An Experimental Study of the Effect of Particle Shape on Force Transmission and Mobilized Strength of Granular Materials

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Force chains have been regarded as an important hallmark of granular materials. Numerous studies have examined their evolution, properties, and statistics in highly idealized, often circular-shaped, granular assemblies. However, particles found in nature and handled in industries come in a wide variety of shapes. In this article, we experimentally investigate the robustness of force chains with respect to particle shape. We present a detailed analysis on the particle- to continuum-scale response of granular materials affected by particle shape, which includes the force transmission and mobilized shear strength. The effect of shape is studied by comparing experimental results collected from shear tests performed on 2D analog circular- and arbitrarily shaped granular assemblies. Particle shapes are directly discretized from X-ray CT images of a real sand sample. By inferring individual contact forces using the granular element method (GEM), we provide a direct visualization of the force network, a statistical characterization of the force transmission and a quantitative description of the shear strength in terms of rolling, sliding, and interlocking contact mechanisms. We report that force chains are less prevalent in assemblies of arbitrarily-shaped particles than in circular-shaped samples. Furthermore, interlocking is identified as the essential contact mechanism that (1) furnishes a stable structure for force chains to emerge and (2) explains the enhanced shear strength observed in the arbitrarily-shaped samples. These findings highlight the importance of accounting for particle shape to capture and predict the complex mechanical behavior of granular materials across scales. [DOI: 10.1115/1.4051818]

Keywords: mechanical properties of materials, micromechanics, stress analysis

1 Introduction

Granular materials are abundant in nature and highly manipulated in industries. Examples range from particles of sand to kernels of corn to pharmaceutical pills. Despite the seeming simplicity of the laws governing their individual constituents—particles, granular assemblies exhibit rich and complex mechanical behavior arising from the interplay between particles [1]. Striking features observed in granular materials are intimately linked to the discrete and heterogeneous character of the media [2,3]. Among these features, there has been a long-standing interest in studying the self-organization of contact forces into an anisotropic and heterogeneous network [4–6]. Under external loading, contact forces tend to align along the direction of maximum compression, eventually forming chain-like structures that serve as preferential pathways for force transmission. Such a phenomenon has become known as force chains [6].

Evidence of force chains was first furnished by photoelastic experiments performed on quasi-2D granular assemblies composed of birefringent disks [4,7]. By providing a direct visualization of stress distributions, photoelasticity has shown that particles participating in force chains are highly engaged in load bearing, whereas between chains, most particles carry little or no load. These strong spatial fluctuations are revealed by the probability density function of contact forces, which typically decays exponentially for forces greater than the mean [8,9]. Numerical simulations performed

with the discrete element method (DEM) [10] confirmed the prevalence of force chains in granular materials subjected to external loadings [6,8]. Force chains have been regarded as a key signature of granular materials and considered of central importance in coupling particle-scale processes to their overall constitutive behavior.

Photoelastic and DEM analysis have provided insights into the force transmission in highly idealized granular assemblies with simple geometries. Nevertheless, granular materials are composed of particles that come in a broad variety of shapes. Experimental results quantifying contact forces and characterizing their properties in arbitrarily shaped granular samples have yet to be collected. As a result, our understanding of the effect of particle shape on the force transmission is still very limited. Although force chains are widely believed to prevail in all granular materials, their existence in granular assemblies that yields realistic shape representation remains to be confirmed.

At the macroscopic scale, the crucial role of particle shape on the behavior of granular materials has been revealed by laboratory tests [11–17] and numerical studies [18–20]. Material characteristics intimately affected by particle shape include the mobilized shear strength, volume change, and permeability. In particular, as particle shapes deviate from idealized circles (in 2D) and spheres (in 3D), an increase in mobilized shear strength is observed [21,22]. It has been established that the mobilized shear strength of granular materials depends on the geometrical arrangement of the particles and on the force transmission [5]. Nonetheless, the particle-scale origin of the increased mobilized shear strength in arbitrarily shaped assemblies has yet to be clarified.

In this article, we raise the following questions: to which extent do the well-known features of force chains evidenced in idealized granular materials apply to assemblies of arbitrarily shaped particles? What is the influence of particle shape on the force

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Contributed by the Applied Mechanics Division of ASME for publication in the JOURNAL OF APPLIED MECHANICS. Manuscript received June 14, 2021; final manuscript received July 13, 2021; published online August 3, 2021. Tech. Editor: Yonggang Huang.

transmission in granular materials? What are the underlying particle-scale mechanisms that contribute to an enhanced mobilized shear strength in assemblies of arbitrarily shaped particles?

