Gravitational-wave research: Current status and future prospects*

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There is a reasonably good change that in the 1980s cosmic gravitational waves will be discovered and will become a powerful tool for astronomy. This prospect has stimulated a three-pronged research effort. *First*, relativity theorists are developing new mathematical tools for the analysis of gravitational radiation—including (i) methods of analyzing the generation of gravity waves by sources with strong self-gravity and large internal velocities (e.g., collisions of black holes), (ii) methods of computing radiation reaction in sources, and (iii) methods of analyzing how gravitational waves propagate through our lumpy curved-space Universe. *Second*, astrophysicists are attempting to identify the most promissing sources of gravitational waves, and are using the relativity theorists' mathematical tools to estimate the characteristics of the waves they emit. *Third*, with the estimated wave characteristics in mind, experimenters are designing and constructing a second generation of gravitational-wave detectors—detectors of three types: Doppler tracking of interplanetary spacecraft, Earth-based laser interferometers, and Earth-based Weber-type resonant bars. This article reviews, in brief, all three prongs of the research effort and gives references to more detailed articles about specialized aspects of gravitational-wave physics.

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I. INTRODUCTION

Prior to 1945 man's knowledge of the distant Universe came almost entirely from light gathered by optical telescopes; and that light painted for us a picture of relative quiescence—a universe made of beautiful, slowly evolving stars and nebulae. Since 1945 technology has opened up one new "window" onto the Universe after another, at an ever increasing pace: radio waves (ca. 1945), x rays (ca. 1963), infrared radiation (ca. 1965), ultraviolet radiation (ca. 1968), γ rays (ca. 1972), millimeter and submillimeter waves (ca. 1973), and extreme ultraviolet radiation (ca. 1975). And these new windows have revolutionized our view: exploding galaxies, quasars, pulsars, neutron stars, black holes, cosmic fireball radiation, star births triggered by supernovae, cosmic masers, organic interstellar molecules.... The Universe has turned out to be far more violent and bizarre than man had ever dreamed.

Now, in 1980, with all the electromagnetic windows open at least a little bit, and with high-energy particles (cosmic rays) becoming an ever more powerful tool for astronomy, there remain only two major unopened windows: gravitational radiation, and neutrinos.

Neutrinos from the Sun have quite likely been detected (Davis, 1979) but are so few in number as to produce a crisis of confidence among theroists (Bahcall and Davis, 1976; Bahcall, 1979). Several observatories have been constructed for neutrinos from supernovae and other distant objects, and larger observatories are under construction and being planned (Markov *et al.*, 1978; Deakyne *et al.*, 1978; Roberts and Wilkins, 1978). However, except for one antineutrino burst event (which could have been spurious), seen by the only observatory operating at that time (Lande *et al.*, 1974), these observatories have detected nothing. Greater sensitivity is required.

Gravitational-wave detection was pioneered by Joseph Weber (1960, 1969). Using one-ton room-temperature aluminum bars as his detectors, Weber reported in 1969 tentative evidence for cosmic gravitational-wave bursts at kilohertz frequencies. This report triggered major gravitational-wave-detection efforts, similar but not identical to Weber's, in Moscow, Glasgow, Frascati, Munich, Bell Labs, Rochester, IBM, Tokyo, Bristol, Reading, Stanford, LSU, Rome, Meudon, and Regina. By 1975 half of these groups had operating detectors. Some saw no evidence of bursts; others saw only marginal evidence, (for reviews see, e.g., Hegyi, 1973; Bertotti, 1974; DeWitt-Morette, 1974; the panel discussion in Rosen and Shaviv, 1975; de Sabbata and Weber, 1977; Drever, 1977; Kafka and Schnupp, 1978.) However, by 1975 it was also evident that various design changes and new technology could improve the energy sensitivities of the detectors a millionfold or more, a sufficient improvement to make reasonable the prospects for success. In this climate, Weber convened a meeting of all the research groups (Erice Sicily, March 1975; see de Sabbata and Weber, 1977). An intense two weeks of interaction put the finishing touches on the world's first-generation (room-temperature aluminum-bar) wave-detection effort, and directed attention to a newly developing second-generation program that includes (i) cryogenically cooled bars

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made from aluminum, from niobium, and from sapphire; (ii) laser interferometers at room temperature; and (iii) Doppler tracking of interplanetary spacecraft.

The first of the second-generation detectors may well start operating in 1980, and others will come on the air over the next several years. In the meantime, various third-generation detectors are taking shape in experimenters' heads.

A recent scrutiny of the American effort, carried out for NSF by a committee of nongravitational-wave experimental physicists, concluded (Deslattes *et al.*, 1979): "In our study of the U.S. program for gravitational radiation detection, we were strongly impressed by the state of the field.... The rate of progress in the recent past has been excellent, both in terms of increased instrumental sensitivity and generally useful high-technology spinoffs... . The ultimate detection of gravitational waves, verification of the properties predicted by theory, and exploitation for observational astronomy, are believable consequences of present research directions. Reaching these goals, however, could take a decade of hard work... ."

Alongside the experimental effort there has been a major push by gravitation theorists to develop improved mathematical tools for the analysis of gravitational radiation (reviews in Smarr, 1979; Thorne, 1977); and there is a major effort by astrophysicists to identify the most promising potential sources of gravitational waves, and to estimate the characteristics of the waves they emit (reviews in Smarr, 1979; Thorne, 1978).

This three-pronged effort (experiment, gravitation theory, astrophysics) has a good chance of success within 5 to 10 years. "Success" means not only the detection of cosmic gravitational waves, but also the identification of their sources, and a deciphering of source properties from the observed waveforms. Indeed, it is not unreasonable to expect that gravitational waves will become a powerful tool for astronomy, revealing features of their sources which one could never learn by electromagnetic, cosmic-ray, or neutrino studies. This expectation is motivated by several key features of gravitational radiation, as predicted by general relativity theory:

(i) Gravitational waves are emitted by coherent bulk motions of matter, in contrast with cosmic electromagnetic and neutrino radiations, which are (usually) incoherent superpositions of emission from individual atoms and charged particles. As a result, the gravitational waveforms $h_+(t-\mathbf{n}\cdot\mathbf{x}/c)$ and $h_\times(t-\mathbf{n}\cdot\mathbf{x}/c)$ (where h_+ and $h_\times$ are the metric perturbations associated with two orthogonal polarization states; Sec. II below) carry information not only about the direction $-\mathbf{n}$ to the source, but also about the detailed bulk motions of the matter that produced the waves.

(ii) Gravitational waves are emitted most strongly in regions of spacetime where gravity is relativistic and where the velocities of bulk motion are near the speed of light (e.g., in the cores of supernovae and in the neighborhoods of black holes). Such regions are important because it is likely they power the most violent phenomena in our Universe (supernovae, quasars, strong radio sources). Today we have little direct observational data about such regions—and no strong hope for studying them in detail except through gravitational waves, because of the following:

(iii) Gravitational waves pass through surrounding matter with impunity, by contrast with electromagnetic waves which are easily absorbed and scattered, and neutrinos which, although they easily penetrate normal matter, probably scatter many times while leaving the core of a supernova.

Examples of phenomena which might be studied with gravitational waves are these: (i) the dynamics of the cores of supernovae; (ii) the dynamical evolution of newborn, rapidly rotating neutron stars (very young pulsars which are probably surrounded by so much matter that their electromagnetic radiation is obscured); (iii) the dynamics of quakes in neutron stars, both young and old (giving information complementary to what one obtains from pulsar timing); (iv) the dynamics of the formation of black holes by stellar collapse, and the pulsations of a newborn hole; (v) collisions between compact objects (black holes and neutron stars) in the nuclei of distant galaxies; and (vi) the internal structures of common-envelope binary stars.

