}{3} > \kappa_C$. *As such, the upper bound in the previous proof shows that* $d^* \neq \hat{d}(101)$ *in any equilibrium. Thus, there are only two possible decentralization levels in equilibrium:* $\{0, \hat{d}(1)\}$.

If C chooses $d^* = 0$, *then* $g^-[0] = 0$ *and* $g^+[0] = 6$. *Because* $g^1 > g^+[0]$, *if C chooses* $d^* = 0$, *then long-term repression is the equilibrium outcome, which implies C's dynamic payoff is* $\frac{\pi - \kappa_C}{1-\delta} = 600$.

If C chooses $d^* = \hat{d}(1) = 5$, *then* $g^-[d^*] = 100$ *and* $g^+[0] = 102$. *Because* $g^1 < g^+[d^*]$, *if C chooses* $d^* = \hat{d}(1)$, *then one period of gambling for unity is the equlibrium path of play, in which case C's expected utility is*

$$-F(g^1)\psi + (1 - F(g^1))\left[\pi - d^* + \delta\frac{\pi - d^*}{1 - \delta}\right] = 635.$$

As such, C chooses to decentralize, $d^* = \hat{d}(1) > 0$ *and gambling for unity occurs along the subsequent equilibrium path of play.*

The next example illustrates that the Center decentralizes in equilibrium (d^*, σ) and the a long-term peace emerges in the subsequent interaction.

Example 3 *The payoff parameters match those from Example 2, but now F takes the form:*

$$F(g) = \begin{cases} 0 & \text{if } g = 0 \\ \frac{1}{10} & \text{if } g \in \{1, \ldots, 101\} \\ 1 & \text{if } g \geq 102. \end{cases}$$

and initial grievances are $g^1 = 101$*. Following the logic in the previous example, there are two potential levels of decentralization in equilibrium:* $\{0, \hat{d}(1)\}$*. If C chooses* $d^* = 0$*, then its payoff is* $\frac{\pi - \kappa_C}{1-\delta} = 600$ *for reasons described above. If C choses* $d^* = \hat{d}(1)$*, then* $g^-[d^*] = 101 = g^1$ *and its equilibrium payoff is* $\frac{\pi - d^*}{1-\delta} = 950$*. As such, C chooses to decentralizes and a long-term peace emerges.*

Supplemental Material (not copyedited or formatted) for: Michael Gibilisco. 2021. "Decentralization, Repression, and Gambling for Unity." The Journal of Politics 83(4). DOI: https://doi.org/10.1086/711626.

Online Appendix

A Continuation Values and Expected Utilities

Let $\bar{V}_i$ denote i's continuation value after a history in which the Periphery was won control of the territory. These values are independent of a strategy profile σ and take the form $\bar{V}_C = 0$ and $\bar{V}_P = \frac{\pi_P^P}{1-\delta}$.

Let $V_i^\sigma(g)$ denotes i's continuation value from beginning the game with grievance g when the Periphery has not won control of its territory and actors subsequently playing according to profile σ. In a similar vein, $U_C^\sigma(r; g)$ and $U_P^\sigma(m; g)$ denote the Center and Periphery's dynamic payoffs from choosing $r \in \{\varnothing, 0, 1\}$ and $m \in \{0, 1\}$ given grievance g when actors subsequently play according to profile σ. For the Center, $U_C^\sigma(r; g)$ takes the following form:

$$U_C^\sigma(r; g) = \begin{cases} 0 & \text{if } = \varnothing \\ \pi_C^C - \kappa_C + \delta V_C^\sigma(g+1) & \text{if } r = 1 \\ -\sigma_P(g)F(g)\psi + (1 - \sigma_P(g)F(g))\left(\pi_C^C + \delta V_C^\sigma(\max\{g-1, 0\})\right) & \text{if } r = 0. \end{cases}$$

For the Periphery, $U_P^\sigma(m; g)$ denotes the its dynamic payoff conditional on having reached its decision node, i.e., the Center chooses $r = 0$, in state g. Thus, $U_P^\sigma(m; g)$ takes the form

$$U_P^\sigma(m; g) = \begin{cases} -\kappa_P + F(g)\bar{V}_P + (1 - F(g))\left(\pi_P^C + \delta V_P^\sigma(\max\{g-1, 0\})\right) & \text{if } m = 1 \\ \pi_P^C + \delta V_P^\sigma(\max\{g-1, 0\}) & \text{if } m = 0. \end{cases} \tag{5}$$

With this notation in hand, the next definition states the equilibrium conditions.

Definition 1 *Strategy profile σ is an equilibrium if the following hold:*

$$\begin{aligned} \sigma_C(r; g) > 0 &\implies U_C^\sigma(r; g) \geq U_C^\sigma(r'; g), \\ \sigma_P(g) > 0 &\implies U_P^\sigma(1; g) \geq U_P^\sigma(0; g), \text{ and} \\ \sigma_P(g) < 1 &\implies U_P^\sigma(0; g) \geq U_P^\sigma(1; g) \end{aligned}$$

for all grievance g and polices $r, r' \in \{\varnothing, 0, 1\}$.

Because the game is a dynamic game with a countable state space and a finite number of actions, an equilibrium from Definition 1 exists in mixed strategies. Notice that for some grievance g, the Center's continuation value, $V_C^\sigma(g)$, takes the form

$$V_C^\sigma(g) = \sum_{r \in \{\varnothing, 0, 1\}} \sigma(r; g) U_C^\sigma(r; g).$$

Thus, if σ is an equilibrium and $\sigma(r; g) > 0$ for some grievance g and action $r \in \{\varnothing, 0, 1\}$, then $V_C^\sigma(g) = U_C^\sigma(r; g)$ or else C has a deviation by playing some $r' \in \{\varnothing, 0, 1\}$.

B Proof of Proposition 1

Proposition 1 *If grievances are small, then the Periphery never mobilizes, the Center neither represses nor grants independence, and grievances dissipate on the equilibrium path. That is, $g \leq g^-$ implies $\sigma_P(g) = 0$ and $\sigma_C(0; g) = 1$ in every equilibrium σ.*

Proof. The proof that $g \leq g^-$ implies the Periphery does not mobilize with positive probability is covered in the main text. We prove that $g \leq g^-$ implies the Center does not repress or grant independence with positive probability. To see this, suppose $\sigma_C(r; g) > 0$ for some $g \leq g^-$, $r \neq 0$, and equilibrium σ. There are two cases.

Case 1: $r = 1$, repression. Then, C's expected utility is

$$\begin{aligned} U_C^\sigma(1; g) &= \pi_C - \kappa_C + \delta V_C^\sigma(g+1) \\ &\leq \pi_C - \kappa_C + \delta \frac{\pi_C^C}{1-\delta} \\ &< \frac{\pi_C^C}{1-\delta}. \end{aligned}$$

However, $\frac{\pi_C^C}{1-\delta}$ is C's continuation value if it takes action $r = 0$ in all future periods because grievances will never increase and P will never mobilize with positive probability along the subsequent path of play. Hence, taking action $r = 0$ in all future periods is a profitable deviation, a contradiction.

Case 2: $r = \emptyset$, independence. Then, C's expected utility is

$$U_C^\sigma(\emptyset; g) = 0 < \frac{\pi_C^C}{1-\delta}.$$

As in Case 1, this inequality implies taking action $r = 0$ in all future periods is a profitable deviation, a contradiction. □

C Properties of $\tilde{V}_C$

We first state and prove three Lemmas concerning properties of $\tilde{V}_C$.

Lemma 1 *1.* $\tilde{V}_C(g) > \frac{-F(g)\psi + (1-F(g))\pi_C^C}{1-(1-F(g))\delta}$ *for all* g *such that* $F(g) > 0$.

2. $\tilde{V}_C(g-1) > \tilde{V}_C(g)$ *for all* $g > g^-$.

3. If Assumption 1 holds, then $\lim_{g\to\infty} \tilde{V}_C(g) = \frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta}$.

