True-amplitude migration through regularized extended linearized waveform inversion

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ABSTRACT

The ability to create subsurface images whose amplitudes are proportional to the elastic wavefield variations recorded within seismic data as a function of reflection angle is fundamental for performing accurate amplitude-variation-with-offset (AVO) analysis and inversion. A process that generates such images is commonly referred to as true-amplitude migration. We have determined how the extended subsurface-offset image space is able to preserve the elastic behavior of the primary reflections when these events are acoustically migrated with a reverse time migration approach performed in a least-squares fashion. Using a single-interface model, we determine how the angle-domain image amplitude variations from an extended-offset acoustically

migrated image closely follow the theoretical elastic Zoeppritz response even at the critical angle. Furthermore, we develop a subsalt synthetic test in which 1C ocean-bottom-node (OBN) data are used within a regularized linearized waveform inversion procedure. In this test, we highlight the ability of the acoustic extended-angle image domain to preserve the correct elastic amplitude variations of the reflected events from three subsalt sand lenses. Our method allows accurate inversion of elastic-wave data for subsurface parameter variations that are critical for reservoir characterization in oil and gas exploration and production. We determine its performance on an OBN field data set recorded in the Gulf of Mexico in which the AVO response of a potential gas-bearing prospect is correctly retrieved.

INTRODUCTION

In seismic exploration, the output of any amplitude-preserving or true-amplitude migration process is commonly displayed as an image volume whose values are proportional, up to a constant factor, to the P-P plane-wave reflection coefficients of the subsurface interfaces (Gray and Bleistein, 2009). Therefore, when we fix the spatial coordinates corresponding to a planar geologic interface within a subsurface image volume, the extracted amplitudes as a function of the subsurface angles must be proportional to the reflection coefficient R_{pp} predicted by the Zoeppritz equation (Aki and Richards, 2002). These kinds of image volumes are then used to estimate the elastic-parameter contrasts present in the subsurface as well as their petrophysical properties (Buland and Omre, 2003; Grana and Della Rossa, 2010).

Prestack Kirchhoff migration methodologies were among the first proposed techniques for forming amplitude-preserving images (Martins et al., 1997; Dellinger et al., 2000; Zhang et al., 2000). These approaches are based on infinite-frequency approximations of the wave equation (i.e., ray theory). However, in complex geology, these methods fail to construct a useful image when multipathing is present (Hill, 2001). To overcome these limitations, beam-based true-amplitude migration algorithms have been proposed (Sun et al., 2000; Albertin et al., 2004; Bleistein et al., 2005). In these techniques, the migration rays are considered to have a finite spatial width that affects their raypath properties, allowing for a more accurate approximation of the wave physics. Moreover, an extension of these methods has been proposed to accommodate the multipathing nature of waves propagating in complex velocity structures (Gray et al., 2002; Liu and Palacharla, 2011). Recent

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advances in the computational power of computer clusters have led to efficient retrieval of solutions of arbitrarily complex wave equations, thereby obviating the need to approximate the true nature of propagating waves. Most methods based on the full solution of the wave equation are commonly referred to as reverse time migration (RTM) (Baysal et al., 1983), and multiple true-amplitude RTM techniques have been proposed over the years (Zhang et al., 2007; Chattopadhyay and McMechan, 2008; Dong et al., 2012).

In this work, we use the extended-image space to form true-amplitude images of subsurface reflecting interfaces. Many authors have used this image domain to invert for the subsurface migration velocity based on optimal focusing of the image when the correct velocity is used during migration (Symes and Kern, 1994; Yang and Sava, 2009; Alkhalifah, 2014; Biondi and Almomin, 2014). Specific forms of the extended imaging condition have been used to generate angle gathers that preserve the amplitude information of the migrated events (Sava et al., 2001; Kuehl and Sacchi, 2002; Wang et al., 2005). Here, we show that the extended subsurface-offset image space is able to preserve the observed energy present in elastic pressure records. We use extended imaging operators based on the acoustic isotropic wave equation to migrate the elastic shot gathers. The acoustic approximation may limit the imaging procedure's applicability when strong elastic conversion effects happen during wave transmission in the overburden. The same limitation affects conventional ray-based true-amplitude migration processes unless an elastic ray tracing is used during the imaging step (Červený, 1987). Similarly, in these scenarios, an elastic wave equation should be used during the extended-image formation in our proposed approach. However, as we show in the reported examples, acoustic wavefields can accurately model transmission effects that significantly influence the recorded reflected events. In fact, in any imaging process, one key component in creating true-amplitude gathers is to correctly approximate transmission phenomena affecting the recorded reflection data.

We pose the imaging procedure as the solution of a least-squares migration process or linearized waveform inversion and also highlight the importance of applying a regularization term in certain exploration scenarios to mitigate imaging artifacts for improving the amplitude response of the angle-domain (AD) gathers. We derive the pseudoinverse operator of the subsurface-offset-to-subsurface-angle transformation (Sava and Fomel, 2003), and we show the importance of using such an operator to convert the subsurface-offset images into the corresponding AD ones correctly. We present two synthetic examples that illustrate how the extended-image space can preserve the true-amplitude nature of elastic-wave data despite the use of an acoustic approximation during the least-squares imaging process. Throughout our discussion, we refer to elastic pressure data as pressure recordings generated by elastic isotropic media. In the first test, we demonstrate the agreement between the AD image and the theoretical amplitude response across a wide range of reflection angles, including the postcritical angle, using a simple single-interface model. In the second experiment, we solve a regularized waveform inversion of elastic pressure data generated from three gas-sand lenses located underneath a complex salt body. We demonstrate the efficacy of the regularized imaging procedure in retaining the elastic amplitude information present in the recorded reflected events from complex geologic scenarios where converted waves, multiples, and incorrect transmission effects are present.

Finally, we apply the proposed extended regularized inversion to pressure data from an ocean-bottom-node (OBN) data set recorded in the Gulf of Mexico (GOM), and we clearly demonstrate the ability of the algorithm to identify a potential gas-bearing sand reservoir in the resulting common-image gathers. In the presented examples, an acoustic wave equation used during the imaging process is able to approximate the transmission effects of the overburden to preserve the elastic amplitude variations. In case strong conversion effects are observed, an elastic wave equation should be used during the image formation.

THEORY

The imaging of scattered energy by subsurface interfaces can be posed as an inverse problem by minimizing the following objective function:

$$\phi(\tilde{\mathbf{m}}) = \frac{1}{2} \|\tilde{\mathbf{B}} \tilde{\mathbf{m}} - \mathbf{d}\|_2^2, \quad (1)$$

where $\tilde{\mathbf{B}}$ represents the extended acoustic wave-equation Born operator, $\tilde{\mathbf{m}}$ is the extended subsurface-offset image, and $\mathbf{d}$ is the recorded data. By minimizing this linear problem, we seek to find an optimal extended image $\tilde{\mathbf{m}}_{\text{opt}}$ that matches the amplitudes of the recorded data in a least-squares sense.