An example of how one might extract information about a source from the waveforms $h_+(t-\mathbf{n}\cdot\mathbf{x}/c)$ and $h_\times(t-\mathbf{n}\cdot\mathbf{x}/c)$ is this: Consider any violent event in which the final state is a black hole (the collapse of the core of a star to form a hole, the swallowing of a neutron star by a hole, etc.). The waveforms from such an event should have the qualitative behavior shown in Fig. 1. The early-time behavior is governed by the bulk motion of the matter and/or black hole during the violent event. It might tell one, for example, some details of the orbit of the infalling neutron star, and whether the neutron star was tidally disrupted before it reached the horizon of the black hole. The final damped oscillations are produced by pulsations of the final hole. From their period P and e-folding time τ, plus the (often reasonable) assumption that the hole's pulsations are predominantly quadrupolar, one can hope to deduce the mass M and angular momentum J

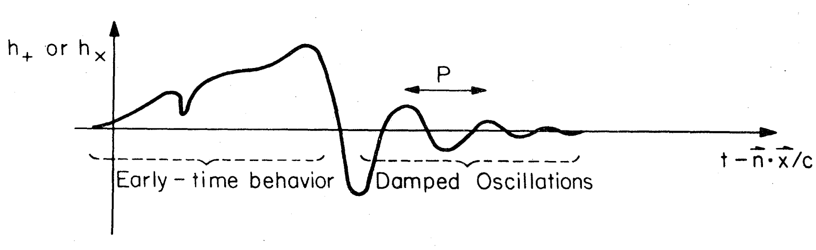

FIG. 1. Schematic diagram of the waveform for the gravitational waves from a violent event which produces a black hole.

of the hole (Davis, Ruffini, and Tiomno, 1972; Chandrasekhar and Detweiler, 1975; Detweiler, 1977; Cunningham *et al.*, 1979). This is valuable information, since all properties of the final hole are determined by M and J. As an example, for a nonrotating hole ($J=0$),

$$P=16.8GM/c^3=0.083(M/M_\odot)\ \text{msec}\ ,$$
$$\tau=11.2GM/c^3=0.055(M/M_\odot)\ \text{msec} \tag{1}$$

(Chandrasekhar and Detweiler, 1975). Here $M_\odot$ is the mass of the Sun.

A number of recent review articles describe special facets of gravitational-wave research. However, there has been no comprehensive review of the entire field. This paper is a brief comprehensive review, designed in part to set the stage for the two more specialized reviews that follow it in this journal.

This paper does not attempt to cover all or even much of the primary literature. Instead, it briefly describes the key points of current research and cites more specialized review articles, where one can find greater detail and citations to the primary literature. In essence, this paper is a review of the review literature—which means that the authors cited in the text (e.g., Thorne, 1978) are often review writers rather than people responsible for major research results.

The first half of this paper consists of four sections on the mathematical theory of gravitational waves: Sec. II on their mathematical description, Sec. III on their generation, Sec. IV on their propagation, and Sec. V on radiation reaction in their sources. These are followed by Sec. VI on astrophysicists' estimates of the waves bathing the Earth, Sec. VII on the experimental search for gravitational waves, and Sec. VIII on quantum limits for gravitational-wave detectors.

The two specialized review articles that follow this one (Thorne, 1980; Caves *et al.*, 1980) deal with multipole-moment formalisms for computing the generation and propagation of gravitational waves (one facet of the material in Sec. III of this paper), and with quantum-mechanical aspects of gravitational-wave detectors (Sec. VIII of this paper). In a sense, however, those specialized review articles are broader than gravitational-wave astronomy: The mathematical tools in the multipole article should be useful wherever one deals with vector, scalar, and tensor spherical harmonics (e.g., in electromagnetism and in nuclear physics). And the research described in the article on quantum aspects of detectors is opening up a new chapter in quantum electronics and in the quantum theory of measurement—a chapter that may have import elsewhere in physics and technology.

II. MATHEMATICAL DESCRIPTION OF GRAVITATIONAL WAVES

Gravitational waves are predicted to exist by all relativistic theories of gravity; all theories predict roughly the same strengths of waves from typical astrophysical sources; and the waves of all theories couple roughly equally strongly to typical gravitational-wave detectors. On the other hand, different theories predict very different polarization properties and propagation speeds for the waves; and for special astrophysical sources (notably the binary pulsar), different theories can predict rather different wave strengths. These differences may give rise to definitive tests of gravitation theories once gravitational-wave astronomy is "on the air." For details see, e.g., Eardley *et al.* (1973), Will (1979), and Hellings (1978).

Throughout this article we shall assume that general relativity is the correct theory of gravity. This assumption is strongly supported by experimental data (Will, 1979). General relativity describes gravitational waves as ripples in the curvature of spacetime, which propagate with the speed of light. These ripples are characterized by two dimensionless *gravitational-wave amplitudes* h_+ and $h_\times$, which can be regarded as scalar fields in spacetime (Kovàcs and Thorne, 1978), and which determine all features of the waves. Once the waves leave their source, they find themselves in regions where their wavelengths ($\lambda\sim$ a few kilometers to a few astronomical units) are tiny compared to the radius of curvature of the background spacetime through which they propagate ($\mathcal{R}\sim 10^{10}$ light-years between galaxies, $\mathcal{R}\sim 10^7$ light-years in a galaxy, $\mathcal{R}\sim 0.1$ light-year in the solar system). In regions such as the solar system, which are small compared to $\mathcal{R}$ but large compared to λ, one can introduce nearly Minkowskii coordinates in which the waves propagate along the z direction, so that

$$h_+=h_+(t-z/c)\ ,\quad h_\times=h_\times(t-z/c)\ , \tag{2}$$

where c is the speed of light. It then is useful to introduce polarization *tensors* $\mathbf{e}^+$ and $\mathbf{e}^\times$

$$e^+_{xx}=-e^+_{yy}=1\ ,\quad e^\times_{xy}=e^\times_{yx}=1\ ,$$
$$\text{all other components vanish}\ , \tag{3}$$

and a *gravitational-wave field*

$$h^{\mathrm{TT}}_{jk}=h_+e^+_{jk}+h_\times e^\times_{jk}\ , \tag{4}$$

which is a symmetric spatial tensor that is trace-free and is transverse to the waves' propagation direction (no z component). (The "TT" means "transverse traceless.") This h^{TT}_{jk} is the analog of the Lorentz-gauge vector potential of electrodynamics. In general relativity, with appropriate choice of gauge, h^{TT}_{jk} is the metric perturbation associated with the waves; and independently of gauge it is related to the Riemann curvature tensor by

$$R_{j0k0}=-\tfrac{1}{2}\ddot{h}^{\mathrm{TT}}_{jk}\ . \tag{5}$$

Here the dots denote time derivatives $\partial/\partial t$. For further details see, e.g., Chap. 35 of Misner, Thorne, and Wheeler (1973)—cited henceforth as "MTW." Other mathematical embodiments of gravitational radiation (e.g., the "Bondi news function"), which are basically equivalent to h^{TT}_{jk} but look different, are reviewed in Sachs (1964), Pirani (1964), Penrose (1964, 1968), Newman and Penrose (1968), and Bardeen and Press (1973). These references also review a number of beautiful theorems about the mathematical properties of gravitational radiation.

When an electromagnetic wave hits a charged particle it produces an acceleration that (i) is transverse to the wave's propagation direction, and (ii) is proportional to e/m, the particle's charge-to-mass ratio. Similarly, when a gravitational wave hits a free particle with "passive gravitational mass" m_P and "inertial mass" m_I, it produces an acceleration that (i) is transverse to the wave's propagation direction, and (ii) is proportional to m_P/m_I. General relativity asserts that m_P/m_I is the same for all particles (equivalence principle). Therefore, all free particles at the same location experience the same transverse acceleration—which means that local inertial frames themselves (which are tied to uncharged, free particles) undergo this same acceleration. Thus, the acceleration is *locally* undetectable. On the other hand, the acceleration is different at different locations in spacetime—which is just another way of saying that the gravitational waves prevent neighboring local inertial frames from meshing to form a global inertial frame; and this, in turn, is another way of saying that the waves produce spacetime curvature.

Consider a fiducial free test particle. Onto it attach a Cartesian coordinate system, with the directions of the axes fixed by gyroscopes that ride on the test particle, and with the scales along the axes fixed by rigid meter sticks (cf. Sec. 13.6 of MTW with $\mathbf{a}=\boldsymbol{\omega}=0$). In this coordinate system denote by ξ_j the vector separation of a second test particle from the fiducial particle. Then a passing gravitational wave will produce a tiny *relative* acceleration of the particles (and of their local inertial frames)

$$\ddot{\xi}_j = \tfrac{1}{2}\ddot{h}^{\mathrm{TT}}_{jk}\,\xi_k\,, \tag{6}$$

which in turn will produce a tiny change

$$\delta\xi_j = \tfrac{1}{2}h^{\mathrm{TT}}_{jk}\xi_k \tag{7}$$

in their separation vector (Sec. 35.5 of MTW). Note that the magnitude of the relative acceleration is proportional to the distance between the test particles (no relative acceleration if particles are at same location). Note also that the relative acceleration is purely transverse in two senses: (i) if ξ_j is along the waves' propagation direction, then $\ddot{h}^{\mathrm{TT}}_{jk}\xi_k$ vanishes and there is no relative acceleration at all; (ii) no matter what direction ξ_j may be, the relative acceleration $\ddot{\xi}_j$ is orthogonal to the propagation direction. The relative acceleration can be described by quadrupole-shaped lines of force (Fig. 2).