In an attempt to answer these questions, we present experimental results of shear tests performed on 2D analogue opaque granular assemblies. The role of shape is investigated by comparing data gathered from circular-shaped and arbitrarily shaped granular assemblies having the same particle size distribution and mechanical properties. Of particular interest are the contact forces, inferred in both assemblies by means of the granular element method (GEM) [23,24], from which the force chains are identified, the force transmission is studied, and strength-related quantities are derived.

For completeness, this article is organized as follows. We first introduce in Sec. 2 the experimental procedure that utilizes the custom-built shear apparatus outlined in Ref. [25]. We describe the design and manufacturing process of the granular assemblies as well as the measurements that are performed. In Sec. 3, we present the particle- to continuum-scale results obtained from shear tests performed at different initial packing densities. We conclude in Sec. 4 with a summary of our findings.

2 Experimental Procedure

2.1 Experimental Setup. Tests are performed with a custom-built shear apparatus described in the study by Marteau and Andrade [25]. A general view of the apparatus is shown in Fig. 1. This mechanical apparatus is designed to subject a two-dimensional analog granular assembly to (quasi-static) shear conditions over large deformation. An electro-mechanical linear actuator is attached to a shear cell and provides the shear mechanism by controlling the shear angle γ .

Figure 2 shows the general state of deformation and imposed boundary conditions on the shear cell. The shear cell consists of a horizontal deformable parallelogram that can undergo shear strain and normal strain in the y -direction (tilting and elongation of faces BC and AD). The sides AB and CD always remain horizontal and at a fixed length $L_x = 525$ mm, while the lateral sides (BC and AD) have a variable length in the y -direction L_y ($450 \text{ mm} < L_y < 630 \text{ mm}$). The shear angle γ is defined as the angle between the y -axis and the sides AD and BC. The length L_y and shear angle γ are measured at each load step using image-processing techniques. From these measurements, the components of the macroscopic strain tensor ϵ are computed as follows: $\epsilon_{xx} = 0$, $\epsilon_{yy} = \Delta L_y / L_y$, and $\epsilon_{xy} = \frac{1}{2} \tan \gamma$, where ΔL_y is the change in length in the y -direction between the initial and deformed configurations. In addition, a constant normal stress σ_N is applied to the side CD of the shear cell using a system of weights and pulley and transmitted to the granular assembly. Images of the granular assembly are acquired using an optical imaging system (Allied Vision Prosilica GT4907 15.7 Megapixel CCD camera attached to a Nikkor AF 105 mm $f/2.8$ lens) that is placed above the apparatus.

Fig. 1 Picture of the experimental setup

Fig. 2 Schematic representation of the state of deformation and prescribed boundary conditions on the shear cell

2.2 Granular Assemblies. Experimental tests are conducted on granular assemblies composed of either circular-shaped particles or arbitrarily shaped particles. While the circular-shaped and arbitrarily shaped granular assemblies differ in shape, both assemblies have the same mechanical properties and particle size distribution. Details on the design and manufacturing, morphology, and particle size distribution of the assemblies are presented below.

A visual schematic of the development process for the arbitrarily shaped granular assembly is depicted in Fig. 3. The arbitrarily shaped granular assembly is devised based on a real sample of a rounded natural sand (Caicos ooids). We start from three-dimensional volume data of sand particles obtained directly from X-ray computed tomography (XRCT) [26,27]. A typical 2D slice image of the specimen obtained with XRCT is shown in the left panel of Fig. 3. Individual sand particle boundaries in the sample are detected using a 2D level set representation as obtained by Lim et al. [26], following the characterization methodology proposed by Vlahinić et al. [28]. From this characterization technique of the particle boundaries, 11 particle shapes are retained for their different morphological properties. Sphericity and roundness (or angularity) are chosen as morphological descriptors due to their widespread use in the fields of geomechanics and geology [29,30]. Figure 4 depicts the selected particle contours as a function of sphericity and roundness. Particle shapes range from elongated to nearly spherical and from rounded to well rounded.

The selected particle shapes are systematically duplicated and scaled such that the particle diameters D follow a log-normal distribution. A log-normal distribution is chosen as it is prevalent in typical granular systems found in industries and in nature [32]. Computer-aided design (CAD) models are created by extending the 2D particle contours into 3D with an extrusion depth set to 20 mm. The generated 3D CAD models are used as input files for 3D printing. The XRCT to 3D printing process introduced in Fig. 3 enables the fabrication of a 2D analog granular assembly that captures the complexity of particle shapes found in real sand.