The application of the extended Born modeling operator onto an extended image is given by the following system of partial differential equations:

$$\left[\frac{\partial^2}{\partial t^2} - v^2(\mathbf{x}) \nabla^2 \right] s(\mathbf{x}, t) = f(\mathbf{x}, t), \quad (2)$$

$$\left[\frac{\partial^2}{\partial t^2} - v^2(\mathbf{x}) \nabla^2 \right] r(\mathbf{x}, t) = f_s(\mathbf{x}, t), \quad (3)$$

where f represents a known source term, v is the subsurface acoustic velocity, and s and r represent two pressure wavefields. Commonly, the two fields s and r are referred to as the source and receiver wavefields, respectively. In these equations, the spatial and time coordinates are denoted by $\mathbf{x}$ and t , respectively. The secondary source field f_s is given by the following expression:

$$f_s(\mathbf{x}, t) = \int s(\mathbf{x} - 2\mathbf{h}, t) \tilde{m}(\mathbf{x} - \mathbf{h}, \mathbf{h}) d\mathbf{h}, \quad (4)$$

where $\tilde{m}$ is the continuous representation of the vector $\tilde{\mathbf{m}}$ and $\mathbf{h}$ is the subsurface-offset extended axis (Rickett and Sava, 2002). Finally, the predicted data for this extended scattering process are obtained by extracting the value of r at the receiver positions.

The solution $\tilde{\mathbf{m}}_{\text{opt}}$ of the linear problem in equation 1 is typically found using iterative conjugate-gradient (CG) schemes due to the large dimensions of the extended Born matrix in typical problems of interest and the computational cost associated with its application to given vectors. To this end, it is necessary to apply the adjoint of the extended Born operator, whose application can be written as follows:

$$\tilde{m}(\mathbf{x}, \mathbf{h}) = \int s(\mathbf{x} - \mathbf{h}, t) r'(\mathbf{x} + \mathbf{h}, t) dt, \quad (5)$$

where r' is given by solving the adjoint wave equation:

$$\left[\frac{\partial^2}{\partial t^2} - v^2(\mathbf{x}) \nabla^2 \right]^* r'(\mathbf{x}, t) = d(\mathbf{x}, t). \quad (6)$$

Here, $*$ denotes the adjoint operation and d represents the pressure data injected from the receiver positions, which is used as a forcing term of the adjoint wave equation.

In field applications, the extended-image volumes obtained by solving the basic linearized waveform inversion problem of equation 1 are usually hampered by artifacts that can greatly diminish the image quality. Sparse acquisition sampling, for example, can produce such artifacts (Marfurt et al., 1998). In fact, Prucha and Biondi (2002) demonstrate the ability of a regularized approach to increase the coherency of the image in subsalt sections. To mitigate such artifacts, we propose to minimize the following regularized linearized waveform inversion problem:

$$\phi'(\tilde{\mathbf{m}}) = \frac{1}{2} \|\tilde{\mathbf{B}} \tilde{\mathbf{m}} - \mathbf{d}\|_2^2 + \frac{\epsilon^2}{2} \|\mathbf{D} \tilde{\mathbf{m}}\|_2^2, \quad (7)$$

where $\mathbf{D}$ represents the regularization operator that enhances the undesired features (which will diminish during the inversion) in the image $\tilde{\mathbf{m}}$ and ϵ is a scalar weight associated with the regularization term. In this work, a differential semblance optimization (DSO) operator is used within the regularization term (Symes and Carrazzone, 1991).

The subsurface-offset images obtained by solving either one of the linearized waveform inversion problems in equations 1 and 7 can be mapped into the subsurface AD as shown by Sava and Fomel (2003). When a single subsurface-offset extension is used, this transformation can be written as follows:

$$\tilde{m}_\gamma(x, z, \gamma) = \int_{-\infty}^{\infty} \tilde{m}(x, z + \tan(\gamma)h, h) dh, \quad (8)$$

where h represents the single subsurface-offset axis variable and γ is the reflection angle (Biondi and Tisserant, 2004). When double horizontal extensions are used within a 3D algorithm, the subsurface AD transform takes the following form:

$$\tilde{m}_\gamma(x, y, z, \gamma, \phi) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{m}_h(x, y, z + \tan(\gamma)[\cos(\phi)h_x - \sin(\phi)h_y], h_x, h_y) dh_x dh_y, \quad (9)$$

where ϕ represent the azimuth angle (Biondi and Tisserant, 2004) and h_i represents the extended axis along the i th direction. In Appendix A, we derive the 2D and 3D forward and adjoint transformations as well as the corresponding pseudoinverse operators, which are crucial for obtaining amplitude-preserving AD images. In addition, in Appendix B, we show that this DSO regularization, which increases the image's focusing at the zero-subsurface offset, also enhances the coherency of the image across reflection angles.

NUMERICAL EXAMPLES

We use two numerical synthetic tests to show the extended image space's ability to preserve the elastic pressure variation of elastic data even when an acoustic propagation mechanism is used during

image formation. In the first example, we use a single-interface model to support this claim and quantitatively compare the agreement between the image amplitudes and the theoretically computed reflection coefficient. We also highlight the importance of mapping subsurface offsets into angles by using the pseudoinverse operator of such a domain transformation. Finally, we solve an extended regularized linearized inversion of elastic pressure data generated by three subsalt lenses located underneath a complex salt body. This second test highlights the importance of using a regularization term during the inversion process to obtain a coherent amplitude response of the AD subsurface image. In both synthetic experiments, we solve the elastic isotropic wave equation by applying a finite-difference (FD) staggered-grid approach (Virieux, 1986). The elastic pressure data can be computed as $P = -K \nabla \cdot \mathbf{u}$, where K is the bulk modulus and $\mathbf{u}$ are the particle displacements. For the imaging inverse problems, we solve the second-order acoustic isotropic wave equation by a second-order-in-time 10th-order-in-space FD method to compute the necessary wavefields for the image formation and extended scattering process.

Single-interface example

In this first numerical example, we use two single-interface elastic models to generate elastic pressure data. The elastic-parameter interface is flat in one case and is tilted by 10° in the other.

Figure 1 shows the compressional wave (P-wave) velocity model for the flat-interface test. The change in elastic parameters is depicted in the three vertical profiles shown in Figure 2, where all three elastic parameters increase across the interface. We generate

Figure 1. P-wave velocity model of the elastic single-interface model used in the first amplitude-preserving numerical test.

Figure 2. Elastic parameter profiles of the single-interface model: (a) P-wave velocity, (b) S-wave velocity, and (c) density profiles.

elastic pressure data using an explosive source whose time signature and amplitude spectrum are displayed in Figure 3.

We record the pressure using 81 sources and 401 receivers placed at the surface and spaced by 50 and 10 m, respectively. A single reflected event is recorded by the receivers for each experiment, in which a clear phase rotation is present as the offset increases between the source and the receiver (Figure 4).

To illustrate the importance of using an extended imaging condition when performing the migration step, we solve two linearized

waveform inversion problems using acoustic Born modeling operators. We use nonextended (i.e., h restricted only to zero-subsurface offset) and extended Born operators and minimize the two objective functions using 500 iterations of the linear CG method (Aster et al., 2018). In both cases, the migration velocity model is set to a constant 2.5 km/s. Figure 5a shows the comparison between the convergence rates of the extended and nonextended linearized waveform inversions of the same elastic pressure data displayed in Figure 4. It is clear that when the extended space is used during

the image formation, the relative objective function decrease reaches almost a numerical level of accuracy for single-precision operators (i.e., 10^{-6}). The nonextended migration can only achieve a level of accuracy of 10% compared to the initial objective function. This observation is also highlighted by comparing the data residuals for the two migration processes for a given shot gather (Figure 5b and 5c). The extended-image space can fully preserve all of the elastic amplitudes present in the recorded data. In contrast, the nonextended Born operator lacks this property, resulting in an inability to model all of the elastic pressure variations of the data. This simple comparison demonstrates that elastic amplitude variations cannot be fully captured by the nonextended image space.

Figure 3. Explosive source used to generate the elastic pressure data for the true-amplitude migration tests. (a and b) The time signature and frequency spectrum, respectively.