When the wave hits an object with internal forces (e.g., a gravitational-wave detector), the various pieces of the object cannot move as free test particles. Instead the object then vibrates in accord with its standard equation of motion—with, however, a gravitational-wave driving force

$$F_j = \tfrac{1}{2}m\ddot{h}^{\mathrm{TT}}_{jk}\,\xi_k \tag{8}$$

acting on each tiny piece of the object. Here m is the piece's mass and ξ_k is its location relative to the object's center of mass (Box 37.1 of MTW).

Actually, Eqs. (6)–(8) for the relative acceleration, displacement, and force are correct only if the separation distance $|\xi|$ is short compared to the wavelength λ of the waves. For $|\xi| \gtrsim \lambda$ retardation effects cause

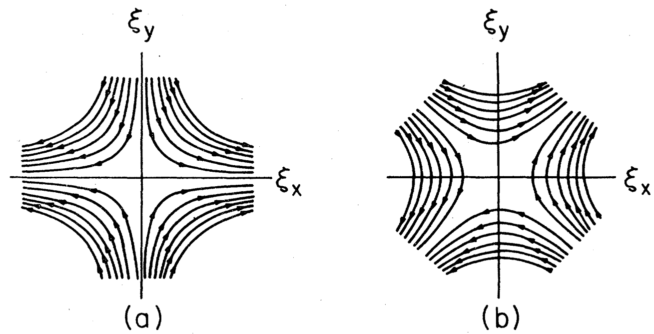

FIG. 2. Lines of force for gravitational waves: (a) with "+" polarization $\ddot{\xi}_x = \frac{1}{2}\ddot{h}_+\xi_x$, $\ddot{\xi}_y = -\frac{1}{2}\ddot{h}_+\xi_y$, and (b) with "×" polarization $\ddot{\xi}_x = \frac{1}{2}\ddot{h}_\times\xi_y$, $\ddot{\xi}_y = \frac{1}{2}\ddot{h}_\times\xi_x$ (Press, 1970); cf. Eqs. (3), (4), and (6).

$\ddot{\xi}_j$, $\delta\xi_j$, and F_j to become oscillatory—roughly as $\sin(2\pi|\xi|/\lambda)$ (e.g., Estabrook and Wahlquist, 1975; Exercise 37.6 of MTW).

Since gravitational waves can exert forces and do work, they must carry energy and momentum. Their density of energy and momentum is described by a stress-energy tensor which has the form, for waves propagating in the z direction [Eqs. (2)–(4)],

$$T^{00} = T^{0z} = T^{zz} = (1/16\pi)(c^2/G)\langle(\dot{h}_+)^2 + (\dot{h}_\times)^2\rangle \tag{9}$$

(Isaacson 1968a, 1968b; MTW Chap. 35). Here G is Newton's gravitation constant, c is the speed of light, and $\langle\ \rangle$ denotes an average over several wavelengths. The equivalence principle prevents one from localizing the energy and momentum any more accurately than a few wavelengths.

In many laboratory and astrophysical situations, electromagnetic waves display quantum-mechanical behavior—e.g., quantization into photons. Thus a purely classical description is inadequate. Not so for the cosmic gravitational waves that play important roles in the evolution of astrophysical systems, and that experimenters hope to detect: Those waves surely are quantized into gravitons (spin-two zero-rest-mass bosons); see, e.g., Feynman (1963), Isham, Penrose, and Sciama (1975), DeWitt (1979). However, because those waves are emitted by the bulk motion of huge amounts of matter, the occupation numbers of their gravitons' quantum-mechanical states are enormous—e.g., $n \simeq 10^{75}$ for the gravitational-wave burst emitted by a supernova [Eqs. (6)–(8) of Thorne *et al.*, 1979]. This means that the waves behave exceedingly classically; quantum-mechanical corrections to the classical theory have fractional magnitude $1/\sqrt{n} \sim 10^{-37}$.

III. GENERATION OF GRAVITATIONAL WAVES

Albert Einstein (1918), in his pioneering analysis of gravitational radiation, derived an expression for the gravitational-wave field in terms of the second time derivative of the quadrupole moment of its source

$$h^{\mathrm{TT}}_{jk}(t,\mathbf{x}) = (2/r)(G/c^4)[\ddot{\mathcal{I}}_{jk}(t-r/c)]^{\mathrm{TT}}\,. \tag{10}$$

Here $\mathbf{x}$ is the location of the observer in Cartesian coordinates centered on the source, $r \equiv |\mathbf{x}|$ is the distance from source to observer, $\mathcal{I}_{jk}$ is the source's mass quadrupole moment

$$\mathcal{I}_{jk}(t') = \int \rho(\mathbf{x}', t') \cdot (x'_j x'_k - \tfrac{1}{3}\delta_{jk} r'^2) d^3x' \tag{11}$$

(with ρ the mass density); and the superscript "TT" means "keep the transverse traceless part and throw the rest away":

$$\mathcal{I}^{\mathrm{TT}}_{jk} \equiv P_{jl} P_{km} \mathcal{I}_{lm} - \tfrac{1}{2} P_{jk}(P_{lm}\mathcal{I}_{lm}),$$
$$P_{jk} \equiv \delta_{jk} - x_j x_k / r^2 \equiv \text{"transverse projector."} \tag{12}$$

Note that, in order of magnitude,

$$h_+ \sim h_\times \sim h^{\mathrm{TT}}_{jk}$$
$$\sim \left(\frac{\begin{array}{l}\text{"Schwarzschild radius" } GM_{\mathrm{kin}}/c^2 \text{ associated with} \\ \text{the mass equivalent } M_{\mathrm{kin}} = E_{\mathrm{kin}}/c^2 \text{ of the source's} \\ \text{kinetic energy of quadrupolar motion}\end{array}}{\text{(distance from source to observer)}} \right). \tag{13}$$

For a source with the mass of the Sun $M_\odot$ and with $M_{\mathrm{kin}} \ll M_\odot$, at the distance of our Galaxy's center $r \simeq 3 \times 10^4$ light-years, this gives

$$h_+ \sim h_\times \sim h^{\mathrm{TT}}_{jk} \ll (GM_\odot/c^2 r) \simeq 5 \times 10^{-18} \tag{14}$$

—a very weak wave indeed!

In deriving the quadrupole-moment formula (10), Einstein made some serious restrictive assumptions: (i) that the internal motion of the source is governed by nongravitational forces ("negligible self-gravity"), (ii) that these forces are associated with stresses T_{jk} small compared to the mass-energy density ρc^2 ("weak stress"), and (iii) that the source is small compared to the characteristic wavelength of the gravitational waves it emits, which implies that its internal velocities are small compared to the speed of light ("slow motion"). These assumptions are valid for laboratory-type sources (rotating rods, people waving their fists, etc.); but all astrophysical sources violate the "negligible self-gravity" assumption.

So far as I know, it was Landau and Lifshitz (1941) who first recognized that the negligible self-gravity assumption is unnecessary. It can be replaced by the demand (i') that the source's internal Newtonian gravitational potential be small compared to c^2 ("weak self-gravity"). This extended the quadrupole-moment formula to sources for which Newton's theory of gravity is fairly accurate, e.g., binary star systems and pulsating stars, but not neutron stars or black holes.

(Actually, Yevgeny Lifshitz, who is responsible for the Landau-Lifshitz prose, writes with such terse elegance that most readers overlook the fact that his derivation is valid for self-gravitating sources. I only discovered it 10 years after first reading Landau and Lifshitz, while writing the corresponding segment of MTW—Secs. 36.9 and 36.10).

In electromagnetism there is a dipole-moment formula for the radiation's vector potential in Lorentz gauge

$$A_j(t, \mathbf{x}) = (1/r)(1/c)[\dot{d}_j(t - r/c)]^T. \tag{15}$$

Here d_j is the electric dipole moment $[\int \rho'_e x'_j d^3x'$, with $\rho'_e = \rho_e(t', \mathbf{x}')$ the charge density], and the superscript "T" means "take the transverse part and throw the rest away"

$$d^T_j \equiv P_{jk} d_k. \tag{16}$$

This electromagnetic analog of the gravitational quadrupole formula (10) is accurate under only one restrictive assumption: "slow motion" (source size small compared to wavelength of emitted waves). I have long suspected by analogy that the gravitational formula (10) should also require only slow motion; it should be valid independent of the magnitudes of self-gravity and internal stress, provided one uses a modified expression for the quadrupole moment (11). That this is indeed so is proved in Secs. VII and XII of the review article which follows (Thorne, 1980).

