Online Appendix

A Continuation Values and Expected Utilities

Let $\tilde{V}_i$ denote $i$’s continuation value after a history in which the Periphery was won control of the territory. These values are independent of a strategy profile $\sigma$ and take the form $\tilde{V}_C = 0$ and $\tilde{V}_P = \pi_P - \delta V_P^{\sigma}$. Let $V_i^{\sigma}(g)$ denotes $i$’s continuation value from beginning the game with grievance $g$ when the Periphery has not won control of its territory and actors subsequently playing according to profile $\sigma$. In a similar vein, $U_C^{\sigma}(r; g)$ and $U_P^{\sigma}(m; g)$ denote the Center and Periphery’s dynamic payoffs from choosing $r \in \{\emptyset, 0, 1\}$ and $m \in \{0, 1\}$ given grievance $g$ when actors subsequently play according to profile $\sigma$. For the Center, $U_C^{\sigma}(r; g)$ takes the following form:

$$U_C^{\sigma}(r; g) = \begin{cases} 0 & \text{if } r = \emptyset \\ \pi_C - \kappa_C + \delta V_C^{\sigma}(g + 1) & \text{if } r = 1 \\ -\sigma_P(g)F(g)\psi + (1 - \sigma_P(g)F(g))\left(\pi_C + \delta V_C^{\sigma}(\max\{g - 1, 0\})\right) & \text{if } r = 0. \end{cases}$$
For the Periphery, \( U_P^r(m; g) \) denotes the its dynamic payoff conditional on having reached its decision node, i.e., the Center chooses \( r = 0 \), in state \( g \). Thus, \( U_P^r(m; g) \) takes the form

\[
U_P^r(m; g) = \begin{cases} 
-\kappa_P + F(g)\tilde{V}_P + (1 - F(g))\left(\pi_P^C + \delta V_P^r(\max\{g - 1, 0\})\right) & \text{if } m = 1 \\
\pi_P^C + \delta V_P^r(\max\{g - 1, 0\}) & \text{if } m = 0.
\end{cases}
\]

With this notation in hand, the next definition states the equilibrium conditions.

**Definition 1** Strategy profile \( \sigma \) is an equilibrium if the following hold:

\[
\begin{align*}
\sigma_C(r; g) > 0 & \implies U_C^r(r; g) \geq U_C^r(r'; g), \\
\sigma_P(g) > 0 & \implies U_P^r(1; g) \geq U_P^r(0; g), \text{ and} \\
\sigma_P(g) < 1 & \implies U_P^r(0; g) \geq U_P^r(1; g)
\end{align*}
\]

for all grievance \( g \) and polices \( r, r' \in \{\emptyset, 0, 1\} \).

Because the game is a dynamic game with a countable state space and a finite number of actions, an equilibrium from Definition 1 exists in mixed strategies. Notice that for some grievance \( g \), the Center’s continuation value, \( V_C^r(g) \), takes the form

\[
V_C^r(g) = \sum_{r \in \{\emptyset, 0, 1\}} \sigma(r; g)U_C^r(r; g).
\]

Thus, if \( \sigma \) is an equilibrium and \( \sigma(r; g) > 0 \) for some grievance \( g \) and action \( r \in \{\emptyset, 0, 1\} \), then \( V_C^r(g) = U_C^r(r; g) \) or else \( C \) has a deviation by playing some \( r' \in \{\emptyset, 0, 1\} \).

**B Proof of Proposition 1**

**Proposition 1** If grievances are small, then the Periphery never mobilizes, the Center neither represses nor grants independence, and grievances dissipate on the equilibrium path. That is, \( g \leq g^- \) implies \( \sigma_P(g) = 0 \) and \( \sigma_C(0; g) = 1 \) in every equilibrium \( \sigma \).

**Proof.** The proof that \( g \leq g^- \) implies the Periphery does not mobilize with positive probability is covered in the main text. We prove that \( g \leq g^- \) implies the Center does not repress or grant independence with positive probability. To see this, suppose \( \sigma_C(r; g) > 0 \) for some \( g \leq g^- \), \( r \neq 0 \), and equilibrium \( \sigma \). There are two cases.
Case 1: $r = 1$, repression. Then, $C$’s expected utility is

$$U_C^r(1; g) = \pi_C - \kappa_C + \delta V_C'(g + 1)$$

$$\leq \pi_C - \kappa_C + \delta \frac{\pi_C}{1 - \delta}$$

$$< \frac{\pi_C}{1 - \delta}.$$

However, $\frac{\pi_C}{1 - \delta}$ is $C$’s continuation value if it takes action $r = 0$ in all future periods because grievances will never increase and $P$ will never mobilize with positive probability along the subsequent path of play. Hence, taking action $r = 0$ in all future periods is a profitable deviation, a contradiction.

Case 2: $r = \emptyset$, independence. Then, $C$’s expected utility is

$$U_C^r(\emptyset; g) = 0 < \pi_C - \kappa_C + \delta \pi_C 1 - \delta.$$

As in Case 1, this inequality implies taking action $r = 0$ in all future periods is a profitable deviation, a contradiction. □

C Properties of $\tilde{V}_C$

We first state and prove three Lemmas concerning properties of $\tilde{V}_C$.

Lemma 1

1. $\tilde{V}_C(g) > \frac{-F(g)\psi + (1 - F(g))\pi_C}{1 - (1 - F(g))\delta}$ for all $g$ such that $F(g) > 0$.

2. $\tilde{V}_C(g - 1) > \tilde{V}_C(g)$ for all $g > g^*$.

3. If Assumption 1 holds, then $\lim_{g \to \infty} \tilde{V}_C(g) = \frac{-p\psi + (1 - p)\pi_C}{1 - (1 - p)\delta}$.

Proof. To show (1), consider some $g$ such that $F(g) > 0$ and $F(g') = 0$ for all $g' < g$. Such a $g$ exists because $F(0) = 0$ and $\lim_{g \to \infty} F(g) = p > 0$. In addition, $F(g) < 1$ because there exists at least one $g$ such that $F(g) \in (0, 1)$ by assumption. Then we have

$$\tilde{V}_C(g) = -F(g)\psi + (1 - F(g))\left(\pi_C + \delta \frac{\pi_C}{1 - \delta}\right)$$

$$= (1 - (1 - F(g))\delta)\frac{-F(g)\psi + (1 - F(g))\pi_C}{1 - (1 - F(g))\delta} + (1 - F(g))\delta \frac{\pi_C}{1 - \delta}$$

$$> \frac{-F(g)\psi + (1 - F(g))\pi_C}{1 - (1 - F(g))\delta},$$

where the strict inequality follows because $F(g) \in (0, 1)$

For induction, consider some $g$ such that $F(g) > 0$ and $F(g - 1) > 0$, which implies $g - 1 > 0$. Suppose the inequality holds for all $g' < g$ such that $F(g') > 0$. Then we have

$$\tilde{V}_C(g) = -F(g)\psi + (1 - F(g))(\pi^C_C + \delta \tilde{V}_C(g - 1))$$

$$> -F(g)\psi + (1 - F(g))\left(\pi^C_C + \delta \frac{-F(g - 1)\psi + (1 - F(g - 1))\pi^C_C}{1 - (1 - F(g - 1))\delta}\right)$$

$$\geq -F(g)\psi + (1 - F(g))\left(\pi^C_C + \delta \frac{-F(g)\psi + (1 - F(g))\pi^C_C}{1 - (1 - F(g))\delta}\right)$$

$$= -F(g)\psi + (1 - F(g))\pi^C_C$$

where the third line follows because the fraction $\frac{-F(g)\psi + (1 - F(g))\pi^C_C}{1 - (1 - F(g))\delta}$ is decreasing in $F(g)$.

To show (2), note that it must hold when $g = g^- + 1$, because $\psi > 0$ and $F(g) > 0$ as $g > g^-$. Now consider some $g > g^- + 1$. For induction, suppose $\tilde{V}_C(g' - 1) > \tilde{V}_C(g')$ for all $g'$ such that $g^- < g' < g$. Then

$$\tilde{V}_C(g) = -F(g)\psi + (1 - F(g))(\pi^C_C + \delta \tilde{V}_C(g - 1))$$

$$\leq -F(g - 1)\psi + (1 - F(g - 1))(\pi^C_C + \delta \tilde{V}_C(g - 1))$$

$$< -F(g - 1)\psi + (1 - F(g - 1))(\pi^C_C + \delta \tilde{V}_C(g - 2))$$

$$= \tilde{V}_C(g - 1),$$

where the second line follows because

$$\tilde{V}_C(g) > \frac{-F(g)\psi + (1 - F(g))\pi^C_C}{1 - (1 - F(g))\delta} \geq -\psi$$

and $F(g)$ is increasing in $g$.

To prove (3), consider a sequence $\{g_n\}_{n=1}^{\infty}$ such that $\lim_{n\to\infty} g_n = \infty$ and $g_n < g_{n+1}$. Then the sequence $\{\tilde{V}_C(g_n)\}_{n=1}^{\infty}$ is weakly decreasing due to above arguments. In addition, $\{\tilde{V}_C(g_n)\}_{n=1}^{\infty}$ is bounded below because $C$'s payoffs are finite and $C$ discounts with rate $\delta < 1$. Thus, $\{\tilde{V}_C(g_n)\}_{n=1}^{\infty}$ has a limit, call it $L$. If the Periphery does value independence, the we have

$$L = \lim_{n\to\infty} \tilde{V}_C(g_n)$$

$$= \lim_{n\to\infty} F(g_n)(-\psi) + \lim_{n\to\infty} (1 - F(g_n))(\pi^C_C + \delta \tilde{V}_C(g_n - 1))$$

$$= -p\psi + (1 - p)(\pi^C_C + \delta L),$$
which implies \( L = \frac{-p\psi + (1-p)\pi_C}{1-(1-p)\delta}. \)

The next Lemma demonstrates that \( C \)'s gambling for unity utility, \( \tilde{V}_C \) is a lower bound on its equilibrium expected utility, \( V^*_C \).

**Lemma 2** For all grievances \( g \), \( V^*_C(g) \geq \tilde{V}_C(g) \) in every equilibrium \( \sigma \).

**Proof.** To see this, suppose not. That is, suppose there exist grievance \( g \) and equilibrium \( \sigma \) such that \( V^*_C(g) < \tilde{V}_C(g) \). Then by the construction of \( \tilde{V}_C \) and Proposition 1, \( g > g^- \), or else \( V^*_C(g) = \frac{\pi_C}{1-\delta} = \tilde{V}_C \).

Next consider a deviation for \( C \), labeled \( \sigma'_C \), such that \( \sigma'_C(0; g') = 1 \) for all \( g' \leq g \). I now demonstrate that \( V^*_C(g') \geq \tilde{V}_C(g') \), where \( \sigma' = (\sigma'_C, \sigma_P) \), which implies \( \sigma'_C \) is a profitable deviation because \( \tilde{V}_C(g) > V^*_C(g) \) by supposition.

The proof is by induction. The inequality, \( V^*_C(g') \geq \tilde{V}_C(g') \), holds when \( g' \leq g^- \) by the construction of \( \tilde{V}_C \) and Proposition 1. Now consider some \( g' > g^- \) and suppose \( V^*_C(g'') \geq \tilde{V}_C(g'') \) for all \( g'' < g' \). Then we have

\[
V^*_C(g') = -\sigma_P(g')F(g')\psi + (1 - \sigma_P(g')F(g'))(\pi_C + \delta V^*_C(g' - 1))
\geq -\sigma_P(g')F(g')\psi + (1 - \sigma_P(g')F(g'))(\pi_C + \delta \tilde{V}_C(g' - 1))
\geq -F(g')\psi + (1 - F(g'))(\pi_C + \delta \tilde{V}_C(g' - 1))
= \tilde{V}_C(g').
\]

Hence, \( V^*_C(g') \geq \tilde{V}_C(g') \) as required. \( \square \)

The final Lemma demonstrates that the cutpoint \( g^+ \) exists if and only if Assumptions 1 and 2 hold.

**Lemma 3** The cutpoint \( g^+ \) solving Equation (3) exists if and only if the Periphery values independence (Assumption 1) and secession is costly (Assumption 2).

**Proof.** For necessity, suppose Assumptions 1 and 2 hold. Then Lemma 1 and Assumption 1 imply that \( \tilde{V}_C(g) \) is weakly decreasing in \( g \) and converges to

\[
\lim_{g \to \infty} \tilde{V}_C(g) = \frac{-p\psi + (1-p)\pi_C}{1-(1-p)\delta}.
\]
Because $\tilde{V}_C(g) = \frac{\pi_C}{1-\delta} > 0$ for all $g \leq g^-$ and $\tilde{V}_C(g)$ is strictly decreasing in $g$ when $g > g^-$, we require

$$-p\psi + (1-p)\pi_C^C < \max \left\{ \frac{\pi_C^C - \kappa_C}{1-\delta}, 0 \right\}.$$  \hspace{1cm} (6)

We now demonstrate that the inequality in Equation (6) holds when $\pi_C^C > \kappa_C$, the proof when $\pi_C^C < \kappa_C$ is identical. Suppose $\pi_C^C - \kappa_C > 0$. Then Equation (6) reduces to

$$-p\psi + (1-p)\pi_C^C < \frac{\pi_C^C - \kappa_C}{1-\delta},$$

which is equivalent to

$$\psi > \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)}.$$  

Because $\pi_C^C - \kappa_C > 0$, Assumption 2 reduces to

$$\psi > \min \left\{ \frac{\pi_C^C(1-p)}{p}, \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)} \right\} = \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)}.$$  

Thus, the inequality in Equation (6) holds, and therefore $g^+$ exists.

For sufficiency, suppose Assumption 1 does not hold, then $\kappa_P \geq F(g)\frac{\pi_P^P - \pi_P^C}{1-\delta}$ for all grievances $g$. Thus, $\tilde{V}_C(g) = \frac{\pi_C}{1-\delta} > \max \left\{ \frac{\pi_C^C - \kappa_C}{1-\delta}, 0 \right\}$ for all grievances $g$. Now suppose Assumption 1 holds but not Assumption 2. Then Lemma 1 implies that, for all $g$

$$\tilde{V}_C(g) \geq \frac{-p\psi + (1-p)\pi_C^C}{1-(1-p)\delta} \geq \max \left\{ \frac{\pi_C^C - \kappa_C}{1-\delta}, 0 \right\}.$$  \hspace{1cm} \(\square\)

### D Preliminary Results

In this section, we state and prove two technical results that are essential to characterize equilibria in the remainder of the paper.

**Lemma 4** If $\sigma_C(1; g) > 0$ and $\sigma_C(0; g + 1) = 1$ for some grievance $g$, then $\sigma_P(g + 1) < 1$ in every equilibrium $\sigma$. 