Both assemblies, circular and arbitrarily shaped, were fabricated using the same manufacturing process. Particles were 3D printed using Stratasys Objet500 Connex3 printer.² This printer uses inkjet 3D printing technology with a microscopic layer resolution and accuracy of $16 \mu\text{m}$ [33]. Printing material is a rubber-like material (Stratasys FLX9895DM) that is obtained by mixing a rigid

²<http://www.stratasys.com>

Fig. 3 XRCT to 3D printing process. The reader is referred to Sec. 2.2 for a detailed discussion of the steps involved.

plastic (Stratasys VeroWhitePlus RGD835) with a flexible elastomer (Stratasys TangoBlackPlus FLX980). Factors such as the orientation of the print, printing mode, support material composition and removal, and environmental conditions of storage of raw materials were kept constant for all 3D-print jobs. Hence, the printed particles constituting the circular-shaped and arbitrarily shaped assemblies have identical mechanical properties [34,35]. In particular, the particles have a Young's modulus $E = 63$ MPa and a Poisson's ratio ν of approximately 0.5.

For both assemblies, the distribution of particle diameters D is fitted to a log-normal distribution with mean μ and standard deviation σ , such that $\ln(D) \sim \mathcal{N}(\mu, \sigma^2)$. The distribution parameters were chosen to be $\sigma = 0.2$ and $\mu = \ln(25)$. For the arbitrarily-shaped granular assembly, the particle diameter D is defined as the diameter of the smallest circumscribed circle D_{cir} , as illustrated in the inset of Fig. 4. The circular-shaped sample is composed of 313 particles, while the arbitrarily shaped sample has a total of 398 particles. In both assemblies, particle diameters D ranges between 14 mm $< D < 42$ mm.

2.3 Measurements. The mechanical apparatus presented in Sec. 2.1 allows for simultaneous measurements from the particle to the continuum scale. A full characterization of the particle-scale quantities that control the mechanical behavior of granular materials is extracted throughout the shear deformation. First, a description of

Fig. 4 Selected particle shapes of real sand (Caicos ooids) as a function of sphericity and roundness. Sphericity and roundness are computed using the numerical method developed in Ref. [31]. Inset: Definition of circumscribed circle and diameter.

the geometrical arrangement, or fabric, of the granular assembly is obtained using image-processing techniques. The watershed segmentation algorithm [36–38] from MATLAB IMAGE PROCESSING toolbox is employed in combination with a priori knowledge of the true particle contours (as shown in Fig. 4) to determine particle and contact positions. Second, a description of particle kinematics, i.e., displacement, rotation, and strain field, is extracted using the digital image correlation (DIC) technique [39,40]. Third, a description of contact forces is obtained using the GEM [23–25]. GEM provides a mathematical framework that, when combined with fabric information and average particle strains, allows for the inference of the contact forces. Unlike photoelasticity, the GEM methodology enables reconstruction of the force network in opaque, arbitrarily shaped granular assemblies.

At the continuum scale, the stress state ($\bar{\sigma}_{xx}$, $\bar{\sigma}_{yy}$, $\bar{\tau}_{xy}$) of the granular assembly subjected to shear deformation is computed. The Cauchy stress tensor $\bar{\sigma}$ is linked to particle-scale quantities and expressed in terms of contact forces and fabric according to the following well-established equation [41]:

$$\bar{\sigma} = \frac{1}{\Omega} \sum_{\alpha=1}^{N_c} \text{sym}(f^\alpha \otimes d^\alpha) \quad (1)$$

where N_c is the total number of contact points, Ω is the total volume of the granular assembly, f^α is the contact force, and d^α is the branch vector at the contact α . The total volume of the granular assembly Ω is equal to the particles thickness ($t = 20$ mm) times the total area, which is obtained from the 2D particle contours. GEM associated with Eq. (1) enables the connection between contact forces in realistic-shaped granular materials and macroscopic stresses.

3 Experimental Results

Quasi-static simple shear tests are performed to investigate how particle shapes affect the force transmission and particle-scale mechanics that ultimately control the mobilized shear strength. The subsequent sections provide experimental results of a series of four shear tests. Two tests are conducted on the circular-shaped assembly, while two other tests are conducted on the arbitrarily shaped assembly. At the beginning of each test, the particles are randomly arranged into the shear cell, so that the sample is initially isotropic with no preferred contact orientation. For both circular-shaped and arbitrarily shaped assemblies, one dense and one loose samples are created by varying the initial packing density.

The particles can assume a virtually infinite number of packing configurations while statistically retaining consistent macroscopic properties.

Prior to starting the shearing stage, a normal stress of $\sigma_N = 26.3$ kPa is applied. The granular assembly is then sheared at a constant

rate $\dot{\gamma} = 0.002 \text{ s}^{-1}$, and image sequences are acquired at a frame rate of seven images per second. During the shear deformation, measurements described in Sec. 2.3 are performed.