In electromagnetism the dipole-moment formula (15) is just the first term in a multipole-moment expansion of the radiation field. Charge conservation prevents the inclusion of a monopole term. For slow-motion sources the electric-dipole term dominates the expansion, except in cases of special symmetry where it is suppressed, leaving other terms (magnetic dipole, electric quadrupole, etc.) to dominate.

Similarly, in general relativity the quadrupole-moment formula (10) is just the first term in a multipole-moment expansion of the radiation field. Conservation of mass-energy prevents the inclusion of a monopole term; conservation of momentum and of angular momentum prevents the inclusion of dipole terms. The "mass quadrupole term" [Eq. (10)] dominates the expansion, except in cases of special symmetry where it is suppressed, leaving others ("current quadrupole", "mass octupole", etc.) to dominate. As in the electromagnetic case, there are two sets of moments in the gravitational-wave expansion: moments of the mass distribution ρ, and moments of the "mass current" distribution ρv_j (with v_j the velocity). The "mass moments" are analogs of "electric moments," the "current moments" are analogs of "magnetic moments."

There is by now an enormous body of literature on gravitational multipole expansions in general relativity—literature using a wide variety of different notations and conventions. The chief purpose of the review which follows (Thorne, 1980) is to bring together, in a single unified formalism, the main results in the literature—and to present formulas for translating from one multipole-moment (vector and tensor spherical-harmonic) notation to another. A much more sketchy review is given in Thorne (1977).

The slow-motion assumption is violated by some of the most interesting astrophysical sources of electromagnetic radiation (e.g., synchrotron radiation from high-energy electrons in cosmic magnetic fields), and gravitational radiation (e.g., the radiation from collisions of black holes). For such sources multipole expansions are rarely a useful tool. Alternative mathematical tools, valid for fast-motion sources of gravity waves, have been developed in recent years. These tools fall into four classes:

Weak-gravity formalisms require only the restriction to weak internal gravity. When internal gravity is totally negligible the appropriate formalism is *linearized theory* (the linearized approximation to general relativity; MTW Chap. 18). When internal gravity is important but still weak, one uses a *post-linear theory* [e.g., Kovacs and Thorne (1978) and references therein]. The entire subject of weak-gravity formalisms is reviewed in Thorne (1977).

Ultra-high-speed formalisms deal with radiation from objects with relative speeds very close to the speed of light $[\gamma \equiv (1 - v^2/c^2)^{-1/2} \gg 1]$, which attract each other gravitationally and thereby radiate. Amazingly, these formalisms do not require weak gravity; they are valid, for example, in the high-velocity head-on collision of two black holes. These formalisms are due to D'Eath (1978) and Curtis (1977) and are reviewed by D'Eath (1979).

Perturbation formalisms deal with sources whose internal motions are weak perturbations of a nonradiating system. Examples are small-amplitude vibrations of neutron stars (e.g., Thorne, 1969a), small objects falling into large black holes (e.g., Detweiler and Szedenitz, 1979), and small nonsphericities in the collapse of a star to form a black hole (e.g., Cunningham *et al.*, 1979; Gaiser and Wagoner, 1980). In the 1970s perturbation analyses have been our most important tool for studying radiation from black-hole events. There is no comprehensive, up-to-date review of these analyses—and there probably will not be soon because they are developing so rapidly. An out-of-date review is given in the last half of Thorne (1978), and descriptions of several specific recent calculations will be found in Smarr (1979).

Computer solutions of the full, nonlinear Einstein field equations are an extremely powerful tool for the future. Pioneering work by Larry Smarr and Kenneth Eppley (building on foundations of Bryce DeWitt and Andrej Čadež) has produced beautiful computer-generated movies of the head-on collision and coalescence of two equal-mass black holes, and of the resulting gravitational radiation. Other researchers are analyzing nonspherical stellar collapse to form a black hole. These computations are all axially symmetric (two nontrivial space dimensions; one time); for reviews see Smarr (1979). It is reasonable to hope that, within about five years, computer codes with three space dimensions will have given us waveforms for non-head-on collisions of black holes—waveforms that can be compared definitively with observational data. Such comparisons would provide powerful tests of general relativity and of its theory of black holes, as well as astrophysical information about the galactic nuclei and quasars where black-hole collisions are likely to occur (e.g., Blandford, 1979).

IV. PROPAGATION OF GRAVITATIONAL WAVES

Between Earth and typical sources of gravitational waves (our Galaxy's nucleus, nearby clusters of galaxies ...) spacetime is very nearly flat, and the waves propagate according to very simple laws: a "$1/r$" falloff of amplitude and no change of polarization

$$h_+ = (1/r)A_+(t - r/c), \quad h_\times = (1/r)A_\times(t - r/c), \qquad (17)$$
$$e^+_{jk} \text{ and } e^\times_{jk} \text{ constant along the "rays" } \mathbf{x} = \mathbf{n}c(t - t_0),$$

where n is the unit radial vector.

In special cases, however, the waves may encounter a large mass concentration (e.g., an intervening Galaxy or the Sun). Although absorption and dispersion will be negligible (cf. Carter and Quintana, 1977), a "gravitational lens effect" may not be: it may strongly amplify the waves. The lens effect can be evaluated using geometric optics (MTW Exercise 35.15; last section of Thorne, 1977) if the wavelength λ is short compared to the scale $\mathcal{L}$ of the intervening object. This will be true for intervening galaxies. However, when $\lambda \sim \mathcal{L}$ (e.g., for Crab-pulsar waves passing through the Sun) geometric optics fails, and one must solve the full wave-propagation equation on a curved-space background (MTW Eq. 35.64)—e.g., using a curved-space Green's function (DeWitt and Brehme, 1960; Robaschik, 1963; Crowley and Thorne, 1977). When $\lambda \gg \mathcal{L}$ the waves will hardly notice the intervening object at all.

Gravitational waves from cosmologically large distances will suffer the same cosmological redshift and "lens-effect of the Universe" as does light. As for light, these effects can be evaluated using geometric optics (last section of Thorne, 1977).

V. RADIATION REACTION IN SOURCES

There are general relativistic conservation laws which guarantee that the energy, linear momentum, and angular momentum of a source must decrease by precisely the amounts carried off in gravitational radiation (MTW Chaps. 19 and 20). The mechanism by which this occurs is radiation reaction in the source. In the special case of weak-field slow-motion sources one can express the radiation-reaction forces as gradients of a Newtonian-type potential:

$$\mathbf{F} = -m\nabla\Phi^{(\mathrm{react})}, \quad \Phi^{(\mathrm{react})} = \frac{1}{5}\frac{G}{c^5}\frac{d^5}{dt^5}\mathcal{I}_{jk}(t)x_j x_k \qquad (18)$$

(Burke, 1969, 1971; Chandrasekhar and Esposito, 1970; MTW Secs. 36.8–36.11). As with other quadrupole formulas, this is only the first term in a multipole expansion for $\Phi^{(\mathrm{react})}$ (Burke, 1971; Thorne, 1969b).

Controversy swirls around Eq. (18) for the radiation-reaction force, and around the conservation laws which predict, more generally, a source's loss of energy, momentum, and angular momentum: One group of gravitation theorists, led by Jurgen Ehlers, Arnold Rosenblum, Joshua Goldberg, and Peter Havas (1976) (see also Rosenblum, 1978), believes that these radiation-reaction results have not been derived with sufficient mathematical rigor to be fully trustworthy. Another group, to which I strongly adhere, believes that the rigor, e.g., of the Burke (1969, 1971) and Chandrasekhar-Esposito (1970) derivations exceeds that of many analyses in mathematical physics which physicists firmly trust. We are happy to let our more mathematical colleagues polish up the derivations; but we have no doubt that in the end the results will remain unchanged. Our colleagues are now working hard, with a level of rigor that is beautiful to behold; see, e.g., Ehlers (1979), Walker and Will (1979), and Christodoulou and Schmidt (1980).

In the meantime, observations of the binary pulsar (Taylor *et al.*, 1979a, b) have become accurate enough to reveal a gradual decrease of the stars' orbital period at a rate 1.12 ± 0.21 times that predicted by the standard radiation-reaction formulae. This is widely regarded as observational evidence for the existence of gravitational waves and for the approximate correctness of the standard radiation-reaction results. Such a conclusion seems premature to me, however. As Smarr and Blandford (1976) have argued, period changes of the observed magnitude could be produced by a reasonable amount of mass loss from the pulsar's companion. It seems to me that before drawing any conclusions we should wait for several more years of data to reveal the period change with accuracy of a few percent. If it still agrees with the radiation-reaction predictions, that would be strong circumstantial evidence for the correctness of the theory.