Figure 4. Elastic pressure waves recorded at the surface for a shot placed at (a) $x = 0$ km and (b) $x = 2.0$ km. The direct arrival has been removed from the observed data. The main reflection and part of the head wave are visible on panel (a).

Figure 5. (a) Comparison of the convergence rate for the extended (the blue curve) and nonextended (the dashed red curve) linearized waveform inversion problems for the data from the flat interface shown in Figure 4. Final data residual gathers for the shot placed at $x = 2.0$ km are shown for the (b) extended and (c) nonextended migration processes. Panels (b and c) have the same dynamic range.

The solution to the extended linearized waveform inversion problem is a function of the spatial coordinates x and z and of the extended subsurface offset h . How the data are mapped within this image volume highly depends on the recorded events and the acquisition geometry used. Figure 6 shows the offset-domain common-image gathers (ODCIGs) for two different x coordinates. For $x = 2.0$ km (Figure 6b), the ODCIG appears to be focused around the zero-offset axis. This behavior is expected because the correct migration velocity has been used during the inversion process (Biondi and Tisserant, 2004). The two linear features below $z = 0.8$ km represent the head waves recorded in the longer offset shot gathers mapped into the image space. The other two faint linear features greater than $z = 0.8$ km are caused by the limited acquisition aperture (i.e., the maximum source-receiver offset of 4 km). However, when an ODCIG is extracted at $x = 0.0$ km (Figure 6a), the image does not appear as focused as for the central-model position in Figure 6b because fewer reflection angles have been illuminated from the surface acquisition.

To highlight the importance of using the pseudoinverse of the offset-to-angle transform to obtain angle-domain common-image gathers (ADCIGs), we use a synthetic example in which a single event with constant-angle response is present in the subsurface (Figure 7a). This ADCIG is transformed into the corresponding ODCIG (Figure 7b) using the transformation derived in Appendix B. When this ODCIG is mapped back to the AD, different results are obtained

depending on whether the adjoint or pseudoinverse operators are used. Figure 7c shows the result of applying the adjoint operator. The shape and amplitude responses are different compared to the original angle response of Figure 7a, especially for large reflection angles γ . In contrast, the pseudoinverse application can better retrieve the original ADCIG response (Figure 7d). The artifacts present above a 60° reflection angles are due to the numerical approximation of the oscillatory integral of equation A-5. In fact, this equation can be written as follows:

$$\begin{aligned} & \int_{-\infty}^{\infty} \hat{I}_h(k_z, h) e^{i2\pi k_z \tan(\gamma) h} dh \\ &= \int_{-\infty}^{\infty} \hat{I}_h(k_z, h) [\cos(2\pi k_z \tan(\gamma) h) - i \sin(2\pi k_z \tan(\gamma) h)] dh, \end{aligned} \quad (10)$$

where it is clear that numerical errors would occur for $|k_z \tan(\gamma)| > k_{h,\max}$, where $k_{h,\max}$ denotes the Nyquist sampling rate of the subsurface-offset axis. In this case, to reduce such numerical errors, the previous integral has been computed for $|k_z \tan(\gamma)| \leq k_{h,\max}$ only.

Figure 6. Subsurface-offset common-image gathers for the flat-interface example extracted at (a) $x = 0$ km and (b) $x = 2.0$ km.

Figure 8. Amplitude response extracted at $z = 1.0$ km from the AD gathers of Figure 7. The red line represents the correct constant-angle response, whereas the black dashed and blue continuous lines denote the responses after applying the adjoint and pseudoinverse operators, respectively. All responses have been scaled to have the same value for $\gamma = 0$.

Figure 7. Single-reflector example with constant-angle response highlighting the importance of using the pseudoinverse operator of the subsurface-offset-to-angle transformation: (a) original ADCIG, (b) ODCIG obtained from the original AD gather, and ADCIGs obtained by applying the (c) adjoint and (d) pseudoinverse operators of the OD-to-AD transform, respectively.

Figure 9. ADCIGs for the flat-interface example extracted at (a) $x = 0$ km and (b) $x = 2.0$ km.

Figure 10. Comparison between the angle response extracted at $z = 0.8$ km from the ADCIG generated by one iteration (the dashed red curve) and 500 iterations (the black dashed curve) of extended linearized waveform inversion of the flat-interface model and the theoretical plane-wave response predicted using the Zoeppritz equation (the blue curve). The ADCIG amplitudes have been scaled so that the three responses match for $\gamma = 0^\circ$. The dashed blue line corresponds to the P-P reflection coefficient computed assuming an acoustic response.

Figure 11. P-wave velocity of the elastic single-interface model with a 10° dipping interface.

Figure 8 compares the correct angle response with the amplitudes of the ADCIGs in Figure 7c and 7d extracted at $z = 1.0$ km. As expected, the adjoint provides an angle response that follows a cosine behavior (equation A-12). Conversely, the pseudoinverse is able to retrieve the original amplitude response up to a 60° angle, above which the discretization of the integral does not allow a correct reconstruction. This test demonstrates the importance of using the pseudoinverse operator of the offset-to-angle transform for obtaining amplitude-preserving angle gathers. In fact, all of the ADCIGs shown next are computed by applying the pseudoinverse of this transformation.

The previous observations drawn from Figure 6 are more evident when the ODCIGs are transformed into the corresponding ADCIGs. The reflection angles that are illuminated for a given x are now obvious in the panels of Figure 9. As expected, for $x = 0.0$ km, the ADCIG displays energy focused close to the zero-reflection-angle axis. However, when the ADCIG is extracted at $x = 2.0$ km, a flat event positioned at the interface depth is present.

To verify that the amplitude response of the ADCIG in Figure 9b represents a true-amplitude image, we extract the image amplitudes at $z = 0.8$ km and compare them against the plane-wave reflection coefficient R_{pp} of the planar interface (Figure 10). A close fit between the ADCIG response and the theoretical R_{pp} is obtained for angles up to 42° (compare the blue and the black dashed curves), which is beyond the critical angle of 40° . The mismatch at higher angular values is due to the limited acquisition aperture as well as the truncation of the iterative inversion process. To highlight the importance of the iterative process, we plot the ADCIG response obtained with a single iteration of the linearized waveform inversion (i.e., by applying an extended RTM process). A close fit is obtained for precritical angles. However, for angles beyond 35° , the response suffers from the missing compensation of the propagation effects, which are accounted for by further iterations of the inversion process. The acoustic Zoeppritz R_{pp} coefficient plotted in the same panel (the dashed blue line) shows a distinct pattern compared to the elastic one (the solid blue line). This comparison highlights even further the preservation of the elastic-amplitude response of the ADCIG.

As described by Sava and Fomel (2003), the transformation from subsurface offsets to angles is independent of the geologic dip of the subsurface reflectors when the correct migration velocity is considered during image formation. To illustrate this property, we perform the same synthetic test as in the previous example, but we use an interface that dips 10° (Figure 11).

By applying the same iterative process to solve the linearized waveform inversion problem based on the dipping-interface data, we obtain the ODCIGs shown in Figure 12a and 12b. In this case, the focusing depth in the gathers coincides with the interface's position for each horizontal coordinate under consideration. As in the previous case, the ODCIG at $x = 0.0$ km is not as focused around $h = 0.0$ km as the one extracted at the center of the model due to the angular coverage of the reflected event. The observations on the ADCIGs in the flat-interface case are also valid when the reflector is tilted (Figure 12c and 12d). In addition, even if the interface is dipping, a similar amplitude variation with angle (AVA) can be observed in the ADCIG of Figure 12d when compared to the one of Figure 9b. When the amplitude response from the central ADCIG is compared to the theoretical R_{pp} , a similarly close match is observed

(Figure 13). The slight dissimilarities between the two responses compared to those in Figure 10 are due to the different illuminated reflection angles from the surface acquisition as well as errors that arise from the Cartesian coordinate-system approximation of the tilted interface in Cartesian coordinates. This synthetic test demonstrates the invariability of the ODCIG-to-ADCIG transform with respect to the geologic dip in the 2D case.