Proof. Suppose not. Then there exists a $g$ such that $\sigma_C(1; g) > 1$, $\sigma_C(0; g + 1) = 1$ and $\sigma_P(g + 1) = 1$ in equilibrium $\sigma$. We can write $V^\sigma_C(g + 1)$ as

\[
V^\sigma_C(g + 1) = -F(g + 1)\psi - (1 - F(g + 1))\left(\pi^C_C + \delta V^\sigma_C(g)\right)
= -F(g + 1)\psi - (1 - F(g + 1))\left(\pi^C_C + \delta U^\sigma_C(1; g)\right)
= -F(g + 1)\psi - (1 - F(g + 1))\left(\pi^C_C + \delta (\pi^C_C - \kappa_C + \delta V^\sigma_C(g + 1))\right).
\]

Solving reveals that

\[
V^\sigma_C(g + 1) = \frac{(1 - F(g + 1))(\pi(1 + \delta) - \delta \kappa_C) - F(g + 1)\psi}{1 - (1 - F(g + 1)\delta)^2}.
\]

By Lemma 2, $V^\sigma_C(g + 1) \geq \tilde{V}_C(g + 1)$. By Lemma 1.1,

\[
\tilde{V}_C(g) > \frac{(1 - F(g + 1))\pi^C_C - F(g + 1)\psi}{1 - (1 - F(g + 1)\delta)}.
\]

Stringing these two inequalities together,

\[
V^\sigma_C(g + 1) > \frac{(1 - F(g + 1))\pi^C_C - F(g + 1)\psi}{1 - (1 - F(g + 1)\delta)}.
\]

Substituting the closed form solution for $V^\sigma_C(g + 1)$ into the inequality above and solving for $\kappa_C$ reveals that

\[
\kappa_C < \frac{F(g + 1)(\pi^C_C + \psi(1 - \delta))}{1 - (1 - F(g + 1)\delta)}.
\]

To derive a contradiction, consider a deviation in which $C$ plays $r = 1$ with probability 1 in all future periods beginning at grievance $g + 1$. This is a profitable deviation if and only if

\[
V^\sigma_C(g + 1) < \frac{\pi^C_C - \kappa_C}{1 - \delta} \iff \kappa_C < \frac{F(g + 1)(\pi^C_C + \psi(1 - \delta))}{1 - (1 - F(g + 1)\delta)}.
\]

However, $\kappa_C < \frac{F(g + 1)(\pi^C_C + \psi(1 - \delta))}{1 - (1 - F(g + 1)\delta)}$ as shown above. Hence, $C$ can profitably deviate by repressing in all future periods. \hfill \Box

Lemma 5 Consider some $g > g^-$ and equilibrium $\sigma$. If (a) $\sigma_C(0; g - 1) = 1$ or $\sigma_C(0; g) = 1$ and (b) $\sigma_C(\emptyset; g') = 0$ for all $g' < g$, then $\sigma_P(g) = 1$.

Proof. Suppose not. That is, consider some equilibrium $\sigma$ and grievance $g > g^-$ such that

(a) $\sigma_C(0; g - 1) = 1$ or $\sigma_C(0; g) = 1$,
(b) \( \sigma_C(\emptyset; g') = 0 \) for all \( g' < g \), and

(c) \( \sigma_P(g) < 1 \).

Because \( \sigma \) is an equilibrium, we require \( U_P^\sigma(0; g) \geq U_P^\sigma(1; g) \) to rule out profitable deviations, which is equivalent to

\[
\kappa_P \geq F(g) \left[ \bar{V}_P - \pi_P^C - \delta V_P^\sigma(g - 1) \right].
\]

Because \( \sigma_C(0; g - 1) = 1 \) or \( \sigma_C(0; g) = 1 \), the path of play will never reach a grievance larger than \( g \). Because \( \sigma_C(\emptyset; g') = 0 \) for all \( g' \leq g \), the Center will never grant independence along the subsequent path of play. Recall that when the \( C \) represses, \( P \) stage payoff is \( \pi_P^C \), which is its payoff if it chooses not to mobilize, and even if \( C \) does repress with positive probability at some \( g' < g \), the subsequent path of play will still never reach a grievance larger than \( g \). Then \( g > g^- \) implies \( V_P^\sigma(g - 1) \) is bounded above by

\[
\frac{F(g)\bar{V}_P + (1 - F(g))\pi_P^C - \kappa_P}{1 - (1 - F(g))\delta},
\]

which is \( P \)'s payoff if its grievance never depreciates along the path of play, \( C \) never represses, and \( P \) always mobilizes. Combining these two inequalities, we require

\[
\kappa_P \geq F(g) \left[ \bar{V}_P - \pi_P^C - \delta V_P^\sigma(g - 1) \right] \\
\geq F(g) \left[ \bar{V}_P - \pi_P^C - \delta \frac{F(g)\bar{V}_P + (1 - F(g))\pi_P^C - \kappa_P}{1 - (1 - F(g))\delta} \right].
\]

Solving for \( \kappa_P \) implies

\[
\kappa_P \geq F(g) \frac{\pi_P^C - \pi_P^C}{1 - \delta},
\]

that is, \( g \leq g^- \). But this contradicts the assumption \( g > g^- \). \( \square \)

### E  Proof of Proposition 2

This section characterizes equilibrium behavior at moderate grievances.

We now prove that \( g < g^+ \) implies \( \sigma_C(0; g) = 1 \) in every equilibrium \( \sigma \), that is, the Center neither represses nor grants independence with moderate grievances. The result requires preliminary lemmas. Notice that if either Assumption 1 or 2 does not hold, \( \bar{V}_C(g) > \max \left\{ \frac{\pi_C^C - \kappa_C}{1 - \delta}, 0 \right\} \) for all \( g \), and we can set \( g^+ = \infty \) in the subsequent results.
Lemma 6 If $g < g^+$, then $\sigma_C(\varnothing; g) = 0$ in every equilibrium $\sigma$.

Proof. If not, then $V_C^\sigma(g) = U_C^\sigma(\varnothing; g) = 0$. If $g < g^+$, this contradicts Lemma 2 because $\hat{V}_C(g) > 0 = V_C^\sigma(g)$. □

Lemma 7 For all $g$, $\sigma(r; g) > 0$ imply $\sigma(\varnothing; g+1) = 0$ in every equilibrium $\sigma$.

Proof. First, if $\kappa_C < \pi_C^c$, then $C$ cannot grant independence with positive probability in any equilibrium. Doing so would result in a payoff of 0, but $C$ could repress for all future periods, giving a payoff of $\pi_C^c - \kappa_C < 0$. Thus, consider the case where $\pi_C^c - \kappa_C < 0$. Suppose $\sigma_C(r; g) > 0$ for some $g$ and $\sigma_C(\varnothing; g+1) > 0$. Then

$$V_C^\sigma(g) = U_C^\sigma(r; g)$$
$$= \pi_C^c - \kappa_C + \delta V_C^\sigma(g+1)$$
$$= \pi_C^c - \kappa_C + \delta U_C^\sigma(\varnothing; g)$$
$$= \pi_C^c - \kappa_C < 0,$$

but this means $C$ can profitably deviate at $g$ by granting independence, i.e., $\sigma$ is not an equilibrium. □

Lemma 8 Fix an equilibrium $\sigma$. Then there does not exist a $g < g^+$ such that $\sigma_C(1; g') > 0$ for all $g' \geq g$.

Proof. Suppose not and consider such a $g < g^+$ where $\sigma_C(1; g') > 0$ for all $g' \geq g$ in equilibrium $\sigma$. Then

$$V_C^\sigma(g) = U_C^\sigma(1; g) = \pi_C^c - \kappa_C + \delta V_C^\sigma(g+1).$$

Because $V_C(g') = U_C^\sigma(r; g')$ for all $g'$ such that $\sigma_C(r; g') > 0$, similar substitutions imply $V_C^\sigma(g) = \frac{\pi_C^c - \kappa_C}{1-\delta}$. However, $g < g^+$ implies

$$\hat{V}_C(g) > \frac{\pi_C^c - \kappa_C}{1-\delta} = V_C^\sigma(g),$$

by Equation (3). However, $\hat{V}_C(g) > V_C^\sigma(g)$ contradicts Lemma 2. □

With these lemmas in hand, we now state the main result of the section.

Proposition 2 If grievances are moderate, then the Periphery always mobilizes, the Center neither represses nor grants independence, and grievances dissipate on the equilibrium path. That is, $g \in (g^-, g^+)$ implies $\sigma_P(g) = 1$ and $\sigma_C(0; g) = 1$ in every equilibrium $\sigma$. 

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Proof. We first prove prove that $\sigma_C(0; g) = 1$ when $g \in (g^-, g^+)$ and $\sigma$ is an equilibrium. Suppose not. By Lemma 6, $\sigma_C(1; g) > 0$. Furthermore, $C$ represses with positive probability for at most some finite $k$ periods by Lemma 8. That is, there exists a $\tilde{g}$ such that $\sigma_C(1; g') > 0$ for $g' = g, ..., \tilde{g}$ and $\sigma_C(1; \tilde{g} + 1) = 0$. By Lemma 7, this implies $\sigma_C(0; \tilde{g} + 1) = 1$. In addition, Proposition 1 and Lemma 7 imply $\sigma_C(\emptyset; g') = 0$ for all $g' < \tilde{g}$. Thus, Lemma 5 and $\sigma_C(1; \tilde{g} + 1) = 0$ imply $P$ mobilizes at $\tilde{g} + 1$ with probability 1. However, $\sigma_C(1; \tilde{g}) > 0, \sigma_C(0; \tilde{g} + 1) = 1$, and $\sigma_P(\tilde{g} + 1) = 1$ contradict Lemma 4. To pin down $P$’s strategy at $g \in (g^-, g^+)$, note that $\sigma_C(0; g') = 1$ for all $g' < g^+$. Then Lemma 5 implies $\sigma_P(g) = 1$. □

F Proof of Proposition 3

We now characterize equilibrium behavior at large grievances ($g \geq g^+$). We consider the generic case in which there does not exist $g \in \mathbb{N}_0$ such that $\tilde{V}_C(g) = \max\left\{\frac{\pi_C - \kappa_C}{1 - \delta}, 0\right\}$, that is $\tilde{V}_C(g^+) < \max\left\{\frac{\pi_C - \kappa_C}{1 - \delta}, 0\right\}$, where the inequality from Equation (3) holds strictly. If this held with equality, the Center would be indifferent leading to trivial indeterminacy. We consider high- and low-capacity regimes separately because the proof techniques vary dramatically between the two cases.

F.1 High repression capacity: $\kappa_C < \pi_C^C$

Lemma 9 In high-capacity regimes, $\sigma_C(\emptyset; g) = 0$ for every grievance $g$ and in every equilibrium $\sigma$.

The proof is straightforward and omitted.

Lemma 10 In high-capacity regimes, $\sigma_C(1; g^+) = 1$ and $\sigma_C(1; g) > 0$ for all $g > g^+$ in every equilibrium $\sigma$.

Proof. The proof is by induction. First, we demonstrate that $\sigma_C(1; g^+) = 1$. To see this, suppose $\sigma_C(1; g^+) < 1$. Then Lemma 9 implies $\sigma_C(0; g^+) > 0$, in which case we have

$$U_C'(0; g^+) = \tilde{V}_C(g^+) < \frac{\pi_C^C - \kappa_C}{1 - \delta}.$$ 

This means $C$ can profitably deviate at grievance $g^+$ by repressing for an infinite number of periods, a contradiction.

For induction, consider some $g > g^+$ and assume $\sigma_C(1; g - 1) > 0$. To derive a contradiction, assume $\sigma_C(1; g) = 0$. By Lemma 9, $\sigma_C(0; g) = 1$. Likewise, Lemma 9 guarantees $C$ does not grant
Lemma 1.1 implies $g < \sigma$ and $g < \bar{\sigma}$. Then we have $\bar{\sigma} = \min(g, \sigma)$. The proof is constructive. Define $\bar{\sigma}$. Proof. Suppose not. Then there exists $g > g^*$ such that $\sigma_C(0; g) > 0$ and $\sigma_P(g) = 1$. Because $g > g^*$, $g - 1 > g^*$. Likewise, $\sigma_C(1; g) > 0$ by Lemma 10, so it must be the case that $U_C^\sigma(0; g) = U_C^\sigma(1; g)$. Then we have

$$U_C^\sigma(0; g) = U_C^\sigma(1; g) \iff -F(g)\psi + (1 - F(g))(\pi_C^\sigma + \delta V_C^\sigma(g - 1)) = \pi - \kappa_C + \delta V_C^\sigma(g + 1)$$

$$\iff -F(g)\psi + (1 - F(g))\left(\pi_C^\sigma + \delta \frac{\pi - \kappa_C}{1 - \delta}\right) = \pi - \kappa_C$$

$$\iff \kappa_C = \frac{F(g)(\pi_C^\sigma + (1 - \delta)\psi)}{1 - (1 - F(g))\delta},$$

where we use Lemma 11 and $g - 1 > g^*$ to substitute for values $V_C^\sigma(g - 1)$ and $V_C^\sigma(g + 1)$.

Because $\sigma$ is an equilibrium, we require $U_C^\sigma(1; g) = V_C^\sigma(g) \geq \tilde{V}_C(g)$, by Lemma 2. Then Lemma 1.1 implies

$$U_C^\sigma(1; g) > \frac{-F(g)\psi + (1 - F(g))\pi_C^\sigma}{1 - (1 - F(g))\delta} \iff \frac{\pi_C^\sigma - \kappa_C}{1 - \delta} > \frac{-F(g)\psi + (1 - F(g))\pi_C^\sigma}{1 - (1 - F(g))\delta}$$

$$\iff \kappa_C < \frac{F(g)(\pi_C^\sigma + (1 - \delta)\psi)}{1 - (1 - F(g))\delta},$$

which establishes the desired contradiction. □

Lemma 12 In high-capacity regimes, $g > g^*$ and $\sigma_C(0; g) > 0$ imply $\sigma_P(g) < 1$ in every equilibrium $\sigma$.

Proof. Suppose not. Then there exists $g > g^*$ such that $\sigma_C(0; g) > 0$ and $\sigma_P(g) = 1$. Because $g > g^*$, $g - 1 > g^*$. Likewise, $\sigma_C(1; g) > 0$ by Lemma 10, so it must be the case that $U_C^\sigma(0; g) = U_C^\sigma(1; g)$. Then we have

$$U_C^\sigma(0; g) = U_C^\sigma(1; g) \iff -F(g)\psi + (1 - F(g))(\pi_C^\sigma + \delta V_C^\sigma(g - 1)) = \pi - \kappa_C + \delta V_C^\sigma(g + 1)$$

$$\iff -F(g)\psi + (1 - F(g))\left(\pi_C^\sigma + \delta \frac{\pi - \kappa_C}{1 - \delta}\right) = \pi - \kappa_C$$

$$\iff \kappa_C = \frac{F(g)(\pi_C^\sigma + (1 - \delta)\psi)}{1 - (1 - F(g))\delta},$$

where we use Lemma 11 and $g - 1 > g^*$ to substitute for values $V_C^\sigma(g - 1)$ and $V_C^\sigma(g + 1)$.

Because $\sigma$ is an equilibrium, we require $U_C^\sigma(1; g) = V_C^\sigma(g) \geq \tilde{V}_C(g)$, by Lemma 2. Then Lemma 1.1 implies

$$U_C^\sigma(1; g) > \frac{-F(g)\psi + (1 - F(g))\pi_C^\sigma}{1 - (1 - F(g))\delta} \iff \frac{\pi_C^\sigma - \kappa_C}{1 - \delta} > \frac{-F(g)\psi + (1 - F(g))\pi_C^\sigma}{1 - (1 - F(g))\delta}$$

$$\iff \kappa_C < \frac{F(g)(\pi_C^\sigma + (1 - \delta)\psi)}{1 - (1 - F(g))\delta},$$

which establishes the desired contradiction. □

Lemma 13 In high-capacity regimes, there exists cutpoint $\tilde{g} \in \mathbb{R}$ such that if $g > \tilde{g}$, then $\sigma_P(g) = 1$ and $\sigma_C(1; g) = 1$ in every equilibrium $\sigma$.