3.1 Macroscopic Measurements. Figure 5 compares the macroscopic mechanical response of circular-shaped and arbitrarily shaped samples with initial dense and loose packing densities. In this figure, stress ratio μ , volumetric strain ϵ_v , void ratio e , and dilatancy β are plotted as a function of shear strain γ with each curve corresponding to a different assembly.

The stress ratio μ represents the mobilized shear strength of cohesionless granular materials and is defined as the ratio of shear stress τ_{xy} to normal stress σ_{yy} . The stress components τ_{xy} and σ_{yy} are computed from particle-scale quantities according to Eq. (1). In Fig. 5(a), the observed stress-strain behaviors are characteristic of dense and loose samples under shear deformation [42–44]. Dense samples (solid lines) exhibit a linear elastic behavior followed by a peak in shear stress and subsequent strain softening, whereas in loose samples (dashed lines), no peak is observed. At large shear strain values, circular-shaped (circle markers) and arbitrarily shaped (asterisk markers) assemblies show a tendency to stabilize around an asymptotic stress ratio value μ_c that is independent on the initial packing density (dense or loose).

Evolution of volumetric strain ϵ_v as a function of shear strain γ is presented in Fig. 5(b). The volumetric strain is simply related to the normal strain, such that $\epsilon_v = \text{tr}(\epsilon) = \epsilon_{yy}$. In terms of volumetric

deformations, we once again observe typical behaviors of dense and loose samples subjected to shear. Dense samples dilate after an initial contraction, while loose samples only contract. For large shear deformations, the volumetric strain of both dense and loose samples reaches a constant value.

As the particles are considered incompressible, the change in volume observed in Fig. 5(b) occurs due to change in the volume of voids, measured by the void ratio e . The void ratio e is defined as the ratio of the volume of voids to the volume of solid particles and is plotted as a function of the shear strain γ in Fig. 5(c). Loose samples (dashed lines) start at an initial void ratio $e_0 = 0.295$, while dense samples (solid lines) start at a lower value of $e_0 = 0.27$. At large shear strain values, loose and dense samples evolve toward an identical and constant value of void ratio e_c that is different for arbitrarily shaped (asterisk markers) and the circular-shaped (circle markers) assemblies.

The Reynolds dilatancy β [45] describes the change in volume associated with the shear deformation of granular materials and relates the volumetric and shear strain rates by $\dot{\epsilon}_v = \beta \dot{\gamma}$. Figure 5(d) displays the evolution of dilatancy β as a function of the shear strain γ . It can be seen that, for all samples, dilatancy β vanishes for $\gamma > 0.25$.

These macroscopic results show that, for shear strains $\gamma > 0.25$, all samples reach critical state, conceptually defined as the equilibrium state in which no further change in volume and stress state occur with increased shear deformation. Nevertheless, the critical state values of stress ratio μ_c and void ratio e_c are different for the

Fig. 5 Macroscopic results: (a) stress ratio μ , (b) volumetric strain ϵ_v , (c) void ratio e , and (d) dilatancy β as a function of shear strain γ

circular-shaped and the arbitrarily shaped assemblies. Under a normal stress of $\sigma_N = 26.3$ kPa, the arbitrarily shaped assembly exhibits a significantly higher mobilized stress ratio of $\mu_c \approx 0.38$, and thus higher shear strength, than the circular-shaped assembly for which $\mu_c \approx 0.27$. The arbitrarily shaped assembly reaches a critical void ratio $e_c \approx 0.275$ that is lower than that of the circular-shaped assembly ($e_c \approx 0.293$).

The macroscopic results presented in Fig. 5 are consistent with previous studies that investigated the effect of shape on the bulk properties of granular materials [11–17]. The overall mechanical behavior of real granular materials (e.g., sand) subjected to shear deformations is reproduced by the conducted tests, even though the particles are quasi-2D and the sample sizes (i.e., the total number of particles) are smaller than typical real granular samples. This qualitative agreement confirms the validity of the proposed experimental capability to investigate the effect of shape on the force transmission and particle-scale mechanics of realistic granular materials.

3.2 Particle-Scale Measurements. By examining the response of granular assemblies with different particle shapes and initial packing densities under shear deformation, differences in the macroscopic mechanical behavior have been revealed. As the mechanical behavior of granular materials is fundamentally encoded at the particle scale, in this section, we gain insights into the particle-scale properties of the different granular assemblies (dense and loose, circular and arbitrarily shaped) from the measurements described in Sec. 2.3. Figures 6–9 present snapshots of 2D-DIC results and contact force networks obtained with GEM for the four different assemblies at a shear strain $\gamma = 0.25$. We emphasize that, as shown in Fig. 5, all samples have reached a critical state at $\gamma = 0.25$.

