Stabilizing a Bosonic Qubit Using Colored Dissipation

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Protected qubits such as the 0-π qubit, and bosonic qubits including cat qubits and Gottesman-Kitaev-Preskill (GKP) qubits offer advantages for fault tolerance. Some of these protected qubits (e.g., 0-π qubit and Kerr-cat qubit) are stabilized by Hamiltonians which have (near-)degenerate ground state manifolds with large energy gaps to the excited state manifolds. Without dissipative stabilization mechanisms the performance of such energy-gap-protected qubits can be limited by leakage to excited states. Here, we propose a scheme for dissipatively stabilizing an energy-gap-protected qubit using colored (i.e., frequency-selective) dissipation without inducing errors in the ground state manifold. Concretely we apply our colored dissipation technique to Kerr-cat qubits and propose colored Kerr-cat qubits which are protected by an engineered colored single-photon loss. When applied to the Kerr-cat qubits our scheme significantly suppresses leakage-induced bit-flip errors (which we show are a limiting error mechanism) while only using linear interactions. Beyond the benefits to the Kerr-cat qubit we also show that our frequency-selective loss technique can be applied to a broader class of protected qubits.

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Introduction.—One standard approach for realizing fault-tolerant quantum computation is to use the surface code [1] (or its similar variants) with two-level systems such as transmons [2,3] or trapped-ion qubits [4]. One promising alternative approach is based on protected qubits [5]. Examples of protected qubits include the 0-π qubit [6–9], and bosonic qubits [10,11] such as cat qubits [12,13] and GKP qubits [14]. Such protected qubits can have an intrinsic robustness against environmental errors because of the structure of the wave functions and/or vanishing energy dispersion. This physical level of error suppression can reduce the hardware overhead for implementing fault-tolerance techniques with protected qubits [15,16].

Certain protected qubits such as the 0-π qubits and Kerr-cat qubits (a type of bosonic qubit) are stabilized by a Hamiltonian. In this case of Hamiltonian protected qubits, the computational basis states are given by (near-) degenerate ground states of a Hamiltonian with an energy gap to the excited state manifolds. Importantly in the absence of dissipative stabilization mechanisms, a leaked population in the excited state manifolds (e.g., from incoherent heating) cannot be returned to the code manifold. As we will illustrate below, depending on the structure of the excited states, such leakage can severely limit the performance of energy-gap-protected qubits.

In this Letter, we present a solution to this problem by adding colored (or frequency-selective) dissipation to energy-gap-protected qubits. In particular, we show that colored dissipation with a suitably engineered bath spectrum can bring the excited states back to the code manifold while not causing logical errors in the code manifold. To make the discussion concrete, we first focus on Kerr-cat qubits and show how they are limited by leakage-induced bit-flip errors. We then propose *colored Kerr-cat qubits*, i.e., Kerr-cat qubits that are protected by colored single-photon loss. Specifically, we propose to engineer the bath spectrum of the colored loss channel by using multiple filter modes. See Fig. 1 for a

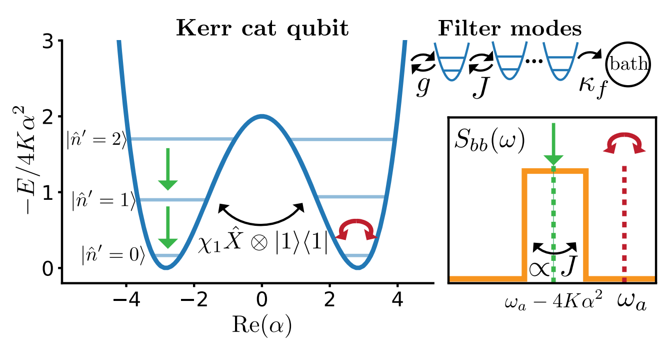

FIG. 1. Schematic representation of a colored Kerr-cat qubit, i.e., a Kerr-cat qubit protected by frequency-selective (colored) single-photon loss. As shown in the inset, the harmonic filter modes are designed such that the colored single-photon loss realizes the desired cooling process (green arrow) while not inducing any additional phase-flip errors in the cat qubit manifold (red arrow). The black arrow in the schematic energy diagram represents the tunneling process between the first excited states, which is important for understanding the bit-flip rate of a Kerr-cat qubit.

schematic diagram. We then provide a general formulation of our colored dissipation technique and explain how it can be applied to a wide class of energy-gap-protected qubits besides Kerr-cat qubits.