Proof. The proof is constructive. Define $\tilde{g} \in \mathbb{N}_0$ to be a number that satisfies

$$g \geq \tilde{g} \implies \kappa_P < F(g) \left[ \tilde{V}_P - \pi_P^C - \delta p \tilde{V}_P + (1 - p)\pi_P^C - \kappa_P \right].$$
Such a $\bar{g}$ exists because $F(g) \left[ \bar{V}_p - \pi^*_p - \delta \frac{p\bar{V}_p + (1 - p)\pi^C_p}{1 - (1 - p)\delta} \right]$ is positive and strictly increasing in $g$. Furthermore,

$$\lim_{g \to \infty} F(g) \left[ \bar{V}_p - \pi^*_p - \delta \frac{p\bar{V}_p + (1 - p)\pi^C_p}{1 - (1 - p)\delta} \right] = \frac{\pi^p_p - \pi^C_p}{1 - \delta},$$

and Assumption 1 implies

$$\kappa_p < \frac{\pi^p_p - \pi^C_p}{1 - \delta}.$$

We first show that $\sigma_p(g) = 1$ for $g \geq \bar{g}$. Suppose not; then there exists $g > \bar{g}$ such that $\sigma_p(g) < 1$. To rule out profitable deviations, we require $U^p_g(0; g) \geq U^p_g(1; g)$, which is equivalent to

$$\kappa_p \geq F(g) \left[ \bar{V}_p - \pi^*_p - \delta V^\sigma_p(g - 1) \right].$$

Because the Center never grants independence in strong regimes, $V^\sigma_p(g - 1)$ is bounded above by $\frac{p\bar{V}_p + (1 - p)\pi^C_p}{1 - (1 - p)\delta}$, which is the Periphery’s dynamic payoff if it mobilizes in every period at maximum capacity, $p$. Combining these two inequalities gives us

$$\kappa_p \geq F(g) \left[ \bar{V}_p - \pi^*_p - \delta V^\sigma_p(g - 1) \right]$$

$$\geq F(g) \left[ \bar{V}_p - \pi^*_p - \delta \frac{p\bar{V}_p + (1 - p)\pi^C_p - \kappa_p}{1 - (1 - p)\delta} \right],$$

but this implies $g < \bar{g}$, which is contradiction. Thus, $\sigma_p(g) = 1$. Then Lemma 10 and the contrapositive of Lemma 12 imply $\sigma_C(1; g) = 1$, as required. \hfill $\square$

**Lemma 14** In high-capacity regimes, if $g \geq g^+$, then $\sigma_p(g) = 1$ in every equilibrium $\sigma$.

**Proof.** Suppose there exists $g \geq g^+$ such that $\sigma_p(g) < 1$. Lemma 13 implies that there exists grievance $\bar{g}^+ \geq g$ such that $\sigma_p(g^+) < 1$ and $\sigma_p(g^+) = \sigma_C(1; g^+) = 1$ for all $g^+ > g^+$. To rule out profitable deviations, we require $U^p_p(0; g^+) \geq U^p_p(1; g^+)$. This implies

$$\kappa_p \geq F(g^+) \left[ \bar{V}_p - \pi^*_p - \delta V^\sigma_p(g^+ - 1) \right].$$

Because $P$ will never be able to mobilize at a larger grievance than $g^+$ along the path of play and $C$ never grants independence, $V^\sigma_p(g^+ - 1)$ is bounded above by

$$\frac{F(g^+)\bar{V}_p + (1 - F(g^+))\pi^C_p - \kappa_p}{1 - (1 - F(g^+)\delta)}.$$
Then we have
\[
\kappa_P \geq F(g^+) \left[ \tilde{V}_P - \pi_C^P - \delta V_P^\sigma (g^+ - 1) \right] \\
\geq F(g^+) \left[ \tilde{V}_P - \pi_C^P - \delta \frac{F(g^+) V_P + (1 - F(g^+)) \pi_C^P - \kappa_P}{1 - (1 - F(g^+)) \delta} \right] \\
= F(g^+) \frac{\pi_C^P - \pi_C^P}{1 - \delta},
\]
which implies \( g^+ \leq g^- \leq g^+ \), a contradiction. \( \square \)

We now prove Proposition 3.1, which characterizes equilibria in regimes with large grievances when \( \pi_C^P > \kappa_C \).

**Proof of Proposition 3.1.** If \( g \geq g^+ \), then Lemma 14 implies \( \sigma_P(g) = 1 \). Because \( g > g^+ \) implies \( \sigma_P(g) = 1 \). Lemma 10 and the contrapositive of Lemma 12 imply \( \sigma_C(1; g) = 1 \), as required. \( \square \)

### F.2 Low repression capacity: \( \kappa_C > \pi_C^P \)

**Lemma 15** Fix an equilibrium \( \sigma \). In low-capacity regimes, the there does not exist grievance \( g \) such that \( \sigma_C(1; g') > 0 \) for all \( g' \geq g \).

**Proof.** The result follows from the inequality \( \pi_C^P - \kappa_C < 0 \) and the argument proving Lemma 8. \( \square \)

**Lemma 16** In low-capacity regimes, \( \sigma_P(g^+) = 1 \), \( \sigma_C(0; g^+ = 0 \), and \( \sigma_C(\emptyset; g^+ > 0 \) in ever equilibrium \( \sigma \).

**Proof.** First, \( P \) mobilizes at \( g^+ \) by Lemma 5 and Proposition 2.

Second, \( \sigma_C(0; g^+) = 0 \). If not, then with positive probability the Center chooses to enter the path of play into moderate grievance levels. That is, \( V_C^\sigma(g^+) = U_C^\sigma(0; g^+) = \tilde{V}_C(g^+) \). But then \( V_C^\sigma(g^+) < 0 \) because the regime has low capacity, so \( C \) can profitably deviate by granting independence at \( g^+ \).

Third, \( \sigma_C(1; g^+) < 1 \). To see this, suppose not, i.e., suppose \( \sigma_C(1; g^+) = 1 \). By Lemma 15, there exists \( g^+ \geq g^+ \) such that \( \sigma_C(1; g^+ + 1) = 0 \) and \( \sigma_C(1; g^+) > 0 \) for all \( g' = g^+; \ldots; g^+ \). Then by Lemma 7, \( \sigma_C(\emptyset; g') = 0 \) for all \( g' = g^+; \ldots; g^+ + 1 \). By Proposition 2, \( \sigma_C(0; g') = 1 \) for all \( g' < g^+ \). Then Lemma 5 implies \( \sigma_P(g^+ + 1) = 1 \). However, \( \sigma_C(1; g^+) > 0 \), \( \sigma_C(0; g^+ + 1) = 1 \), and \( \sigma_P(g^+ + 1) = 1 \) contradict Lemma 4. Thus, \( \sigma_C(1; g^+) < 1 \), which implies \( \sigma_C(\emptyset; g^+) > 0 \) by the previous paragraph. \( \square \)
Before proving the last technical lemma of this section, consider the following definitions. The set \( G \subseteq N_0 \) is an absorbing set with respect to profile \( \sigma \) if once the path of play enters grievance level \( g \) such that \( g \in G \), it never transitions to a grievance \( g' \) such that \( g' \notin G \) with positive probability. The set \( G \) is an irreducible absorbing set with respect to \( \sigma \) if \( G \) is an absorbing set with respect to \( \sigma \) and there does not exist a proper subset \( G' \subseteq G \) such that \( G' \) is an absorbing set with respect to \( \sigma \).

**Lemma 17** Consider an equilibrium \( \sigma \) and some grievance \( g \geq g^+ \). Then the following hold:

1. beginning at grievance \( g \), the path of play enters an irreducible absorbing set \( G \) with respect to \( \sigma \),
2. \( \max G \) exists,
3. \( g^+ \leq \min G \), and
4. there exists \( g' \in G \) such that \( \sigma_C(\emptyset; g) > 0 \).

**Proof.** To prove (1), consider \( g \geq g^+ \) and two cases. If \( \sigma_C(1; g) = 0 \), then the path of play enters the set \( \{g^+, ..., g\} \), which is an absorbing set because \( \sigma_C(0; g^+) = 0 \) by Lemma 16. So the set \( \{g^+, ..., g\} \) has an irreducible absorbing set, \( G \). If \( \sigma_C(1; g) > 0 \), then Lemma 15 implies there exists \( g^* \geq g \) such that \( \sigma_C(1; g') > 0 \) for all \( g' = g, ..., g^* \) and \( \sigma_C(1; g^* + 1) = 0 \) from Lemma 7. Then the path of play enters the set \( \{g^+, ..., g^* + 1\} \), which is an absorbing set as well.

The proof of (2) and (3) follow immediately from the existence of \( G \) and Lemmas 15 and 16, respectively.

To prove (4), suppose not. Suppose \( \sigma_C(\emptyset; g') = 0 \) for all \( g' \in G \). I first claim that it must be the case that \( \#G > 1 \). Suppose the contrary. Then \( G = \{g'\} \), and \( C \) cannot be repressing with positive probability at \( g \), or else \( G \) is not absorbing. Also, if \( G = \{g'\} \) and \( \sigma_C(0; g') > 0 \), then \( F(g) = 1 \) and \( \sigma_C(0; x) = 1 \) or else the path of play would transition to \( g - 1 \) with positive probability. In this case, \( UC(0; g') = -\psi < 0 \), but this means \( C \) has a profitable deviation by granting independence at \( g' \). Thus, \( \#G > 2 \) and as such \( \max G - 1 \in G \).

Second, because \( G \) is irreducible, \( \sigma_C(1; \max G - 1) > 0 \), or else \( G \setminus \{\max G\} \) would be absorbing as well. Furthermore, \( \sigma_C(1; \max G) = 0 \) or else the path of play would transition with positive probability to \( \max G + 1 \). Because \( \sigma_C(1; \max G - 1) > 0 \) and \( \sigma_C(1; \max G) = 0 \), Lemma 7 implies \( \sigma_C(0; \max G) = 1 \) because the path of play never leaves \( G \) nor transitions to grievance \( g' > \max G \) and \( C \) never grants independence along the path of play starting from \( \max G \), then \( \sigma_P(\max G) = 1 \), which follows from an identical argument as the one in Lemma 5. However, this contradicts Lemma 4.

The proof of Proposition 3.2 follows from Lemma 17.
G Proof of Proposition 4

First, the result in Proposition 4.1 follows immediately form Lemma 15. Second, the result in Proposition 4.2 is proved below in Lemma 18. Third, I construct an equilibrium that supports cycles of repression and mobilization, as described in Proposition 4.3, in Example 1. As part of this construction, I need a new result in Lemma 19.

Lemma 18 If $\kappa_C > (1+\delta)\pi^C_C$, then the Center never represses in any equilibrium $\sigma$, i.e., $\sigma_C(1; g) = 0$ for every grievances $g$ and every equilibrium $\sigma$.

Proof. To derive a contradiction, suppose the contrary. That is, suppose $\kappa_C > (1+\delta)\pi^C_C$ and the Center represses in equilibrium $\sigma$. Thus, the regime is is low capacity, and there exist some $g$ such that $\sigma_C(1; g) > 0$. By Lemma 15, there exists $g^\dagger \geq g$ such that $\sigma_C(1; g^\dagger + 1) = 0$ and $\sigma_C(1; g') > 0$ for all $g' = g, ..., g^\dagger$. Then by Lemma 7, $\sigma_C(\emptyset; g') = 0$ for all $g' = g + 1, ..., g^\dagger + 1$. Hence, $\sigma_C(0; g^\dagger + 1) = 1$. We can compute $C$’s continuation value at $g^\dagger$ as

$$V^C_C(g^\dagger) = \sigma_C(1; g^\dagger)U^C_C(1; g^\dagger) + \sigma_C(0; g^\dagger)U^C_C(0; g^\dagger) = U^C_C(1; g^\dagger)$$

$$= \pi^C_C - \kappa_C + \delta V^C_C(g^\dagger + 1)$$

$$= \pi^C_C - \kappa_C + \delta \left[ \sigma_C(g^\dagger + 1)(-F(g^\dagger + 1)\psi + (1 - F(g^\dagger + 1))\pi^C_C + \delta V^C_C(g^\dagger)) + (1 - \sigma_C(g^\dagger + 1))\pi^C_C + \delta V^C_C(g^\dagger) \right],$$

where the second equality follows because $\sigma$ is an equilibrium and $\sigma_C(1; g^\dagger) > 0$. Solving for $V^C_C(g^\dagger)$ reveals that

$$V^C_C(g^\dagger) = \frac{\pi^C_C(1 + (1 - F(g^\dagger + 1)\sigma_C(g^\dagger + 1))\delta) - \kappa_C - F(g^\dagger + 1)\sigma_C(g^\dagger + 1)\delta\psi}{1 - (1 - \sigma_C(g^\dagger + 1)F(g^\dagger + 1))\delta^2},$$

which is decreasing in $\sigma_C(g^\dagger + 1)$. Because $\sigma_C(g^\dagger + 1) \geq 0$, then

$$V^C_C(g^\dagger) \leq \frac{\pi^C_C(1 + \delta) - \kappa_C}{1 - \delta^2}$$

Thus, $\kappa_C > (1+\delta)\pi^C_C$ implies $V^C_C(g^\dagger) < 0$. But this implies $C$ can profitably deviate at $g^\dagger$ by granting independence and guaranteeing itself a payoff of zero.

Lemma 19 In low-capacity regimes, if $F(g)(\pi^P_P - \pi^C_C) > \kappa_P$ and $g \geq g^\dagger$, then $\sigma_P(g') = 1$ and $\sigma_C(1; g') = 0$ for all $g' \geq g$ in every equilibrium $\sigma$. 

\[\square\]
Proof. By Equation (1), \( P \) mobilizes at \( g' \) if
\[
\kappa_C < F(g') \left( \bar{V}_P - \pi_P^C - \delta V_P^P(g' - 1) \right).
\]
An upper bound on \( V_P^P(g' - 1) \) is \( \frac{\pi_P^C}{1 - \delta} \), which is the discounted sum of \( P \)'s largest per-period payoff. Combining these two inequalities implies \( P \) mobilizes when \( F(g') (\pi_P^C - \pi_P^C) > \kappa_P \), which holds because \( F(g') (\pi_P^C - \pi_P^C) > \kappa_P \), and \( F \) is increasing.

Second, I claim that \( \sigma_C(1; g') = 0 \) for all \( g' \geq g \). Suppose not. Then there exists a \( g^+ \) such that \( \sigma_C(1; g^+) > 0 \) and \( \sigma_C(0; g^+ + 1) = 1 \) by Lemmas 7 and 15. The previous paragraph demonstrates that \( P \) mobilizes with probability 1 with grievance \( g^+ + 1 \). But this contradicts Lemma 4. \( \square \)

**Example 1** In this example, I assume \( \pi_C^C = \pi_P^P = 1 \), and \( \pi_C^P = 0 \). In addition, \( \kappa_C = 1.2 \) and \( \kappa_P = .25 \). This implies that the regime has low repression capacity. Finally, \( \delta = .9 \), \( \psi = 6 \), and \( F \) takes the form:
\[
F(g) = \begin{cases} 
0 & \text{if } g = 0 \\
\frac{g}{700} + \frac{33}{175} & \text{if } g \geq 1 \text{ and } g \leq 8 \\
1 & \text{otherwise.}
\end{cases}
\]

Thus, \( g^- = 0 \), and \( g^+ = 7 \), because \( \tilde{V}_C(6) \approx .33 \) and \( \tilde{V}_C(7) \approx -.15 \). By Proposition 2, the Periphery mobilizes with probability one for all \( g \in \{1, 2, ..., 7\} \) and the Center neither represses nor grants independence for all \( g \in \{0, 1, 2, ..., 6\} \). Note that \( F(9) (\pi_P^P - \pi_P^C) > \kappa_P \), so Lemma 19 implies the Periphery mobilizes for all grievances \( g \geq 9 \) and the Center does not repress at grievance \( g \geq 9 \).