VI. ASTROPHYSICAL ESTIMATES OF THE WAVES BATHING EARTH

Astrophysicists are putting much effort into estimates of the gravitational waves that bathe the Earth. Their estimates are based on gravitation theorists' wave-generation calculations (Sec. III), plus currently fashionable astrophysical models for the nature of the Universe around us. The astrophysical models are the weak link in the estimates. For example, recent observational data have made it fashionable to believe that large black holes ($M \sim 10^5$ to $10^9 M_\odot$) reside in the nuclei of most galaxies, including our own (Blandford, 1979; Blandford and Thorne, 1979). Three years ago such speculations, while common, were not terribly fashionable. As another example, four years ago x-ray observations made it fashionable to believe that moderately large black holes ($M \sim 10^3$ to $10^4 M_\odot$) reside at the centers of globular clusters. However, more recent observations and theory have thrown this model somewhat out of fashion (for a review see Lightman and Shapiro, 1978). As yet another example, a supernova produces gravitational radiation in amounts that depend critically on the speed of collapse of the stellar core, and its deviations from sphericity. Recently, computer predictions of collapse speeds and nonsphericities have been oscillating with a period of about four years—and, as a result, the most fashionable estimates of the supernova energy carried off by gravitational waves have oscillated between $0.03 M_\odot c^2$ and $10^{-5} M_\odot c^2$ (articles by Arnett, Wilson, Shapiro, and Kazanas and Schramm, in Smarr, 1979; see also the article by Turner and Wagoner for estimates far smaller than $10^{-5} M_\odot c^2$).

These fluctuations of fashion are not due to theoretical incompetence. Rather, they are due to the great complexity of astrophysical systems, plus extremely rapid advances in observational astronomy.

Given the near orthogonality between the kinds of information carried by electromagnetic waves and by gravitational waves (cf. Sec. I), I doubt that we astrophysicists can make our gravity-wave estimates much firmer in the next few years than they are today. A firmer knowledge must probably await the actual discovery and measurement of gravity waves.

Figure 3 sketches current astrophysical estimates of the gravitational waves bathing Earth. This figure is my own attempt to boil down into a single graph the enormous number of models, estimates, and scenarios that are currently fashionable. More detailed summaries will be found in Epstein and Clark (1979) and in Thorne (1978). Figure 3 deals with three types of gravity waves: bursts, periodic waves, and stochastic background.

Bursts

Broad-band bursts, with durations $\tau \sim 1/f \equiv 1/(\text{frequency})$, should be produced by: the collapsing and bouncing cores of supernovae ($f \sim 10$ to 10^4 Hz); neutrinos pouring out of a supernova ($f \sim 1$ to 100 Hz); corequakes in neutron stars ($f \sim 100$ to 10^4 Hz); the births of black holes [$f \sim 10^4 \text{ Hz} (M/M_\odot)^{-1}$, Eq. (1), with $M \sim 3$ to $100 M_\odot$ for holes born in the collapse of a normal star; M as large as $10^9 M_\odot$ for holes born in galactic nuclei and quasars]; collisions between black holes, and between black holes and neutron stars—which may occur in globular clusters ($f \sim 1$ to 10^3 Hz) and in galactic nuclei and quasars ($f \sim 10^{-4}$ to 10^3 Hz); and the final inspiral, coalescence, and destruction of compact binaries such as the binary pulsar ($f \sim 100$ to 3000 Hz). For discussion and references see Epstein and Clark (1979) and Rees (1977). Such bursts, arriving at Earth once per month, could have amplitudes $h (= h_+ \text{ or } h_\times)$ as large as the topmost line in Fig. 3 without violating any conventional "cherished beliefs" about the nature of gravity or the astrophysical structure of our Universe (Zimmermann and Thorne, 1980). However, currently or recently fashionable models for

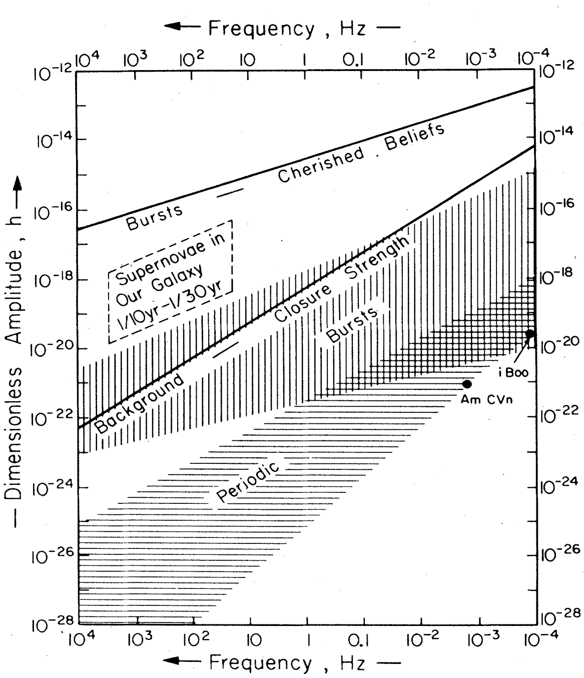

FIG. 3. Estimates of the strengths of the gravitational waves that bathe the Earth. See text for explanation of the lines and hatched regions.

the Universe predict that the strongest *once-per-month bursts* should lie far below the "cherished belief line"—somewhere in the vertically hatched region of Fig. 3. The great vertical extent of the hatched region (2.5 to 5 orders of magnitude in h) is a measure of our extreme ignorance about the strengths of potential gravitational-wave sources. The first burst to be discovered could well lie above the "once-per-month" hatched region. For example, with luck one might catch gravitational waves from a supernova in our own Galaxy (estimated to occur once each 10 to 30 years, with $h \sim 10^{-17}$ to 10^{-20}; Fig. 3).

Periodic waves

Periodic gravitational waves should be produced by binary star systems ($f \sim 10^{-2}$ to 10^{-4} Hz), rotating, deformed neutron stars ($f \sim 10^{-2}$ to 10^{4} Hz), rotating, deformed white dwarfs ($f \sim 10^{-2}$ to 1 Hz), and pulsations of white dwarfs that may follow nova outbursts ($f \sim 10^{-2}$ to 1 Hz). For discussion and references see Epstein and Clark (1979). Fashionable models for the Universe predict that the strongest periodic sources should lie in the horizontally-hatched region of Fig. 3. The predicted waves from two specific binary stars, i Boo and Am CVn, are shown explicitly.

Stochastic background

There may exist a stochastic background of gravitational radiation produced, for example, by the deaths-to-form-black-holes of "Population III stars" (stars that were born before galaxies formed), or by the big-bang explosion in which the Universe presumably originated. In principle, such a stochastic background could be so strong that its energy density, in one or two special decades of frequency, is adequate to close the Universe (line marked "Background—closure strength" in Fig. 3, where the quantity plotted is $[(\text{frequency})\times(\text{spectral density of } h)]^{1/2}$). On the other hand, the stochastic background could be many many orders of magnitude weaker than the closure strength. For further details see Epstein and Clark (1979), Carr (1980), and Bertotti and Carr (1980).

VII. EXPERIMENTAL SEARCH FOR GRAVITATIONAL WAVES

Current and near-future efforts to detect gravitational waves are reviewed by Weiss (1979), Epstein (1979), Tyson and Giffard (1978), Weber (1979), Douglass and Braginsky (1979), and Braginsky and Rudenko (1978). These efforts involve three distinct types of detectors: Doppler tracking of spacecraft, laser interferometers, and Weber-type resonant bars. Other types of detectors look promising but have not yet been constructed.

Doppler tracking of spacecraft

In addition to the above references, see Vessot and Levine, 1978; Estabrook *et al.*, 1979; Bertotti and Carr, 1979. A gravitational wave produces tiny relative motions of the Earth and a distant interplanetary spacecraft [Eq. (7), generalized to the case $\lambda \lesssim |\xi|$ $\equiv$ (earth-spacecraft distance); Estabrook and Wahlquist, 1975]. These motions in turn produce fluctuations in the Doppler shift of radio tracking signals—fluctuations of magnitude

$$\delta\nu/\nu \simeq \tfrac{1}{2}h \equiv \tfrac{1}{2}h_{jk}^{\mathrm{TT}}\xi_j\xi_k/|\xi|^2\,, \tag{19}$$

where "$\simeq$" means that numerical factors of order unity have been ignored. (The radio signals are transmitted from Earth to the spacecraft, received by the spacecraft which amplifies them and transmits them back to Earth, where they are received and their frequency $\nu+\delta\nu$ is compared with the transmitted frequency ν.) High-accuracy measurements of the Doppler shift $\delta\nu/\nu$ can be made over time scales τ between ~100 sec and ~10^4 sec (gravity-wave frequencies $f \simeq 1/\tau \sim 10^{-2}$ to 10^{-4} Hz). For $\tau \ll 100$ sec clock noise and noise in the Doppler readout system become prohibitive. For $\tau \gg 10^4$ sec the Earth's rotation prevents continuous tracking from a single antenna site.