Proof. To show (1), consider some g such that $F(g) > 0$ and $F(g') = 0$ for all $g' < g$. Such a g exists because $F(0) = 0$ and $\lim_{g\to\infty} F(g) = p > 0$. In addition, $F(g) < 1$ because there exists at least one g such that $F(g) \in (0, 1)$ by assumption. Then we have

$$\begin{aligned} \tilde{V}_C(g) &= -F(g)\psi + (1-F(g))\left(\pi_C^C + \delta \frac{\pi_C^C}{1-\delta}\right) \\ &= (1-(1-F(g))\delta)\frac{-F(g)\psi + (1-F(g))\pi_C^C}{1-(1-F(g))\delta} + (1-F(g))\delta\frac{\pi_C^C}{1-\delta} \\ &> \frac{-F(g)\psi + (1-F(g))\pi_C^C}{1-(1-F(g))\delta}, \end{aligned}$$

where the strict inequality follows because $F(g) \in (0, 1)$

For induction, consider some g such that $F(g) > 0$ and $F(g-1) > 0$, which implies $g - 1 > 0$. Suppose the inequality holds for all $g' < g$ such that $F(g') > 0$. Then we have

$$\begin{aligned}\tilde{V}_C(g) &= -F(g)\psi + (1 - F(g))(\pi_C^C + \delta\tilde{V}_C(g-1)) \\ &> -F(g)\psi + (1 - F(g))\left(\pi_C^C + \delta\frac{-F(g-1)\psi + (1 - F(g-1))\pi_C^C}{1 - (1 - F(g-1))\delta}\right) \\ &\geq -F(g)\psi + (1 - F(g))\left(\pi_C^C + \delta\frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta}\right) \\ &= \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta},\end{aligned}$$

where the third line follows because the fraction $\frac{-F(g)\psi+(1-F(g))\pi_C^C}{1-(1-F(g))\delta}$ is decreasing in $F(g)$.

To show (2), note that it must hold when $g = g^- + 1$, because $\psi > 0$ and $F(g) > 0$ as $g > g^-$. Now consider some $g > g^- + 1$. For induction, suppose $\tilde{V}_C(g' - 1) > \tilde{V}_C(g')$ for all g' such that $g^- < g' < g$. Then

$$\begin{aligned}\tilde{V}_C(g) &= -F(g)\psi + (1 - F(g))(\pi_C^C + \delta\tilde{V}_C(g-1)) \\ &\leq -F(g-1)\psi + (1 - F(g-1))(\pi_C^C + \delta\tilde{V}_C(g-1)) \\ &< -F(g-1)\psi + (1 - F(g-1))(\pi_C^C + \delta\tilde{V}_C(g-2)) \\ &= \tilde{V}_C(g-1),\end{aligned}$$

where the second line follows because

$$\tilde{V}_C(g) > \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta} \geq -\psi$$

and $F(g)$ is increasing in g.

To prove (3), consider a sequence $\{g_n\}_{n=1}^\infty$ such that $\lim_{n\to\infty} g_n = \infty$ and $g_n < g_{n+1}$. Then the sequence $\left\{\tilde{V}_C(g_n)\right\}_{n=1}^\infty$ is weakly decreasing due to above arguments. In addition, $\left\{\tilde{V}_C(g_n)\right\}_{n=1}^\infty$ is bounded below because C's payoffs are finite and C discounts with rate $\delta < 1$. Thus, $\left\{\tilde{V}_C(g_n)\right\}_{n=1}^\infty$ has a limit, call it L. If the Periphery does value independence, the we have

$$\begin{aligned}L &= \lim_{n\to\infty} \tilde{V}_C(g_n) \\ &= \lim_{n\to\infty} F(g_n)(-\psi) + \lim_{n\to\infty}(1 - F(g_n))(\pi_C^C + \delta\tilde{V}_C(g_n - 1)) \\ &= -p\psi + (1 - p)\left(\pi_C^C + \delta L\right),\end{aligned}$$

Supplemental Material (not copyedited or formatted) for: Michael Gibilisco. 2021. "Decentralization, Repression, and Gambling for Unity." The Journal of Politics 83(4). DOI: https://doi.org/10.1086/711626.

which implies $L = \frac{-p\psi+(1-p)\pi_C^C}{1-(1-p)\delta}$. □

The next Lemma demonstrates that C's gambling for unity utility, $\tilde{V}_C$ is a lower bound on its equilibrium expected utility, V_C^σ.

Lemma 2 *For all grievances g, $V_C^\sigma(g) \geq \tilde{V}_C(g)$ in every equilibrium σ.*

Proof. To see this, suppose not. That is, suppose there exist grievance g and equilibrium σ such that $V_C^\sigma(g) < \tilde{V}_C(g)$. Then by the construction of $\tilde{V}_C$ and Proposition 1, $g > g^-$, or else $V_C^\sigma(g) = \frac{\pi_C^C}{1-\delta} = \tilde{V}_C$.

Next consider a deviation for C, labeled σ'_C, such that $\sigma'_C(0; g') = 1$ for all $g' \leq g$. I now demonstrate that $V_C^{\sigma'}(g) \geq \tilde{V}_C(g)$, where $\sigma' = (\sigma'_C, \sigma_P)$, which implies σ'_C is a profitable deviation because $\tilde{V}_C(g) > V_C^\sigma(g)$ by supposition.

The proof is by induction. The inequality, $V_C^{\sigma'}(g') \geq \tilde{V}_C(g')$, holds when $g' \leq g^-$ by the construction of $\tilde{V}_C$ and Proposition 1. Now consider some $g' > g^-$ and suppose $V_C^{\sigma'}(g'') \geq \tilde{V}_C(g'')$ for all $g'' < g'$. Then we have

$$\begin{aligned}
V_C^{\sigma'}(g') &= -\sigma_P(g')F(g')\psi + (1 - \sigma_P(g')F(g'))\left(\pi_C^C + \delta V_C^{\sigma'}(g'-1)\right)\\
&\geq -\sigma_P(g')F(g')\psi + (1 - \sigma_P(g')F(g'))\left(\pi_C^C + \delta \tilde{V}_C(g'-1)\right)\\
&\geq -F(g')\psi + (1 - F(g'))\left(\pi_C^C + \delta \tilde{V}_C(g'-1)\right)\\
&= \tilde{V}_C(g').
\end{aligned}$$

Hence, $V_C^{\sigma'}(g) \geq \tilde{V}_C(g)$ as required. □

The final Lemma demonstrates that the cutpoint g^+ exists if and only if Assumptions 1 and 2 hold.

Lemma 3 *The cutpoint g^+ solving Equation (3) exists if and only if the Periphery values independence (Assumption 1) and secession is costly (Assumption 2).*

Proof. For necessity, suppose Assumptions 1 and 2 hold. Then Lemma 1 and Assumption 1 imply that $\tilde{V}_C(g)$ is weakly decreasing in g and converges to

$$\lim_{g\to\infty} \tilde{V}_C(g) = \frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta}.$$

Because $\tilde{V}_C(g) = \frac{\pi_C^C}{1-\delta} > 0$ for all $g \leq g^-$ and $\tilde{V}_C(g)$ is strictly decreasing in g when $g > g^-$, we require

$$\frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta} < \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}. \tag{6}$$

We now demonstrate that the inequality in Equation (6) holds when $\pi_C^C > \kappa_C$, the proof when $\pi_C^C < \kappa_C$ is identical. Suppose $\pi_C^C - \kappa_C > 0$. Then Equation (6) reduces to

$$\frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta} < \frac{\pi_C^C - \kappa_C}{1-\delta},$$

which is equivalent to

$$\psi > \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)}.$$

Because $\pi_C^C - \kappa_C > 0$, Assumption 2 reduces to

$$\psi > \min\left\{\frac{\pi_C^C(1-p)}{p}, \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)}\right\} = \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)}.$$

Thus, the inequality in Equation (6) holds, and therefore g^+ exists.