Particle rotations $\bar{\theta}$ at $\gamma = 0.25$ obtained from 2D-DIC are plotted in Fig. 6. Positive $\bar{\theta}$ values correspond to counterclockwise rotations, negative values to clockwise rotations, while the center contour depicts nearly zero rotation ($-0.5^\circ < \bar{\theta} < 0.5^\circ$). It can be observed that, for all samples, the particle rotations are in majority counterclockwise, which is consistent with the imposed (counterclockwise) shear angle γ . Particles with counterclockwise rotation form random clusters, whereas particles with clockwise rotation are more isolated. The intraparticle strain field ϵ_{xy} measured using 2D-DIC is represented in Fig. 7. Strain fields ϵ_{xx} and ϵ_{yy} are also measured but not depicted in this article. The full-field intraparticle stress distribution (σ_{xx} , σ_{yy} , and σ_{xy}) and associated principal stresses (σ_1 and σ_2) is then deduced from the full-field intraparticle strain measurements (ϵ_{xx} , ϵ_{yy} , and ϵ_{xy}) assuming linear elasticity. The difference in principal stresses $\sigma_1 - \sigma_2$ is plotted in Fig. 8. Figures 7 and 8 show that 2D-DIC measurements of strains and stresses provide qualitative assessments and insights into the force

Fig. 6 Particle rotation at $\gamma = 0.25$

Fig. 7 Contour distribution of strain component ϵ_{xy} at $\gamma = 0.25$ obtained from DIC measurements

Fig. 8 Contour distribution of difference in principal stresses $\sigma_1 - \sigma_2$ at $\gamma = 0.25$

distribution in circular-shaped and arbitrarily shaped granular assemblies under shear. Full-field intraparticle stresses (σ_{xx} , σ_{yy} , and σ_{xy}) and fabric information are used as inputs into the GEM mathematical framework to infer normal and tangential contact forces. The reconstructed force network is plotted in Fig. 9. Figures 9(b) and 9(d) offer the first visualization of contact forces in a granular assembly composed of particles whose shapes are directly extracted from a real sand sample. As one can see in

Fig. 9 Contact forces inferred with GEM superimposed on difference of principal stresses $\sigma_1 - \sigma_2$ at $\gamma = 0.25$

Figs. 9(b) and 9(d), in the arbitrarily shaped assembly, we observe the formation of chain-like structures that resemble force chains. Furthermore, at first glance, the force transmission is different in each assembly. Particle shape appears to play a role in the development of force chains as the pattern of force transmission looks less heterogeneous and more uniformly spread in the arbitrarily shaped samples (Figs. 9(b) and 9(d)) when compared to the circular-shaped samples of similar initial packing density (Figs. 9(a) and 9(c)). We report two notable differences between the force transmission in circular-shaped and arbitrarily shaped assemblies. First, in the circular-shaped assemblies, most particles carry little or no load, whereas in the arbitrarily shaped assemblies, a larger proportion of particles belongs to the load-bearing network. Second, the more localized force chains in the circular-shaped samples lead to significantly higher contact forces. These aspects are analyzed in more detailed in the following section.

3.3 Evolution of Particle-Scale Mechanics

3.3.1 Particle Rotations. The evolution of mean and standard deviation (std) of particle rotations $\bar{\theta}$ as a function of the shear strain γ for the four different assemblies is plotted in Fig. 10. Figure 10 shows that the mean value of particle rotation $\bar{\theta}$ linearly increases as the granular assembly is sheared and is equal for all samples. Hence, the mean value of particle rotation $\bar{\theta}$ is independent of particle shape and initial packing fraction. In addition, the mean rotation is compared with the rigid rotation angle Ω associated with the simple shear deformation of the cell. For any given deformation, the deformation gradient $\mathbf{F}$ can be decomposed into the product of a stretch tensor $\mathbf{U}$ and a rotation matrix $\mathbf{R}$ using the polar decomposition concept, such that $\mathbf{F} = \mathbf{U} \cdot \mathbf{R}$. The previous expression separates deformation (described by $\mathbf{U}$) and rigid body rotation (described by $\mathbf{R}$). The rigid rotation Ω corresponds to the angle of the rotation matrix $\mathbf{R}$. In Fig. 10, we observe that the mean value of particle rotation $\bar{\theta}$ agrees with the non-zero rigid body rotation Ω associated with the macroscopic deformation of the shear cell. Thus, the mean value of particle rotation $\bar{\theta}$ is dependent on the imposed rotational component of deformation. In other words, the essence of particle rotation is related to the macroscopic rigid rotation. It can be seen in Fig. 10 that the standard deviation (std) of particle rotation $\bar{\theta}$ also increases linearly as the granular assembly is sheared. Nevertheless, at the end of the shearing process, the standard deviation is larger for the circular samples than for the arbitrarily shaped samples, indicating more variations from the mean value. This wider rotation distribution suggests that the circular assemblies are subjected to more rotational motion. Under the same conditions, a circular particle can rotate more easily compared to an arbitrarily shaped particle inside their respective assemblies. As it has been previously reported [30], a decrease in sphericity (e.g., more elongated particles) increases the tendency of rotational frustration. Figure 10 shows that the standard deviation is the same for dense and loose circular-shaped assemblies and dense and loose arbitrarily shaped assemblies. Hence, the ability of particles to rotate depends