Subsalt example

When imaging targets are present underneath a salt body, the correct recovery of a true-amplitude migrated volume becomes challenging for multiple reasons (Muerdter and Ratcliff, 2001a; Prucha and Biondi, 2002):

- 1) Any imaging method would be affected by shadow zones below the salt body, where little energy is transmitted.
- 2) Amplitude variations caused by salt structures that focus seismic energy would hamper the constructed subsalt image.
- 3) The subsalt image is significantly impacted by the choice of the acquisition geometry, making the survey design a crucial step in subsalt seismic exploration scenarios (Muerdter and Ratcliff, 2001b).

In this example, we show how a regularized linearized waveform inversion can correctly retrieve the subsalt AVAs of sand lenses located underneath a large salt body. Figure 14 displays the true P-wave velocity model in which a salt body presenting a rugose top interface is placed above three sediment lenses. To understand the variability of other elastic parameters, we plot a vertical profile for $x = 10.0$ km (Figure 15). A substantial increase in the P- and shear wave (S-wave) velocities is encountered within the salt body relative to the surrounding sediments, whereas a mild decrease in the density is present. The sand lenses present a moderate increase in the S-wave velocity, with small variations in the other two elastic parameters. The elastic-parameter changes of the lenses correspond to a gas-bearing sand (Castagna and Swan, 1997). This example also shows that usage of acoustic wavefields during the imaging step for a complex scenario can produce accurate amplitude responses within the formed ADCIGs, despite the presence of a sharp elastic-parameter contrast due to a salt body.

To generate the observed elastic pressure data and simulate a realistic long-offset acquisition geometry, we place 250 OBNs at the water-sediment contact (i.e., $z = 0.3$ km in Figure 14), starting from $x = 2.5$ km up to $x = 17.44$ km in 60 m increments. In total, 1001

sources are placed at the surface and span the entire model width in even increments of 20 m. A representative receiver gather at $x = 10.0$ km is shown in Figure 16a. Multiple scattered events are present underneath the direct arrival due to the presence of the salt body and sand lenses. To show the effect of the data used during the linearized waveform inversion on the final migrated im-

Figure 13. Comparison between the angle response extracted at $z = 0.64$ km from the ADCIG (the dashed red curve) of Figure 12d and the theoretical response predicted using Zoeppritz (the blue curve). The ADCIG amplitudes have been scaled so that the two responses match for $\gamma = 0$.

Figure 14. P-wave velocity of the elastic model used in the subsalt amplitude-preserving numerical test for the three sand lenses.

Figure 12. Subsurface-offset common-image gathers for the tilted-interface example extracted at (a) $x = 0$ km and (b) $x = 1.0$ km. Panels (c) and (d) display the previous offset gathers mapped into the angle domain.

age, we perform two tests using the observed elastic pressure. In the first test, we linearly invert the data in which the direct arrival has been removed by computing the elastic pressure due only to the water layer and sediments (i.e., without the salt body and the sand lenses) (Figure 16b). In the second test, only the events related to the

presence of the sand lenses are used during the imaging process (Figure 16c). To obtain the second test events, we subtract from the observed data the elastic pressure modeled using an elastic subsurface that does not contain the sand lenses. In both tests, nonlinear effects are still present (e.g., amplitude variations, converted events, multiples, and primary-wave multipathing).

These two data sets are used within an acoustic extended linearized waveform inversion procedure and are solved using 500 iterations of the linear CG method. For the acoustic migration velocity, we use the true P-wave model velocity after the removal of the sand lenses. The convergence rates of the two processes are displayed in Figure 17a. Contrary to the results from the single-interface example, the optimizations do not achieve a perfect match. However, they both reach an approximately flat convergence rate for the chosen number of iterations. To understand which events could not be mapped into the extended image space, we plot the final data residual for the receiver gather extracted at $x = 10.0$ km for both data sets (Figure 17b and 17c). In both panels, most of the events have been mapped into the image space. However, some energy possibly corresponding to converted waves cannot be modeled using the maximum subsurface offset of 0.5 km. In fact, by analyzing three ODCIGs passing through the three lenses (Figure 18a–18c), it is clear that a larger subsurface-offset extent is necessary to match the data. The goal of this test is not to match all of the recorded

Figure 15. Elastic parameter profiles of the salt model in Figure 14 extracted at $x = 5.0$ km: (a) P-wave velocity, (b) S-wave velocity, and (c) density profiles.

Figure 16. Elastic pressure waves recorded by a receiver placed at $x = 9.94$ km on the seafloor. (a) Observed pressure. (b) Observed pressure where the direct arrival has been removed. (c) Observed pressure resulting only from the presence of the subsalt lenses. Panels (a) and (b) have the same clipping, whereas panel (c) has a narrower dynamic range, which is approximately two orders of magnitude smaller.

Figure 17. (a) Comparison of the convergence rate for the extended linearized waveform inversion when the data shown in Figure 16b (the blue curve) and 16c (the dashed red curve) are used during the migration process. Each curve is scaled separately. (b) and (c) The final residual gathers when the data shown in Figure 16b and 16c, respectively, are iteratively imaged.

events but only the ones corresponding to P-P reflections. These events focus on the proximity of the zero-subsurface offset axis because the correct migration velocity has been used during the migration process. As a matter of fact, when these image gathers are transformed into the corresponding ADCIGs (Figure 18d–18f), flat events at depths corresponding to the geologic interfaces of the elastic model are clearly present. Amplitude continuity across the reflection angles is hampered by the relatively sparse receiver acquisition geometry, as well as the presence of converted waves recorded in the elastic pressure data.

Similar conclusions can be drawn from the image gathers obtained by migrating only the events due to the sand lenses (Figure 19). However, in this case, we observe clearly noticeable flat events corresponding to the lens interfaces in the ADCIGs, which highlights the importance of performing a data-selection pre-processing step before migration when dealing with field applications.

As shown by Prucha and Biondi (2002), image artifacts related to shadow zones and incoherent illumination can be mitigated by forcing coherency across reflection angles during the migration step. To impose such a condition in the current test, we add a DSO regularization term to each waveform objective function. We solve the regu-

larized imaging problems using 500 iterations of the linear CG approach. The regularization weight ϵ is chosen based on trial and error. An initial guess for this variable is represented by a value that balances the two terms in each objective function for a nonnull reflectivity model. The regularization weights chosen for the two data sets are different. A larger weight has been used for the data set shown in Figure 16b than for the one in Figure 16c.

The regularized extended-image solutions in both cases are more focused in the offset domain (OD) and present a more coherent angle behavior in the resulting ADCIGs compared to the inversion problems that are not regularized (Figures 20 and 21). Moreover, most of the imaging artifacts are greatly diminished especially for the migration of the data set containing the salt-related events (Figure 16b).