For sufficiency, suppose Assumption 1 does not hold, then $\kappa_P \geq F(g)\frac{\pi_P^P - \pi_P^C}{1-\delta}$ for all grievances g. Thus, $\tilde{V}_C(g) = \frac{\pi_C^C}{1-\delta} > \max\left\{\frac{\pi_C^C-\kappa_C}{1-\delta}, 0\right\}$ for all grievances g. Now suppose Assumption 1 holds but not Assumption 2. Then Lemma 1 implies that, for all g

$$\tilde{V}_C(g) \geq \frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta} \geq \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}.$$ □

D Preliminary Results

In this section, we state and prove two technical results that are essential to characterize equilibria in the remainder of the paper.

Lemma 4 *If $\sigma_C(1; g) > 0$ and $\sigma_C(0; g+1) = 1$ for some grievance g, then $\sigma_P(g+1) < 1$ in every equilibrium σ.*

Proof. Suppose not. Then there exists a g such that $\sigma_C(1; g) > 1$, $\sigma_C(0; g+1) = 1$ and $\sigma_P(g+1) = 1$ in equilibrium σ. We can write $V_C^\sigma(g+1)$ as

$$\begin{aligned} V_C^\sigma(g+1) &= -F(g+1)\psi - (1 - F(g+1))\left(\pi_C^C + \delta V_C^\sigma(g)\right) \\ &= -F(g+1)\psi - (1 - F(g+1))\left(\pi_C^C + \delta U_C^\sigma(1; g)\right) \\ &= -F(g+1)\psi - (1 - F(g+1))\left(\pi_C^C + \delta\left(\pi_C^C - \kappa_C + \delta V_C^\sigma(g+1)\right)\right). \end{aligned}$$

Solving reveals that

$$V_C^\sigma(g+1) = \frac{(1 - F(g+1))(\pi(1+\delta) - \delta\kappa_C) - F(g+1)\psi}{1 - (1 - F(g+1))\delta^2}.$$

By Lemma 2, $V_C^\sigma(g+1) \geq \tilde{V}_C(g+1)$. By Lemma 1.1,

$$\tilde{V}_C(g) > \frac{(1 - F(g+1))\pi_C^C - F(g+1)\psi}{1 - (1 - F(g+1)\delta)}.$$

Stringing these two inequalities together,

$$V_C^\sigma(g+1) > \frac{(1 - F(g+1))\pi_C^C - F(g+1)\psi}{1 - (1 - F(g+1)\delta)}.$$

Substituting the closed form solution for $V_C^\sigma(g+1)$ into the inequality above and solving for κ_C reveals that

$$\kappa_C < \frac{F(g+1)(\pi_C^C + \psi(1-\delta))}{1 - (1 - F(g+1))\delta}.$$

To derive a contradiction, consider a deviation in which C plays $r = 1$ with probability 1 in all future periods beginning at grievance $g+1$. This is a profitable deviation if and only if

$$V_C^\sigma(g+1) < \frac{\pi_C^C - \kappa_C}{1-\delta} \iff \kappa_C < \frac{F(g+1)(\pi_C^C + \psi(1-\delta))}{1 - (1 - F(g+1))\delta}.$$

However, $\kappa_C < \frac{F(g+1)(\pi_C^C + \psi(1-\delta))}{1-(1-F(g+1))\delta}$ as shown above. Hence, C can profitably deviate by repressing in all future periods. □

Lemma 5 *Consider some $g > g^-$ and equilibrium σ. If (a) $\sigma_C(0; g-1) = 1$ or $\sigma_C(0; g) = 1$ and (b) $\sigma_C(\varnothing; g') = 0$ for all $g' < g$, then $\sigma_P(g) = 1$.*

Proof. Suppose not. That is, consider some equilibrium σ and grievance $g > g^-$ such that

(a) $\sigma_C(0; g-1) = 1$ or $\sigma_C(0; g) = 1$,

(b) $\sigma_C(\emptyset; g') = 0$ for all $g' < g$, and

(c) $\sigma_P(g) < 1$.

Because σ is an equilibrium, we require $U_P^\sigma(0; g) \geq U_P^\sigma(1; g)$ to rule out profitable deviations, which is equivalent to

$$\kappa_P \geq F(g)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g-1)\right].$$

Because $\sigma_C(0; g-1) = 1$ or $\sigma_C(0; g) = 1$, the path of play will never reach a grievance larger than g. Because $\sigma_C(\emptyset; g') = 0$ for all $g' \leq g$, the Center will never grant independence along the subsequent path of play. Recall that when the C represses, P stage payoff is π_P^C, which is its payoff if it chooses not to mobilize, and even if C does repress with positive probability at some $g' < g$, the subsequent path of play will still never reach a grievance larger than g. Then $g > g^-$ implies $V_P^\sigma(g-1)$ is bounded above by

$$\frac{F(g)\bar{V}_P + (1 - F(g))\pi_P^C - \kappa_P}{1 - (1 - F(g))\delta},$$

which is P's payoff if its grievance never depreciates along the path of play, C never represses, and P always mobilizes. Combining these two inequalities, we require

$$\begin{aligned}\kappa_P &\geq F(g)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g-1)\right]\\ &\geq F(g)\left[\bar{V}_P - \pi_P^C - \delta\frac{F(g)\bar{V}_P + (1 - F(g))\pi_P^C - \kappa_P}{1 - (1 - F(g))\delta}\right].\end{aligned}$$

Solving for κ_P implies

$$\kappa_P \geq F(g)\frac{\pi_P^P - \pi_P^C}{1 - \delta},$$

that is, $g \leq g^-$. But this contradicts the assumption $g > g^-$. □

E Proof of Proposition 2

This section characterizes equilibrium behavior at moderate grievances.

We now prove that $g < g^+$ implies $\sigma_C(0; g) = 1$ in every equilibrium σ, that is, the Center neither represses nor grants independence with moderate grievances. The result requires preliminary lemmas. Notice that if either Assumption 1 or 2 does not hold, $\tilde{V}_C(g) > \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}$ for all g, and we can set $g^+ = \infty$ in the subsequent results.

Lemma 6 *If $g < g^+$, then $\sigma_C(\varnothing; g) = 0$ in every equilibrium σ.*

Proof. If not, then $V_C^\sigma(g) = U_C^\sigma(\varnothing; g) = 0$. If $g < g^+$, this contradicts Lemma 2 because $\tilde{V}_C(g) > 0 = V_C^\sigma(g)$. □

Lemma 7 *For all g, $\sigma(r; g) > 0$ imply $\sigma(\varnothing; g+1) = 0$ in every equilibrium σ.*

Proof. First, if $\kappa_C < \pi_C^C$, then C cannot grant independence with positive probability in any equilibrium. Doing so would result in a payoff of 0, but C could repress for all future periods, giving a payoff of $\frac{\pi_C^C - \kappa_C}{1-\delta} > 0$. Thus, consider the case where $\pi_C^C - \kappa_C < 0$. Suppose $\sigma_C(r; g) > 0$ for some g and $\sigma_C(\varnothing; g+1) > 0$. Then

$$\begin{aligned} V_C^\sigma(g) &= U_C^\sigma(r; g) \\ &= \pi_C^C - \kappa_C + \delta V_C^\sigma(g+1) \\ &= \pi_C^C - \kappa_C + \delta U_C^\sigma(\varnothing; g) \\ &= \pi_C^C - \kappa_C < 0, \end{aligned}$$

but this means C can profitably deviate at g by granting independence, i.e., σ is not an equilibrium. □

Lemma 8 *Fix an equilibrium σ. Then there does not exist a $g < g^+$ such that $\sigma_C(1; g') > 0$ for all $g' \geq g$.*

Proof. Suppose not and consider such a $g < g^+$ where $\sigma_C(1; g') > 0$ for all $g' \geq g$ in equilibrium σ. Then

$$V_C^\sigma(g) = U_C^\sigma(1; g) = \pi_C^C - \kappa_C + \delta V_C^\sigma(g+1).$$

Because $V_C(g') = U_C^\sigma(r; g')$ for all g' such that $\sigma_C(r; g') > 0$, similar substitutions imply $V_C^\sigma(g) = \frac{\pi_C^C - \kappa_C}{1-\delta}$. However, $g < g^+$ implies

$$\tilde{V}_C(g) > \frac{\pi_C^C - \kappa_C}{1-\delta} = V_C^\sigma(g),$$

by Equation(3). However, $\tilde{V}_C(g) > V_C^\sigma(g)$ contradicts Lemma 2. □

With these lemmas in hand, we now state the main result of the section.