Cat qubits.—Before proceeding, we briefly summarize the idea behind cat qubits. Two-component cat qubits [12,13,17–24] encode information into an oscillator mode using the $|\pm\alpha\rangle$ coherent states as their approximate computational basis states. These qubits benefit from an exponential suppression of bit-flip errors with $|\alpha|^2$ due to the large phase space separation between $|\pm\alpha\rangle$ with $|\alpha|^2 \gg 1$. This bias against bit-flip (X) errors can be maintained during the execution of gates [15,25] allowing us to focus on correcting the dominant phase-flip (Z) errors. This can reduce the hardware overhead of error correction compared with that of bare two-level qubits [15,16,25–31].

When studying cat qubits, we make use of the shifted-Fock basis [16]: a subsystem decomposition which breaks the Hilbert space of our harmonic modes into two sectors. The sectors capture the encoded logical information and gauge information about the system. The shifted Fock basis states are spanned by the displaced Fock states $\hat{D}(\pm\alpha)|\hat{n} = n\rangle$. As shown in Ref. [16], we can express the annihilation operator in this basis as follows:

$$\hat{a} = \hat{Z} \otimes (\hat{a}' + \alpha) + \mathcal{O}(e^{-2|\alpha|^2}). \tag{1}$$

In this subsystem decomposition, $\hat{Z}$ is a 2×2 Pauli Z operator acting on a qubit sector which describes the cat qubit logical information. The qubit sector of $\hat{a}$ is given by $\hat{Z}$ because single-photon loss changes the parity of the cat qubit and hence causes a phase-flip (or Z) error on the logical information in our basis convention. $\hat{a}'$ is an annihilation operator acting on a gauge sector which lowers the cat qubit to the ground state manifold (where $\hat{a}'^\dagger\hat{a}' = 0$). In what follows, we assume that α is real.

Kerr-cat qubits.—Kerr-cat qubits are an implementation of two-component cat codes that stabilize the $|\pm\alpha\rangle$ manifold using a Hamiltonian with a Kerr nonlinearity and two-photon drive $\hat{H}_{\mathrm{KC}} = -K(\hat{a}^{\dagger 2} - \alpha^2)(\hat{a}^2 - \alpha^2)$. Rewriting this Hamiltonian in the shifted Fock basis, we find

$$\begin{aligned}\hat{H}_{\mathrm{KC}} = {} & -4K\alpha^2\hat{I} \otimes \hat{a}'^\dagger\hat{a}' - K\hat{I} \otimes \hat{a}'^{\dagger 2}\hat{a}'^2 \\ & - 2K\alpha\hat{I} \otimes (\hat{a}'^{\dagger 2}\hat{a}' + \hat{a}'^\dagger\hat{a}'^2) + \mathcal{O}(e^{-2\alpha^2}).\end{aligned} \tag{2}$$

In the limit of small excitations in the gauge sector (i.e., $\langle\hat{a}'^\dagger\hat{a}'\rangle \ll \alpha$), all but the first term in Eq. (2) can be neglected, and the Kerr-cat Hamiltonian is approximately reduced to that of a harmonic oscillator with an energy spacing $-4K\alpha^2$. This nonzero energy gap protects Kerr-cat qubits against coherent perturbations by making them off resonant. However, Kerr-cat qubits are not robust against some incoherent perturbations (e.g., heating) due to the absence of a dissipative stabilization mechanism.

Heating-induced bit-flip errors.—Heating of an oscillator can be modeled by the dissipator $\kappa_1 n_{\mathrm{th}}\mathcal{D}[\hat{a}^\dagger]$. Since the creation operator $\hat{a}^\dagger$ is approximately given by $\hat{a}^\dagger \simeq \hat{Z} \otimes (\hat{a}'^\dagger + \alpha)$ in the shifted-Fock basis, heating induces phase flips and importantly leakage outside the code space due to the $\hat{a}'^\dagger$ term in the gauge sector [see Fig. 2(a)] [32].