We specify remaining behavior as follows.

1. At grievance \( g = 7 \), the Periphery mobilizes with probability 1 and the Center mixes between repression and granting independence, \( \sigma_C(1; 7) + \sigma_C(\emptyset; 7) = 1 \)

2. At grievance \( g = 8 \), the Center neither represses nor grants independence, i.e., \( \sigma_C(0; 8) = 1 \) and the Periphery mobilizes with probability \( \sigma_P(8) \).

We first characterize mixing probabilities, \( \sigma_C(1; 7) \), \( \sigma_C(\emptyset; 7) \), and \( \sigma_P(8) \), such that the following hold:
\[
\sigma_C(1; 7) + \sigma_C(\emptyset; 7) = 1 \\
U_C^C(1; 7) = U_C^C(\emptyset; 7) \\
U_P^P(1; 8) = U_P^P(0; 8).
\]

The first equation says the Center mixes between repression and granting independence at \( g = 7 = g^+ \). The second and third equations are \( C \) and \( P \)'s indifference conditions, respectively. Because

\[ U_C^e(\emptyset; 7) = 0, \]  
\[ C \text{'s indifference equations takes the form:} \]
\[ \pi_C^C - \kappa_C + \delta V_C^e(8) = 0, \quad (7) \]

where
\[ V_C^e(8) = \sigma_P(8) \left[ -F(8)\psi + (1 - F(8)) \left( \pi_C^C + \delta V_C^e(7) \right) \right] + (1 - \sigma_P(8)) \left[ \pi_C^C + \delta V_C^e(7) \right]. \]

In equilibrium, \[ V_C^e(7) = U_C^e(\emptyset; 7) = 0. \] Thus, we have
\[ V_C^e(8) = \sigma_P(8) \left[ -F(8)\psi + (1 - F(8))\pi_C^C \right] + (1 - \sigma_P(8))\pi_C^C. \]

Substituting the above equality into Equation (7), \( C \)'s indifference condition takes the form:
\[ \pi_C^C - \kappa_C + \delta \left( \sigma_P(8) \left[ -F(8)\psi + (1 - F(8))\pi_C^C \right] + (1 - \sigma_P(8))\pi_C^C \right) = 0. \quad (8) \]

Next, consider \( P \)'s indifference equation, \[ U_P^e(1; 8) = U_P^e(0; 8), \] which takes the form
\[ -\kappa_P + F(8) \frac{\pi_P^P}{1 - \delta} + (1 - F(8)) \left( \pi_P^C + \delta V_P^e(7) \right) = \pi_P^C + \delta V_P^e(7), \quad (9) \]

where
\[ V_P^e(7) = \sigma_C(\emptyset; 7) \frac{\pi_P^P}{1 - \delta} + \sigma_C(1; 7) \left[ \pi_P^C + \delta V_P^e(8) \right] \]
\[ = \sigma_C(\emptyset; 7) \frac{\pi_P^P}{1 - \delta} + \sigma_C(1; 7) \left[ \pi_P^C + \delta U_P^e(0; 8) \right] \]
\[ = \sigma_C(\emptyset; 7) \frac{\pi_P^P}{1 - \delta} + \sigma_C(1; 7) \left[ \pi_P^C + \delta \left( \pi_P^C + \delta V_P^e(7) \right) \right]. \]

Here the second equality follows because \( \sigma_C(0; 8) = 1. \) Solving Equations (8) and (9) with the constraint \( \sigma_C(\emptyset; 7) + \sigma_C(1; 7) = 1 \) reveals that
\[ \sigma_P(8) = \frac{(1 + \delta)\pi_C^C - \kappa_C}{\pi_C^C + \psi}\delta F(8) \approx .56 \]
and
\[ \sigma_C(1; 7) = \frac{\kappa_P - F(8)(\pi_P^P - \pi_P^C)}{\delta^2 \kappa_P + \delta F(8)(\pi_P^P - \pi_P^C)} \approx .13. \]

Finally, we check profitable deviations. First, \( P \)'s indifference condition precludes profitable deviations at \( g = 8. \) Second, \( C \) does not have a profitable deviation at \( g = 7 \) due to its indifference
equation and because \( U_C^\sigma(0; 7) = \tilde{V}_C(7) < 0 \). Also, C has no profitable deviation at \( g = 8 \), because \( V_C(8) > 0 \). To see this, note that \( U_C^\sigma(1; 7) = \pi_C^C - \kappa_C + \delta V_C^\sigma(8) = 0 \) by Equation (7), and \( \pi_C^C - \kappa_C < 0 \). If C deviates by granting independence at \( g = 8 \), then its payoff is zero. Likewise, if C deviates by repressing, its payoff is \( \pi_C^C - \kappa_C + \delta V_C^\sigma(9) \), which reduces to \( \pi_C^C - \kappa_C < 0 \) because C is granting independence when \( g = 9 \). Lemma 19 implies that C cannot profitably deviate by using repression, at grievances \( g \geq 9 \). Thus, we only need to verify that C cannot profitably deviate by choosing to refrain from repression or granting independence, at grievances \( g \geq 9 \). Because the Periphery mobilizes at \( g \geq 9 \) and \( F(g) = 1 \), mobilization surely succeeds, implying \( U_C^\sigma(0; g) = -\psi \) for all \( g \geq 9 \) which is strictly less than C’s utility from following its equilibrium strategy of granting independence.

### H Exogenous Decentralization

In this section, I continue to analyze the numerical example in Figure 4 and prove Proposition 5.

From, the example in Figure 4, I compute the probability that the country breaks apart due to secessionist mobilization —labeled probability of secession hereafter—as a function of decentralization. For a fixed \( d \), three potential paths of play emerge at initial grievance \( g^1 \) in equilibrium. First, if \( g^1 < g^+[d] \), the Center neither represses nor grants independence, and the probability secession is

\[
\begin{cases}
0 & \text{if } g^1 \leq g^-[d] \\
1 - \prod_{g^1 \leq g' \leq g^1} (1 - F(g')) & \text{otherwise}.
\end{cases}
\]

Second, if \( g^1 \geq g^+[d] \) and the regime has high capacity (\( \pi - d > \kappa_C \)), then the Center represses in all future periods, and the probability of secession is zero. Third, if \( g^1 \geq g^+[d] \) and the regime has low capacity (\( \pi - d < \kappa_C \)), the probability of secession is undefined. Although the Periphery will eventually gain control of its territory (Proposition 3.2), this may arise either from secessionist mobilization or Center-granted independence. This third case does not arise in the numerical example. As seen in Figure 4, if \( g^1 \geq g^+[d] \) for some \( d \), then the regime has high capacity.

Figure 6 graphs the probability of secession decentralization varies. When \( d \) is small, \( g^1 > g^+[d] \) so the high-capacity regime represses and the probability of secession is zero. When \( d \) is large, \( g^1 < g^- \), so grievances are small and a lasting peace emerges. When \( d \) is moderate, then the Center gambles for unity and secession occurs with positive probability. When \( \psi \) is large (left panel), all decentralization levels below \( d = 44 \) result in long-term repression and a zero probability of secession. When \( \psi \) is small (right panel), all decentralization levels below \( d = 38 \) result in long-term repression and a zero probability of secession.
Notes: The panels graph the probability of secession (vertical axis) for a fixed decentralization level (horizontal axis) with a large cost of secession $\psi = \frac{3}{2}$ (left) and a small cost $\psi = \frac{π}{2}$ (right). The remaining parameters take on the following values: $\pi = 100$, $\kappa_C = 50$, $\kappa_P = 50$, $\delta = 0.95$, and $F(g) = 1 - \left(0.01g + 0.001g^2 - 1\right)^{-1}$.

**Proposition 5** Assume the regime has a high capacity for repression ($\kappa_C < \pi$) and initial grievances are large ($g^1 \geq g^+[0]$). There exist cutpoints $d$ and $\bar{d}$ such that $0 \leq d < \bar{d} < 1$ and secession occurs with positive probability on the equilibrium path only if decentralization is moderate, i.e., $d < d < \bar{d}$.

**Proof.** Set $d = 0$. The regime has high repression capacity by assumption, and $g^1 \geq g^+[d]$ implies that $C$ represses with probability one in all future periods when the game begins at grievance $g^1$. As such the probability of secession is zero.

In addition, we can set $\bar{d}$ as follows

$$\bar{d} = \hat{d}(g^1) + \epsilon$$

where $\hat{d}$ is defined in Equation (4) above and $\epsilon \in \mathbb{R}$ is such that $0 < \epsilon < \max\{\frac{(1-\delta)\kappa_P}{F(g^1)}, 1\}$. Note that the fraction $\frac{(1-\delta)\kappa_C}{F(g^1)}$ is well defined because $F(g^1) \neq 0$. If $F(g^1) = 0$ then $g^1 \leq g^-[0] < g^+[0]$, a contradiction.

It suffices to show that $g^1 \leq g^-[\bar{d}]$ because this inequality implies that $g^1$ is small at decentralization level $\bar{d}$ and $g^-$ is strictly increasing in $d$. As such, $g^1$ is small at decentralization levels $d > \bar{d}$. In addition, when $g^1 \leq g^-$ no mobilization occurs along the path of play by Proposition 1.
When \(\pi_C^p = d\) and \(\pi_P^p = \pi\), then we can write \(g^-[d]\) as

\[g^-[d] = \max \left\{ g \in \mathbb{N}_0 \mid \kappa_P > F(g) \frac{\pi - d}{1 - \delta} \right\}.
\]

Thus, \(g^1 \leq g^-[\bar{d}]\), as required. \(\square\)

\section{Proof of Proposition 6}

We first prove Proposition 6.1 and then present two numerical examples that establish Propositions 6.2 and 6.3.

\textit{Proof of 6.1.} Consider equilibrium \((d^*, \sigma)\). We first prove that \(d^* \leq \min[\hat{d}(g^1), \kappa_C]\). First, \(d^* \leq \kappa_C\). To see this, note that \(V_{\sigma C}(g^1; d^*) \leq \pi - d^* - \delta\). Thus, if \(C\) chooses \(d^* > \kappa_C\), then \(V_{\sigma C}(g^1; d^*) < \frac{\pi - \kappa_C}{1 - \delta}\), which means \(C\) can profitably deviate by choosing \(d^* = 0\) and repressing in all future periods.

Second, \(d^* \leq \hat{d}(g^1)\). When \(C\) chooses \(d^* > \hat{d}(g^1)\), then \(g^1 \leq g^-[d^*]\), which implies that \(V_{\sigma C}(g^1; d^*) = \pi - d^* - \delta\), which is strictly decreasing in \(d^*\). So \(C\) has a profitable deviation by choosing decentralization \(d = d^* - \epsilon\) for \(\epsilon > 0\) but close to zero. This establishes the desired result.

Finally, we prove that if \(\kappa_C < \max \left\{ \frac{\pi}{2}, \pi - \hat{d}(g^1) \right\}\) and \(d^* > 0\), then \(g^1 < g^+[d^*]\), i.e., \(C\) never represses nor grants independence along the subsequent path of play. To do this suppose not and consider two relevant cases.

\textit{Case 1:} \(\pi - d^* - \kappa_C > 0\). Then \(V_{\sigma C}(g^1; d^*) = \frac{\pi - d^* - \kappa_C}{1 - \delta}\), and \(C\) can profitably deviate by choosing \(d^* = 0\) and repressing in all future periods.

\textit{Case 2:} \(\pi - d^* - \kappa_C \leq 0\). If \(\kappa_C < \frac{\pi}{2}\), then

\[d^* \geq \pi - \kappa_C > \pi - \frac{\pi}{2} > \kappa_C,
\]

which contradicts the upper bound described above. If \(\kappa_C < \pi - \hat{d}(g^1)\), then we have

\[d^* \geq \pi - \kappa_C
\]

\[> \pi - \left(\pi - \hat{d}(g^1)\right)
\]

\[= \pi - \left(\frac{1 - \delta}{1 - \delta} \kappa_P\right)
\]

\[= \hat{d}(g^1),
\]

which contradicts the upper bound described above. \(\square\)
The next example illustrates that the Center decentralizes in equilibrium \((d^*, \sigma)\) and the subsequent interaction entails gambling for unity.

**Example 2** For the exogenous parameters, we consider \(\pi = 100, \psi = 100, \kappa_C = 40, \kappa_P = 95\) and \(\delta = 0.9\). In addition, \(F\) takes the form:

\[
F(g) = \begin{cases} 
0 & \text{if } g = 0 \\
\frac{1}{10} & \text{if } g \in \{1, \ldots, 100\} \\
\frac{3}{10} & \text{if } g = 101 \\
1 & \text{if } g \geq 102.
\end{cases}
\]

and initial grievances are \(g^1 = 101\).

Note that \(\kappa_C < \frac{\pi}{2}\), so Proposition 6.1 implies that if \(C\) decentralizes in an equilibrium \((d^*, \sigma)\), then it chooses to neither repress nor grant independence in all future periods, in which case, \(C\)’s expected utility is \(\tilde{V}_C(g^1; d^*)\). Thus, if \(C\) chooses \(d^* > 0\), it will choose a \(d^*\) that solves

\[
F(g') \frac{\pi - d^*}{1 - \delta} - \kappa_P = 0
\]

for some \(g' > g^-[0]\) and \(g' \leq g^1\). In words, if \(C\) decentralizes, it will choose a decentralization level that makes the Periphery (at some grievance level \(g^1\)) indifferent between mobilizing and not along the subsequent path of play. If not, \(C\) can profitably deviate by offering slightly less decentralization without changing the Periphery’s strategy in states \(g \leq g^1\).

Given this discussion and the construction of \(F\), there are three possible decentralization levels to consider: \(\{0, \hat{d}(1), \hat{d}(101)\}\). Note that \(\hat{d}(101) = \frac{205}{3} > \kappa_C\). As such, the upper bound in the previous proof shows that \(d^* \neq \hat{d}(101)\) in any equilibrium. Thus, there are only two possible decentralization levels in equilibrium: \(\{0, \hat{d}(1)\}\).

If \(C\) chooses \(d^* = 0\), then \(g^-[0] = 0\) and \(g^+[0] = 6\). Because \(g^1 > g^+[0]\), if \(C\) chooses \(d^* = 0\), then long-term repression is the equilibrium outcome, which implies \(C\)’s dynamic payoff is \(\frac{\pi - \kappa_C}{1 - \delta} = 600\).

If \(C\) chooses \(d^* = \hat{d}(1) = 5\), then \(g^-[d^*] = 100\) and \(g^+[0] = 102\). Because \(g^1 < g^+[d^*]\), if \(C\) chooses \(d^* = \hat{d}(1)\), then one period of gambling for unity is the equilibrium path of play, in which case \(C\)’s expected utility is

\[
-F(g^1)\psi + (1 - F(g^1)) \left[ \pi - d^* + \delta \frac{\pi - d^*}{1 - \delta} \right] = 635.
\]
As such, C chooses to decentralize, $d^* = \hat{d}(1) > 0$ and gambling for unity occurs along the subsequent equilibrium path of play.