Proposition 2 *If grievances are moderate, then the Periphery always mobilizes, the Center neither represses nor grants independence, and grievances dissipate on the equilibrium path. That is, $g \in (g^-, g^+)$ implies $\sigma_P(g) = 1$ and $\sigma_C(0; g) = 1$ in every equilibrium σ.*

Proof. We first prove prove that $\sigma_C(0; g) = 1$ when $g \in (g^-, g^+)$ and σ is an equilibrium. Suppose not. By Lemma 6, $\sigma_C(1; g) > 0$. Furthermore, C represses with positive probability for at most some finite k periods by Lemma 8. That is, there exists a $\bar{g}$ such that $\sigma_C(1; g') > 0$ for $g' = g, ..., \bar{g}$ and $\sigma_C(1; \bar{g} + 1) = 0$. By Lemma 7, this implies $\sigma_C(0; \bar{g} + 1) = 1$. In addition, Proposition 1 and Lemma 7 imply $\sigma_C(\varnothing; g') = 0$ for all $g' < \bar{g}$. Thus, Lemma 5 and $\sigma_C(1; \bar{g} + 1) = 0$ imply P mobilizes at $\bar{g} + 1$ with probability 1. However, $\sigma_C(1; \bar{g}) > 0$, $\sigma_C(0; \bar{g} + 1) = 1$, and $\sigma_P(\bar{g} + 1) = 1$ contradict Lemma 4. To pin down P's strategy at $g \in (g^-, g^+)$, note that $\sigma_C(0; g') = 1$ for all $g' < g^+$. Then Lemma 5 implies $\sigma_P(g) = 1$. □

F Proof of Proposition 3

We now characterize equilibrium behavior at large grievances ($g \geq g^+$). We consider the generic case in which there does not exist $g \in \mathbb{N}_0$ such that $\tilde{V}_C(g) = \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}$, that is $\tilde{V}_C(g^+) < \max\left\{\frac{\pi_C^C - \kappa_C}{1-\delta}, 0\right\}$, where the inequality from Equation (3) holds strictly. If this held with equality, the Center would be indifferent leading to trivial indeterminacy. We consider high- and low-capacity regimes separately because the proof techniques vary dramatically between the two cases.

F.1 High repression capacity: $\kappa_C < \pi_C^C$

Lemma 9 *In high-capacity regimes, $\sigma_C(\varnothing; g) = 0$ for every grievance g and in every equilibrium σ.*

The proof is straightforward and omitted.

Lemma 10 *In high-capacity regimes, $\sigma_C(1; g^+) = 1$ and $\sigma_C(1; g) > 0$ for all $g > g^+$ in every equilibrium σ.*

Proof. The proof is by induction. First, we demonstrate that $\sigma_C(1; g^+) = 1$. To see this, suppose $\sigma_C(1; g^+) < 1$. Then Lemma 9 implies $\sigma_C(0; g^+) > 0$, in which case we have

$$U_C^\sigma(0; g^+) = \tilde{V}_C(g^+) < \frac{\pi_C^C - \kappa_C}{1 - \delta}.$$

This means C can profitably deviate at grievance g^+ by repressing for an infinite number of periods, a contradiction.

For induction, consider some $g > g^+$ and assume $\sigma_C(1; g - 1) > 0$. To derive a contradiction, assume $\sigma_C(1; g) = 0$. By Lemma 9, $\sigma_C(0; g) = 1$. Likewise, Lemma 9 guarantees C does not grant independence in any equilibrium, so Lemma 5 implies P mobilizes at g with probability 1. But then this contradicts Lemma 4. □

Lemma 11 *In high-capacity regimes, $g \geq g^+$ implies $V_C^\sigma(g) = \frac{\pi_C^C - \kappa_C}{1-\delta}$ in every equilibrium σ.*

Proof. If $g \geq g^+$, then Lemma 10 implies $\sigma_C(1; g') > 0$ for all $g' \geq g$. The remainder of the proof follows from an identical argument as the one in Lemma 8. □

Lemma 12 *In high-capacity regimes, $g > g^+$ and $\sigma_C(0; g) > 0$ imply $\sigma_P(g) < 1$ in every equilibrium σ.*

Proof. Suppose not. Then there exists $g > g^+$ such that $\sigma_C(0; g) > 0$ and $\sigma_P(g) = 1$. Because $g > g^+$, $g - 1 \geq g^+$. Likewise, $\sigma_C(1; g) > 0$ by Lemma 10, so it must be the case that $U_C^\sigma(0; g) = U_C^\sigma(1; g)$. Then we have

$$\begin{aligned} U_C^\sigma(0; g) = U_C^\sigma(1; g) &\iff -F(g)\psi + (1 - F(g))(\pi_C^C + \delta V_C^\sigma(g - 1)) = \pi - \kappa_C + \delta V_C^\sigma(g + 1) \\ &\iff -F(g)\psi + (1 - F(g))\left(\pi_C^C + \delta\frac{\pi - \kappa_C}{1-\delta}\right) = \frac{\pi - \kappa_C}{1-\delta} \\ &\iff \kappa_C = \frac{F(g)(\pi_C^C + (1-\delta)\psi)}{1(1 - F(g))\delta}, \end{aligned}$$

where we use Lemma 11 and $g - 1 \geq g^+$ to substitute for values $V_C^\sigma(g - 1)$ and $V_C^\sigma(g + 1)$.

Because σ is an equilibrium, we require $U_C^\sigma(1; g) = V_C^\sigma(g) \geq \tilde{V}_C(g)$, by Lemma 2. Then Lemma 1.1 implies

$$\begin{aligned} U_C^\sigma(1; g) > \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta} &\iff \frac{\pi_C^C - \kappa_C}{1-\delta} > \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta} \\ &\iff \kappa_C < \frac{F(g)(\pi_C^C + (1-\delta)\psi)}{1(1 - F(g))\delta}, \end{aligned}$$

which establishes the desired contradiction. □

Lemma 13 *In high-capacity regimes, there exists cutpoint $\bar{g} \in \mathbb{R}$ such that if $g > \bar{g}$, then $\sigma_P(g) = 1$ and $\sigma_C(1; g) = 1$ in every equilibrium σ.*

Proof. The proof is constructive. Define $\bar{g} \in \mathbb{N}_0$ to be a number that satisfies

$$g \geq \bar{g} \implies \kappa_P < F(g)\left[\bar{V}_P - \pi_P^C - \delta\frac{p\bar{V}_P + (1-p)\pi_P^C - \kappa_P}{1 - (1-p)\delta}\right].$$

Such a $\bar{g}$ exists because $F(g)\left[\bar{V}_P - \pi_P^C - \delta\frac{p\bar{V}_P+(1-p)\pi_P^C}{1-(1-p)\delta}\right]$ is positive and strictly increasing in g. Furthermore,

$$\lim_{g\to\infty} F(g)\left[\bar{V}_P - \pi_P^C - \delta\frac{p\bar{V}_P + (1-p)\pi_P^C}{1-(1-p)\delta}\right] = p\frac{\pi_P^P - \pi_P^C}{1-\delta},$$

and Assumption 1 implies

$$\kappa_P < p\frac{\pi_P^P - \pi_P^C}{1-\delta}.$$

We first show that $\sigma_P(g) = 1$ for $g \geq \bar{g}$. Suppose not; then there exists $g \geq \bar{g}$ such that $\sigma_P(g) < 1$. To rule out profitable deviations, we require $U_P^\sigma(0;g) \geq U_P^\sigma(1;g)$, which is equivalent to

$$\kappa_P \geq F(g)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g-1)\right].$$