Indeed, in the first experimental realization of a Kerr-cat qubit [24], significant heating occurred, and only a modest noise bias factor of ~40 was achieved. Thus, realizing the

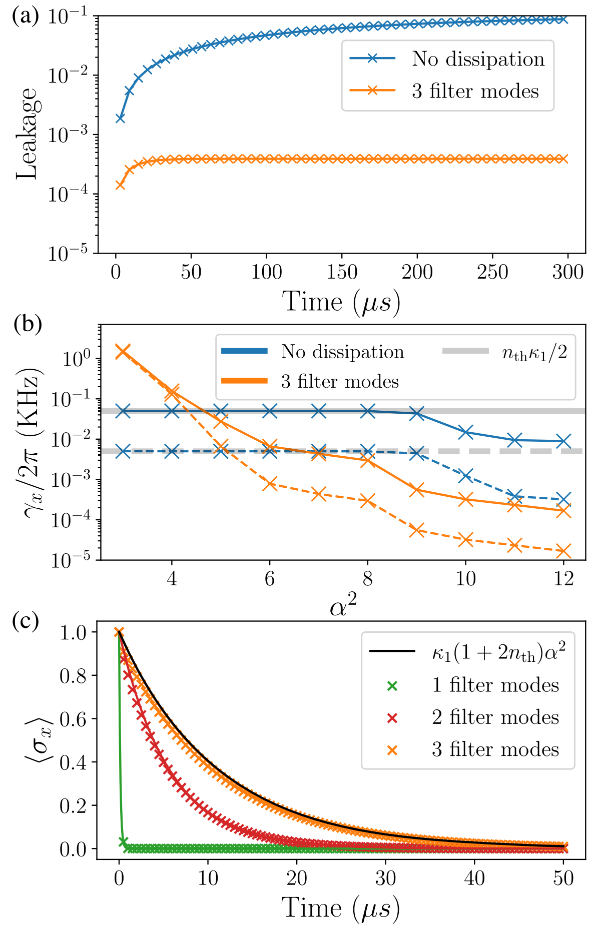

FIG. 2. (a) Leakage accumulation over time in a Kerr-cat qubit without any engineered dissipation (blue) and a colored Kerr-cat qubit with three filter modes (orange) from the initial state $|\alpha\rangle$. (b) Bit-flip error rate of a Kerr-cat qubit (blue) and a colored Kerr-cat qubit with three filter modes (orange) as a function of the average photon number α^2 for $n_{\mathrm{th}} = 0.1$ (solid) and 0.01 (dotted). Gray lines represent the analytical prediction $\gamma_X = \kappa_1 n_{\mathrm{th}}/2$ for the regime $\chi_1 \gg \kappa_1 + \kappa_{1,\mathrm{eng}}$. (c) Decay of the parity of a colored Kerr-cat qubit starting from $|+\rangle$ with one (green), two (red), and three (orange) filter modes. Xs represent numerical data and solid lines represent analytical predictions. The black line shows the baseline decay of the parity at a rate $2\kappa_1(1+n_{\mathrm{th}})\alpha^2$, i.e., twice the phase-flip rate. In all three plots, we use the parameters $K = 2\pi\times 10$ MHz and $\kappa_1 = 2\pi\times 1$ kHz. In (a) and (c), we further assume $n_{\mathrm{th}} = 0.1$ and $\alpha^2 = 6$. See the Supplemental Material [33] for details.

full potential of Kerr-cat qubits requires counteracting the leakage caused by heating.

To be used as a biased-noise qubit, Kerr-cat qubits need to have strongly suppressed bit-flip errors with α^2. Previous works [24,36] have suggested that Kerr-cat qubits can be made robust to leakage-induced bit-flip errors by ensuring that higher excited states reached through heating are below the energy barrier so that that tunneling [37,38] between them is suppressed. Although this argument is qualitatively correct, we show that it does not apply to near term experiments and fault-tolerant quantum computation proposals where heating poses a limit on achievable bit-flip times in both regimes.