The next example illustrates that the Center decentralizes in equilibrium $(d^*, \sigma)$ and the a long-term peace emerges in the subsequent interaction.

**Example 3** The payoff parameters match those from Example 2, but now $F$ takes the form:

$$F(g) = \begin{cases} 
0 & \text{if } g = 0 \\
\frac{1}{10} & \text{if } g \in \{1, \ldots, 101\} \\
1 & \text{if } g \geq 102.
\end{cases}$$

and initial grievances are $g^1 = 101$. Following the logic in the previous example, there are two potential levels of decentralization in equilibrium: $[0, \hat{d}(1)]$. If C chooses $d^* = 0$, then its payoff is $\frac{\pi - \kappa C}{1 - \delta} = 600$ for reasons described above. If C choses $d^* = \hat{d}(1)$, then $g^{-}[d^*] = 101 = g^1$ and its equilibrium payoff is $\frac{\pi - d^*}{1 - \delta} = 950$. As such, C chooses to decentralizes and a long-term peace emerges.
Online Appendix

A Continuation Values and Expected Utilities

Let \( \overline{V}_i \) denote \( i \)'s continuation value after a history in which the Periphery was won control of the territory. These values are independent of a strategy profile \( \sigma \) and take the form \( \overline{V}_C = 0 \) and \( \overline{V}_P = \frac{\pi_{P}^C}{1-\delta} \).

Let \( V^\sigma_i(g) \) denotes \( i \)'s continuation value from beginning the game with grievance \( g \) when the Periphery has not won control of its territory and actors subsequently playing according to profile \( \sigma \). In a similar vein, \( U^\sigma_C(r; g) \) and \( U^\sigma_P(m; g) \) denote the Center and Periphery’s dynamic payoffs from choosing \( r \in \{\emptyset, 0, 1\} \) and \( m \in \{0, 1\} \) given grievance \( g \) when actors subsequently play according to profile \( \sigma \). For the Center, \( U^\sigma_C(r; g) \) takes the following form:

\[
U^\sigma_C(r; g) = \begin{cases} 
\pi_{C}^C - \kappa_C + \delta V^\sigma_C(g + 1) & \text{if } r = 1 \\
-\sigma_P(g)F(g)\psi + (1 - \sigma_P(g)F(g))\left(\pi_{C}^C + \delta V^\sigma_C(\max\{g - 1, 0\})\right) & \text{if } r = 0.
\end{cases}
\]
For the Periphery, $U^\sigma_P(m; g)$ denotes the its dynamic payoff conditional on having reached its decision node, i.e., the Center chooses $r = 0$, in state $g$. Thus, $U^\sigma_P(m; g)$ takes the form

$$U^\sigma_P(m; g) = \begin{cases} -\kappa_P + F(g)\bar{V}_P + (1 - F(g))\left(\pi^C_P + \delta V^\sigma_P(\max\{g - 1, 0\})\right) & \text{if } m = 1 \\ \pi^C_P + \delta V^\sigma_P(\max\{g - 1, 0\}) & \text{if } m = 0. \end{cases}$$

With this notation in hand, the next definition states the equilibrium conditions.

**Definition 1** Strategy profile $\sigma$ is an equilibrium if the following hold:

$$\sigma_C(r; g) > 0 \implies U^\sigma_C(r; g) \geq U^\sigma_C(r'; g),$$

$$\sigma_P(g) > 0 \implies U^\sigma_P(1; g) \geq U^\sigma_P(0; g), \text{ and}$$

$$\sigma_P(g) < 1 \implies U^\sigma_P(0; g) \geq U^\sigma_P(1; g)$$

for all grievance $g$ and polices $r, r' \in \{\emptyset, 0, 1\}$.

Because the game is a dynamic game with a countable state space and a finite number of actions, an equilibrium from Definition 1 exists in mixed strategies. Notice that for some grievance $g$, the Center’s continuation value, $V^\sigma_C(g)$, takes the form

$$V^\sigma_C(g) = \sum_{r \in \{\emptyset, 0, 1\}} \sigma(r; g) U^\sigma_C(r; g).$$

Thus, if $\sigma$ is an equilibrium and $\sigma(r; g) > 0$ for some grievance $g$ and action $r \in \{\emptyset, 0, 1\}$, then $V^\sigma_C(g) = U^\sigma_C(r; g)$ or else $C$ has a deviation by playing some $r' \in \{\emptyset, 0, 1\}$.

**B  Proof of Proposition 1**

**Proposition 1** If grievances are small, then the Periphery never mobilizes, the Center neither represses nor grants independence, and grievances dissipate on the equilibrium path. That is, $g \leq g^-$ implies $\sigma_P(g) = 0$ and $\sigma_C(0; g) = 1$ in every equilibrium $\sigma$.

**Proof.** The proof that $g \leq g^-$ implies the Periphery does not mobilize with positive probability is covered in the main text. We prove that $g \leq g^-$ implies the Center does not repress or grant independence with positive probability. To see this, suppose $\sigma_C(r; g) > 0$ for some $g \leq g^-$, $r \neq 0$, and equilibrium $\sigma$. There are two cases.
Case 1: $r = 1$, repression. Then, $C$’s expected utility is

$$
\begin{align*}
U_C^r(1; g) &= \pi_C - \kappa_C + \delta V_C^r(g + 1) \\
&\leq \pi_C - \kappa_C + \delta \frac{\pi_C}{1 - \delta} \\
&< \frac{\pi_C}{1 - \delta}.
\end{align*}
$$

However, $\frac{\pi_C}{1 - \delta}$ is $C$’s continuation value if it takes action $r = 0$ in all future periods because grievances will never increase and $P$ will never mobilize with positive probability along the subsequent path of play. Hence, taking action $r = 0$ in all future periods is a profitable deviation, a contradiction.

Case 2: $r = \emptyset$, independence. Then, $C$’s expected utility is

$$
U_C^r(\emptyset; g) = 0 < \frac{\pi_C}{1 - \delta}.
$$

As in Case 1, this inequality implies taking action $r = 0$ in all future periods is a profitable deviation, a contradiction. □

C Properties of $\tilde{V}_C$

We first state and prove three Lemmas concerning properties of $\tilde{V}_C$.

**Lemma 1**

1. $\tilde{V}_C(g) > \frac{-F(g)\psi + (1 - F(g))\pi_C}{1 - (1 - F(g))\delta}$ for all $g$ such that $F(g) > 0$.

2. $\tilde{V}_C(g - 1) > \tilde{V}_C(g)$ for all $g > g^*$.

3. If Assumption 1 holds, then $\lim_{g \to \infty} \tilde{V}_C(g) = \frac{-p\psi + (1 - p)\pi_C}{1 - (1 - p)\delta}$.

**Proof.** To show (1), consider some $g$ such that $F(g) > 0$ and $F(g') = 0$ for all $g' < g$. Such a $g$ exists because $F(0) = 0$ and $\lim_{g \to \infty} F(g) = p > 0$. In addition, $F(g) < 1$ because there exists at least one $g$ such that $F(g) \in (0, 1)$ by assumption. Then we have

$$
\begin{align*}
\tilde{V}_C(g) &= -F(g)\psi + (1 - F(g))\left(\pi_C + \delta \frac{\pi_C}{1 - \delta}\right) \\
&= (1 - (1 - F(g))\delta) \frac{-F(g)\psi + (1 - F(g))\pi_C}{1 - (1 - F(g))\delta} + (1 - F(g))\delta \frac{\pi_C}{1 - \delta} \\
&> \frac{-F(g)\psi + (1 - F(g))\pi_C}{1 - (1 - F(g))\delta}.
\end{align*}
$$

where the strict inequality follows because $F(g) \in (0, 1)$

For induction, consider some $g$ such that $F(g) > 0$ and $F(g - 1) > 0$, which implies $g - 1 > 0$. Suppose the inequality holds for all $g' < g$ such that $F(g') > 0$. Then we have

$$
\hat{V}_C(g) = -F(g)\psi + (1 - F(g))(\pi_C^C + \delta\hat{V}_C(g - 1))
$$

$$
> -F(g)\psi + (1 - F(g))\left(\pi_C^C + \delta\frac{-F(g - 1)\psi + (1 - F(g - 1))\pi_C^C}{1 - (1 - F(g - 1))\delta}\right)
$$

$$
\geq -F(g)\psi + (1 - F(g))\left(\pi_C^C + \delta\frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta}\right)
$$

$$
= \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta},
$$

where the third line follows because the fraction $\frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta}$ is decreasing in $F(g)$.

To show (2), note that it must hold when $g = g^+ + 1$, because $\psi > 0$ and $F(g) > 0$ as $g > g^-$. Now consider some $g > g^- + 1$. For induction, suppose $\hat{V}_C(g' - 1) > \hat{V}_C(g')$ for all $g'$ such that $g^- < g' < g$. Then

$$
\hat{V}_C(g) = -F(g)\psi + (1 - F(g))(\pi_C^C + \delta\hat{V}_C(g - 1))
$$

$$
\leq -F(g - 1)\psi + (1 - F(g - 1))(\pi_C^C + \delta\hat{V}_C(g - 1))
$$

$$
< -F(g - 1)\psi + (1 - F(g - 1))(\pi_C^C + \delta\hat{V}_C(g - 2))
$$

$$
= \hat{V}_C(g - 1),
$$

where the second line follows because

$$
\hat{V}_C(g) > \frac{-F(g)\psi + (1 - F(g))\pi_C^C}{1 - (1 - F(g))\delta} \geq -\psi
$$

and $F(g)$ is increasing in $g$.

To prove (3), consider a sequence $\{g_n\}_{n=1}^{\infty}$ such that $\lim_{n \to \infty} g_n = \infty$ and $g_n < g_{n+1}$. Then the sequence $\{\hat{V}_C(g_n)\}_{n=1}^{\infty}$ is weakly decreasing due to above arguments. In addition, $\{\hat{V}_C(g_n)\}_{n=1}^{\infty}$ is bounded below because $C$’s payoffs are finite and $C$ discounts with rate $\delta < 1$. Thus, $\{\hat{V}_C(g_n)\}_{n=1}^{\infty}$ has a limit, call it $L$. If the Periphery does value independence, the we have

$$
L = \lim_{n \to \infty} \hat{V}_C(g_n)
$$

$$
= \lim_{n \to \infty} F(g_n)(-\psi) + \lim_{n \to \infty}(1 - F(g_n))(\pi_C^C + \delta\hat{V}_C(g_n - 1))
$$

$$
= -p\psi + (1 - p)\left(\pi_C^C + \delta L\right).
$$
which implies \( L = \frac{-p\psi + (1-p)p_C}{1-(1-p)\delta} \). \( \square \)

The next Lemma demonstrates that \( C \)'s gambling for unity utility, \( \tilde{V}_C \) is a lower bound on its equilibrium expected utility, \( V_C^\sigma \).

**Lemma 2** For all grievances \( g \), \( V_C^\sigma(g) \geq \tilde{V}_C(g) \) in every equilibrium \( \sigma \).

**Proof.** To see this, suppose not. That is, suppose there exist grievance \( g \) and equilibrium \( \sigma \) such that \( V_C^\sigma(g) < \tilde{V}_C(g) \). Then by the construction of \( \tilde{V}_C \) and Proposition 1, \( g > g^- \), or else \( V_C^\sigma(g) = \frac{\pi_C}{1-\delta} = \tilde{V}_C \).

Next consider a deviation for \( C \), labeled \( \sigma'_C \), such that \( \sigma'_C(0; g') = 1 \) for all \( g' \leq g \). I now demonstrate that \( V_C^\sigma'(g') \geq \tilde{V}_C(g) \), where \( \sigma' = (\sigma'_C, \sigma_P) \), which implies \( \sigma'_C \) is a profitable deviation because \( \tilde{V}_C(g) > V_C^\sigma(g) \) by supposition.

The proof is by induction. The inequality, \( V_C^\sigma'(g') \geq \tilde{V}_C(g') \), holds when \( g' \leq g^- \) by the construction of \( \tilde{V}_C \) and Proposition 1. Now consider some \( g' > g^- \) and suppose \( V_C^\sigma'(g'') \geq \tilde{V}_C(g'') \) for all \( g'' < g' \). Then we have

\[
V_C^\sigma'(g') = -\sigma_P(g')F(g')\psi + (1 - \sigma_P(g')F(g'))\left(\pi_C + \delta V_C^\sigma(g' - 1)\right) \\
\geq -\sigma_P(g')F(g')\psi + (1 - \sigma_P(g')F(g'))\left(\pi_C + \delta\tilde{V}_C(g' - 1)\right) \\
\geq -F(g')\psi + (1 - F(g'))\left(\pi_C + \delta\tilde{V}_C(g' - 1)\right) \\
= \tilde{V}_C(g').
\]

Hence, \( V_C^\sigma'(g') \geq \tilde{V}_C(g') \) as required. \( \square \)

The final Lemma demonstrates that the cutpoint \( g^+ \) exists if and only if Assumptions 1 and 2 hold.

**Lemma 3** The cutpoint \( g^+ \) solving Equation (3) exists if and only if the Periphery values independence (Assumption 1) and secession is costly (Assumption 2).

**Proof.** For necessity, suppose Assumptions 1 and 2 hold. Then Lemma 1 and Assumption 1 imply that \( \tilde{V}_C(g) \) is weakly decreasing in \( g \) and converges to

\[
\lim_{g \to \infty} \tilde{V}_C(g) = \frac{-p\psi + (1-p)p_C}{1-(1-p)\delta}.
\]
Because $\tilde{V}_C(g) = \frac{\sigma_C}{1-\delta} > 0$ for all $g \leq g^-$ and $\tilde{V}_C(g)$ is strictly decreasing in $g$ when $g > g^-$, we require

$$\frac{-p\psi + (1-p)\pi_C^C}{1 - (1-p)\delta} < \max \left\{ \frac{\pi_C^C - \kappa_C}{1 - \delta}, 0 \right\}.$$  \hspace{1cm} (6)

We now demonstrate that the inequality in Equation (6) holds when $\pi_C^C > \kappa_C$, the proof when $\pi_C^C < \kappa_C$ is identical. Suppose $\pi_C^C - \kappa_C > 0$. Then Equation (6) reduces to

$$\frac{-p\psi + (1-p)\pi_C^C}{1 - (1-p)\delta} < \frac{\pi_C^C - \kappa_C}{1 - \delta},$$

which is equivalent to

$$\psi > \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)}.$$

Because $\pi_C^C - \kappa_C > 0$, Assumption 2 reduces to

$$\psi > \min \left\{ \frac{\pi_C^C(1-p)}{p}, \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)} \right\} = \frac{(1-\delta)\kappa_C - p(\pi_C^C - \delta\kappa_C)}{p(1-\delta)}.$$

Thus, the inequality in Equation (6) holds, and therefore $g^+$ exists.

For sufficiency, suppose Assumption 1 does not hold, then $\kappa_p \geq F(g)\frac{\pi_C^P - \pi_C^P}{1-\delta}$ for all grievances $g$. Thus, $\tilde{V}_C(g) = \frac{\pi_C}{1-\delta} > \max \left\{ \frac{\pi_C^C - \kappa_C}{1-\delta}, 0 \right\}$ for all grievances $g$. Now suppose Assumption 1 holds but not Assumption 2. Then Lemma 1 implies that, for all $g$

$$\tilde{V}_C(g) \geq \frac{-p\psi + (1-p)\pi_C^C}{1 - (1-p)\delta} \geq \max \left\{ \frac{\pi_C^C - \kappa_C}{1 - \delta}, 0 \right\}.$$

\[\square\]

D Preliminary Results

In this section, we state and prove two technical results that are essential to characterize equilibria in the remainder of the paper.