Because the Center never grants independence in strong regimes, $V_P^\sigma(g-1)$ is bounded above by $\frac{p\bar{V}_P+(1-p)\pi_P^C-\kappa_P}{1-(1-p)\delta}$, which is the Periphery's dynamic payoff if it mobilizes in every period at maximum capacity, p. Combining these two inequalities gives us

$$\begin{aligned}\kappa_P &\geq F(g)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g-1)\right]\\ &\geq F(g)\left[\bar{V}_P - \pi_P^C - \delta\frac{p\bar{V}_P + (1-p)\pi_P^C - \kappa_P}{1-(1-p)\delta}\right],\end{aligned}$$

but this implies $g < \bar{g}$, which is contradiction. Thus, $\sigma_P(g) = 1$. Then Lemma 10 and the contrapositive of Lemma 12 imply $\sigma_C(1;g) = 1$, as required. □

Lemma 14 *In high-capacity regimes, if $g \geq g^+$, then $\sigma_P(g) = 1$ in every equilibrium σ.*

Proof. Suppose there exists $g \geq g^+$ such that $\sigma_P(g) < 1$. Lemma 13 implies that there exists grievance $g^\dagger \geq g$ such that $\sigma_P(g^\dagger) < 1$ and $\sigma_P(g') = \sigma_C(1;g') = 1$ for all $g' > g^\dagger$. To rule out profitable deviations, we require $U_P^\sigma(0;g^\dagger) \geq U_P^\sigma(1;g^\dagger)$. This implies

$$\kappa_P \geq F(g^\dagger)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g^\dagger - 1)\right].$$

Because P will never be able to mobilize at a larger grievance than $g^\dagger$ along the path of play and C never grants independence, $V_C^\sigma(g^\dagger - 1)$ is bounded above by

$$\frac{F(g^\dagger)\bar{V}_P + (1-F(g^\dagger))\pi_P^C - \kappa_P}{1-(1-F(g^\dagger)\delta)}.$$

Then we have

$$\begin{aligned}\kappa_P &\geq F(g^\dagger)\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g^\dagger - 1)\right]\\ &\geq F(g^\dagger)\left[\bar{V}_P - \pi_P^C - \delta\frac{F(g^\dagger)\bar{V}_P + (1 - F(g^\dagger))\pi_P^C - \kappa_P}{1 - (1 - F(g^\dagger)\delta)}\right]\\ &= F(g^\dagger)\frac{\pi_P^P - \pi_P^C}{1 - \delta},\end{aligned}$$

which implies $g^\dagger \leq g^- \leq g^+$, a contradiction. □

We now prove Proposition 3.1, which characterizes equilibria in regimes with large grievances when $\pi_C^C > \kappa_C$.

Proof of Proposition 3.1. If $g \geq g^+$, then Lemma 14 implies $\sigma_P(g) = 1$. Because $g > g^+$ implies $\sigma_P(g) = 1$. Lemma 10 and the contrapositive of Lemma 12 imply $\sigma_C(1; g) = 1$, as required. □

F.2 Low repression capacity: $\kappa_C > \pi_C^C$

Lemma 15 *Fix an equilibrium σ. In low-capacity regimes, the there does not exist grievance g such that $\sigma_C(1; g') > 0$ for all $g' \geq g$.*

Proof. The result follows from the inequality $\pi_C^C - \kappa_C < 0$ and the argument proving Lemma 8. □

Lemma 16 *In low-capacity regimes, $\sigma_P(g^+) = 1$, $\sigma_C(0; g^+) = 0$, and $\sigma_C(\varnothing; g^+) > 0$ in ever equilibrium σ.*

Proof. First, P mobilizes at g^+ by Lemma 5 and Proposition 2.

Second, $\sigma_C(0; g^+) = 0$. If not, then with positive probability the Center chooses to enter the path of play into moderate grievance levels. That is, $V_C^\sigma(g^+) = U_C^\sigma(0; g^+) = \tilde{V}_C(g^+)$. But then $V_C^\sigma(g^+) < 0$ because the regime has low capacity, so C can profitably deviate by granting independence at g^+.

Third, $\sigma_C(1; g^+) < 1$. To see this, suppose not, i.e., suppose $\sigma_C(1; g^+) = 1$. By Lemma 15, there exists $g^\dagger \geq g^+$ such that $\sigma_C(1; g^\dagger + 1) = 0$ and $\sigma_C(1; g^\dagger) > 0$ for all $g' = g^+, ..., g^\dagger$. Then by Lemma 7, $\sigma_C(\varnothing; g') = 0$ for all $g' = g^+, .., g^\dagger + 1$. By Proposition 2, $\sigma_C(0; g') = 1$ for all $g' < g^+$. Then Lemma 5 implies $\sigma_P(g^\dagger + 1) = 1$. However, $\sigma_C(1; g^\dagger) > 0$, $\sigma_C(0; g^\dagger + 1) = 1$, and $\sigma_P(g^\dagger + 1) = 1$ contradict Lemma 4. Thus, $\sigma_C(1; g^+) < 1$, which implies $\sigma_C(\varnothing; g^+) > 0$ by the previous paragraph. □

Before proving the last technical lemma of this section, consider the following definitions. The set $\mathcal{G} \subseteq \mathbb{N}_0$ is *an absorbing set with respect to profile* σ if once the path of play enters grievance level g such that $g \in \mathcal{G}$, it never transitions to a grievance g' such that $g' \notin \mathcal{G}$ with positive probability. The set $\mathcal{G}$ is *an irreducible absorbing set with respect to* σ if $\mathcal{G}$ is an absorbing set with respect to σ and there does not exist a proper subset $\mathcal{G}' \subsetneq \mathcal{G}$ such that $\mathcal{G}'$ is an absorbing set with respect to σ.

Lemma 17 *Consider an equilibrium σ and some grievance $g \geq g^+$. Then the following hold:*

1. *beginning at grievance g, the path of play enters an irreducible absorbing set $\mathcal{G}$ with respect to σ,*
2. $\max \mathcal{G}$ *exists,*
3. $g^+ \leq \min \mathcal{G}$, *and*
4. *there exists $g' \in \mathcal{G}$ such that $\sigma_C(\varnothing; g) > 0$.*

Proof. To prove (1), consider $g \geq g^+$ and two cases. If $\sigma_C(1; g) = 0$, then the path of play enters the set $\{g^+, ..., g\}$, which is an absorbing set because $\sigma_C(0; g^+) = 0$ by Lemma 16. So the set $\{g^+, ..., g\}$ has a irreducible absorbing set, $\mathcal{G}$. If $\sigma_C(1; g) > 0$, then Lemma 15 implies there exists $g^\dagger \geq g$ such that $\sigma_C(1; g') > 0$ for all $g' = g, ..., g^\dagger$ and $\sigma_C(1; g^\dagger + 1) = 0$ from Lemma 7. Then the path of play enters the set $\{g^+, ..., g^\dagger + 1\}$, which is an absorbing set as well.

The proof of (2) and (3) follow immediately from the existence of $\mathcal{G}$ and Lemmas 15 and 16, respectively.

To prove (4), suppose not. Suppose $\sigma_C(\varnothing; g') = 0$ for all $g' \in \mathcal{G}$. I first claim that it must be the case that $\#\mathcal{G} > 1$. Suppose the contrary. Then $\mathcal{G} = \{g'\}$, and C cannot be repressing with positive probability at g, or else $\mathcal{G}$ is not absorbing. Also, if $\mathcal{G} = \{g'\}$ and $\sigma_C(0; g') > 0$, then $F(g) = 1$ and $\sigma_P(g) = 1$ or else the path of play would transition to $g - 1$ with positive probability. In this case, $U_C(0; g') = -\psi < 0$, but this means C has a profitable deviation by granting independence at g'. Thus, $\#\mathcal{G} > 2$ and as such $\max \mathcal{G} - 1 \in \mathcal{G}$.