In Fig. 2, we consider a set of experimentally relevant parameters: $K = 2\pi \times 10$ MHz, $\kappa_1 = 2\pi \times 1$ kHz (corresponding to the lifetime of $1/\kappa_1 = 159$ μs), and a thermal population of $n_{\rm th} = 0.1$ and $n_{\rm th} = 0.01$ [39]. As indicated by the blue line in Fig. 2(b), the bit-flip error rate γ_X of a Kerr-cat qubit stays constant throughout the range $3 \le \alpha^2 \lesssim 9$, which is most experimentally relevant. This contrasts with expectations for exponential suppression of the bit-flip error rate γ_X with α^2 used throughout the literature for biased noise cat qubits.

To understand why the bit-flip error rate γ_X of a Kerr-cat qubit does not improve as we increase α^2 up to 9, we need to consider the $\mathcal{O}(e^{-2\alpha^2})$ contributions in Eq. (2). In particular, we need to consider the terms in $\hat{H}_{\rm KC}$ of the form $\chi_n \hat{X} \otimes |\hat{n}' = n\rangle\langle\hat{n}' = n|$. Here, χ_n can be understood as the tunneling rate between the states $|0\rangle \otimes |\hat{n}' = n\rangle$ and $|1\rangle \otimes |\hat{n}' = n\rangle$ (see the schematic in Fig. 1). In the Supplemental Material [33], we show that the tunneling rate χ_1 in the first excited state manifold is perturbatively given by

$$\chi_1 \simeq 16K\alpha^4 e^{-2\alpha^2}, \tag{3}$$

which agrees with the exact numerical results for all $\alpha^2 \ge 3$. Although χ_1 decreases exponentially in α^2, the large prefactor $16K\alpha^4$ can still make this χ_1 (induced by the Kerr-cat Hamiltonian $\hat{H}_{\rm KC}$) limiting in practice.

We now explain why the bit-flip error rate γ_X [blue line in Fig. 2(b)] plateaus in the range $3 \le \alpha^2 \lesssim 9$. Heating excites the system to the first excited state manifold. Here it persists for a time $\Delta t \sim 1/\kappa_1$ until it decays back to the cat state manifold. During this period, if $\chi_1 \gg \kappa_1$, rapid oscillations occur between the states $|0\rangle \otimes |\hat{n}' = 1\rangle$ and $|1\rangle \otimes |\hat{n}' = 1\rangle$. In this regime, a bit-flip error happens with 50% probability whenever heating creates an excitation. As a result, the bit-flip error rate is given by half the heating rate, i.e., $\gamma_X = \kappa_1 n_{\rm th}/2$ in the regime of $\chi_1 \gg \kappa_1$. With our parameters (yielding $K/\kappa_1 = 10^4$), $\chi_1 = 16K\alpha^4 e^{-2\alpha^2}$ is at least 10 times larger than κ_1 for all $3 \le \alpha^2 \le 6.75$ and $\chi_1 = \kappa_1$ at $\alpha^2 = 8.08$. This explains why the bit-flip error rate γ_X is independent of α^2 and given by $\kappa_1 n_{\rm th}/2$ in the range $3 \le \alpha^2 \lesssim 9$. Above $\alpha^2 \sim 9$ heating to higher excited states becomes the important error mechanism because tunneling between the first excited states is sufficiently suppressed (see the Supplemental Material [33]). A similar mechanism can limit other energy-gap-protected qubits if the transition rates within the excited state manifold are significant.

Colored Kerr-cat qubits.—As shown by our numerical and analytical results, the heating-induced bit-flip errors can be even more detrimental than previously anticipated. Here, we propose to counteract the heating and leakage by adding frequency-selective (i.e., colored [40,41]) single-photon loss to Kerr-cat qubits, hence making them *colored Kerr-cat qubits*. Our scheme fundamentally differs from the previous proposals based on two-photon dissipation [24,31,36] as we only require single-photon loss. Intrinsic single-photon loss $\kappa_1\mathcal{D}[\hat{a}]$ is harmful for cat qubits because the $+\alpha\hat{Z} \otimes \hat{I}$ term in the shifted-Fock basis representation of the annihilation operator $\hat{a} \simeq \hat{Z} \otimes (\hat{a}' + \alpha)$ causes phase-flip (or Z) errors in their ground state manifold [12,23]. The other term (i.e., $\hat{Z} \otimes \hat{a}'$) is useful for suppressing leakage as it brings the excited states back to the code space via $\hat{a}'$.