To show the importance of forcing coherency across reflection angles, we compare the amplitude responses of the ADCIGs extracted at depths corresponding to the top of each sand lens against the theoretical Zoeppritz Rpp coefficients (Figure 22). The unregularized process produces an image whose amplitudes are affected by the inconsistent illumination and migration artifacts. In contrast, the regularized ADCIGs present smoother angle responses that are consistent with the theoretical Rpp. The mismatch at angles greater than

Figure 18. Final ODCIGs obtained after iteratively migrating the data shown in Figure 16b and extracted at (a) $x = 5.5$ km, (b) $x = 10.0$ km, and (c) $x = 14.0$ km. The lower panels show ADCIGs extracted at (d) $x = 5.5$ km, (e) $x = 10.0$ km, and (f) $x = 14.0$ km.

Figure 19. Final ODCIGs obtained after iteratively migrating the data shown in Figure 16c and extracted at (a) $x = 5.5$ km, (b) $x = 10.0$ km, and (c) $x = 14.0$ km. The lower panels show ADCIGs extracted at (d) $x = 5.5$ km, (e) $x = 10.0$ km, and (f) $x = 14.0$ km.

24° is due to the lack of illumination from the surface acquisition geometry.

When events related only to the sand-lenses are linearly inverted, the ADCIGs' angle responses are less impacted by image artifacts, due primarily to the absence of salt-related multiples and converted events. In this case, the regularization provides a mild improvement (Figure 23). Nevertheless, the addition of regularization permits the formation of an image whose amplitudes show a near-perfect matching of the theoretical response up to a 24° reflection angle. A comparison of Figures 22 and 23 illustrates even further the dependency of the image amplitudes on the events considered during the migration process.

Figure 24 shows the zero-angle image when the entire data volume is inverted without any regularization term (i.e., using the data of Figure 16b). Use of the regularization term within the migration problem greatly diminishes the image artifacts. A simple stacking procedure across angles for the unregularized inversion result can also diminish such artifacts (Figure 24c). Within the stacked and regularized images, the salt shape is perfectly captured and the contours of the subsalt sand lenses are visible. The regularized zero-angle image is more interpretable than the corresponding angle-stacked image. The image is generally less affected by artifacts at locations where the bottom interface of the middle lens can be clearly recognized. Some artifacts that affect image quality could

be further reduced with filtering, but this step is beyond the scope of this study. Nonetheless, a useful subsurface image can be obtained without complex workflows using the approach described here and can be used to perform a structural analysis of the subsurface interfaces.

The subsalt lens test clearly demonstrates the ability of the extended-image space to form true-amplitude migrated volumes even when Born acoustic operators are used during linearized inversion of elastic pressure data. In this test, where realistic elastic parameters are used, the regularized linearized waveform inversion produces amplitude-preserved gathers despite the presence of converted waves and multiples.

DISCUSSION: VELOCITY ERRORS

We discuss the sensitivity of the proposed method to the accuracy of the velocity model used during the linearized inversion. We start by using the data obtained from the flat-layered model of Figure 1. We perform the same inversion procedure, but we use two incorrect velocity values, namely, 2.625 and 2.375 km/s. These two values correspond to 5% velocity errors with respect to the true velocity of 2.5 km/s. As described by Biondi and Symes (2004), the expected behavior would result in ADCIGs that present a negative or positive curvature depending on the velocity error. The former

Figure 20. Final DSO-regularized ODCIGs obtained after iteratively migrating the data shown in Figure 16b and extracted at (a) $x = 5.5$ km, (b) $x = 10.0$ km, and (c) $x = 14.0$ km. The lower panels show ADCIGs extracted at (d) $x = 5.5$ km, (e) $x = 10.0$ km, and (f) $x = 14.0$ km.

Figure 21. Final DSO-regularized ODCIGs obtained after iteratively migrating the data shown in Figure 16c and extracted at (a) $x = 5.5$ km, (b) $x = 10.0$ km, and (c) $x = 14.0$ km. The lower panels show ADCIGs extracted at (d) $x = 5.5$ km, (e) $x = 10.0$ km, and (f) $x = 14.0$ km.

corresponds to a higher velocity value with respect to the correct migration velocity, whereas the latter corresponds to the opposite scenario. We observe this behavior in this synthetic example by analyzing the ADCIGs displayed in Figure 25. In the case of constant velocity, the vertical moveout Δz due to the velocity error is given by the following equation (Biondi and Symes, 2004):

$$\Delta z = (\rho_{\text{mig}} - 1) \tan^2(\gamma) z_0, \quad (11)$$

where ρ_{mig} represents the velocity ratio between the correct and used (incorrect) migration velocities, γ is the reflection angle, and z_0 is the depth of the considered reflector. Because we know these parameters, we can plot the moveout curve for the lower velocity case and overlay it on the ADCIG (Figure 26a). We can also extract the amplitude behavior along this curve and compare it to the theoretical Rpp predicted by the Zoeppritz equation (see the blue and red curves in Figure 26b). The mismatch between the two ADCIG curves is due to the incorrect relation between subsurface offsets and angles. In fact, we can account for this error by remapping the angle axis using Snell's law in the following expression:

$$\gamma_{\text{corr}} = \sin^{-1}(\sin(\gamma) * \rho_{\text{mig}}), \quad (12)$$

where γ_{corr} is the remapped angle value for the corresponding reflection angle γ . This remapping process is able to compensate for the migration velocity error and correctly predict the Zoeppritz amplitude variation. In real applications, the value of ρ_{mig} is unknown, but it can be estimated from the ADCIGs (Yang and Sava, 2010).

Incorrectly interpreted salt body

To show the importance of correctly estimating a migration velocity model for use in the image formation, we perform a regularized linearized inversion of the subsalt data shown in Figure 16b using the incorrectly interpreted salt body shown in Figure 27a. We introduce positive and negative velocity errors and incorrectly interpret most of the salt base and some of the top salt structure (Figure 27b). The incorrect interpretation leads to a subsurface image of the salt body with incorrect base positions and to some migration artifacts. In addition, the images of the left and right lenses are negatively affected by the velocity errors, causing them to be hardly interpretable. Despite using the incorrect salt interpretation, however, we can still distinguish the central lens (see Figure 28). This observation also applies to the ADCIGs in Figure 29 extracted from

Figure 22. Amplitude-response comparison of the P-P reflection coefficients at the top of the (a) left, (b) middle, and (c) right sand lenses after migrating all of the events in Figure 16b. Each panel shows the angle responses from the Zoeppritz equation (the blue curve) and the ADCIG amplitudes without (red dashed) and with (black dashed) DSO regularization extracted at one location on the top interface near the middle of each lens.

Figure 23. The same as Figure 22, but using only the lens-related events in Figure 16c.

the extended image volume. Finally, when we compare the amplitude behavior at the lens' top interface with the true Zoeppritz response, only the amplitudes at angles greater than 14° are affected by the incorrect velocity used during the image formation (Figure 30). Clearly, a different erroneous salt-body interpretation could lead to different conclusions. However, our purpose here is to highlight the importance of correctly estimating a migration velocity model to form the subsurface image volume.

FIELD-DATA EXAMPLE

The field data set used in this example was acquired in the GOM by Shell Exploration and Production Company in 2010. The acquisition area lies within the Garden Banks region, approximately 362 km southwest of New Orleans, Louisiana. We select 255 nodes from the entire data set that recorded multicomponent seismic data generated by 41,000 air-gun sources covering an area of 100 km^2 . Figure 31 shows the x - y locations of the sources and nodes overlaid on a depth slice of the initial velocity model containing a salt diapir (Murray, 1966). The data were acquired using a flip-flop acquisition geometry with a source interval of approximately 25 m and a source depth of 9.8 m. The sail lines are aligned with the x -axis, and their interval on the crossline or y -axis is approximately 100 m. The multicomponent nodes are placed on the seabed at an interval of approximately 250 m in the x - and y -directions, and their depth

varies between 0.83 and 1.0 km. Their spatial x - y interval is approximately 250 m in both directions. In this application, we use only the observed pressure and perform the acoustic Green's function

Figure 25. ADCIGs extracted at $x = 2 \text{ km}$ obtained using incorrect migration velocities for the flat-interface case. (a) A 5% faster velocity with respect to the true one is used during the imaging process. (b) A slower velocity of 5% with respect to the true one is used during the imaging process.