Fig. 10 Mean and standard deviation of particle rotations $\bar{\theta}$ as a function of shear strain γ for the four different samples

Fig. 11 Evolution of coordination number as a function of shear strain γ

on the particle shape but seems to be independent of the initial packing density for the relatively dense systems studied in this article. More investigation is warranted into granular fluids and gasses.

3.3.2 Coordination Number. The average number of contacts per particle, defined as the coordination number z , is an important particle-scale quantity that influences the stability and force transmission of a granular assembly [46]. In Fig. 11, the coordination number z is plotted as a function of the shear strain γ . The coordination number z provides information on the creation and loss of contacts. Initially (at $\gamma = 0$), as expected, the coordination number z of loose samples (dashed curves) is smaller than the one of dense samples (solid curves). As the granular assemblies are sheared, the coordination number z of dense samples decreases, corresponding to a loss of contacts. Simultaneously, the coordination number z of loose samples increases, owing to the creation of new contacts. We emphasize that the samples with initial void ratio e_0 far from their critical void ratio e_c , i.e., the dense circular-shaped and loose arbitrarily shaped samples (respectively, solid line with circle markers and dashed line with asterisk markers), manifest larger variation of coordination number z . Once critical state is reached, for $\gamma > 0.25$, the coordination number z tends to stabilize to a constant value that is larger for the arbitrarily shaped assemblies than for the circular assemblies. This constant value of coordination number z is an indication that the contact network has evolved toward a steady structure. The fact that the coordination number z at critical state is higher for the arbitrarily shaped assemblies than for the circular-shaped assemblies has many implications for the particle-scale mechanics of granular materials. A higher coordination number z suggests that more paths are available for force propagation, which ultimately influences the distribution of forces toward strongly less heterogeneous force networks. A higher coordination number z also increases the stability of the contact network structure as there are more mechanical constraints.

3.3.3 Force Distribution. Although the force networks of both the circular-shaped and arbitrarily shaped assemblies are heterogeneous and characterized by the formation of force chains, Fig. 9 suggests that the distribution of contact forces is affected by the particle shape. In this section, the force transmission is studied by computing the probability density function of normalized forces $f_n/\langle f_n \rangle$, where $\langle f_n \rangle$ is the mean value of normal forces. Figure 12 shows the probability density function $P(f_n)$ of normal forces f_n at a critical state in the dense and loose samples in a log-linear scale. The data are averaged from several snapshots at critical state, i.e., for $\gamma > 0.25$. Following Radji et al. [47], we designate the forces below the mean normal force ($f_n < \langle f_n \rangle$) as the “weak” network and the forces above the mean normal force ($f_n > \langle f_n \rangle$) as the “strong” network. The concept of “weak” and “strong” networks provides a quantitative description of force network that will be used in the remaining of this article. As is often reported in the literature on granular materials [8,9,48,49], we find that the

Fig. 12 Probability density function $P(f_n)$ of normal forces f_n in a log-linear scale at critical state ($\gamma > 0.25$) for the (a) dense and (b) loose samples

Fig. 13 Possible mechanisms that can occur at a contact α

probability density function is characterized by an exponential decay for the forces above the mean force $\langle f_n \rangle$ (i.e., in the “strong” network), such that

$$P(f_n) \propto \exp(-\beta_n f_n) \quad (2)$$

where β_n is a positive constant. The results presented in Fig. 12 clearly show a broader uniformity of the normal force distribution in the assemblies composed of arbitrarily shaped particles than in the circular-shaped assemblies. In other words, there is a significantly lower chance of an individual contact force largely exceeding the mean force in the arbitrarily shaped samples than in the circular-shaped samples. This broader uniformity is consistent with the higher coordination number z measured in the arbitrarily-shaped assemblies, as it provides more pathways for force transmission. In the circular-shaped assemblies, while the number of “strong” contacts declines, stronger force chains occur. In close connection with the shear strength, the force transmission is strongly influenced by particle shapes.