Our key idea is to engineer the frequency spectrum of the bath of the extrinsic single-photon loss such that we can take advantage of the beneficial decay term ($\hat{Z} \otimes \hat{a}'$) while filtering out the parasitic term ($+\alpha\hat{Z} \otimes \hat{I}$) from the single-photon loss $\hat{a}$. Since only the extrinsic single-photon loss is engineered with this technique, the intrinsic single-photon loss rate κ_1 should still be kept as small as possible.

To demonstrate how our scheme works, we introduce a concrete setup where a Kerr-cat qubit is coupled to an engineered bath through a set of harmonic filter modes with nearest-neighbor hopping, forming a colored Kerr-cat qubit (diagram in Fig. 1). Specifically, we consider the following Lindblad equation in the rotating frame of a Kerr-cat qubit ($\hat{a}$ with frequency ω_a) and filter modes ($\hat{f}_1, \ldots, \hat{f}_M$ with frequency ω_f):

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}, \hat{\rho}(t)] + \kappa_1(1 + n_{\rm th})\hat{D}[\hat{a}]\hat{\rho}(t) + \kappa_1 n_{\rm th}\hat{D}[\hat{a}^\dagger]\hat{\rho}(t) + \kappa_f\mathcal{D}[\hat{f}_M]\hat{\rho}(t), \tag{4}$$

where the Hamiltonian $\hat{H}$ is given by

$$\hat{H} = \hat{H}_{\rm KC} + \left[g\hat{a}\hat{f}_1^\dagger e^{i\Delta t} + J\sum_{j=1}^{M-1}\hat{f}_j\hat{f}_{j+1}^\dagger + \text{H.c.}\right]. \tag{5}$$

Here, $\Delta \equiv \omega_f - \omega_a$ is the detuning between the filter modes $\hat{f}_1, \ldots, \hat{f}_N$ and the mode $\hat{a}$ which hosts the Kerr-cat qubit. Also, $\mathcal{D}[\hat{A}]\hat{\rho} \equiv \hat{A}\hat{\rho}\hat{A}^\dagger - \frac{1}{2}\{\hat{A}^\dagger\hat{A}, \hat{\rho}\}$ is the Lindblad dissipator. Besides having the intrinsic loss and heating processes, the Kerr-cat qubit can lose an excitation to the first filter mode at a rate g. Such an excitation is then transported to the last filter mode at a hopping rate J where it decays to a cold bath at a rate κ_f. It is important that this bath and the filter modes have a temperature much lower than the Kerr-cat qubit so as to not induce additional heating (here $n_{\mathrm{th,filter}} = 0$). In practice, strong pump tones are necessary to realize Kerr-cat qubits while the filter modes and their baths are passive and undriven. Thus it is plausible that the filter modes would be colder than the Kerr-cat qubits. We take $\kappa_f = 2J$ so that the filter modes act as an ideal bandpass filter (centered at the frequency ω_f and with a bandwidth $4J$) as $N \to \infty$. See the Supplemental Material [33] for more details.

