Supporting Information for "Data-Driven Inference of the Mechanics of Slip Along Glacier Beds Using Physics-Informed Neural Networks: Case study on Rutford Ice Stream, Antarctica"

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S1. 2D Shallow Ice stream Simulations

To incorporate the effects of lateral shear stress on ice flow and inferences of basal drag, we generate 2D SSA simulations using $\dot{U}_a$ for a glacier 20 km wide and 100 km long, dimensions that are comparable to Rutford Ice Stream, West Antarctica, the study area discussed in the main text. The sliding-law parameters are set to be spatially and temporally constant with $m = 3$ and $c_b = 800$. We use a laterally-varying profile for the flow rate parameter $A = A(y)$ to represent softening of the ice at the margins due to heating and melting through viscous dissipation and formation of crystallographic fabric:

$$A(y) = A_0 \left[ 1 - \frac{1}{2} \cos \left( \frac{2\pi y}{L_y} \right) \right],$$

(1)
where $A_0 = 3.6 \times 10^{-24}\text{s}^{-1}\text{Pa}^{-3}$ is a reference flow rate parameter for ice near the melting temperature (Cuffey & Paterson, 2010) and $L_y$ is the full width of the simulated ice stream. We use triangular elements roughly 2 km in size with cubic interpolation within the elements. The glacier bed slopes downward from -50 m elevation at the upstream boundary to -100 m at the grounding line. We prescribe a uniform initial ice thickness of 1 km and run the simulation for 300 years to reach steady-state conditions. Then, for a 1-year simulation period, we simulate periodically-varying ocean elevations with a period of 90 days and an amplitude of 10 meters in order to impose periodic terminus stress perturbations. This tide-like forcing results in time-varying velocity fields similar to the 1D examples discussed in the previous section. Since velocity solutions are generated at the element nodes, we use the nodal velocities and thickness values to train the network $f_\theta$ (Figure S3). In the 2D case, $f_\theta$ is now trained to predict both velocity components $u$ and $v$ as well as the ice thickness $h$. For the physics-based loss functions, we compute basal drag components using the 2D momentum balance in Equations 1a and 1b in the main text, which we then project into the along-flow direction prior to computing the smoothness and sign penalties. The inputs to the network are velocity magnitude, $x$ and $y$ coordinates, and time.

After training $f_\theta$, we generate stochastic spatiotemporal predictions for basal drag and estimate $m$ and $c_b$ and their corresponding uncertainties (Figure S4). Uncertainties in the parameters are generally higher near the shear margins where the velocity gradients are not well-modeled by $f_\theta$. Additionally, similar to the 1D simulations, parameter uncertainties increase in the upstream areas of the glacier where the velocity fluctuations are more limited and the stochastic $\hat{\tau}_b$ predictions have larger uncertainties.
S2. Inference of Basal Drag Variations from Longitudinal Stresses

Here, we formulate a simplified version of time-varying stresses in glaciers under tidal loading. Firstly, at fortnightly to sub-annual timescales, ice thickness variations will generally be minimal, so driving stress will not vary significantly during a forcing period. Secondly, assuming that lateral shear stress due to the margins is always resistive to positive velocity variations, then the stress balance will simply involve trade-offs between longitudinal normal stresses and basal drag (under an implicit assumption that lateral shear stress cannot provide sufficient resistive stress for a given stress perturbation). Therefore, examination of the change in longitudinal stresses (more specifically, their gradients), can provide evidence of the underlying basal drag variations with minimal modeling assumptions. To that end, we compute the along-flow gradient of longitudinal stresses as:

\[ \hat{\tau}_{xx} = (\nabla \vec{\tau}) \cdot \frac{\mathbf{u}}{||\mathbf{u}||}, \] (2)

where \( \vec{\tau} = [\tau_{xx}, \tau_{yy}] \). The above quantity is actually proportional to the along-flow dot product of the first terms in Equations 1a and 1b in the main text under the assumption of spatially uniform ice thickness, \( h \), and minimal longitudinal variations in shear strain. In other words, \( \hat{\tau}_{xx} \) is proportional to the longitudinal stresses that resist or drive ice flow. By computing the change in \( \hat{\tau}_{xx} \) before and after an increase in velocity, we can then infer the sign of the corresponding increase in basal drag.