3.3.4 Contact Deformation Mechanisms. To accommodate an imposed shear deformation, the granular structure must rearrange through particle-to-particle interactions at contact points [50]. We

Fig. 14 Contribution of contact mechanisms to the total number of contacts N_{tot} . Contact mechanisms are identified as interlocking, rolling, and sliding.

Fig. 15 Normal component of contact forces at $\gamma = 0.25$ for the dense (a) circular-shaped and (b) arbitrarily shaped assemblies. Forces are categorized according to their dominant contact mechanisms: sliding, rolling, and interlocking.

identify three possible rearrangement mechanisms at a contact point α between two particles p and q . As illustrated in Fig. 13, particles can interlock or move past each other by rolling or sliding. Definitions, solely based on geometrical criteria, are provided for each contact mechanism.

Interlocking occurs when two particles rotate (or translate) together as a single rigid body. The distance between any arbitrary chosen points on the two particles remain constant during the motion and rotate by the same amount $\Delta\theta$. Hence, there is no relative movement between the two particles. Rolling occurs when two particles rotate over each other in a gear-like manner such that the contact point changes its relative location from the particles centroid. When particles are rolling, the arc lengths representing the displacement of the contact point α have opposite signs. In other words, while one particle rotates clockwise, the other one rotates counterclockwise. Sliding occurs when both particles rotate through the same angle $\Delta\theta$ and the arcs representing the displacement of the contact point α have the same signs. When sliding,

the particles rotate both clockwise or counterclockwise. A fourth category is defined as a combination of these mechanisms and occurs when one contact mechanism is not prevalent, i.e., when the geometrical criteria defined for either interlocking, rolling, or sliding are not strictly met. Given these definitions, we identify the population of contacts for which each mechanism is dominant. The total number of contacts can be decomposed as follows:

$$N_{tot} = N_I + N_R + N_S + N_* \quad (3)$$

where N_{tot} is the total number of contacts in the assembly and N_I , N_R , and N_S are the number of contacts where, respectively, interlocking, rolling, and sliding are the dominant mechanisms. N_* corresponds to the number of contacts for which the mechanism is not clearly identified (e.g., combination of contact mechanisms, creation, or destruction of contact). The contact contribution of each mechanism is simply obtained by dividing each population by the total number of contacts N_{tot} and is plotted in Fig. 14 as a function of the shear strain γ . In this figure, it can be seen that rolling (gold shade) is the prevalent particle-scale deformation mechanism for the circular-shaped granular assembly with about 50% of N_{tot} throughout shear deformation. Interlocking contribution (black shade) is 20–25% for the circular-shaped samples and about 40% for the arbitrarily shaped samples. In the arbitrarily shaped assembly, no dominant deformation mechanism is identified as rolling and interlocking have approximately the same contribution. An assembly composed of arbitrarily shaped particles exhibits less rolling and more interlocking than one composed of circular-shaped particles. The difference in rolling contribution between the circular-shaped and arbitrarily shaped assemblies is consistent with the results obtained from standard deviation of particle rotation $\bar{\theta}$ presented in Fig. 10. Owing to the prevalence of the rolling mechanism, circular-shaped particles experience a wider range of particle rotation. Correspondingly, particle shape induces a resistance to particle rotation that results in a lower contribution from rolling contacts. For all samples, relatively few contacts are sliding (red shade), accounting for 12–15% of N_{tot} . The white shade corresponds to the contacts for which a mechanism is not clearly identified, which is composed of approximately 10% of N_{tot} .

Fig. 16 Contribution of contact mechanisms to the total shear strength μ

Figure 15 offers another visualization of the force network in the dense circular-shaped and arbitrarily shaped assemblies at $\gamma = 0.25$. In this figure, the normal component of forces is categorized according to its dominant contact mechanism: sliding, rolling, or interlocking. The thickness of the segments joining the particle centroids is proportional to the magnitude of the normal forces. Figure 15 shows that, at interlocking contacts (black lines), relatively large forces are transmitted, even more so in the circular-shaped assembly (see Fig. 15(a)).