For $f \sim 10^{-2}$ to 10^{-4} Hz the chief sources of noise in NASA's Doppler tracking system are: (i) fluctuations in the index of refraction of the interplanetary plasma (solar wind), through which the tracking signal passes ($\delta\nu/\nu \sim 3\times10^{-13}$ to 3×10^{-14} for tracking signals at S band, $\nu \simeq 2\times10^9$ Hz; $\delta\nu/\nu \sim 3\times10^{-14}$ to 3×10^{-15} at X band, $\nu \simeq 1\times10^{10}$ Hz; Armstrong *et al.*, 1979); (ii) index-of-refraction fluctuations in the Earth's troposphere ($\delta\nu/\nu \lesssim 5\times10^{-14}$; Armstrong *et al.*, 1979); (iii) frequency fluctuations in the hydrogen-maser clock that regulates the tracking signal ($\delta\nu/\nu \simeq 2\times10^{-15}$); (iv) buffeting of the spacecraft by fluctuations in the solar wind and radiation pressure, and by the jet effect of leaking gas ($\delta\nu/\nu$ not well known, but $\lesssim 5\times10^{-14}$ on the Viking spacecraft). All of these noise sources can be reduced in strength or monitored if sufficient effort and money are expended. Especially effective in reducing the noise would be a "four-link" Doppler tracking system that uses two clocks—one on the spacecraft and one on the Earth (Vessot and Levine, 1978).

The first serious Doppler-tracking search for gravitational waves will be by the American Solar-Polar Mission (launch February 1983, swing by Jupiter May 1984, pass over the Sun's pole November 1986). This spacecraft will probably carry an X-band tracking capability and may be able to detect gravitational wave bursts as weak as $h \sim (\text{a few})\times10^{-15}$ at $f \sim 10^{-2}$ to 10^{-4} Hz. A twin European Solar-Polar Spacecraft may have much worse sensitivity because the Europeans have tentatively chosen to not equip their spacecraft with a capability for receiving X-band tracking signals; they must suffer along with S-band plasma-dispersion noise.

Other missions in the late 1980s (e.g., the "Solar Probe"; Neugebauer and Davies, 1978) may achieve sensitivities of $h \sim 1\times10^{-16}$.

Here and below all estimated sensitivities (values of h) are for gravitational wave bursts with duration roughly one period. For periodic sources and stochastic background one can integrate over a long time, thereby obtaining a sensitivity much better than for bursts (Hough *et al.*, 1975; Hirakawa and Narihara, 1975; Weiss, 1979).

Laser interferometers

A prototype laser-interferometer detector for gravitational waves has been operated with modest sensitivity ($h \sim 10^{-15}$ rms for $f \simeq 100$ Hz) by Forward (1978) at Hughes Research Laboratories. Laser detectors of greater sensitivity are under construction in Munich, Germany (H. Billing); Glasgow, Scotland (R. Drever); MIT (R. Weiss); and Caltech (R. Drever). The next decade will likely be devoted to ground-based interferometers, with frequencies limited (for current designs) to $f \lesssim 10^4$ Hz by photon-counting statistics, and to $f \gtrsim 30$ Hz by seismic noise and by the fluctuating gravity gradients of passing automobiles or other moving objects. A reasonable goal for this ~10 year effort is $h \sim 10^{-21}$. In the more distant future experimenters may fly in space similar interferometers—or closely related "optical heterodyne" (optical Doppler tracking) systems. Such detectors might achieve $h \lesssim 10^{-21}$ at the low frequencies $f \sim 30$ to 10^{-4} Hz where gravity-gradient noise and seismic noise debilitate ground-based systems; see, e.g., Weiss (1979).

In its simplest variant, a laser-interferometer gravity-wave detector has three test masses—one at the corner of the interferometer; the other two at the ends of its arms (Fig. 4). In an Earth-based interferometer these test masses are suspended as pendula from overhead supports, but they behave like free masses for horizontal motions at frequencies $f \gg$ (pendulum swing frequency) $\simeq 1$ Hz. In a space-based interferometer, the test masses could be truly free, except for tiny noise forces (e.g., due to cosmic-ray impacts); the test masses would be shielded from solar wind and radiation pressure by surrounding shells. A passing gravity wave at frequency f will push the masses of one arm together, and those of the other arm apart ($\delta l/l = \frac{1}{2}h_{xx}^{TT} = \frac{1}{2}h_+$ for arm along x direction; $\delta l/l = \frac{1}{2}h_{yy}^{TT} = -\frac{1}{2}h_+$ for arm along y direction). This will cause a change in the path-length difference for laser beams in the two arms (Fig. 4)—and will thereby cause a relative phase shift and consequent intensity change of the recombined light, as measured by the photodiode readout system. One can increase the phase shift, and thereby improve one's sensitivity, by bouncing the

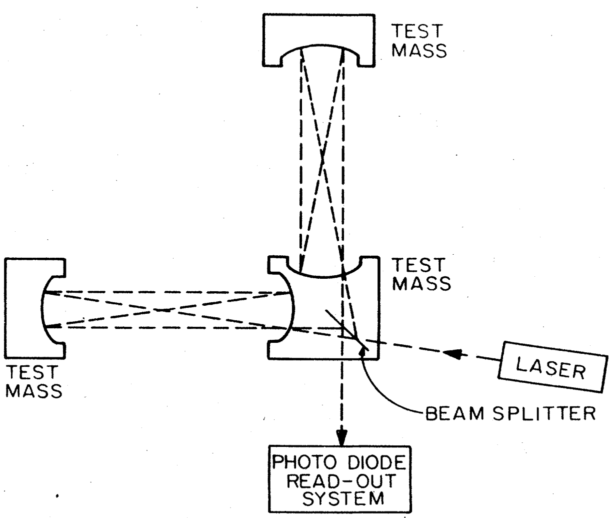

FIG. 4. Schematic diagram of laser-interferometer gravitational-wave detector.

light back and forth in the arms a large number of times. At each bounce a fraction $(1-R)$ of the photons are lost, where R is the mirror reflectivity. For b bounces the light traverses the arms $b+1$ times, producing a relative phase shift $\delta\phi = 2(b+1)\cdot(\delta l/\lambda\!\!\!^{-})$, which can be measured to accuracy $1/\sqrt{N} = (\hbar c/\lambda\!\!\!^{-} W \varepsilon e^{-(1-R)b}\tau)^{1/2}$. Here $\lambda\!\!\!^{-}$ is the reduced wavelength of the light (wavelength/2π), N is the number of photons collected in an averaging time τ, W is the laser output power, ε is the photodiode collecting efficiency, $e^{-(1-R)b}$ is the fraction of the emitted photons that reach the photodiode, and $\hbar$ is Planck's constant. The limiting sensitivity due to photon counting statistics is thus

$$\Delta l \simeq \frac{\lambda\!\!\!^{-}}{2(b+1)\sqrt{N}} = \frac{\lambda\!\!\!^{-}}{2(b+1)}\left(\frac{\hbar c}{\lambda\!\!\!^{-}\varepsilon e^{-(1-R)b} W\tau}\right)^{1/2}$$

$$= \frac{\lambda\!\!\!^{-}}{2(b+1)}\left(\frac{\hbar c e^2}{\lambda\!\!\!^{-}\varepsilon W\tau}\right)^{1/2} \quad \text{(rms noise)}, \tag{20}$$

where an optimal number of bounces, $b = 2(1-R)^{-1}$ has been assumed. The reflectivity of the best available mirrors, $R \simeq 0.997$, limits b to $\lesssim 500$; the best laser powers readily available are for an argon-ion laser ($\lambda\!\!\!^{-} \simeq 1\times10^{-5}$ cm), $W \simeq 1$ W; a reasonable photodiode efficiency is $\varepsilon \simeq 0.5$; and τ cannot exceed $1/2f$, where f is the gravitational-wave frequency. For $f \simeq 100$ Hz this gives $\Delta l \simeq 3\times10^{-16}$ cm for the rms noise. The second-generation detectors, now under construction, have arm lengths up to tens of meters, corresponding to a photon-limited gravitational-wave sensitivity $h \simeq 2\Delta l/l \sim 2\times10^{-19}$ (rms). In a search for bursts that occur only once per month, one must face amplitude noise five times larger than the rms noise—corresponding to

$$h \sim 1\times10^{-18} \text{ for once-per-month bursts at } f \simeq 100 \text{ Hz}. \tag{21}$$

To actually achieve this "photon-counting" limit, an experimenter must surmount a huge number of other noise sources—laser frequency fluctuations, laser amplitude fluctuations, seismic vibrations, thermal noise in the pendulum supports, fluctuations of index of refraction of residual gas in the evacuated interferometer arms .... These noise sources all look beatable, but only with a rather complex experimental setup that may include many feedback loops (Weiss, 1972, 1979; Billing *et al.*, 1979, Drever *et al.*, 1980). Prototypes with much complexity have been operating in Munich and Glasgow for about one year now (July 1979), but have not yet reached sensitivities near the photon-counting limit; see Billing *et al.* (1979), Drever *et al.* (1980). On the other hand, it is not unreasonable to hope for a burst sensitivity $h \sim 10^{-21}$ (with $l \sim 5$ km and $W \sim 100$ W) within the next decade. Sensitivities to periodic sources and stochastic background might be several orders of magnitude better than 10^{-21}.