We compute \( \hat{\tau}_{xx} \) at times \( t = 0 \) and 5 days (same time values as Figure 6) and extract the change in \( \hat{\tau}_{xx} \) along a centerline for RIS. We repeat this process for 2D simulations forced by changes in ocean elevation (same outputs as in Section S1), which effectively models longitudinal stress perturbations at the terminus by reducing hydrostatic backpressure. Additionally, we simulate a pressure wave (analogous to the pressure wave for the 1D
simulations in the main text) in order to model decreases in basal drag associated with subglacial hydrology. For both RIS and the simulation forced by a pressure wave, the change in $\hat{\tau}_{xx}$ is negative for a significant portion of the ice upstream from the grounding line (Figure S8). On the other hand, a positive change in $\hat{\tau}_{xx}$ is associated with velocity changes due only to changes in ocean elevation. A negative change signifies that the longitudinal stress gradients become more resistive due to the decrease in basal drag whereas a positive change is representative of an extensional perturbation, which must be accommodated by an increase in basal drag. These results provide additional evidence that an upstream-propagating decrease in basal drag is responsible for the increases in velocity over RIS for the grounded ice.

References


Figure S1. Simulated time-dependent velocity (A, B) and thickness (C, D) profiles for the two different simulation scenarios. For Case I (varying prefactor; A and C), the ice velocities show slightly stronger periodic variations than Case II (varying prefactor and exponent; B and D), and the ice thicknesses are nearly identical.
Figure S2. Illustration of effect of phase lag between two hypothetical periodic variables $X$ and $Y$. A) Time series for $X$ and $Y$ where thick black line corresponds to $X$ values. $Y$ time series are identical to $X$ but with increasing phase lags indicated by cool to warm colors. B) Plot of $Y$ vs. $X$ for same phase lags shown in (A). Phase lags of 0 and $T/2$, where $T$ is the period, correspond to direct proportional and inversely proportional variations, respectively. Phase lags between $[0, 0.5]$ result in elliptical variations.
Figure S3. Geometry and mean velocity (A) and thickness (B) for 2D ice flow simulations generated with Úa. Each marker corresponds to a single node within the triangular finite elements.
**Figure S4.** Maps of estimated basal parameters for 2D simulation outputs. For the estimated exponent (A) and prefactor (B), the marker colors indicate the difference between the true parameter values ($m = 3$ and $c_b = 800$ everywhere, for all times) and the parameter predictions. Maps of estimated one-standard-deviation uncertainties for the exponent (C) and prefactor (D) show a correspondence between sliding law parameter misfits and uncertainties.
**Figure S5.** L-curve showing basal drag negative log-likelihood (NLL) vs. mean Laplacian smoothing factor over validation data for Rutford Ice Stream, Antarctica, for different values of the Laplacian smoothing hyperparameter, $\alpha$. The black circle indicates the hyperparameter value used for the results in the main text.

**Figure S6.** Bed elevation for Rutford Ice Stream from BedMachine V1 (Morlighem et al., 2020).
Figure S7. Uncertainties for neural-network-predicted secular velocity magnitude, ice thickness, and surface elevation (top plots) and residuals between the predictions and observed values (bottom plots) for Rutford Ice Stream, Antarctica. Velocity observations are from Minchew et al. (2017) and ice thickness and surface elevation are from BedMachine V1 (Morlighem et al., 2020). Generally, predicted high uncertainties are co-located with larger data misfits.
Figure S8. Profiles of normalized changes in longitudinal stresses corresponding to *increases* in ice velocity in response to different types of forcing. Orange line corresponds to tidal forcing (changes in hydrostatic backpressure), green line corresponds to pressure wave forcing, and blue line corresponds to observed data at Rutford Ice Stream.