**Lemma 4** If $\sigma_C(1; g) > 0$ and $\sigma_C(0; g + 1) = 1$ for some grievance $g$, then $\sigma_p(g + 1) < 1$ in every equilibrium $\sigma$. 

Proof. Suppose not. Then there exists a $g$ such that $\sigma_C(1; g) > 1$, $\sigma_C(0; g + 1) = 1$ and $\sigma_P(g + 1) = 1$ in equilibrium $\sigma$. We can write $V^\sigma_C(g + 1)$ as

$$V^\sigma_C(g + 1) = -F(g + 1)\psi - (1 - F(g + 1))\left(\pi_C^\sigma + \delta V^\sigma_C(g)\right)$$

$$= -F(g + 1)\psi - (1 - F(g + 1))\left(\pi_C^\sigma + \delta U^\sigma_C(1; g)\right)$$

$$= -F(g + 1)\psi - (1 - F(g + 1))\left(\pi_C^\sigma + \delta (\pi_C^\sigma - \kappa_C + \delta V^\sigma_C(g + 1))\right).$$

Solving reveals that

$$V^\sigma_C(g + 1) = \frac{(1 - F(g + 1))(\pi(1 + \delta) - \delta\kappa_C) - F(g + 1)\psi}{1 - (1 - F(g + 1))\delta^2}.$$

By Lemma 2, $V^\sigma_C(g + 1) \geq \tilde{V}_C(g + 1)$. By Lemma 1.1,

$$\tilde{V}_C(g) > \frac{(1 - F(g + 1))\pi_C^\sigma - F(g + 1)\psi}{1 - (1 - F(g + 1))\delta}.$$

Stringing these two inequalities together,

$$V^\sigma_C(g + 1) > \frac{(1 - F(g + 1))\pi_C^\sigma - F(g + 1)\psi}{1 - (1 - F(g + 1))\delta}.$$

Substituting the closed form solution for $V^\sigma_C(g + 1)$ into the inequality above and solving for $\kappa_C$ reveals that

$$\kappa_C < \frac{F(g + 1)(\pi_C^\sigma + \psi(1 - \delta))}{1 - (1 - F(g + 1))\delta}.$$

To derive a contradiction, consider a deviation in which $C$ plays $r = 1$ with probability 1 in all future periods beginning at grievance $g + 1$. This is a profitable deviation if and only if

$$V^\sigma_C(g + 1) < \frac{\pi_C^\sigma - \kappa_C}{1 - \delta} \iff \kappa_C < \frac{F(g + 1)(\pi_C^\sigma + \psi(1 - \delta))}{1 - (1 - F(g + 1))\delta}.$$

However, $\kappa_C < \frac{F(g + 1)(\pi_C^\sigma + \psi(1 - \delta))}{1 - (1 - F(g + 1))\delta}$ as shown above. Hence, $C$ can profitably deviate by repressing in all future periods. \qed

Lemma 5 Consider some $g > g^-$ and equilibrium $\sigma$. If (a) $\sigma_C(0; g - 1) = 1$ or $\sigma_C(0; g) = 1$ and (b) $\sigma_C(\emptyset; g') = 0$ for all $g' < g$, then $\sigma_P(g) = 1$.

Proof. Suppose not. That is, consider some equilibrium $\sigma$ and grievance $g > g^-$ such that

(a) $\sigma_C(0; g - 1) = 1$ or $\sigma_C(0; g) = 1$,
(b) $\sigma_C(\emptyset; g') = 0$ for all $g' < g$, and

(c) $\sigma_P(g) < 1$.

Because $\sigma$ is an equilibrium, we require $U_P^r(0; g) \geq U_P^r(1; g)$ to rule out profitable deviations, which is equivalent to

$$\kappa_P \geq F(g) \left[ \hat{V}_P - \pi_C^P - \delta V_P^r(g - 1) \right].$$

Because $\sigma_C(0; g - 1) = 1$ or $\sigma_C(0; g) = 1$, the path of play will never reach a grievance larger than $g$. Because $\sigma_C(\emptyset; g') = 0$ for all $g' \leq g$, the Center will never grant independence along the subsequent path of play. Recall that when the $C$ represses, $P$ stage payoff is $\pi_C^C$, which is its payoff if it chooses not to mobilize, and even if $C$ does repress with positive probability at some $g' < g$, the subsequent path of play will still never reach a grievance larger than $g$. Then $g > g^-$ implies $V_P^r(g - 1)$ is bounded above by

$$\frac{F(g)\hat{V}_P + (1 - F(g))\pi_C^C - \kappa_P}{1 - (1 - F(g))\delta},$$

which is $P$'s payoff if its grievance never depreciates along the path of play, $C$ never represses, and $P$ always mobilizes. Combining these two inequalities, we require

$$\kappa_P \geq F(g) \left[ \hat{V}_P - \pi_C^C - \delta V_P^r(g - 1) \right] \geq F(g) \left[ \hat{V}_P - \pi_C^C - \delta \frac{F(g)\hat{V}_P + (1 - F(g))\pi_C^C - \kappa_P}{1 - (1 - F(g))\delta} \right].$$

Solving for $\kappa_P$ implies

$$\kappa_P \geq F(g) \frac{\pi_P^P - \pi_C^C}{1 - \delta},$$

that is, $g \leq g^-$. But this contradicts the assumption $g > g^-$. $\square$

## E  Proof of Proposition 2

This section characterizes equilibrium behavior at moderate grievances.

We now prove that $g < g^+$ implies $\sigma_C(0; g) = 1$ in every equilibrium $\sigma$, that is, the Center neither represses nor grants independence with moderate grievances. The result requires preliminary lemmas. Notice that if either Assumption 1 or 2 does not hold, $\hat{V}_C(g) > \max \left\{ \frac{\pi_C^C - \kappa_C}{1 - \delta}, 0 \right\}$ for all $g$, and we can set $g^+ = \infty$ in the subsequent results.
Lemma 6 If $g < g^+$, then $\sigma_C(\emptyset; g) = 0$ in every equilibrium $\sigma$.

Proof. If not, then $V^C_\sigma(g) = U^C_\sigma(\emptyset; g) = 0$. If $g < g^+$, this contradicts Lemma 2 because $\tilde{V}_C(g) > 0 = V^C_\sigma(g)$.

Lemma 7 For all $g$, $\sigma(r; g) > 0$ imply $\sigma(\emptyset; g + 1) = 0$ in every equilibrium $\sigma$.

Proof. First, if $\kappa_C < \pi_C^C$, then $C$ cannot grant independence with positive probability in any equilibrium. Doing so would result in a payoff of 0, but $C$ could repress for all future periods, giving a payoff of $\pi_C^C - \kappa_C - \delta > 0$. Thus, consider the case where $\pi_C^C - \kappa_C < 0$. Suppose $\sigma_C(r; g) > 0$ for some $g$ and $\sigma_C(\emptyset; g + 1) > 0$. Then

\[
V^C_\sigma(g) = U^C_\sigma(r; g) = \pi_C^C - \kappa_C + \delta V^C_\sigma(g + 1) = \pi_C^C - \kappa_C + \delta U^C_\sigma(\emptyset; g) = \pi_C^C - \kappa_C < 0,
\]

but this means $C$ can profitably deviate at $g$ by granting independence, i.e., $\sigma$ is not an equilibrium.

Lemma 8 Fix an equilibrium $\sigma$. Then there does not exist a $g < g^+$ such that $\sigma_C(1; g') > 0$ for all $g' \geq g$.

Proof. Suppose not and consider such a $g < g^+$ where $\sigma_C(1; g') > 0$ for all $g' \geq g$ in equilibrium $\sigma$. Then

\[
V^C_\sigma(g) = U^C_\sigma(1; g) = \pi_C^C - \kappa_C + \delta V^C_\sigma(g + 1) = \pi_C^C - \kappa_C + \delta U^C_\sigma(\emptyset; g).
\]

Because $V_C(g') = U^C_\sigma(r; g')$ for all $g'$ such that $\sigma_C(r; g') > 0$, similar substitutions imply $V^C_\sigma(g) = \frac{\pi_C^C - \kappa_C}{1 - \delta}$. However, $g < g^+$ implies

\[
\tilde{V}_C(g) > \frac{\pi_C^C - \kappa_C}{1 - \delta} = V^C_\sigma(g),
\]

by Equation(3). However, $\tilde{V}_C(g) > V^C_\sigma(g)$ contradicts Lemma 2.

With these lemmas in hand, we now state the main result of the section.

Proposition 2 If grievances are moderate, then the Periphery always mobilizes, the Center neither represses nor grants independence, and grievances dissipate on the equilibrium path. That is, $g \in (g^-, g^+)$ implies $\sigma_P(g) = 1$ and $\sigma_C(0; g) = 1$ in every equilibrium $\sigma$. 

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Proof. We first prove that $\sigma_C(0; g) = 1$ when $g \in (g^-, g^+)$ and $\sigma$ is an equilibrium. Suppose not. By Lemma 6, $\sigma_C(1; g) > 0$. Furthermore, $C$ represses with positive probability for at most some finite $k$ periods by Lemma 8. That is, there exists a $\bar{g}$ such that $\sigma_C(1; g') > 0$ for $g' = g, ..., \bar{g}$ and $\sigma_C(1; \bar{g} + 1) = 0$. By Lemma 7, this implies $\sigma_C(0; \bar{g} + 1) = 1$. In addition, Proposition 1 and Lemma 7 imply $\sigma_P(\bar{g} + 1) = 1$.

□

F Proof of Proposition 3

We now characterize equilibrium behavior at large grievances ($g \geq g^+$). We consider the generic case in which there does not exist $g \in \mathbb{N}_0$ such that $\tilde{V}_C(g) = \max \left\{ \frac{\pi_C - \kappa_C}{1 - \delta}, 0 \right\}$, that is $\tilde{V}_C(g^+) < \max \left\{ \frac{\pi_C - \kappa_C}{1 - \delta}, 0 \right\}$, where the inequality from Equation (3) holds strictly. If this held with equality, the Center would be indifferent leading to trivial indeterminacy. We consider high- and low-capacity regimes separately because the proof techniques vary dramatically between the two cases.

F.1 High repression capacity: $\kappa_C < \pi_C^C$

Lemma 9 In high-capacity regimes, $\sigma_C(\emptyset; g) = 0$ for every grievance $g$ and in every equilibrium $\sigma$.

The proof is straightforward and omitted.

Lemma 10 In high-capacity regimes, $\sigma_C(1; g^+) = 1$ and $\sigma_C(1; g) > 0$ for all $g > g^+$ in every equilibrium $\sigma$.

Proof. The proof is by induction. First, we demonstrate that $\sigma_C(1; g^+) = 1$. To see this, suppose $\sigma_C(1; g^+) < 1$. Then Lemma 9 implies $\sigma_C(0; g^+) > 0$, in which case we have

\[ U_C^C(0; g^+) = \tilde{V}_C(g^+) < \frac{\pi_C^C - \kappa_C}{1 - \delta}. \]

This means $C$ can profitably deviate at grievance $g^+$ by repressing for an infinite number of periods, a contradiction.

For induction, consider some $g > g^+$ and assume $\sigma_C(1; g - 1) > 0$. To derive a contradiction, assume $\sigma_C(1; g) = 0$. By Lemma 9, $\sigma_C(0; g) = 1$. Likewise, Lemma 9 guarantees $C$ does not grant...
Lemma 11. In high-capacity regimes, \( g \geq g^+ \) implies \( V_C^\sigma(g) = \frac{\pi_C \cdot \kappa_C}{1 - \delta} \) in every equilibrium \( \sigma \).

Proof. If \( g \geq g^+ \), then Lemma 10 implies \( \sigma_C(1; g') > 0 \) for all \( g' \geq g \). The remainder of the proof follows from an identical argument as the one in Lemma 8.

Lemma 12. In high-capacity regimes, \( g > g^+ \) and \( \sigma_C(0; g) > 0 \) imply \( \sigma_P(g) < 1 \) in every equilibrium \( \sigma \).

Proof. Suppose not. Then there exists \( g > g^+ \) such that \( \sigma_C(0; g) > 0 \) and \( \sigma_P(g) = 1 \). Because \( g > g^+ \), \( g - 1 \geq g^+ \). Likewise, \( \sigma_C(1; g) > 0 \) by Lemma 10, so it must be the case that \( U_C^\sigma(0; g) = U_C^\sigma(1; g) \).

Then we have

\[
U_C^\sigma(0; g) = U_C^\sigma(1; g) \iff -F(g)\psi + (1 - F(g))(\pi_C(1) + \delta V_C^\sigma(g - 1)) = \pi - \kappa_C + \delta V_C^\sigma(g + 1)
\]

\[
\iff -F(g)\psi + (1 - F(g))\left(\pi_C + \delta \frac{\pi - \kappa C}{1 - \delta}\right) = \pi - \kappa_C
\]

\[
\iff \kappa_C = \frac{-F(g)(\pi_C + (1 - \delta)\psi)}{1 - (1 - F(g))\delta},
\]

where we use Lemma 11 and \( g - 1 \geq g^+ \) to substitute for values \( V_C^\sigma(g - 1) \) and \( V_C^\sigma(g + 1) \).

Because \( \sigma \) is an equilibrium, we require \( U_C^\sigma(1; g) = V_C^\sigma(g) \geq \tilde{V}_C(g) \), by Lemma 2. Then Lemma 1.1 implies

\[
U_C^\sigma(1; g) > \frac{-F(g)\psi + (1 - F(g))\pi_C}{1 - (1 - F(g))\delta} \iff \pi_C \cdot \kappa_C \cdot \pi_C < \frac{-F(g)\psi + (1 - F(g))\pi_C}{1 - (1 - F(g))\delta}
\]

\[
\iff \kappa_C < \frac{-F(g)(\pi_C(1) + (1 - \delta)\psi)}{1 - (1 - F(g))}\delta.
\]

which establishes the desired contradiction.

Lemma 13. In high-capacity regimes, there exists cutpoint \( \bar{g} \in \mathbb{R} \) such that if \( g > \bar{g} \), then \( \sigma_P(g) = 1 \) and \( \sigma_C(1; g) = 1 \) in every equilibrium \( \sigma \).