Recall that in the shifted Fock basis, the Kerr-cat Hamiltonian is approximately given by $\hat{H}_{\mathrm{KC}} \simeq -4K\alpha^2 \hat{I} \otimes \hat{a}'^{\dagger}\hat{a}'$. Transforming to the shifted-Fock basis, and moving into the rotating frame of the $\hat{a}'$ mode the coupling term $g\hat{a}\hat{f}_1^{\dagger}e^{i\Delta t}$ becomes $g(\hat{Z} \otimes \hat{a}')\hat{f}_1^{\dagger}e^{i(\Delta+4K\alpha^2)t} + g\alpha(\hat{Z} \otimes \hat{I})\hat{f}_1^{\dagger}e^{i\Delta t}$. The first term realizes a desired cooling effect through $\hat{a}'$ whereas the second term causes undesired phase-flip (Z) errors in the cat qubit manifold. By choosing $\Delta = -4K\alpha^2$ (or equivalently $\omega_f = \omega_a - 4K\alpha^2$), we can make the desired first term resonant while making the undesired second term off resonant. Furthermore, by ensuring that the half bandwidth $\kappa_f = 2J$ is smaller than the detuning $|\Delta|$, we can place the undesired second term outside the filter passband and filter it out (see Fig. 1). In particular, through adiabatic elimination (see the Supplemental Material [33]), the induced phase-flip error rate due to the second term is given by $(4g^2\alpha^2/\kappa_f) \times (J/\Delta)^{2M}$ in the $\Delta \gg J$ limit and hence decreases exponentially in the number of the filter modes M. On the other hand, the resonant desired term realizes an engineered cooling process $\kappa_{1,\mathrm{eng}}\mathcal{D}[\hat{Z} \otimes \hat{a}']$ with an effective cooling rate $\kappa_{1,\mathrm{eng}} = 4g^2/\kappa_f$.

In Fig. 2, we study the performance of a bare Kerr-cat qubit and colored Kerr-cat qubits with a varying number of filter modes. For colored Kerr-cat qubits, we choose $\kappa_f = 2J = \Delta/5$ and $g = \kappa_f/5$ to filter out the induced phase-flip errors and guarantee the validity of the adiabatic elimination, respectively. We tune $\Delta = -3.6K\alpha^2$ (vs $\Delta = -4K\alpha^2$) by accounting for higher order contributions to more closely target the $0 \leftrightarrow 1$ transition of the Kerr excited states [33]. With these parameters, we get a large engineered cooling rate of $\kappa_{1,\mathrm{eng}} = 2\pi \times 1.15\alpha^2$ MHz (e.g., $\kappa_{1,\mathrm{eng}} = 2\pi \times 6.9$ MHz at $\alpha^2 = 6$). As indicated by the orange line in Fig. 2(a), the leakage population of a Kerr-cat qubit (of size $\alpha^2 = 6$) can be made orders of magnitude smaller by adding a frequency-selective single-photon loss with three filter modes. Additionally, the idling bit-flip error rate is reduced by at least 1 order of magnitude for all $\alpha^2 \geq 6$ [see Fig. 2(b)]. This is because the large engineered cooling rate dramatically reduces the lifetime of excited states (especially the first excited states) so that the condition $\kappa_1 + \kappa_{1,\mathrm{eng}} \gg \chi_1$ is satisfied at lower values of α^2 [42].

In Fig. 2(c) we show the parity as a function of time with 1, 2, and 3 filter modes and $\alpha^2 = 6$. With only one or two filter modes, the induced phase-flip rate is much larger than the intrinsic phase-flip rate of $\approx\kappa_1(1 + 2n_{\mathrm{th}})\alpha^2$ (green and red lines). With three filter modes, however, the induced phase-flip rate is negligible, and the total phase-flip probability is close to the intrinsic rate (orange line). The simulated (Xs) parity decays are consistent with our analytical prediction (solid lines) on the induced phase-flip rate $(4g^2\alpha^2/\kappa_f) \times (J/\Delta)^{2M}$ in the $\Delta \gg J$ limit. Hence, Fig. 2(c) demonstrates that with a properly engineered single-photon loss spectrum we can benefit from the desired cooling effects without inducing additional phase-flip errors.