Figure 24. Zero-angle image when the entire data volume is imaged (a) with and (b) without regularization used during the inversion process. (c) Stacked image obtained by stacking the unregularized linearized waveform inversion result across the reflection angles for $\gamma \leq 10^\circ$.

computation using a free-surface condition at the water surface during the imaging procedure (Robertsson, 1996). This choice avoids the necessity of performing an up-down separation step (Schalkwijk et al., 2003).

The preprocessing steps and velocity model building are described in detail in Biondi (2021). We display a representative processed common-receiver gather in Figure 32. In this gather, scattered energy due to the presence of the salt body is clearly visible within the observed events. Moreover, it is clearly evident that the data processing removed the reverberation due to the air-gun-bubble oscillations (Watson et al., 2019). The processed data are used in all steps of this application: velocity model building, RTM, and extended linearized waveform inversion. Two representative cross- and inline sections from the velocity model resulting from applying an acoustic full-waveform inversion (FWI) process are shown in Figure 33. The FWI workflow has introduced a low-velocity anomaly on the left side of the diapir (Figure 33a). This velocity decrease could be associated with salt-encased sediment packages present during the diapir formation (Fernandez et al., 2017). An additional low-velocity anomaly is visible in the sediments adjacent to the salt flank at $z = 2.8$ km (Figure 33b). This decrease in velocity could be related to gas accumulation at the top of a hydrocarbon reservoir sealed by the salt body.

To further characterize this area and identify potential targets for an amplitude-variation-with-offset (AVO) analysis, we form an RTM image using data up to 30 Hz. In this imaging step, we apply a source-side illumination compensation to reduce any acquisition artifacts within the subsurface image volume (Kaelin and Guitton, 2006). Three representative depth slices extracted from the image volume are shown in Figure 34. In the shallowest depth section at $z = 0.9$ km (Figure 34a), the top edges of the diapir are recognizable and multiple radially distributed features extend around it. These structures are commonly observed in areas where diapirism is present and are due to the presence of radial faults caused by the upward growth of the salt diapir (Stewart, 2006; Coleman et al., 2018). Furthermore, within deeper sections, multiple structures associated

with turbidite sequences can be observed (Berg, 1982). The panel of Figure 34b shows an example of such structures on the bottom-right corner of the depth slice. Finally, in the depth section extracted

Figure 27. (a) Incorrectly interpreted velocity model for the subsalt example. (b) Velocity errors obtained by subtracting the incorrect model from the correct one.

Figure 28. Zero-angle image obtained using the incorrect interpreted salt model and using regularization during the inversion process.

Figure 26. (a) The same figure as Figure 25b where the dashed line indicates the computed ADCIG moveout due to the incorrect migration velocity. (b) Comparison between the theoretical Rpp coefficient (solid blue) and the incorrect (red dashed) and remapped (black dashed) ADCIG amplitude behaviors.

Figure 29. Final DSO-regularized ADCIGs obtained after iterative migration of the data shown in Figure 16b and extracted at (a) $x = 5.5$ km, (b) $x = 10.0$ km, and (c) $x = 14.0$ km. These gathers are obtained using the incorrectly interpreted salt model.

Figure 30. Amplitude-response comparison of the P-P reflection coefficient at the top of the middle sand lens after migrating all of the events in Figure 16b using the incorrect subsalt velocity. The amplitude of the image is extracted from the ADCIG of Figure 29b.

Figure 31. (a) Source and (b) receiver x - y locations overlaid on a depth slice extracted at $z = 2.5$ km from the initial velocity model. The high-velocity portion is associated with the presence of a salt diapir.

at $z = 2.865$ km, high-amplitude features are visible close to the diapir flanks, in which a noticeable faulted structure is present. Such image features are potentially associated with hydrocarbon prospects (Harding and Lowell, 1979; Tiapkina et al., 2008).

From analysis of the RTM image volume (Figure 35), we identify a high-amplitude event positioned at the same location as the low-velocity anomaly shown in Figure 33b. Such an amplitude response could be related to gas accumulation at the top of a hydrocarbon reservoir (Mazzotti, 1990). To characterize its amplitude response as a function of the reflection angle and identify its AVO class, we apply an extended acoustic linearized inversion of the observed data to obtain an extended 3D image volume. The observed data's maximum frequency is limited to 12 Hz to make the least-squares process feasible with the available computational resources. The FD grid is set to 35 m in each direction. Moreover, given the acquisition's full-azimuth nature, we use a subsurface-offset extent in h_x and h_y of nine grid points in each direction, resulting in a maximum absolute subsurface offset of 140 m. Finally, a DSO regularization term is added to improve the image focusing, and its weight is

Figure 32. Representative processed common-receiver gather. The maximum frequency of the plotted data is 20 Hz.

chosen on a heuristic basis. We also focus the iterative process on inverting the data component stemming from the target area by using a mask tailored for the potential prospect.

After 30 iterations of a linear CG algorithm, an acceptable numerical minimum of the objective function is reached given the selected parameters. The inversion process leads to a clear identification of the target and of the related geologic structures by analyzing the zero-subsurface offset image (Figure 36a). An ODCIG extracted at the x - and y -coordinates of the bright spot shows a clear focusing of the energy, highlighting the correctness of the migration velocity model used during the inversion process. In fact, the corresponding ADCIG, obtained using the 3D pseudoinverse trans-

form from subsurface offset to angle, presents a clear flat moveout. Finally, the amplitude reflection-angle behavior of the ADCIG extracted within the prospect for a fixed azimuth is shown in Figure 37. We observe a clear decrease in amplitude as the reflection angle increases, which corresponds to a class-3 AVO signature (Castagna, 1993). This AVO class is commonly associated with the top of gas-bearing sand reservoirs (Goodway et al., 2008). This hypothesis is also supported by the associated velocity decrease resulting from applying acoustic FWI in the velocity model-building step.

The field example demonstrates the efficacy of the proposed extended linearized waveform inversion to form true-amplitude image volumes. This test also shows how identified targets from the subsurface image can be correctly characterized from an AVO perspective using the computed ADCIG image amplitudes.

Figure 33. (a) Cross- and (b) inline slices extracted from the inverted acoustic FWI velocity model at $x = 214.8$ km and $y = 51.5$ km, respectively.

Figure 35. A 30 Hz RTM image volume obtained using the FWI model showing sections passing through a potential reservoir indicated by the red arrow. In these panels, negative values are plotted in white.

Figure 34. Depth slices from the 30 Hz RTM volume. The slices are extracted at (a) $z = 0.9$ km, (b) $z = 1.69$ km, and (c) $z = 2.865$ km.

Figure 36. Close-ups of extended least-squares migration results of the field data example. (a) Zero-subsurface offset. (b) A 3D ODCIG extracted at $x = 216.1$ km and $y = 51.5$ km. (c) A 3D ADCIG extracted at the same spatial coordinates as in (b). In these panels, negative values are plotted in white.

Figure 37. Amplitude behavior of the 3D ADCIG of Figure 36c extracted at $z = 2.7$ km and $\phi = 45^\circ$.

