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Heavy Quarks and e^+e^- Annihilation*

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The effects of new, heavy quarks are examined in a colored quark-gluon model. The e^+e^- total cross section scales for energies far above any quark mass. However, it is much greater than the scaling prediction in a domain about the nominal two-heavy-quark threshold, despite $\sigma_{e^+e^-}$ being a weak-coupling problem above 2 GeV. We expect spikes at the low end of this domain and a broad enhancement at the upper end.

We report some theoretical work on e^+e^- annihilation in asymptotically free, colored quark-gluon models of hadronic matter. Our fundamental assumption is that in addition to the light quarks that make up ordinary hadrons, there is a heavy quark, such as the charmed $\mathcal{P}'$. This has been suggested in several other contexts[1] and is consistent with the observed scaling and successful sum rules of inelastic lepton-hadron scattering. We argue that at energies well above the $\bar{\mathcal{P}}'\mathcal{P}'$ threshold ("threshold" and "mass" having technical definitions which in no way imply the existence of physical quarks), the total hadronic cross section scales as in the free-quark model because of the smallness of the asymptotic effective coupling. Scaling also holds in a region well above the $\bar{\lambda}\lambda$ threshold and well below the $\bar{\mathcal{P}}'\mathcal{P}'$ threshold, with the magnitude set by the light-quark charges. However, there are large enhancements in a finite region above and below the $\bar{\mathcal{P}}'\mathcal{P}'$ threshold. We examine the behavior in this region and the approach to the asymptotic region above it.

Consider the Lagrangian $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\bar{\Psi}(i\not{D}-m)\Psi$, where $F_{\mu\nu}$ is the non-Abelian gauge-covariant curl; Ψ is several quark color multiplets: e.g., $\Psi=\mathcal{P}_i, n_i, \lambda_i, \mathcal{P}_i'$, where i runs over colors; D_μ is the gauge-covariant derivative; and m is the quark mass matrix. We take the color gauge symmetry to be exact, giving rise to strong forces at large distances. Hence the gauge fields are massless, and each quark color multiplet has a given mass. We imagine $m_{\mathcal{P}}$, $m_{\mathfrak{N}}$, and m_λ to be small (<1 GeV) while $m_{\mathcal{P}'}>1$ GeV.

In renormalizing the theory, we define g in terms of the two- and three-point functions at some Euclidean momentum configuration of scale M. If asymptotic freedom is to explain Bjorken scaling, then for $M=2$ GeV, $\alpha_s=g^2/4\pi$ must be small. m is related to the bare mass matrix m_0 by $m=Zm_0$, where Z is adjusted so that the $\mathcal{P}'$ propagator has a pole at $\not{p}=m_{\mathcal{P}'}$ to any finite order of perturbation theory.

The renormalization-group apparatus implies that in the one-photon approximation $\sigma(e^+e^-\to\text{hadrons})$ is of the form $\sigma(s,g,m,M)=\sigma(s,\bar{g}(s),\hat{m}(s),s^{1/2})$, where s is the square of the center-of-mass energy, $\bar{g}\approx g[1+g^2b\ln(s/M^2)]^{-1/2}$, and $\hat{m}\approx m[1+g^2b\times\ln(s/M^2)]^d$ for small $\bar{g}$, with b and d positive group-theoretic constants. In particular, the total cross section, a function of a *single* energy, is governed by $\bar{g}(s)$. Such is not the case for any partial rate. If we are interested in a range of s such that $\ln(s/M^2)=O(1)$, perturbation theory in

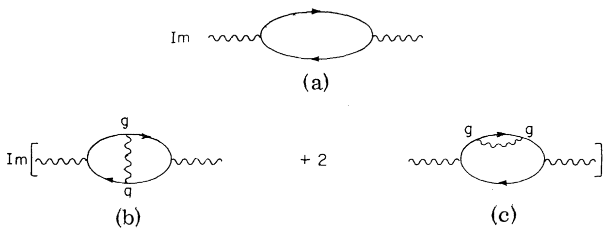

FIG. 1. Contributions to the total e^+e^- cross section, to be summed over light and heavy quarks.

g and in $\bar{g}$ are equivalent, and $\hat{m} \approx m$. *However, perturbation expansions are only to be trusted when successive terms are systematically smaller.*

The first two orders in α_s contributing to $\sigma_{e^+e^-}$ involve the graphs of Fig. 1. For light quarks, the calculation can be done with the masses scaled to zero. The mass-singularity theorem of Kinoshita[2] assures us that no singularities exist to any order of perturbation theory since we are calculating a total transition probability. Thus large logarithms do not invalidate the perturbation expansion. We emphasize that it is only the total cross section that can be calculated in this way. Partial rates, details of final states, and the question of whether quarks exist as physical particles involve logarithmic singularities that cancel in the total cross section.