General formulation.—We now present how our colored dissipation technique can be generally applied to a wide class of energy-gap-protected qubits. Specifically we consider energy-gap-protected qubits whose Hamiltonian is given by $\hat{H}_{\mathrm{PQ}} = \Delta\hat{I} \otimes |1\rangle\langle 1|$. Here, $|0\rangle$ and $|1\rangle$ in the gauge sector respectively correspond to the ground and first-excited state manifolds of the qubit, and Δ is the energy gap. The Hamiltonian including the filter is given by $\hat{H} = \hat{H}_{\mathrm{PQ}} + [g\hat{c}\hat{f}_1^{\dagger}e^{i\Delta t} + J\sum_{j=1}^{M-1}\hat{f}_j\hat{f}_{j+1}^{\dagger} + \mathrm{H.c.}]$, where $\hat{c}$ is a coupling operator acting on the protected qubit. In the subsystem decomposition this coupling operator generically takes the form $\hat{c} = \sum_{i,j\in\{0,1\}}\hat{c}_{ij} \otimes |i\rangle\langle j|$. The limit of interest is when $|\Delta| \gg J$ such that any induced dissipation can be selective on the $|1\rangle \to |0\rangle$ decay. Similarly as above, adiabatic elimination of the filter modes yields a Lindblad term $(4g^2/\kappa_f)\mathcal{D}[\hat{c}_{01} \otimes |0\rangle\langle 1|]$ realizing the desired $|1\rangle \to |0\rangle$ decay. Crucially the incoherent errors induced on the ground state manifold are exponentially suppressed with the number of filter modes. Specifically these errors are described by the Lindblad term $(4g^2/\kappa_f) \times (J/\Delta)^{2M}\mathcal{D}[\hat{c}_{00}]$ upon adiabatic elimination of the gauge mode as well as the filter modes. Thus if one uses sufficiently many filter modes with $|\Delta| \gg J$ and $\hat{c}$ has an appreciable matrix element for $\hat{c}_{01}$ one can realize dissipative confinement to the ground state manifold without inducing incoherent errors on the logical information even when $\hat{c}_{00}$ is nontrivial and acts as a logical error in the ground state manifold [43].

Discussion and outlook.—An interesting future direction is to apply our colored dissipation scheme to other energy-gap-protected qubits. An example is the Hamiltonian-stabilized finite-energy GKP qubit where a gap opens up relative to the infinite-energy case [14,44–47]. In this case the subsystem decomposition would be given by a finite energy version of the modular bosonic subsystem decomposition [48]. The 0-π qubit similarly [6–9] would be an interesting case with near degenerate ground states enabling frequency-selective loss. The use of colored loss can also be extended to the application of gates. We consider this for Kerr-cat qubits in the Supplemental Material [33].

In practice the optimal choice of the filter may not be a bandpass filter centered around the gap frequency (as in Fig. 1). Other filter geometries such as wider bandpass filters with the frequency $\omega - \Delta$ near the edge of the passband or low pass filters may allow for higher dissipation rates while still rejecting signals at ω and are interesting areas for future work. These filters can be implemented experimentally in superconducting circuits using quantum metamaterials [49].

In the context of Kerr-cat qubits our proposal takes advantage of the energy structure of the gauge mode as opposed to the parity symmetry of two-photon dissipation ($\kappa_2\mathcal{D}[\hat{a}^2 - \alpha^2]$ [24,25,31,36]) such that it only requires single-photon loss. In particular, this means nonlinear interactions are not needed to implement the dissipation potentially enabling larger engineered cooling rates.

Additionally unlike two-photon dissipation, our engineered cooling process $\kappa_{1,\mathrm{eng}}\mathcal{D}[\hat{Z} \otimes \hat{a}']$ comes with a phase-flip $\hat{Z}$ in the qubit sector. This phase-flip is not problematic because it is only triggered in the excited state manifold. Moreover, some leakage processes such as those associated with heating ($\hat{a}^\dagger$) and the Z gate [$\hat{H}_Z = \epsilon_Z(\hat{a}^\dagger + \hat{a})$] come with a phase-flip in the qubit sector. There having the phase flip in the cooling process is a feature because the phase flip from the leakage process is canceled out when the system is brought back to the ground state manifold via the colored dissipation. Nonadiabatic gate errors can also be directly suppressed by the frequency selectivity of the filter if the leakage processes they are associated with are off resonant from the filter. We remark that there is a complementary approach for suppressing bit-flip error rates by reducing the effective tunneling rates $\chi_1, \chi_2, \cdots$, which can be done by adding a linear drive to the Kerr-cat Hamiltonian. These interesting areas for future work are discussed in the Supplemental Material [33].

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