Proof. The proof is constructive. Define \( \bar{g} \in \mathbb{N}_0 \) to be a number that satisfies

\[
g \geq \bar{g} \iff \kappa_P < F(g) \left[ \tilde{V}_P - \pi_P^C - \delta p \tilde{V}_P + (1 - p)\pi_P^C - \kappa_P \right].
\]
Such a $\bar{g}$ exists because $F(g) \left[ \bar{V}_P - \pi_p^C - \delta p\bar{V}_P + (1 - p)\pi_p^C \right]$ is positive and strictly increasing in $g$. Furthermore,
\[
\lim_{g \to \infty} F(g) \left[ \bar{V}_P - \pi_p^C - \delta p\bar{V}_P + (1 - p)\pi_p^C \right] = p\frac{\pi_p^C - \pi_p^C}{1 - \delta},
\]
and Assumption 1 implies
\[
\kappa_p < p\frac{\pi_p^C - \pi_p^C}{1 - \delta}.
\]

We first show that $\sigma_p(g) = 1$ for $g \geq \bar{g}$. Suppose not; then there exists $g \geq \bar{g}$ such that $\sigma_p(g) < 1$. To rule out profitable deviations, we require $U_p^\sigma(0; g) \geq U_p^\sigma(1; g)$, which is equivalent to
\[
\kappa_p \geq F(g) \left[ \bar{V}_P - \pi_p^C - \delta V_p^\sigma(g - 1) \right].
\]
Because the Center never grants independence in strong regimes, $V_p^\sigma(g - 1)$ is bounded above by $\frac{\rho\bar{V}_P + (1 - p)\pi_p^C - \kappa_p}{1 - (1 - p)\delta}$, which is the Periphery’s dynamic payoff if it mobilizes in every period at maximum capacity, $p$. Combining these two inequalities gives us
\[
\kappa_p \geq F(g) \left[ \bar{V}_P - \pi_p^C - \delta V_p^\sigma(g - 1) \right] = F(g) \left[ \bar{V}_P - \pi_p^C - \delta \frac{p\bar{V}_P + (1 - p)\pi_p^C - \kappa_p}{1 - (1 - p)\delta} \right],
\]
but this implies $g < \bar{g}$, which is contradiction. Thus, $\sigma_p(g) = 1$. Then Lemma 10 and the contrapositive of Lemma 12 imply $\sigma_C(1; g) = 1$, as required. \(\square\)

**Lemma 14** In high-capacity regimes, if $g \geq g^+$, then $\sigma_p(g) = 1$ in every equilibrium $\sigma$.

**Proof.** Suppose there exists $g \geq g^+$ such that $\sigma_p(g) < 1$. Lemma 13 implies that there exists grievance $g^\dagger \geq g$ such that $\sigma_p(g^\dagger) < 1$ and $\sigma_p(g^\dagger) = \sigma_C(1; g^\dagger) = 1$ for all $g > g^\dagger$. To rule out profitable deviations, we require $U_p^\sigma(0; g^\dagger) \geq U_p^\sigma(1; g^\dagger)$. This implies
\[
\kappa_p \geq F(g^\dagger) \left[ \bar{V}_P - \pi_p^C - \delta V_p^\sigma(g^\dagger - 1) \right].
\]
Because $P$ will never be able to mobilize at a larger grievance than $g^\dagger$ along the path of play and $C$ never grants independence, $V_C^\sigma(g^\dagger - 1)$ is bounded above by
\[
\frac{F(g^\dagger)\bar{V}_P + (1 - F(g^\dagger))\pi_p^C - \kappa_p}{1 - (1 - F(g^\dagger))\delta}.
\]
Then we have

\[
\kappa_p \geq F(g^\dagger) \left[ \tilde{V}_p - \pi_p^C - \delta V_p^\tau (g^\dagger - 1) \right] \\
\geq F(g^\dagger) \left[ \tilde{V}_p - \pi_p^C - \frac{F(g^\dagger)\tilde{V}_p + (1 - F(g^\dagger))\pi_p^C - \kappa_p}{1 - (1 - F(g\dagger))\delta} \right] \\
= F(g^\dagger) \frac{\pi_p^C - \pi_p^C}{1 - \delta},
\]

which implies \( g^\dagger \leq g^- \leq g^+, \) a contradiction. \( \square \)

We now prove Proposition 3.1, which characterizes equilibria in regimes with large grievances when \( \pi_C^C > \kappa_C. \)

**Proof of Proposition 3.1.** If \( g \geq g^+, \) then Lemma 14 implies \( \sigma_p(g) = 1. \) Because \( g > g^+ \) implies \( \sigma_p(g) = 1. \) Lemma 10 and the contrapositive of Lemma 12 imply \( \sigma_C(1; g) = 1, \) as required. \( \square \)

**F.2 Low repression capacity: \( \kappa_C > \pi_C^C \)**

**Lemma 15** Fix an equilibrium \( \sigma. \) In low-capacity regimes, the there does not exist grievance \( g \) such that \( \sigma_C(1; g') > 0 \) for all \( g' \geq g. \)

**Proof.** The result follows from the inequality \( \pi_C^C - \kappa_C < 0 \) and the argument proving Lemma 8. \( \square \)

**Lemma 16** In low-capacity regimes, \( \sigma_p(g^+) = 1, \sigma_C(0; g^+) = 0, \) and \( \sigma_C(\emptyset; g^+) > 0 \) in ever equilibrium \( \sigma. \)

**Proof.** First, \( P \) mobilizes at \( g^+ \) by Lemma 5 and Proposition 2.

Second, \( \sigma_C(0; g^+) = 0. \) If not, then with positive probability the Center chooses to enter the path of play into moderate grievance levels. That is, \( V_C^\sigma(g^+) = U_C^\sigma(0; g^+) = \tilde{V}_C(g^+). \) But then \( V_C^\sigma(g^+) < 0 \) because the regime has low capacity, so \( C \) can profitably deviate by granting independence at \( g^+. \)

Third, \( \sigma_C(1; g^+) < 1. \) To see this, suppose not, i.e., suppose \( \sigma_C(1; g^+) = 1. \) By Lemma 15, there exists \( g^\dagger \geq g^+ \) such that \( \sigma_C(1; g^\dagger + 1) = 0 \) and \( \sigma_C(1; g^\dagger) > 0 \) for all \( g' = g^+, ..., g^\dagger. \) Then by Lemma 7, \( \sigma_C(\emptyset; g') = 0 \) for all \( g' = g^+, ..., g^\dagger + 1. \) By Proposition 2, \( \sigma_C(0; g') = 1 \) for all \( g' < g^+. \) Then Lemma 5 implies \( \sigma_p(g^\dagger + 1) = 1. \) However, \( \sigma_C(1; g^\dagger) > 0, \sigma_C(0; g^\dagger + 1) = 1, \) and \( \sigma_p(g^\dagger + 1) = 1 \) contradict Lemma 4. Thus, \( \sigma_C(1; g^+) < 1, \) which implies \( \sigma_C(\emptyset; g^+) > 0 \) by the previous paragraph. \( \square \)
Before proving the last technical lemma of this section, consider the following definitions. The set $G \subseteq \mathbb{N}_0$ is an absorbing set with respect to profile $\sigma$ if once the path of play enters grievance level $g$ such that $g \in G$, it never transitions to a grievance $g'$ such that $g' \notin G$ with positive probability. The set $G$ is an irreducible absorbing set with respect to $\sigma$ if $G$ is an absorbing set with respect to $\sigma$ and there does not exist a proper subset $G' \subset G$ such that $G'$ is an absorbing set with respect to $\sigma$.

**Lemma 17** Consider an equilibrium $\sigma$ and some grievance $g \geq g^+$. Then the following hold:

1. beginning at grievance $g$, the path of play enters an irreducible absorbing set $G$ with respect to $\sigma$,
2. max $G$ exists,
3. $g^+ \leq \min G$, and
4. there exists $g' \in G$ such that $\sigma_C(\emptyset; g') > 0$.

**Proof.** To prove (1), consider $g \geq g^+$ and two cases. If $\sigma_C(1; g) = 0$, then the path of play enters the set $\{g^+, ..., g\}$, which is an absorbing set because $\sigma_C(0; g^+) = 0$ by Lemma 16. So the set $\{g^+, ..., g\}$ has a irreducible absorbing set, $G$. If $\sigma_C(1; g) > 0$, then Lemma 15 implies there exists $g^+ \geq g$ such that $\sigma_C(1; g') > 0$ for all $g' = g, ..., g^+$ and $\sigma_C(1; g^+ + 1) = 0$ from Lemma 7. Then the path of play enters the set $\{g^+, ..., g^+ + 1\}$, which is an absorbing set as well.

The proof of (2) and (3) follow immediately from the existence of $G$ and Lemmas 15 and 16, respectively.

To prove (4), suppose not. Suppose $\sigma_C(\emptyset; g') = 0$ for all $g' \in G$. I first claim that it must be the case that $\#G > 1$. Suppose the contrary. Then $G = \{g'\}$, and $C$ cannot be repressing with positive probability at $g$, or else $G$ is not absorbing. Also, if $G = \{g'\}$ and $\sigma_C(0; g') > 0$, then $F(g) = 1$ and $\sigma_P(g) = 1$ or else the path of play would transition to $g - 1$ with positive probability. In this case, $U_C(0; g') = -\psi < 0$, but this means $C$ has a profitable deviation by granting independence at $g'$. Thus, $\#G > 2$ and as such max $G - 1 \in G$.

Second, because $G$ is irreducible, $\sigma_C(1; \max G - 1) > 0$, or else $G \backslash \{\max G\}$ would be absorbing as well. Furthermore, $\sigma_C(1; \max G) = 0$ or else the path of play would transition with positive probability to $\max G + 1$. Because $\sigma_C(1; \max G - 1) > 0$ and $\sigma_C(1; \max G) = 0$, Lemma 7 implies $\sigma_C(0; \max G) = 1$. Because the path of play never leaves $G$ nor transitions to grievance $g' > \max G$ and $C$ never grants independence along the path of play starting from $\max G$, then $\sigma_P(\max G) = 1$, which follows from an identical argument as the one in Lemma 5. However, this contradicts Lemma 4.

The proof of Proposition 3.2 follows from Lemma 17. □
G  Proof of Proposition 4

First, the result in Proposition 4.1 follows immediately from Lemma 15. Second, the result in Proposition 4.2 is proved below in Lemma 18. Third, I construct an equilibrium that supports cycles of repression and mobilization, as described in Proposition 4.3, in Example 1. As part of this construction, I need a new result in Lemma 19.

Lemma 18  If $\kappa_C > (1+\delta)\pi_C^C$, then the Center never represses in any equilibrium $\sigma$, i.e., $\sigma_C(1; g) = 0$ for every grievances $g$ and every equilibrium $\sigma$.

Proof. To derive a contradiction, suppose the contrary. That is, suppose $\kappa_C > (1+\delta)\pi_C^C$ and the Center represses in equilibrium $\sigma$. Thus, the regime is has low capacity, and there exist some $g$ such that $\sigma_C(1; g) > 0$. By Lemma 15, there exists $g^\dagger \geq g$ such that $\sigma_C(1; g^\dagger + 1) = 0$ and $\sigma_C(1; g') > 0$ for all $g' = g, ..., g^\dagger$. Then by Lemma 7, $\sigma_C(\emptyset; g') = 0$ for all $g' = g + 1, ..., g^\dagger + 1$. Hence, $\sigma_C(0; g^\dagger + 1) = 1$. We can compute $C$’s continuation value at $g^\dagger$ as

$$V^C_C(g^\dagger) = \sigma_C(1; g^\dagger)U^C_C(1; g^\dagger) + \sigma_C(0; g^\dagger)U^C_C(0; g^\dagger) = V^\sigma_C(1; g^\dagger)$$

$$= \pi_C^C - \kappa_C + \delta V^C_C(g^\dagger + 1)$$

$$= \pi_C^C - \kappa_C + \delta \left[ \sigma_P(g^\dagger + 1) \left( -F(g^\dagger + 1)\psi + (1 - F(g^\dagger + 1))\pi_C^C + \delta V^\sigma_C(g^\dagger) \right) + (1 - \sigma_P(g^\dagger + 1)) \left( \pi_C^C + \delta V^\sigma_C(g^\dagger) \right) \right],$$

where the second equality follows because $\sigma$ is an equilibrium and $\sigma_C(1; g^\dagger) > 0$. Solving for $V^\sigma_C(g^\dagger)$ reveals that

$$V^\sigma_C(g^\dagger) = \frac{\pi_C^C(1 + (1 - F(g^\dagger + 1)\sigma_P(g^\dagger + 1))\delta) - \kappa_C - F(g^\dagger + 1)\sigma_P(g^\dagger + 1)\delta \psi}{1 - (1 - \sigma_P(g^\dagger + 1)F(g^\dagger + 1))\delta^2},$$

which is decreasing in $\sigma_P(g^\dagger + 1)$. Because $\sigma_P(g^\dagger + 1) \geq 0$, then

$$V^\sigma_C(g^\dagger) \leq \frac{\pi_C^C(1 + \delta) - \kappa_C}{1 - \delta^2}.$$  

Thus, $\kappa_C > (1 + \delta)\pi_C^C$ implies $V^\sigma_C(g^\dagger) < 0$. But this implies $C$ can profitably deviate at $g^\dagger$ by granting independence and guaranteeing itself a payoff of zero.

Lemma 19  In low-capacity regimes, if $F(g)(\pi_P^C - \pi_C^C) > \kappa_P$ and $g \geq g^\dagger$, then $\sigma_P(g') = 1$ and $\sigma_C(1; g') = 0$ for all $g' \geq g$ in every equilibrium $\sigma$.  


Proof. By Equation (1), \( P \) mobilizes at \( g' \) if

\[ \kappa_C < F(g') \left[ \tilde{V}_P - \pi_C^P - \delta V'_P(g' - 1) \right]. \]

An upper bound on \( V'_P(g' - 1) \) is \( \frac{\pi_P}{1 - \delta} \), which is the discounted sum of \( P \)'s largest per-period payoff. Combining these two inequalities implies \( P \) mobilizes when \( F(g') (\pi_C^P - \pi_P^P) > \kappa_P \), which holds because \( F(g') (\pi_C^P - \pi_P^P) > \kappa_P \), and \( F \) is increasing.

Second, I claim that \( \sigma_C(1; g') = 0 \) for all \( g' \geq g \). Suppose not. Then there exists a \( g^* \) such that \( \sigma_C(1; g^*) > 0 \) and \( \sigma_C(0; g^* + 1) = 1 \) by Lemmas 7 and 15. The previous paragraph demonstrates that \( P \) mobilizes with probability 1 with grievance \( g^* + 1 \). But this contradicts Lemma 4. \( \square \)

**Example 1** In this example, I assume \( \pi_C^P = \pi_P^P = 1 \), and \( \pi_C^P = 0 \). In addition, \( \kappa_C = 1.2 \) and \( \kappa_P = .25 \). This implies that the regime has low repression capacity. Finally, \( \delta = .9 \), \( \psi = 6 \), and \( F \) takes the form:

\[
F(g) = \begin{cases} 
0 & \text{if } g = 0 \\
\frac{g}{700} + \frac{33}{175} & \text{if } g \geq 1 \text{ and } g \leq 8 \\
1 & \text{otherwise.}
\end{cases}
\]

Thus, \( g^- = 0 \), and \( g^+ = 7 \), because \( \tilde{V}_C(6) \approx .33 \) and \( \tilde{V}_C(7) \approx -.15 \). By Proposition 2, the Periphery mobilizes with probability one for all \( g \in \{1, 2, ..., 7\} \) and the Center neither represses nor grants independence for all \( g \in \{0, 1, 2, ..., 6\} \). Note that \( F(9)(\pi_P^P - \pi_C^P) > \kappa_P \), so Lemma 19 implies the Periphery mobilizes for all grievances \( g \geq 9 \) and the Center does not repress at grievance \( g \geq 9 \).

We specify remaining behavior as follows.

1. At grievance \( g = 7 \), the Periphery mobilizes with probability 1 and the Center mixes between repression and granting independence, \( \sigma_C(1; 7) + \sigma_C(\emptyset; 7) = 1 \)

2. At grievance \( g = 8 \), the Center neither represses nor grants independence, i.e., \( \sigma_C(0; 8) = 1 \) and the Periphery mobilizes with probability \( \sigma_P(8) \).