CONCLUSION

We demonstrate the ability of the extended subsurface-offset image space to produce true-amplitude migrated images by preserving the elastic behavior of primary reflections when these events are acoustically migrated. Using acoustic wavefields during the image formation can correctly approximate transmission effects as long as strong P-S conversions in the overburden do not occur. In the case of strong conversion effects, the use of elastic wavefields must be considered during the migration step. We pose the imaging step as an inverse problem that we implement as an extended linearized, least-squares waveform inversion in which a regularization term enforces the focus of the image gathers, resulting in coherency over reflection angles. We highlight the importance of applying the pseudoinverse transformation to correctly map the subsurface-offset axis into the corresponding reflection angle and show the superiority of these results with those obtained with the corresponding adjoint operator.

Using two synthetic tests, we quantitatively verify the excellent agreement between the elastic amplitude variations of the AD images and those theoretically predicted by the Zoeppritz P-P reflection coefficient. In the first test, we show that ADCIGs obtained by an acoustic least-squares inversion of elastic pressure data fully retain the elastic-amplitude response even at and beyond the critical reflection angle. In the second numerical test, we solve an extended linearized waveform inversion problem using 1C data from an elastic subsalt model. We illustrate the importance of data selection and the use of a DSO regularization term during the linear inversion process. In this case, we show the robust and accurate amplitude-preserving nature of the extended image space by comparing the ADCIG responses with theoretically predicted AVA amplitudes for three subsalt sand lenses.

The described amplitude-preserving imaging process is based entirely on wave-equation operators, making its applicability to complex geologic scenarios, such as the subsalt test described here, appealing for field-data applications in which ray-based methods fail to create useful true-amplitude image volumes. We demonstrate the ability of our method to produce amplitude-preserved images in field data by applying it to the pressure component from an OBN data set recorded in the GOM. The resulting 3D ADCIGs clearly

reveal class-3 AVO behavior, which is in agreement with the presence of a gas-bearing sand prospect.

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DATA AND MATERIALS AVAILABILITY

Data associated with this research are confidential and cannot be released.

APPENDIX A

2D AND 3D SUBSURFACE-OFFSET-TO-ANGLE TRANSFORMATIONS

Forward and adjoint operators

The transformation from the subsurface OD into the AD for the 2D case can be written as follows (Sava and Fomel, 2003):

$$I_\gamma(x, z, \gamma) = \int_{-\infty}^{\infty} I_h(x, z + \tan(\gamma)h, h)dh, \quad (\text{A-1})$$

where x and z represent the spatial dimensions, γ is the angle, and h is the subsurface offset. Without loss of generality, we drop the dependency on x of the images I_γ and I_h . Let us refer to the transformation of equation A-1 as the adjoint operator a^* (i.e., mapping the OD into the AD). Then, the goal of this appendix is to find the forward operator a , which maps the AD to the OD, such that

$$\begin{aligned} \langle a^*(I_h), I_\gamma \rangle_{z,\gamma} &= \int_{-\pi/2}^{\pi/2} \int_{-\infty}^{\infty} [a^*(I_h)]^* I_\gamma(z, \gamma) dz d\gamma = \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (I_h)^* a(I_\gamma)(z, h) dz dh &= \langle I_h, a(I_\gamma) \rangle_{z,h}, \end{aligned} \quad (\text{A-2})$$

where $*$ represents the complex conjugate operation. To derive such an operator, we use the Plancherel theorem, which can be stated as follows:

$$\begin{aligned} \langle a^*(I_h), I_\gamma \rangle_{z,\gamma} &= \langle A^*(\hat{I}_h), \hat{I}_\gamma \rangle_{k_z,\gamma} = \\ \langle \hat{I}_h, A(\hat{I}_\gamma) \rangle_{k_z,h} &= \langle I_h, a(I_\gamma) \rangle_{z,h}, \end{aligned} \quad (\text{A-3})$$

where $\hat{I}_h$ and $\hat{I}_\gamma$ are the Fourier-transformed images in the OD and AD, respectively, along the z dimension, whereas A and A^* are for-

ward and adjoint operators, respectively, of the angle-to-offset transform in the Fourier domain. Therefore, to find the forward operator a , we can derive the corresponding operator A and apply the inverse Fourier transform to go back to the spatial domain z .

First, we derive the operator A^* by substituting the Fourier-transformed I_h (i.e., $I_h(z, h) = \int_{-\infty}^{\infty} \hat{I}_h(k_z, h) e^{i2\pi k_z z} dk_z$) into equation A-1:

$$\int_{-\infty}^{\infty} I_h(z + \tan(\gamma)h, h)dh = \int_{-\infty}^{\infty} e^{i2\pi k_z z} \int_{-\infty}^{\infty} \hat{I}_h(k_z, h) e^{i2\pi k_z \tan(\gamma)h} dh dk_z, \quad (\text{A-4})$$

which shows that

$$\hat{I}_\gamma(k_z, \gamma) = A^*(\hat{I}_h)(k_z, \gamma) = \int_{-\infty}^{\infty} \hat{I}_h(k_z, h) e^{i2\pi k_z \tan(\gamma)h} dh. \quad (\text{A-5})$$

We find operator A by applying the definition in equation A-3 with equation A-5 in the following simple algebraic steps:

$$\begin{aligned} \int_{-\pi/2}^{\pi/2} \int_{-\infty}^{\infty} \hat{I}_\gamma(k_z, \gamma) \int_{-\infty}^{\infty} [\hat{I}_h(k_z, h) e^{i2\pi k_z \tan(\gamma)h}]^* dh dk_z d\gamma = \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}_h^*(k_z, h) \int_{-\pi/2}^{\pi/2} \hat{I}_\gamma(k_z, \gamma) e^{-i2\pi k_z \tan(\gamma)h} d\gamma dk_z dh. \end{aligned} \quad (\text{A-6})$$

Hence, the forward operator A is given by the following:

$$A(\hat{I}_\gamma)(k_z, h) = \int_{-\pi/2}^{\pi/2} \hat{I}_\gamma(k_z, \gamma) e^{-i2\pi k_z \tan(\gamma)h} d\gamma, \quad (\text{A-7})$$

with the adjoint operator given by the inverse Fourier transform of equation A-5. Finally, by applying the inverse Fourier transform along the z -axis in equation A-7, we can write the following expression:

$$\tilde{I}_h(x, z, h) = \int_{-\pi/2}^{\pi/2} I_\gamma(x, z - \tan(\gamma)h, \gamma) d\gamma, \quad (\text{A-8})$$

which shows that the forward operator a of equation A-1 is a slant-stack operation performed in the AD.