Second, because $\mathcal{G}$ is irreducible, $\sigma_C(1; \max \mathcal{G} - 1) > 0$, or else $\mathcal{G} \backslash \{\max \mathcal{G}\}$ would be absorbing as well. Furthermore, $\sigma_C(1; \max \mathcal{G}) = 0$ or else the path of play would transition with positive probability to $\max \mathcal{G} + 1$. Because $\sigma_C(1; \max \mathcal{G} - 1) > 0$ and $\sigma_C(1; \max \mathcal{G}) = 0$, Lemma 7 implies $\sigma_C(0; \max \mathcal{G}) = 1$ Because the path of play never leaves $\mathcal{G}$ nor transitions to grievance $g' > \max \mathcal{G}$ and C never grants independence along the path of play starting from $\max \mathcal{G}$, then $\sigma_P(\max \mathcal{G}) = 1$, which follows from an identical argument as the one in Lemma 5. However, this contradicts Lemma 4. □

The proof of Proposition 3.2 follows from Lemma 17.

G Proof of Proposition 4

First, the result in Proposition 4.1 follows immediately form Lemma 15. Second, the result in Proposition 4.2 is proved below in Lemma 18. Third, I construct an equilibrium that supports cycles of repression and mobilization, as described in Proposition 4.3, in Example 1. As part of this construction, I need a new result in Lemma 19.

Lemma 18 *If* $\kappa_C > (1+\delta)\pi_C^C$*, then the Center never represses in any equilibrium* σ*, i.e.,* $\sigma_C(1; g) = 0$ *for every grievances* g *and every equilibrium* σ*.*

Proof. To derive a contradiction, suppose the contrary. That is, suppose $\kappa_C > (1+\delta)\pi_C^C$ and the Center represses in equilibrium σ. Thus, the regime is has low capacity, and there exist some g such that $\sigma_C(1; g) > 0$. By Lemma 15, there exists $g^\dagger \geq g$ such that $\sigma_C(1; g^\dagger + 1) = 0$ and $\sigma_C(1; g') > 0$ for all $g' = g, ..., g^\dagger$. Then by Lemma 7, $\sigma_C(\varnothing; g') = 0$ for all $g' = g+1, .., g^\dagger + 1$. Hence, $\sigma_C(0; g^\dagger + 1) = 1$. We can compute C's continuation value at $g^\dagger$ as

$$\begin{aligned}V_C^\sigma(g^\dagger) &= \sigma_C(1; g^\dagger)U_C^\sigma(1; g^\dagger) + \sigma_C(0; g^\dagger)U_C^\sigma(0; g^\dagger) = U_C^\sigma(1; g^\dagger)\\ &= \pi_C^C - \kappa_C + \delta V_C^\sigma(g^\dagger + 1)\\ &= \pi_C^C - \kappa_C + \delta\Big[\sigma_P(g^\dagger + 1)\Big(-F(g^\dagger + 1)\psi + (1 - F(g^\dagger + 1))(\pi_C^C + \delta V_C^\sigma(g^\dagger)\Big) + \\ &\qquad (1 - \sigma_P(g^\dagger + 1))\Big(\pi_C^C + \delta V_C^\sigma(g^\dagger)\Big)\Big],\end{aligned}$$

where the second equality follows because σ is an equilibrium and $\sigma_C(1; g^\dagger) > 0$. Solving for $V_C^\sigma(g^\dagger)$ reveals that

$$V_C^\sigma(g^\dagger) = \frac{\pi_C^C(1 + (1 - F(g^\dagger + 1)\sigma_P(g^\dagger + 1))\delta) - \kappa_C - F(g^\dagger + 1)\sigma_P(g^\dagger + 1)\delta\psi}{1 - (1 - \sigma_P(g^\dagger + 1)F(g^\dagger + 1))\delta^2},$$

which is decreasing in $\sigma_P(g^\dagger + 1)$. Because $\sigma_P(g^\dagger + 1) \geq 0$, then

$$V_C^\sigma(g^\dagger) \leq \frac{\pi_C^C(1 + \delta) - \kappa_C}{1 - \delta^2}$$

Thus, $\kappa_C > (1 + \delta)\pi_C^C$ implies $V_C^\sigma(g^\dagger) < 0$. But this implies C can profitably deviate at $g^\dagger$ by granting independence and guaranteeing itself a payoff of zero. □

Lemma 19 *In low-capacity regimes, if* $F(g)(\pi_P^P - \pi_P^C) > \kappa_P$ *and* $g \geq g^+$*, then* $\sigma_P(g') = 1$ *and* $\sigma_C(1; g') = 0$ *for all* $g' \geq g$ *in every equilibrium* σ*.*

Proof. By Equation (1), P mobilizes at g' if

$$\kappa_C < F(g')\left[\bar{V}_P - \pi_P^C - \delta V_P^\sigma(g'-1)\right].$$

An upper bound on $V_P^\sigma(g'-1)$ is $\frac{\pi_P^P}{1-\delta}$, which is the discounted sum of P's largest per-period payoff. Combining these two inequalities implies P mobilizes when $F(g')(\pi_P^P - \pi_P^C) > \kappa_P$, which holds because $F(g)(\pi_P^P - \pi_P^C) > \kappa_P$, and F is increasing.

Second, I claim that $\sigma_C(1; g') = 0$ for all $g' \geq g$. Suppose not. Then there exists a $g^\dagger$ such that $\sigma_C(1; g^\dagger) > 0$ and $\sigma_C(0; g^\dagger + 1) = 1$ by Lemmas 7 and 15. The previous paragraph demonstrates that P mobilizes with probability 1 with grievance $g^\dagger + 1$. But this contradicts Lemma 4. □

Example 1 *In this example, I assume $\pi_C^C = \pi_P^P = 1$, and $\pi_P^C = 0$. In addition, $\kappa_C = 1.2$ and $\kappa_P = .25$. This implies that the regime has low repression capacity. Finally, $\delta = .9$, $\psi = 6$, and F takes the form:*

$$F(g) = \begin{cases} 0 & \text{if } g = 0 \\ \frac{g}{700} + \frac{33}{175} & \text{if } g \geq 1 \text{ and } g \leq 8 \\ 1 & \text{otherwise.} \end{cases}$$

Thus, $g^- = 0$, and $g^+ = 7$, because $\tilde{V}_C(6) \approx .33$ and $\tilde{V}_C(7) \approx -.15$. By Proposition 2, the Periphery mobilizes with probability one for all $g \in \{1, 2, ..., 7\}$ and the Center neither represses nor grants independence for all $g \in \{0, 1, 2, ..., 6\}$. Note that $F(9)(\pi_P^P - \pi_P^C) > \kappa_P$, so Lemma 19 implies the Periphery mobilizes for all grievances $g \geq 9$ and the Center does not repress at grievance $g \geq 9$.

We specify remaining behavior as follows.

1. *At grievance $g = 7$, the Periphery mobilizes with probability 1 and the Center mixes between repression and granting independence, $\sigma_C(1; 7) + \sigma_C(\varnothing; 7) = 1$*

2. *At grievance $g = 8$, the Center neither represses nor grants independence, i.e., $\sigma_C(0; 8) = 1$ and the Periphery mobilizes with probability $\sigma_P(8)$.*

We first characterize mixing probabilities, $\sigma_C(1; 7)$, $\sigma_C(\varnothing; 7)$, and $\sigma_P(8)$, such that the following hold:

$$\sigma_C(1;7) + \sigma_C(\varnothing;7) = 1$$
$$U_C^\sigma(1;7) = U_C^\sigma(\varnothing;7)$$
$$U_P^\sigma(1;8) = U_P^\sigma(0;8).$$

The first equation says the Center mixes between repression and granting independence at $g = 7 = g^+$. The second and third equations are C and P's indifference conditions, respectively. Because

$U_C^\sigma(\varnothing;7) = 0$, C's indifference equations takes the form:

$$\pi_C^C - \kappa_C + \delta V_C^\sigma(8) = 0, \tag{7}$$

where

$$V_C^\sigma(8) = \sigma_P(8)\left[-F(8)\psi + (1-F(8))\left(\pi_C^C + \delta V_C^\sigma(7)\right)\right] + (1-\sigma_P(8))\left[\pi_C^C + \delta V_C^\sigma(7)\right].$$

In equilibrium, $V_C^\sigma(7) = U_C^\sigma(\varnothing;7) = 0$. Thus, we have

$$V_C^\sigma(8) = \sigma_P(8)\left[-F(8)\psi + (1-F(8))\pi_C^C\right] + (1-\sigma_P(8))\pi_C^C.$$