The perturbation expansion could be invalidated as a result of the presence of multiple-light-quark thresholds in higher orders with their attendant small subenergies. In a forthcoming paper, we will examine this problem and argue that these contributions will remain small corrections to the dominant graphs of Fig. 1.

The $\mathcal{P}'$ mass is $O(\sqrt{s})$ and so it must be retained. We make use of identical electrodynamic calculations[3,4] and exhibit their contributions along with the light-quark contribution:

$$R(s) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{4\pi\alpha^2/3s} = \sum_{\substack{\text{light}\\ \text{quarks}}} Q_i^2 \left(1 + a\frac{3\alpha_s}{4\pi} + \ldots\right) + \sum_{\substack{\text{heavy}\\ \text{quarks}}} Q_i^2 \theta(s - 4m_{\mathcal{P}'}^2) v \frac{3 - v^2}{2}[1 + a\alpha_s f(v) + \ldots], \tag{1}$$

where $v = (1 - 4m_{\mathcal{P}'}^2/s)^{1/2}$, $a = \frac{4}{3}$ for an SU(3) color group, and the two sums over squares of quark charges are 2 and $\frac{4}{3}$, respectively, in the three-quartet model. $f(v)$ is approximated to $\pm 1\%$ by[4]

$$f(v) = \frac{\pi}{2v} - \frac{3+v}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right). \tag{2}$$

As $v \to 0$, $f(v) \propto 1/v$. This behavior comes from Fig. 1(b) and is a consequence of a Coulomb-like final-state interaction. In nth order, n gluon ladder exchanges give n factors of $1/v$, and this breakdown of perturbation theory for small v is connected to a breakdown below $4m_{\mathcal{P}'}^2$, responsible for the formation of positroniumlike bound states. For s large enough so that, say, $\frac{4}{3}\alpha_s f(v) \lesssim \frac{1}{2}$, second-order perturbation theory can be used to calculate the approach to free-quark-model scaling. The approach is more rapid than the $1/\ln s$ approach that follows from $\bar{g} \to 0$, this effect only becoming relevant when $\ln(s/M^2) \sim \pi/\alpha_s$. For reasonable values of α_s (including the value we determine shortly), the final-state interaction should produce a 15–20% drop in R from $s \approx 25$ GeV^2 to the SPEAR-II limit of $s \approx 81$ GeV^2. Careful estimates will be presented in a future paper.

Another breakdown of the perturbation expansion arises from the non-Abelian structure of the theory. As $s \to 4m_{\mathcal{P}'}^2$, the typical momentum flowing through a gluon line in Fig. 1 goes to zero. A measure of this momentum is $s/4 - m_{\mathcal{P}'}^2$ and when this becomes less than about 1 GeV^2, higher-order effects will be large. The effect of $\mathcal{P}'$ below $4m_{\mathcal{P}'}^2$ is first seen in order α_s^3, e.g., Fig. 2. This is a small correction to $R(s)$ until $4m_{\mathcal{P}'}^2 - s \lesssim 4$ GeV^2 when higher-order corrections typically involve soft gluons. Altogether, for about 8 GeV^2 centered on $4m_{\mathcal{P}'}^2$, perturbation theory breaks down as a result of non-Abelian effects.

The integral of $R(s)$ over this region can be bounded since $R(s)$ is related to the spacelike vacuum polarization $\Pi(-q^2)$ via dispersion relations. The enhancement ΔR of R above Eq. (1) with $\alpha_s = 0$, when dispersed, must correspond to an $O(\alpha_s)$ addition to Π. Perturbation theory is reliable for $-q^2 \gtrsim 4$ GeV^2 because there are no mass singularities in the Euclidean region. We find that $\int \Delta R\, ds \lesssim 4m_{\mathcal{P}'}^2 \pi \alpha_s$. For reasonable values of α_s, this is a stringent bound that will be tested

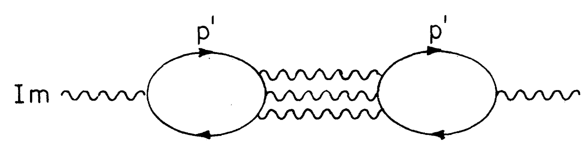

FIG. 2. A $\mathcal{P}'$ contribution to $\sigma_{e^+e^-}$ below $4m_{\mathcal{P}'}^2$.

in the immediate future.