When gravitational wave bursts are ultimately detected, laser systems will be able to measure the details of their waveforms $h(t)$. This is because by their very nature laser systems have a "broad-band" capability (sensitivity to all frequencies from $f \sim 30$ Hz to $f \sim 10^4$ Hz).

Weber-type resonant bars

Weber-type resonant-bar detectors have the advantage of being somewhat less complex than a laser-interferometer detector. However, this lesser complexity is purchased at the price of (i) having much shorter lengths l (~1 m vs. 10 to 10 000 m), and therefore requiring much more accurate position readouts δl in order to achieve a given sensitivity $h \simeq 2\delta l/l$; and (ii) not being broad-band detectors, and therefore not being able to measure the details of $h(t)$—unless certain serious technical problems can be overcome (see below).

In a resonant-bar detector one monitors electronically the complex amplitude $X = X_1 + iX_2$ of one of the normal modes of oscillation of a solid object ("antenna")—usually a cylinder. If the normal mode's eigenfrequency is $f_0 \equiv \omega/2\pi$, then that normal mode will produce a displacement

$$x(t) = \mathrm{Re}(Xe^{-i\omega t}) = X_1 \cos\omega t + X_2 \sin\omega t \tag{22}$$

at the end of the antenna. The force of a passing gravitational-wave burst [Eq. (8)] drives the normal mode, changing its complex amplitude. The change δX is permanent—until another wave burst comes along. Consequently, one can (and usually does) try to detect the change by averaging the electronic signal over many cycles. Averaging is advantageous because it reduces the bandwidth $B \equiv 1/2\tau$ ($\tau \equiv$ averaging time) through which electrical noise of the measuring system can sneak. The electrical noise produces an rms error (Tyson and Giffard, 1978; Weiss, 1979; Braginsky *et al.*, 1980)

$$\Delta X_1 = \Delta X_2 \simeq \left(\frac{kT_n/\Omega}{m\omega}\right)^{1/2}\left(\frac{1}{\beta\omega\tau}\right)^{1/2} \tag{23}$$

in one's measurement of the amplitude $X = X_1 + iX_2$. Here β is a dimensionless coupling constant (denoted $(\omega\tau_0)^{-1}$ by Tyson and Giffard; denoted β by Weiss and by Braginsky *et al.*), which characterizes the strength of the coupling between the mechanical antenna and the electronic measuring system. (The coupling takes place in a transducer which is sometimes a piezoelectric crystal, and sometimes a capacitor or inductor with capacitance C or inductance L modulated by the antenna's vibrations.) Also, in Eq. (23), T_n is the "noise temperature" of the measuring system which, in most second-generation detectors, will probably be equal to the noise temperature of the first electronic amplifier); $\Omega/2\pi$ is the frequency at which the amplifier operates (equal to $\omega/2\pi \sim 1$ kHz if a "SQUID" is used; of order 10^8–10^{10} Hz if the transducer upconverts the signal and a maser amplifier or FET is then used); k is Boltzmann's constant; and m is the mass of the antenna.

It would seem in Eq. (23) that by making the averaging time τ arbitrarily long one could make arbitrarily accurate measurements. This is not so. One must also contend with "back-action forces" which the measuring system exerts on the antenna (Sec. VIII), and with frictional forces ("Nyquist forces") inside the antenna. The Nyquist forces couple the normal mode of interest to all of the antenna's other normal modes—and especially to thermal phonons. Energy exchange with thermal phonons causes the complex amplitude to random walk, during the averaging time τ, by an rms amount

$$\Delta X_1 = \Delta X_2 \simeq (kT_a/m\omega^2)^{1/2}(\omega\tau/Q)^{1/2}. \tag{24}$$

Here T_a is the physical temperature of the antenna, i.e., of its thermal phonons, and Q is the antenna's "quality factor" (the number of radians of oscillation required for its energy to damp by $1/e$ if it is excited to oscillation energies $E \gg kT_a$). Note that the longer one makes the averaging time τ, the larger will be the antenna's Nyquist noise (24).

The competition between electrical noise (23) and Nyquist noise (24) dictates an optimal averaging time, which is usually large compared to the oscillation period $2\pi/\omega$, and a resulting minimum noise:

$$\omega\tau_{\mathrm{opt}} \simeq \left(\frac{T_n}{T_a}\right)^{1/2}\left(\frac{\omega}{\Omega}\right)^{1/2}\left(\frac{Q}{\beta}\right)^{1/2}, \tag{25a}$$

$$(\Delta X_1)_{\min} = (\Delta X_2)_{\min} \simeq \left(\frac{(kT_a/\omega)^{1/2}(kT_n/\Omega)^{1/2}}{m\omega}\right)^{1/2}\left(\frac{1}{\beta Q}\right)^{1/4}. \tag{25b}$$

High sensitivity (small ΔX) requires the following: (i) large antenna mass m; (ii) large antenna quality factor Q; (iii) low antenna temperature T_a; (iv) low electrical noise parameter kT_n/Ω; and (v) strong coupling between antenna and electronics, i.e., large β.

The amplifiers with the lowest noise have $kT_n/\Omega \sim (10 \text{ to } 100)\hbar$, where $\hbar$ is Planck's constant. Whereas first-generation antennas were operated at room temperatures, second-generation antennas will be cooled to liquid-helium temperatures, $T_a \simeq 1$ to 4 °K, to reduce Nyquist noise. Some antennas (e.g., Stanford, LSU, and Rome) have been made from aluminum alloys which are available in large masses $m \sim 6$ tons but have modest quality factors $Q \sim 10^6$ at 4 °K. The Moscow antenna has been made from sapphire, which comes in smaller masses $m \sim 10$ to 100 kg but with larger quality factors, $Q \sim 4\times10^9$ at 4 °K. The Perth antenna is made from niobium, which can be floated superconductively and has an m of approximately a few hundred kg, and $Q \sim 10^8$ at 4 °K. Other groups (Tokyo, Maryland, Rochester) have not yet firmly chosen their antenna materials, but are leaning toward a particular aluminum alloy ("5056") recently discovered by the Tokyo group (Suzuki *et al.*, 1978) to have a Q of 4×10^7 at 4 °K. All of these antennas (except a Tokyo antenna intended for Crab pulsar radiation) will have frequencies $\omega/2\pi$ in the range 700 to 5000 Hz. Each research group is working hard on its own clever design for the transducer and measuring electronics, trying to achieve a coupling constant β as large as possible without substantially reducing the Q of the antenna. The largest value we can hope for in the next few years is $\beta \sim 10^{-2}$. (For details of some recent prototype transducers see, e.g., Hoffman *et al.*, 1976; Paik, 1976; Richard, 1976; Adami *et al.*, 1976; Braginsky *et al.*, 1977; Tsubono *et al.*, 1977; and Blair, 1979.)

With the above parameters, second-generation Weber-type bars may achieve rms noises (Eq. 25b) $\Delta l \equiv \Delta X_1 = \Delta X_2 \sim 1\times10^{-17}$ cm, compared to 3×10^{-15} cm for the best first-generation bars. This corresponds to a gravitational-wave sensitivity $h \simeq 2\delta l/l \sim 2\times 10^{-19}$ rms, or

$$h \sim 1\times 10^{-18} \text{ for once-per-month bursts at } f \simeq 1000 \text{ Hz}. \tag{26}$$

This is comparable to the sensitivity goals for second-generation laser-interferometer detectors [Eq. (21)].