Substituting the above equality into Equation (7), C's indifference condition takes the form:

$$\pi_C^C - \kappa_C + \delta\left(\sigma_P(8)\left[-F(8)\psi + (1-F(8))\pi_C^C\right] + (1-\sigma_P(8))\pi_C^C\right) = 0. \tag{8}$$

Next, consider P's indifference equation, $U_P^\sigma(1;8) = U_P^\sigma(0;8)$, which takes the form

$$-\kappa_P + F(8)\frac{\pi_P^P}{1-\delta} + (1-F(8))\left(\pi_P^C + \delta V_P^\sigma(7)\right) = \pi_P^C + \delta V_P^\sigma(7), \tag{9}$$

where

$$\begin{aligned} V_P^\sigma(7) &= \sigma_C(\varnothing;7)\frac{\pi_P^P}{1-\delta} + \sigma_C(1;7)\left[\pi_P^C + \delta V_P^\sigma(8)\right] \\ &= \sigma_C(\varnothing;7)\frac{\pi_P^P}{1-\delta} + \sigma_C(1;7)\left[\pi_P^C + \delta U_P^\sigma(0;8)\right] \\ &= \sigma_C(\varnothing;7)\frac{\pi_P^P}{1-\delta} + \sigma_C(1;7)\left[\pi_P^C + \delta\left(\pi_P^C + \delta V_P^\sigma(7)\right)\right] \end{aligned}$$

Here the second equality follows because $\sigma_C(0;8) = 1$. Solving Equations (8) and (9) with the constraint $\sigma_C(\varnothing;7) + \sigma_C(1;7) = 1$ reveals that

$$\sigma_P(8) = \frac{(1+\delta)\pi_C^C - \kappa_C}{(\pi_C^C + \psi)\delta F(8)} \approx .56$$

and

$$\sigma_C(1;7) = \frac{\kappa_P - F(8)(\pi_P^P - \pi_P^C)}{\delta^2\kappa_P + \delta F(8)(\pi_P^P - \pi_P^C)} \approx .13.$$

Finally, we check profitable deviations. First, P's indifference condition precludes profitable deviations at $g = 8$. Second, C does not have a profitable deviation at $g = 7$ due to its indifference *equation and because* $U_C^\sigma(0;7) = \tilde{V}_C(7) < 0$. *Also, C has no profitable deviation at* $g = 8$, *because* $V_C(8) > 0$. *To see this, note that* $U_C^\sigma(1;7) = \pi_C^C - \kappa_C + \delta V_C^\sigma(8) = 0$ *by Equation (7), and* $\pi_C^C - \kappa_C < 0$. *If C deviates by granting independence at* $g = 8$, *then its payoff is zero. Likewise, if C deviates by repressing, its payoff is* $\pi_C^C - \kappa_C + \delta V_C^\sigma(9)$, *which reduces to* $\pi_C^C - \kappa_C < 0$ *because C is granting independence when* $g = 9$. *Lemma 19 implies that C cannot profitably deviate by using repression, at grievances* $g \geq 9$. *Thus, we only need to verify that C cannot profitably deviate by choosing to refrain from repression or granting independence, at grievances* $g \geq 9$. *Because the Periphery mobilizes at* $g \geq 9$ *and* $F(g) = 1$, *mobilization surely succeeds, implying* $U_C^\sigma(0;g) = -\psi$ *for all* $g \geq 9$ *which is strictly less than C's utility from following its equilibrium strategy of granting independence.*

H Exogenous Decentralization

In this section, I continue to analyze the numerical example in Figure 4 and prove Proposition 5.

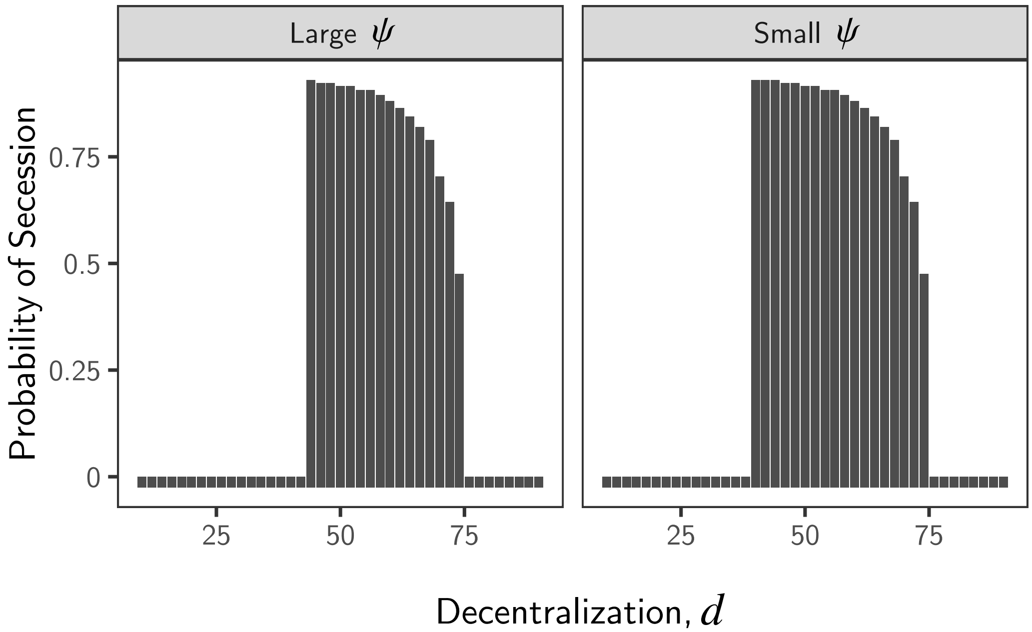
From, the example in Figure 4, I compute the probability that the country breaks apart due to secessionist mobilization —labeled probability of secession hereafter—as a function of decentralization. For a fixed d, three potential paths of play emerge at initial grievance g^1 in equilibrium. First, if $g^1 < g^+[d]$, the Center neither represses nor grants independence, and the probability secession is

$$\begin{cases} 0 & \text{if } g^1 \leq g^-[d] \\ 1 - \prod_{g' : g^\dagger[d] < g' \leq g^1} (1 - F(g')) & \text{otherwise.} \end{cases}$$

Second, if $g^1 \geq g^+[d]$ and the regime has high capacity ($\pi - d > \kappa_C$), then the Center represses in all future periods, and the probability of secession is zero. Third, if $g^1 \geq g^+[d]$ and the regime has low capacity ($\pi - d < \kappa_C$), the probability of secession is undefined. Although the Periphery will eventually gain control of its territory (Proposition 3.2), this may arise either from secessionist mobilization or Center-granted independence. This third case does not arise in the numerical example. As seen in Figure 4, if $g^1 \geq g^+[d]$ for some d, then the regime has high capacity.

Figure 6 graphs the probability of secession decentralization varies. When d is small, $g^1 > g^+[d]$ so the high-capacity regime represses and the probability of secession is zero. When d is large, $g^1 < g^-$, so grievances are small and a lasting peace emerges. When d is moderate, then the Center gambles for unity and secession occurs with positive probability. When ψ is large (left panel), all decentralization levels below $d = 44$ result in long-term repression and a zero probability of secession. When ψ is small (right panel), all decentralization levels below $d = 38$ result in long-term repression and a zero probability of secession.

Figure 6: Decentralization and comparative statics



Notes: The panels graph the probability of secession (vertical axis) for a fixed decentrlaization level (horizontal axis) with a large cost of secession $\psi = \frac{3\pi}{2}$ (left) and a small cost $\psi = \frac{\pi}{2}$ (right). The remaining parameters take on the following valules: $\pi = 100$, $\kappa_C = 50$, $\kappa_P = 50$, $\delta = 0.95$, and and $F(g) = 1 - \left(0.01g + 0.001g^2 - 1\right)^{-1}$.