By following the same reasoning, using Biondi and Tisserant (2004), we write the forward and adjoint 3D OD-to-AD transformations, respectively, as follows:

$$\tilde{I}_h(x, y, z, h_x, h_y) = \int_{-\pi/2}^{\pi/2} \int_0^\pi \tilde{I}_\gamma(x, y, z - \tan(\gamma)[\cos(\phi)h_x - \sin(\phi)h_y], \gamma, \phi) d\phi d\gamma, \quad (\text{A-9})$$

$$\tilde{I}_\gamma(x, y, z, \gamma, \phi) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{I}_h(x, y, z + \tan(\gamma)[\cos(\phi)h_x - \sin(\phi)h_y], h_x, h_y) dh_x dh_y. \quad (\text{A-10})$$

2D and 3D pseudoinverse operators

Whenever dealing with a domain transformation (e.g., Fourier transform), it is useful to find its pseudoinverse operator. Under certain conditions, for the OD-to-AD transform, we can define the pseudoinverse in the Fourier domain as follows:

$$A^\dagger = (A^*A)^{-1}A^*. \quad (\text{A-11})$$

Hence, the derivation of the pseudoinverse operator of the subsurface offset-to-angle transform requires knowledge of the inverse of A^*A . For the operators described in the previous section, we can write the following expression:

$$\begin{aligned} A^*A(\hat{I}_\gamma)(k_z, \gamma') &= \int_{-\infty}^{\infty} e^{i2\pi k_z \tan(\gamma')h} \int_{-\pi/2}^{\pi/2} \hat{I}_\gamma(k_z, \gamma) e^{-i2\pi k_z \tan(\gamma)h} d\gamma dh \\ &= \int_{-\infty}^{\infty} e^{i2\pi p' h} \int_{-\infty}^{\infty} \frac{\cos^2(f(p))}{k_z} \hat{I}_\gamma(k_z, f(p)) e^{-i2\pi p h} dp dh \\ &= \frac{\cos^2(\gamma')}{k_z} \hat{I}_\gamma(k_z, \gamma'), \end{aligned} \quad (\text{A-12})$$

where we define $p = k_z \tan(\gamma)$, $dp = d\gamma k_z / \cos^2(\gamma)$, and $f(p) = \arctan(p/k_z)$, and we have used the Fourier inversion theorem. If we assume that the image $\hat{I}_\gamma$ is equal to zero for $\gamma = \{-\pi/2, \pi/2\}$ and $k_z = 0$, we can write the pseudoinverse operator as

$$A^\dagger(\hat{I}_h)(k_z, \gamma) = \frac{k_z}{\cos^2(\gamma)} \int_{-\infty}^{\infty} \hat{I}_h(k_z, h) e^{i2\pi k_z \tan(\gamma)h} dh. \quad (\text{A-13})$$

Finally, it is trivial to show that, in the space domain, the pseudoinverse can be written as follows:

$$a^\dagger = F^*(A^*A)^{-1}A^*F, \quad (\text{A-14})$$

where F represents the Fourier transform operator along the z -axis. The same result can be obtained using the ray parameter within the transform integrals (Liu and Luo, 2020).

The derivation of the pseudoinverse operator for the 3D OD-to-AD transform can be derived in a similar fashion as the 2D case. If A_{3D} denotes the Fourier-domain 3D AD-to-OD transform, then equation A-12 can be written as follows:

$$\begin{aligned} A_{3D}^\dagger A_{3D}(\hat{I}_\gamma)(k_z, \gamma', \phi') &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i2\pi k_z \tan(\gamma')h_x - \sin(\phi')h_y} \left[\int_{-\pi/2}^{\pi/2} \int_0^\pi \hat{I}_\gamma(k_z, \gamma, \phi) e^{-i2\pi k_z \tan(\gamma)[\cos(\phi)h_x - \sin(\phi)h_y]} d\phi d\gamma \right] dh_x dh_y \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i2\pi p' h_x - i2\pi q' h_y} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\cos^2(f(p, q))}{k_z^2 \tan^2(\gamma)} \hat{I}_\gamma(k_z, f(p, q), g(p, q)) e^{-i2\pi p h_x - i2\pi q h_y} dp dq \right] dh_x dh_y \\ &= \frac{\cos^2(\gamma')}{k_z^2 \tan^2(\gamma')} \hat{I}_\gamma(k_z, \gamma', \phi'). \end{aligned} \quad (\text{A-15})$$

where $p = k_z \tan(\gamma) \cos(\phi)$, $q = k_z \tan(\gamma) \sin(\phi)$, $f(p, q) = \arctan(\sqrt{p^2 + q^2}/k_z)$, $g(p, q) = \arctan(q/p)$, and $dp dq = k_z^2 \tan(\gamma) / \cos^2(\gamma) d\phi d\gamma$. Therefore, the pseudoinverse operator can be written as follows:

$$A_{3D}^\dagger(\hat{I}_h)(k_z, \gamma, \phi) = \frac{k_z^2 \tan(\gamma)}{\cos^2(\gamma)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{I}_h(k_z, h_x, h_y) e^{i2\pi k_z \tan(\gamma)[\cos(\phi)h_x - \sin(\phi)h_y]} dh_x dh_y. \quad (\text{A-16})$$

APPENDIX B

EQUIVALENCE OF DSO REGULARIZATION TO SMOOTHING ALONG ANGLES

In this section, we show how the minimization of a DSO-regularized extended image corresponds to enforcing a smoothing constraint along the angle in the AD image. To simplify the discussion, we define the OD-to-AD transform in terms of $q = \tan(\gamma)$, an invertible change of variables in the domain of γ , and we consider the 2D case. We will now show that the DSO smoothing constraint can be written as an equivalent expression as follows:

$$\left\langle \frac{\partial \hat{I}_q(k_z, q)}{\partial q}, \frac{\partial \hat{I}_q(k_z, q)}{\partial q} \right\rangle_{k_z, q} \propto \langle h \hat{I}_h(k_z, h), h \hat{I}_h(k_z, h) \rangle_{k_z, h}, \quad (\text{B-1})$$

where $\hat{I}_q(k_z, q)$ is given by (see equation A-5)

$$\hat{I}_q(k_z, q) = A^*(\hat{I}_h)(k_z, q) = \int_{-\infty}^{\infty} \hat{I}_h(k_z, h) e^{i2\pi k_z q h} dh. \quad (\text{B-2})$$

The derivative along the variable q is then written as follows:

$$\begin{aligned} \frac{\partial \hat{I}_q(k_z, q)}{\partial q} &= i2\pi k_z \int_{-\infty}^{\infty} h \hat{I}_h(k_z, h) e^{i2\pi k_z q h} dh \\ &= i2\pi k_z A^*(h \hat{I}_h)(k_z, q). \end{aligned} \quad (\text{B-3})$$

Using equations B-3 and A-3, we write the left side of equation B-1 as

$$\begin{aligned} \left\langle \frac{\partial \hat{I}_q(k_z, q)}{\partial q}, \frac{\partial \hat{I}_q(k_z, q)}{\partial q} \right\rangle_{k_z, q} &= \langle i2\pi k_z A^*(h \hat{I}_h)(k_z, q), i2\pi k_z A^*(h \hat{I}_h)(k_z, q) \rangle_{k_z, q} \\ &= \langle h \hat{I}_h(k_z, h), (2\pi k_z)^2 A A^*(h \hat{I}_h)(k_z, h) \rangle_{k_z, h}. \end{aligned} \quad (\text{B-4})$$

From equation B-2, the application of the operator AA^* can be written as follows:

$$\begin{aligned} AA^*(\hat{I}_h)(k_z, h) &= \int_{-\infty}^{\infty} e^{-i2\pi k_z q h} \int_{-\infty}^{\infty} \hat{I}_h(k_z, h') e^{i2\pi k_z q h'} dh' dq \\ &= k_z^{-1} \hat{I}_h(k_z, h). \end{aligned} \quad (\text{B-5})$$

Hence, equation B-4 yields

$$\left\langle \frac{\partial \hat{I}_q(k_z, q)}{\partial q}, \frac{\partial \hat{I}_q(k_z, q)}{\partial q} \right\rangle_{k_z, q} = 4\pi^2 \langle \hat{I}_h(k_z, h), |k_z| \hat{I}_h(k_z, h) \rangle_{k_z, h} \\ \propto \langle \hat{I}_h(k_z, h), \hat{I}_h(k_z, h) \rangle_{k_z, h}, \quad (\text{B-6})$$

which shows the proportionality between the two norms and thus their equivalence when minimized.

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Biographies and photographs of the authors are not available.