Unfortunately, the second-generation resonant-bar antennas will be narrow band: $\omega\tau_{\rm opt} \sim 100$. To achieve broad-band measurements and a monitoring of the waveform, one will need much stronger transducer coupling (e.g., $\beta \sim 1$). This *might* be accomplished by a third generation of antennas, which are now taking shape in experimenters' heads with milli-Kelvin temperatures and "back-action-evading" measuring systems. Such antennas may achieve burst sensitivities $h \sim 10^{-21}$ at kilohertz frequencies.

Other types of detectors

Many other types of gravitational-wave detectors have been proposed (see, e.g., Pegoraro *et al.*, 1978; Caves, 1979; Braginsky and Rudenko, 1978; Grishchuk and Polnarev, 1979; and references therein). Some of these detectors, using cryogenic microwave cavities, are under semiserious consideration for construction.

VIII. BACK ACTION AND THE QUANTUM LIMIT FOR GRAVITATIONAL-WAVE DETECTORS

The above discussion ignores one noise source in gravity-wave detectors that has been negligible in most past experiments, but will be crucially important in the future. This is the back-action force of the electronic measuring system on the antenna (Braginsky, 1970).

For laser interferometers (Fig. 4) the beam splitter produces rms fluctuations $\Delta N \simeq \sqrt{N}$ in the *difference* of the number of photons going into the two arms (Caves, 1980). Each time a photon bounces off a mirror, it imparts a momentum $2\hbar/\lambdabar$ to the mirror. Consequently, in b bounces the $\sqrt{N}$ photons will produce a momentum difference between the two arms $\Delta p \simeq \sqrt{N}(2\hbar/\lambdabar)b$—which is the back-action momentum change required by the Heisenberg uncertainty principle, $\Delta l \Delta p \simeq \hbar$ [cf. Eq. (20)]. In an averaging time τ this Δp will produce a relative change in the arm lengths

$$\Delta l \simeq \left(\frac{\Delta p}{m}\right)\tau \simeq \left(\frac{2\hbar}{\lambdabar}\right)\left(\frac{\sqrt{N}b\tau}{m}\right) \simeq \left(\frac{2\hbar b\tau}{m\lambdabar}\right)\left(\frac{\lambdabar W\tau}{\hbar c}\right)^{1/2}, \tag{27}$$

where we have assumed few enough bounces that mirror losses can be ignored. If the laser power W is too large, this back-action effect will be the dominant noise source. If W is too small, the photon counting statistics (20) will be the dominant noise source. The optimal laser power

$$W_{\rm opt} \simeq m\lambdabar c/4b^2\tau^2 \simeq 30 \text{ W for } m \simeq 100 \text{ kg},$$
$$\lambdabar \simeq 10^{-5} \text{ cm}, \quad b \simeq 500, \quad \tau \simeq 10^{-2} \text{ sec} \tag{28a}$$

leads to a minimum combined noise ["quantum limit" (ql); Braginsky and Vorontsov, 1974; Drever *et al.*, 1977; Caves *et al.*, 1980; Caves, 1980]

$$\Delta l_{\rm ql} \simeq (\hbar\tau/m)^{1/2}$$
$$\simeq 1\times 10^{-17} \text{ cm for } m \simeq 100 \text{ kg}, \ \tau \simeq 10^{-2} \text{ sec}. \tag{28b}$$

Because laser powers as large as 30 W are not yet available, back-action noise is not yet a problem in the search for gravitational-wave bursts of frequency $f \sim \tau^{-1} \sim 100$ Hz. However, in future experiments it may become so, and then one must face up to the limit (28b) which is of quantum-mechanical origin (Heisenberg uncertainty principle; Caves *et al.*, 1980).

Similarly for resonant bar antennas: if the coupling constant β becomes large enough, one must worry about back-action forces of the amplifier and readout system on the bar. These, when counterbalanced against the electronic readout noise (23), lead to an optimal coupling constant

$$(\beta\omega\tau)_{\rm opt} \simeq 1, \tag{29a}$$

which may be achieved by some second-generation detectors, and to a limiting sensitivity (the "amplifier limit"; Braginsky, 1970; Giffard, 1976; Thorne *et al.*, 1979; Caves *et al.*, 1980; Braginsky *et al.*, 1980)

$$(\Delta X_1)_{\rm min} = (\Delta X_2)_{\rm min} \simeq \left(\frac{kT_n/\Omega}{m\omega}\right)^{1/2}. \tag{29b}$$

No linear amplifier can ever achieve a noise temperature less than $kT_n/\Omega \simeq \hbar$; quantum mechanics, as applied to the internal workings of an amplifier, forbids lower noise temperatures (e.g., Weber, 1959; Heffner, 1962). Thus, even with an ideal amplifier one cannot beat the "quantum limit" (Braginsky, 1970; Giffard, 1976)

$$(\Delta X_1)_{\rm ql} = (\Delta X_2)_{\rm ql} \simeq (\hbar/m\omega)^{1/2}$$
$$\simeq 4\times 10^{-19} \text{ cm for } m \simeq 1 \text{ ton},$$
$$\omega \simeq 6\times 10^3 \text{ sec}^{-1}. \tag{30}$$

This quantum limit for resonant-bar detectors, like that for laser systems [Eq. (28b)], is a direct consequence of the Heisenberg uncertainty principle (Thorne *et al.*, 1978): In quantum mechanics the antenna's amplitudes $\hat{X}_1$ and $\hat{X}_2$ are Hermitian operators related to position $\hat{x}$ and momentum $\hat{p}$ by

$$\hat{X}_1 = \hat{x}\cos\omega t - (\hat{p}/m\omega)\sin\omega t,$$
$$\hat{X}_2 = \hat{x}\sin\omega t + (\hat{p}/m\omega)\cos\omega t. \tag{31}$$

The commutation relation $[\hat{x},\hat{p}] = i\hbar$ implies $[\hat{X}_1,\hat{X}_2] = i\hbar/m\omega$, which in turn implies the uncertainty relation

$$\Delta X_1 \Delta X_2 \geq \hbar/2m\omega. \tag{32}$$

Since the standard electronic readout techniques reveal X_1 and X_2 with equal accuracies, the uncertainty relation (32) implies the quantum limit (30).

For resonant-bar detectors at kilohertz frequencies, the amplifier limit (29b) and quantum limit (30) forbid the achievement of gravitational-waveburst sensitivities better than $h \sim 10^{-20}$. Fortunately, there exist techniques of monitoring a bar detector which can circumvent these limits. Those techniques—called "quantum nondemolition" and "back-action evasion"—are the subject of the second accompanying review article

(Caves *et al.*, 1980; see also Thorne *et al.*, 1979 and Braginsky *et al.*, 1980).

IX. CONCLUSION

Weber's original 1969 gravitational-wave detector had an rms noise of $\Delta l \sim 2\times10^{-14}$ cm, corresponding to a gravity-wave burst sensitivity $h \simeq 5\cdot 2\cdot \Delta l/l \sim 2\times10^{-15}$. By the end of the first generation (ca. 1975), rms noises had been reduced to $\Delta l \simeq 3\times10^{-15}$ cm, corresponding to a burst sensitivity of $h \simeq 3\times10^{-16}$. The second-generation bars and laser systems may achieve burst sensitivities $h\sim1\times10^{-18}$; and the third generation, with the help of quantum nondemolition techniques in the case of the bars, may reach $h\sim1\times10^{-21}$.

These sensitivities, at $f\sim 30$ to 10^4 Hz, look promising when compared to the theoretical estimates of the waves bathing Earth (Fig. 3): In passing from first generation to second, the experiments are pushing through the "cherished belief line" and into a realm (i) where serendipity could give many events but probably will not, and (ii) where the detectors can likely see a supernova with a highly asymmetric core anywhere in our galaxy. The third generation could have a good chance of picking up bursts once per month. By integrating up the signal for many days, the third generation may also have a good chance of detecting periodic sources—e.g., young pulsars.

At lower frequencies, $f\simeq10^{-2}$ to 10^{-4} Hz, Doppler tracking of spacecraft in the 1980s will probably be below the cherished-belief line and may reach the region, $h\sim10^{-16}$, where occasional bursts from supermassive black holes at the Hubble distance can be detected. However, the low-frequency region in the longer term probably belongs to optical tracking systems—laser interferometers or optical heterodyne systems in space ($h\sim10^{-21}$). Such systems would be able to detect a variety of different kinds of sources, including the waves from known binary star systems.

The future looks promising—but by no means certain! The search for gravitational waves is a game requiring long, hard effort with a definite risk of total failure—but with very great payoff if it succeeds.

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