Proposition 5 *Assume the regime has a high capacity for repression ($\kappa_C < \pi$) and initial grievances are large ($g^1 \geq g^+[0]$). There exist cutpoints $\underline{d}$ and $\overline{d}$ such that $0 \leq \underline{d} < \overline{d} < 1$ and secession occurs with positive probability on the equilibrium path only if decentralization is moderate, i.e., $\underline{d} < d < \overline{d}$.*

Proof. Set $\underline{d} = 0$. The regime has high repression capacity by assumption, and $g^1 \geq g^+[\underline{d}]$ implies that C represses with probability one in all future periods when the game begins at grievance g^1. As such the probability of secession is zero.

In addition, we can set $\bar{d}$ as follows

$$\bar{d} = \hat{d}(g^1) + \epsilon$$

where $\hat{d}$ is defined in Equation (4) above and $\epsilon \in \mathbb{R}$ is such that $0 < \epsilon < \max\{\frac{(1-\delta)\kappa_P}{F(g^1)}, 1\}$. Note that the fraction $\frac{(1-\delta)\kappa_P}{F(g^1)}$ is well defined because $F(g^1) \neq 0$. If $F(g^1) = 0$ then $g^1 \leq g^-[0] < g^+[0]$, a contradiction.

It suffices to show that $g^1 \leq g^-\left[\bar{d}\right]$ because this inequality implies that g^1 is small at decentralization level $\bar{d}$ and g^- is strictly increasing in d. As such, g^1 is small at decentralization levels $d > \bar{d}$. In addition, when $g^1 \leq g^-$ no mobilization occurs along the path of play by Proposition 1.

When $\pi_P^C = d$ and $\pi_P^P = \pi$, then we can write $g^-[d]$ as

$$g^-[d] = \max\left\{ g \in \mathbb{N}_0 \;\middle|\; \kappa_P > F(g)\frac{\pi - d}{1-\delta} \right\}.$$

Thus, $g^1 \leq g^-\left[\bar{d}\right]$, as required. □

I Proof of Proposition 6

We first prove Proposition 6.1 and then present two numerical examples that establish Propositions 6.2 and 6.3.

Proof of 6.1. Consider equilibrium (d^*, σ). We first prove that $d^* \leq \min\{\hat{d}(g^1), \kappa_C\}$. First, $d^* \leq \kappa_C$. To see this, note that $V_C^\sigma(g; d^*) \leq \frac{\pi - d^*}{1-\delta}$. Thus, if C chooses $d^* > \kappa_C$, then $V_C^\sigma(g; d^*) < \frac{\pi - \kappa_C}{1-\delta}$, which means C can profitably deviate by choosing $d^* = 0$ and repressing in all future periods.

Second, $d^* \leq \hat{d}(g^1)$. When C chooses $d^* > \hat{d}(g^1)$, then $g^1 \leq g^-[d^*]$, which implies that $V_C^\sigma(g^1; d^*) = \frac{\pi - d^*}{1-\delta}$, which is strictly decreasing in d^*. So C has a profitable deviation by choosing decentralization $d = d^* - \epsilon$ for $\epsilon > 0$ but close to zero. This establishes the desired result.

Finally, we prove that if $\kappa_C < \max\left\{\frac{\pi}{2}, \pi - \hat{d}(g^1)\right\}$ and $d^* > 0$, then $g^1 < g^+[d^*]$, i.e., C never represses nor grants independence along the subsequent path of play. To do this suppose not and consider two relevant cases.

Case 1: $\pi - d^* - \kappa_C > 0$. Then $V_C^\sigma(g^1; d^*) = \frac{\pi - d^* - \kappa_C}{1-\delta}$, and C can profitably deviate by choosing $d^* = 0$ and repressing in all future periods.

Case 2: $\pi - d^* - \kappa_C \leq 0$. If $\kappa_C < \frac{\pi}{2}$, then

$$d^* \geq \pi - \kappa_C > \pi - \frac{\pi}{2} > \kappa_C,$$

which contradicts the upper bound described above. If $\kappa_C < \pi - \hat{d}(g^1)$, then we have

$$\begin{aligned} d^* &\geq \pi - \kappa_C \\ &> \pi - \left(\pi - \hat{d}(g^1)\right) \\ &= \pi - \left(\frac{(1-\delta)\kappa_P}{1-\delta}\right) \\ &= \hat{d}(g^1), \end{aligned}$$

which contradicts the upper bound described above. □

The next example illustrates that the Center decentralizes in equilibrium (d^*, σ) and the subsequent interaction entails gambling for unity.

Example 2 *For the exogenous parameters, we consider* $\pi = 100$, $\psi = 100$, $\kappa_C = 40$, $\kappa_P = 95$ *and* $\delta = 0.9$. *In addition, F takes the form:*

$$F(g) = \begin{cases} 0 & \textit{if } g = 0 \\ \frac{1}{10} & \textit{if } g \in \{1, \ldots, 100\} \\ \frac{3}{10} & \textit{if } g = 101 \\ 1 & \textit{if } g \geq 102. \end{cases}$$

and initial grievances are $g^1 = 101$.

Note that $\kappa_C < \frac{\pi}{2}$, *so Proposition 6.1 implies that if C decentralizes in an equilibrium* (d^*, σ), *then it chooses to neither repress nor grant independence in all future periods, in which case, C's expected utility is* $\tilde{V}_C(g^1; d^*)$. *Thus, if C chooses* $d^* > 0$, *it will choose a* d^* *that solves*

$$F(g')\frac{\pi - d^*}{1 - \delta} - \kappa_P = 0$$

for some $g' > g^-[0]$ *and* $g' \leq g^1$. *In words, if C decentralizes, it will choose a decentralization level that makes the Periphery (at some grievance level* g^1*) indifferent between mobilizing and not along the subsequent path of play. If not, C can profitably deviate by offering slightly less decentralization without changing the Periphery's strategy in states* $g \leq g^1$.

Given this discussion and the construction of F, there are three possible decentralization levels to consider: $\{0, \hat{d}(1), \hat{d}(101)\}$. *Note that* $\hat{d}(101) = \frac{205}{3} > \kappa_C$. *As such, the upper bound in the previous proof shows that* $d^* \neq \hat{d}(101)$ *in any equilibrium. Thus, there are only two possible decentralization levels in equilibrium:* $\{0, \hat{d}(1)\}$.

If C chooses $d^* = 0$, *then* $g^-[0] = 0$ *and* $g^+[0] = 6$. *Because* $g^1 > g^+[0]$, *if C chooses* $d^* = 0$, *then long-term repression is the equilibrium outcome, which implies C's dynamic payoff is* $\frac{\pi - \kappa_C}{1 - \delta} = 600$.

If C chooses $d^* = \hat{d}(1) = 5$, *then* $g^-[d^*] = 100$ *and* $g^+[0] = 102$. *Because* $g^1 < g^+[d^*]$, *if C chooses* $d^* = \hat{d}(1)$, *then one period of gambling for unity is the equlibrium path of play, in which case C's expected utility is*

$$-F(g^1)\psi + (1 - F(g^1))\left[\pi - d^* + \delta\frac{\pi - d^*}{1 - \delta}\right] = 635.$$

As such, C chooses to decentralize, $d^* = \hat{d}(1) > 0$ *and gambling for unity occurs along the subsequent equilibrium path of play.*

The next example illustrates that the Center decentralizes in equilibrium (d^*, σ) and the a long-term peace emerges in the subsequent interaction.

Example 3 *The payoff parameters match those from Example 2, but now F takes the form:*

$$F(g) = \begin{cases} 0 & \text{if } g = 0 \\ \frac{1}{10} & \text{if } g \in \{1, \ldots, 101\} \\ 1 & \text{if } g \geq 102. \end{cases}$$

and initial grievances are $g^1 = 101$. *Following the logic in the previous example, there are two potential levels of decentralization in equilibrium:* $\{0, \hat{d}(1)\}$. *If C chooses* $d^* = 0$, *then its payoff is* $\frac{\pi - \kappa_C}{1-\delta} = 600$ *for reasons described above. If C choses* $d^* = \hat{d}(1)$, *then* $g^-[d^*] = 101 = g^1$ *and its equilibrium payoff is* $\frac{\pi - d^*}{1-\delta} = 950$. *As such, C chooses to decentralizes and a long-term peace emerges.*