In an attempt to connect particle-scale processes to the continuum behavior of granular materials, we investigate the contribution of each contact mechanism to the shear strength μ that is described by the stress ratio $\mu = \tau_{xy}/\sigma_{yy}$. The contribution of each contact mechanism to the total shear stress τ_{xy} is obtained from the general equation of the stress tensor (Eq. (1)) by limiting the summation to the contacts belonging to each deformation mechanism [51]. Accordingly, partial shear strength μ_I , μ_R , μ_S , and μ_* are calculated for each deformation mechanism, such that the following additive decomposition holds:

$$\mu = \mu_I + \mu_R + \mu_S + \mu_* \quad (4)$$

The contribution of each deformation mechanism to the shear strength μ as a function of shear strain γ is presented in Fig. 16. This figure reveals that the additional shear strength measured in the arbitrarily shaped assembly is furnished by the interlocking mechanism. It can be seen that, at critical state ($\gamma > 0.25$), the partial shear strength μ_I due to the interlocking in the arbitrarily shaped assembly is almost double the value of μ_I in the circular-shaped assembly. In the circular-shaped assembly, even though rolling is the prevailing deformation mechanism in terms of contact contribution, the shear strength μ_R associated with rolling is similar to the shear strength μ_I associated with interlocking. Correspondingly, in the arbitrarily shaped assembly, although rolling and interlocking have approximately the same contribution to N_{tot} , rolling does not provide as much shear strength as interlocking. These results alongside the force networks depicted in Fig. 15 suggest that interlocking is a stable mechanism for larger contact forces to develop, which accordingly enhances the load-bearing capacity of the granular assembly. Moreover, as shown in Fig. 16, the shear strength gain associated with sliding is relatively small. Sliding occurs when the tangential force f_t reaches a critical value governed by the Coulomb friction law, such that $|f_t| = \mu_p |f_n|$, where μ_p is the interparticle friction coefficient. A visual inspection of Fig. 15 reveals that most sliding contacts belong to the “weak” network ($f_n < \langle f_n \rangle$), which results in a small value of the partial shear strength μ_S . Finally, by adding the shear strength of nonidentified contacts μ_* , the macroscopic stress–strain curve is reconstructed. Figure 16 offers insights into the particle-scale origin of shear strength in granular materials.

4 Summary and Closure

In this article, we carried out a detailed study of the effect of particle shape on the mechanical behavior of sheared granular materials across scales. Particle shapes were directly extracted from a real sand sample to engineer a 2D analog granular assembly. At the particle scale, we provided unprecedented visualizations and quantitative descriptions of the force network in assemblies composed of arbitrarily shaped particles. Particular attention was given to the influence of particle shape on particle-scale quantities that are known to control the mechanical response of granular materials, i.e., the average particle rotation, the coordination number, and the distribution of contact forces. At the macroscopic scale, experimental results of mobilized shear strength and volumetric strain exhibit a typical shear response of dense and loose samples. Once a critical state is reached, we report an enhanced mobilized shear strength and a reduced void ratio in arbitrarily shaped assemblies when compared to corresponding results obtained from circular-shaped assemblies.

The proposed experimental approach lead to significant new insights into the force transmission and shear strength of granular materials. We addressed one of the central question of the paper: to what degree do the force chains observed in idealized granular assemblies pertain to assemblies with arbitrary and complex particle shapes? The results showcased that, in the arbitrarily shaped assembly, the contact forces tend to form heterogeneous chain-like structures that are characteristic signatures of the emergence of force chains. Nonetheless, as particle shapes derive from idealized circles, contact forces are more uniformly distributed and have a significantly lower chance to reach large values. As a result, force chains seem to be less prevalent in arbitrarily shaped assemblies. This finding raises the interesting possibility that this trend may be even more accentuated in 3D granular samples and that the existence of force chains may be biased by the technique used to observe them (e.g., photoelasticity in 2D circular-shaped assemblies). Further studies exploring the results accuracy and repeatability in assemblies with different initial particle positions, particle shapes, and a larger number of particles are warranted.

The force transmission in the circular-shaped and arbitrarily shaped assemblies was further investigated by categorizing the contacts depending on their rolling, sliding, and interlocking mechanisms. In circular-shaped assemblies, there seems to be a clear structuring of interlocking contact forces into filaments, which favors the emergence of force chains. Our experimental results suggest that force chains travel through interlocking contacts that seem to drive strength. The enhanced mobilized shear strength in arbitrarily shaped assemblies was elucidated by analyzing the relative contribution of rolling, sliding, and interlocking contacts to the total shear strength. We report that, while the fraction of interlocking contacts remains fairly constant, their contribution to mobilized strength increases as shear deformation progresses. Interlocking contacts enable the development of a stable force network, which results in greater shear strength. Rolling and sliding are not as favorable contact mechanisms to driving mobilized strength.

Our findings demonstrate that a simplified model of circular-shaped grains cannot reflect the complex interactions between grains in real arbitrarily-shaped granular assemblies. By extracting and understanding realistic particle-scale data, this research lays the groundwork to develop general, physic-based constitutive models for effectively capturing the behavior of granular materials.

Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The datasets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request.

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