Below $4m_{\mathcal{P}'}^2$, ladder exchanges between the quarks in Fig. 2 lead to a breakdown as they did above $4m_{\mathcal{P}'}^2$ and produce "orthocharmonium" bound states. From the Balmer formula, the ground state is at $s = 4m_{\mathcal{P}'}^2 - (\frac{4}{3}\alpha_s)^2 m_{\mathcal{P}'}^2$ which is likely well inside the 8-GeV^2 region. Another way of seeing that a Coulomb-like picture is not completely correct is to estimate the size of the "charmonium" atom. The Bohr radius is $2/\frac{4}{3}\alpha_s m_{\mathcal{P}'}$ which, for $\alpha_s \leq 0.3$ and $m_{\mathcal{P}'} \leq 2$ GeV, is greater than 3 GeV^{-1}, probably too large for use of a Coulomb-like potential.

If this problem is neglected, the width into hadrons is given by the three-gluon discontinuity. The result involves the matrix element and the wave function at the origin:

$$\Gamma_h = |M_h|^2 |\Psi(0)|^2 = (2/9\pi)(\pi^2 - 9)\tfrac{5}{18}\alpha_s^3(\tfrac{4}{3}\alpha_s)^3 m_{\mathcal{P}'}. \quad (3)$$

The leptonic width via one photon into $\bar{l}l$ is

$$\Gamma_l = |M_l|^2 |\Psi(0)|^2 = \tfrac{1}{2}(\tfrac{2}{3}\alpha)^2(\tfrac{4}{3}\alpha_s)^3 m_{\mathcal{P}'}, \quad (4)$$

where $\alpha \approx \frac{1}{137}$. Although separately these calculations are not trustworthy, the ratio

$$\frac{\Gamma_l}{\Gamma_h} = \frac{\frac{2}{9}\alpha^2}{(2/9\pi)(\pi^2 - 9)5/\alpha_s^3} \quad (5)$$

is independent of wave-function effects.

If we assume that the recently announced resonance[5] with mass ≈ 3 GeV is orthocharmonium, Eq. (5) fixes α_s. Preliminary estimates give[6] $\Gamma_l \approx 3$ keV and $\Gamma_h \approx 75$ keV. Their ratio gives $\alpha_s \approx 0.26$. This, along with $m_{\mathcal{P}'} \approx 1.5$ GeV, implies $\Gamma_l \approx 0.8$ keV and $\Gamma_h \approx 20$ keV, surely low estimates since the Coulomb wave function will be enhanced at the origin by stronger forces at large distances. This explains the large width of the resonance.

The existence of excited states is strongly suggested but Coulomb-like calculations are even less believable since the Bohr radius increases like n^2. All but the lowest few states will be broadened, shifted, and smeared together into a broad enhancement which connects smoothly onto the approach to the asymptotic region already discussed.

"Paracharmonium" (0^-) should also exist, with a mass slightly less than that of orthocharmonium, the hyperfine splitting being of order $\alpha_s^4 m_{\mathcal{P}'}$ (≈ 10 MeV with $m_{\mathcal{P}'} = 1.5$ GeV). The ground-state width in the Coulomb approximation is

$$\Gamma_h(\text{para}) = |M_h(\text{para})|^2 |\Psi(0)|^2 = \tfrac{1}{3}\alpha_s^2(\tfrac{4}{3}\alpha_s)^3 m_{\mathcal{P}'}. \quad (6)$$

With $\alpha_s = 0.26$ and $m_{\mathcal{P}'} = 1.5$ GeV, $\Gamma_h(\text{para}) \approx 1.3$ MeV. More reliably,

$$\frac{\Gamma_h(\text{ortho})}{\Gamma_h(\text{para})} = \frac{5}{6}\frac{2}{9\pi}(\pi^2 - 9)\alpha_s \approx 0.013, \quad (7)$$

so that $\Gamma_h(\text{para}) \approx 6$ MeV. The existence of paracharmonium with this width and a mass on the order of 3 GeV is one of our most unambiguous predictions and it is important to look for it experimentally. A more careful estimate of the ortho-para mass splitting will be given in a future paper.

Any two-quark system in which the sum of the masses corresponds to a small $\bar{g}$ can be studied analogously. (Whether it couples to a particular leptonic current is inessential.) Thus we can identify charmed hadrons, whose masses are simply $m_{\mathcal{P}'}$ plus a light-quark mass minus an $O(\alpha_s^2)$ binding energy. Other aspects of charm phenomenology will follow from the smallness of $\bar{g}$. The existence of other heavy quarks would not complicate matters in principle; their effects are calculable. However, specific predictions based on a single $\mathcal{P}'$ may be altered.

In a forthcoming paper, we shall expand on these ideas and discuss the theoretical infrastructure more thoroughly.

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