We first characterize mixing probabilities, \( \sigma_C(1; 7) \), \( \sigma_C(\emptyset; 7) \), and \( \sigma_P(8) \), such that the following hold:

\[
\sigma_C(1; 7) + \sigma_C(\emptyset; 7) = 1 \\
U_C^C(1; 7) = U_C^C(\emptyset; 7) \\
U_P^P(1; 8) = U_P^P(0; 8).
\]

The first equation says the Center mixes between repression and granting independence at \( g = 7 = g^+ \). The second and third equations are \( C \) and \( P \)'s indifference conditions, respectively. Because
\[ U_C^\sigma(\emptyset; 7) = 0, \text{ C's indifference equations takes the form:} \]

\[ \pi_C^C - \kappa_C + \delta V_C^\sigma(8) = 0, \quad (7) \]

where

\[ V_C^\sigma(8) = \sigma_P(8) \left[ -F(8)\psi + (1 - F(8)) \left( \pi_C^C + \delta V_C^\sigma(7) \right) \right] + (1 - \sigma_P(8)) \left[ \pi_C^C + \delta V_C^\sigma(7) \right]. \]

In equilibrium, \( V_C^\sigma(7) = U_C^\sigma(\emptyset; 7) = 0. \) Thus, we have

\[ V_C^\sigma(8) = \sigma_P(8) \left[ -F(8)\psi + (1 - F(8))\pi_C^C \right] + (1 - \sigma_P(8))\pi_C^C. \]

Substituting the above equality into Equation (7), C's indifference condition takes the form:

\[ \pi_C^C - \kappa_C + \delta \left( \sigma_P(8) \left[ -F(8)\psi + (1 - F(8))\pi_C^C \right] + (1 - \sigma_P(8))\pi_C^C \right) = 0. \quad (8) \]

Next, consider P's indifference equation, \( U_P^\sigma(1; 8) = U_P^\sigma(0; 8), \) which takes the form

\[ -\kappa_P + F(8) \frac{\pi_p^P}{1 - \delta} + (1 - F(8)) \left( \pi_P^P + \delta V_P^\sigma(7) \right) = \pi_P^P + \delta V_P^\sigma(7), \quad (9) \]

where

\[ V_P^\sigma(7) = \sigma_C(\emptyset; 7) \frac{\pi_p^P}{1 - \delta} + \sigma_P(1; 7) \left[ \pi_P^P + \delta V_P^\sigma(8) \right] \]

\[ = \sigma_C(\emptyset; 7) \frac{\pi_p^P}{1 - \delta} + \sigma_P(1; 7) \left[ \pi_P^P + \delta U_P^\sigma(0; 8) \right] \]

\[ = \sigma_C(\emptyset; 7) \frac{\pi_p^P}{1 - \delta} + \sigma_P(1; 7) \left[ \pi_P^P + \delta \left( \pi_P^P + \delta V_P^\sigma(7) \right) \right] \]

Here the second equality follows because \( \sigma_C(0; 8) = 1. \) Solving Equations (8) and (9) with the constraint \( \sigma_C(\emptyset; 7) + \sigma_C(1; 7) = 1 \) reveals that

\[ \sigma_P(8) = \frac{(1 + \delta)\pi_C^C - \kappa_C}{(\pi_C^C + \psi)\delta F(8)} \approx .56 \]

and

\[ \sigma_C(1; 7) = \frac{\kappa_P - F(8)(\pi_P^P - \pi_C^C)}{\delta^2\kappa_P + \delta F(8)(\pi_P^P - \pi_C^P)} \approx .13. \]

Finally, we check profitable deviations. First, P's indifference condition precludes profitable deviations at \( g = 8. \) Second, C does not have a profitable deviation at \( g = 7 \) due to its indifference
equation and because \( U_C'(0; 7) = \tilde{V}_C(7) < 0 \). Also, \( C \) has no profitable deviation at \( g = 8 \), because \( V_C(8) > 0 \). To see this, note that \( U_C'(1; 7) = \pi_C - \kappa_C + \delta V_C'(8) = 0 \) by Equation (7), and \( \pi_C - \kappa_C < 0 \). If \( C \) deviates by granting independence at \( g = 8 \), then its payoff is zero. Likewise, if \( C \) deviates by repressing, its payoff is \( \pi_C - \kappa_C + \delta V_C'(9) \), which reduces to \( \pi_C - \kappa_C < 0 \) because \( C \) is granting independence when \( g = 9 \). Lemma 19 implies that \( C \) cannot profitably deviate by using repression, at grievances \( g \geq 9 \). Thus, we only need to verify that \( C \) cannot profitably deviate by choosing to refrain from repression or granting independence, at grievances \( g \geq 9 \). Because the Periphery mobilizes at \( g \geq 9 \) and \( F(g) = 1 \), mobilization surely succeeds, implying \( U_C'(0; g) = -\psi \) for all \( g \geq 9 \) which is strictly less than \( C \)'s utility from following its equilibrium strategy of granting independence.

H Exogenous Decentralization

In this section, I continue to analyze the numerical example in Figure 4 and prove Proposition 5.

From, the example in Figure 4, I compute the probability that the country breaks apart due to secessionist mobilization ---labeled probability of secession hereafter---as a function of decentralization. For a fixed \( d \), three potential paths of play emerge at initial grievance \( g^1 \) in equilibrium. First, if \( g^1 < g^+[d] \), the Center neither represses nor grants independence, and the probability secession is

\[
\begin{cases} 
0 & \text{if } g^1 \leq g^-[d] \\
1 - \prod_{g^1 < g' \leq g^1} (1 - F(g')) & \text{otherwise.}
\end{cases}
\]

Second, if \( g^1 \geq g^+[d] \) and the regime has high capacity \((\pi - d > \kappa_C)\), then the Center represses in all future periods, and the probability of secession is zero. Third, if \( g^1 \geq g^+[d] \) and the regime has low capacity \((\pi - d < \kappa_C)\), the probability of secession is undefined. Although the Periphery will eventually gain control of its territory (Proposition 3.2), this may arise either from secessionist mobilization or Center-granted independence. This third case does not arise in the numerical example. As seen in Figure 4, if \( g^1 \geq g^+[d] \) for some \( d \), then the regime has high capacity.

Figure 6 graphs the probability of secession decentralization varies. When \( d \) is small, \( g^1 > g^+[d] \) so the high-capacity regime represses and the probability of secession is zero. When \( d \) is large, \( g^1 < g^- \), so grievances are small and a lasting peace emerges. When \( d \) is moderate, then the Center gambles for unity and secession occurs with positive probability. When \( \psi \) is large (left panel), all decentralization levels below \( d = 44 \) result in long-term repression and a zero probability of secession. When \( \psi \) is small (right panel), all decentralization levels below \( d = 38 \) result in long-term repression and a zero probability of secession.
Figure 6: Decentralization and comparative statics

Notes: The panels graph the probability of secession (vertical axis) for a fixed decentralization level (horizontal axis) with a large cost of secession $\psi = \frac{\pi}{2}$ (left) and a small cost $\psi = \frac{\pi}{4}$ (right). The remaining parameters take on the following values: $\pi = 100$, $\kappa_C = 50$, $\kappa_P = 50$, $\delta = 0.95$, and and $F(g) = 1 - \left(0.01g + 0.001g^2 - 1\right)^{-1}$.

Proposition 5 Assume the regime has a high capacity for repression ($\kappa_C < \pi$) and initial grievances are large ($g^1 \geq g^+[0]$). There exist cutpoints $d$ and $\bar{d}$ such that $0 \leq d < \bar{d} < 1$ and secession occurs with positive probability on the equilibrium path only if decentralization is moderate, i.e., $d < d < \bar{d}$.

Proof. Set $d = 0$. The regime has high repression capacity by assumption, and $g^1 \geq g^+[d]$ implies that $C$ represses with probability one in all future periods when the game begins at grievance $g^1$. As such the probability of secession is zero.

In addition, we can set $\bar{d}$ as follows

$$\bar{d} = \hat{d}(g^1) + \epsilon$$

where $\hat{d}$ is defined in Equation (4) above and $\epsilon \in \mathbb{R}$ is such that $0 < \epsilon < \max\left\{\frac{(1-\delta)\kappa_C}{F(g^1)}, 1\right\}$. Note that the fraction $\frac{(1-\delta)\kappa_C}{F(g^1)}$ is well defined because $F(g^1) \neq 0$. If $F(g^1) = 0$ then $g^1 \leq g^-[0] < g^+[0]$, a contradiction.

It suffices to show that $g^1 \leq g^-[\bar{d}]$ because this inequality implies that $g^1$ is small at decentralization level $\bar{d}$ and $g^-$ is strictly increasing in $d$. As such, $g^1$ is small at decentralization levels $d > \bar{d}$. In addition, when $g^1 \leq g^-$ no mobilization occurs along the path of play by Proposition 1.
When $\pi_C^P = d$ and $\pi_P^P = \pi$, then we can write $g^{-}[d]$ as

$$g^{-}[d] = \max \left\{ g \in \mathbb{N}_0 \mid \kappa_P > F(g) \frac{\pi - d}{1 - \delta} \right\}.$$  

Thus, $g^1 \leq g^{-}[d]$, as required. $\square$

I Proof of Proposition 6

We first prove Proposition 6.1 and then present two numerical examples that establish Propositions 6.2 and 6.3.

Proof of 6.1. Consider equilibrium $(d^*, \sigma)$. We first prove that $d^* \leq \min\{\hat{d}(g^1), \kappa_C\}$. First, $d^* \leq \kappa_C$. To see this, note that $V_C^{\sigma}(g^1; d^*) \leq \pi - d^* - \kappa_C$. Thus, if $C$ chooses $d^* > \kappa_C$, then $V_C^{\sigma}(g^1; d^*) < \frac{\pi - \kappa_C}{1 - \delta}$, which means $C$ can profitably deviate by choosing $d^* = 0$ and repressing in all future periods.

Second, $d^* \leq \hat{d}(g^1)$. When $C$ chooses $d^* > \hat{d}(g^1)$, then $g^1 \leq g^{-}[d^*]$, which implies that $V_C^{\sigma}(g^1; d^*) = \frac{\pi - d^*}{1 - \delta}$, which is strictly decreasing in $d^*$. So $C$ has a profitable deviation by choosing decentralization $d = d^* - \epsilon$ for $\epsilon > 0$ but close to zero. This establishes the desired result.

Finally, we prove that if $\kappa_C < \max\{\frac{\pi}{2}, \pi - \hat{d}(g^1)\}$ and $d^* > 0$, then $g^1 < g^+[d^*]$, i.e., $C$ never represses nor grants independence along the subsequent path of play. To do this suppose not and consider two relevant cases.

Case 1: $\pi - d^* - \kappa_C > 0$. Then $V_C^{\sigma}(g^1; d^*) = \frac{\pi - d^* - \kappa_C}{1 - \delta}$, and $C$ can profitably deviate by choosing $d^* = 0$ and repressing in all future periods. 

Case 2: $\pi - d^* - \kappa_C \leq 0$. If $\kappa_C < \frac{\pi}{2}$, then

$$d^* \geq \pi - \kappa_C > \pi - \frac{\pi}{2} > \kappa_C,$$

which contradicts the upper bound described above. If $\kappa_C < \pi - \hat{d}(g^1)$, then we have

$$d^* \geq \pi - \kappa_C$$
$$> \pi - (\pi - \hat{d}(g^1))$$
$$= \pi - \left( \frac{(1 - \delta)\kappa_P}{1 - \delta} \right)$$
$$= \hat{d}(g^1),$$

which contradicts the upper bound described above. $\square$
The next example illustrates that the Center decentralizes in equilibrium \((d^*, \sigma)\) and the subsequent interaction entails gambling for unity.

**Example 2** For the exogenous parameters, we consider \(\pi = 100\), \(\psi = 100\), \(\kappa_C = 40\), \(\kappa_P = 95\) and \(\delta = 0.9\). In addition, \(F\) takes the form:

\[
F(g) = \begin{cases} 
0 & \text{if } g = 0 \\
\frac{1}{10} & \text{if } g \in \{1, \ldots, 100\} \\
\frac{3}{10} & \text{if } g = 101 \\
1 & \text{if } g \geq 102.
\end{cases}
\]

and initial grievances are \(g^1 = 101\).

Note that \(\kappa_C < \frac{\pi}{2}\), so Proposition 6.1 implies that if C decentralizes in an equilibrium \((d^*, \sigma)\), then it chooses to neither repress nor grant independence in all future periods, in which case, C’s expected utility is \(\hat{V}_C(g^1; d^*)\). Thus, if C chooses \(d^* > 0\), it will choose a \(d^*\) that solves

\[
F(g') \pi - d^* \frac{\pi - d^*}{1 - \delta} - \kappa_P = 0
\]

for some \(g' > g^-[0]\) and \(g' \leq g^1\). In words, if C decentralizes, it will choose a decentralization level that makes the Periphery (at some grievance level \(g^1\)) indifferent between mobilizing and not along the subsequent path of play. If not, C can profitably deviate by offering slightly less decentralization without changing the Periphery’s strategy in states \(g \leq g^1\).

Given this discussion and the construction of \(F\), there are three possible decentralization levels to consider: \(\{0, \hat{d}(1), \hat{d}(101)\}\). Note that \(\hat{d}(101) = \frac{205}{3} > \kappa_C\). As such, the upper bound in the previous proof shows that \(d^* \neq \hat{d}(101)\) in any equilibrium. Thus, there are only two possible decentralization levels in equilibrium: \(\{0, \hat{d}(1)\}\).

If C chooses \(d^* = 0\), then \(g^-[0] = 0\) and \(g^+[0] = 6\). Because \(g^1 > g^+[0]\), if C chooses \(d^* = 0\), then long-term repression is the equilibrium outcome, which implies C’s dynamic payoff is \(\frac{\pi - \kappa_C}{1 - \delta} = 600\).

If C chooses \(d^* = \hat{d}(1) = 5\), then \(g^-[d^*] = 100\) and \(g^+[0] = 102\). Because \(g^1 < g^+[d^*]\), if C chooses \(d^* = \hat{d}(1)\), then one period of gambling for unity is the equilibrium path of play, in which case C’s expected utility is

\[-F(g^1)\psi + (1 - F(g^1)) \left(\pi - d^* + \delta \frac{\pi - d^*}{1 - \delta}\right) = 635.\]
As such, C chooses to decentralize, $d^* = \hat{d}(1) > 0$ and gambling for unity occurs along the subsequent equilibrium path of play.

The next example illustrates that the Center decentralizes in equilibrium $(d^*, \sigma)$ and the a long-term peace emerges in the subsequent interaction.

**Example 3** The payoff parameters match those from Example 2, but now $F$ takes the form:

$$F(g) = \begin{cases} 
0 & \text{if } g = 0 \\
\frac{1}{10} & \text{if } g \in \{1, \ldots, 101\} \\
1 & \text{if } g \geq 102.
\end{cases}$$

and initial grievances are $g^1 = 101$. Following the logic in the previous example, there are two potential levels of decentralization in equilibrium: $[0, \hat{d}(1)]$. If C chooses $d^* = 0$, then its payoff is $\frac{\pi - \kappa C}{1 - \delta} = 600$ for reasons described above. If C choses $d^* = \hat{d}(1)$, then $g^{-}[d^*] = 101 = g^1$ and its equilibrium payoff is $\frac{\pi - d^*}{1 - \delta} = 950$. As such, C chooses to decentralizes and a long-term peace emerges.