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Inelastic Behavior of Tungsten Carbide at High Pressures

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Abstract

Tungsten carbide (WC) is commonly used in plate impact experiments for generating high pressures due to its combination of high impedance and high strength. In pressure-shear plate impact (PSPI) experiments, WC is used as anvil plates, which sandwich a thin sample of the material of interest. At pressures up to its HEL (~5 GPa), an elastic response may be assumed for the anvil plate, enabling a straightforward analysis to extract the shear strength of the material. At higher pressures, exceeding the HEL, the inelastic behavior of WC must be taken into account to extract the shearing resistance of the material of interest. We report the results of pressure-shear plate impact experiments conducted in the pressure range of 9-37 GPa. Two modeling approaches are considered to account for the inelastic behavior of WC: (1) an elasto-plastic response, accounting for strain and strain-rate hardening, and (2) a brittle-like response, which incorporates pressure hardening and damage. The material models were calibrated using numerical simulations to reconstruct the measured free surface velocity components in the normal and transverse directions. Both models effectively simulate the measured velocity profiles. Above 20 GPa, the brittle model appears to better capture the experimental trends. The calibrated models for WC enable the reconstruction of the experimental data from the current work, as well as from results reported by others, for a broad range of pressures.

Keywords

Pressure-shear, Tungsten carbide, Dynamic strength, High pressure, High strain rate

1. Introduction

Tungsten carbide (WC) based ceramics are widely used due to their unique combination of extreme and desirable properties, such as high density, high strength and high impedance, as well as high melting temperature, chemical stability, and wear resistance. Major applications of tungsten carbide are in the defense industry for dense cores of armor-piercing ammunition and penetrators. WC is also used for cutting tools for machining and rock cutting. An additional important application of WC is in high-pressure mechanics of materials experiments, both in the quasi-static regime, such as for diamond anvil cell configurations, and the dynamic regime, for impact loaded samples. The high impedance of the material coupled with its high strength enables impact testing and shock experiments, at very high pressures.

Because WC is a brittle material, different metals are frequently added, most commonly cobalt (Co) in the range of 3%-20%, to increase fracture toughness and failure strain.

There is an extensive body of literature on the strength, damage, and failure properties of WC extending over orders of magnitude in pressures, strain rates, and grain sizes. Quasi-static and indentation tests have been used to examine the influence of cobalt content and grain sizes on the

mechanical strength and deformation mechanisms [1-3]. The hardness of WC has been shown to follow a Hall-Petch type relation (i.e. dependence of strength on inverse square root of grain size) [1], Its compressive strength was found to increase with decreasing metal binder content, up to a maximum content of ~5-10% cobalt [2,3]. The strain rate sensitivity of WC was investigated by Mandel et al. [4] by dynamically loading samples with different cobalt contents in a Kolsky (Split-Hopkinson) pressure bar. They observed a significant rise in strain rate sensitivity above $\sim 10^2 \text{ s}^{-1}$ for all cobalt contents in the range of 6%-12%. Tungsten carbide exhibits the material behavior of a high-density heavy metal, on the one hand, and a brittle-like ceramic on the other. In the current work, the applicability of both ductile and brittle material models are considered for describing the dynamic mechanical behavior of WC.

At high strain rates and pressures, inelastic behavior and failure mechanisms become dominant in WC. Holmquist et al. [5] examined the behavior of a 6% Co WC heavy alloy penetrator in ballistic tests. A calibrated material model developed for the application combined a J_2 plasticity approach with a Johnson-Cook strength model [6] and the Johnson-Cook failure model [7]. To best match the measured results, they needed to incorporate the dependence of failure on triaxiality. Strain rate dependence was found to be negligible in this regime of high strain rates. A similar conclusion is reported in the work of Hazell et al. [8] from ballistic tests conducted on WC alloys with 6.5% and 8.5% Co binder.

The use of ballistic tests is not ideal for calibrating or validating material models as these tests involve multiaxial loading, inhomogeneous states of deformation, and failure states. A more controlled methodology for investigating and characterizing the inelastic behavior of tungsten carbide involves plane wave loading from plate impact. In these experiments, velocity-time profiles are acquired by interferometric techniques, which allow the continuous observation of material response, as elastic and plastic waves sweep through the sample. Dandekar and Grady [9] studied the mechanical equation of state and the strength of WC through an extensive series of shock compression experiments at stresses in the range of 9-80 GPa. The results led to the construction of a shock velocity vs. particle velocity (U_s - u_p) relation characterized by a Hugoniot Elastic Limit (HEL) of 6.6 GPa, followed by a linear dependence of U_s on u_p . Dandekar [10], in a later work, updated the HEL value to be 7.2 GPa and presented additional shock data on this material in spall experiments up to 24 GPa. The measured spall strength of ~2 GPa at low pressures was shown to decrease significantly with increasing normal stress until reaching an average value of ~0.85 GPa at an impact stress of 24 GPa. The noted decrease in spall strength is possibly due to compressive, stress-induced micro-cracks, or, by pore collapse, as suggested by Frutsky et al. [11]. Dandekar and Grady concluded [9] that at high pressures (9-80 GPa) the material behaves as an elastic-plastic solid. A similar conclusion was presented by Millet et al. [12], based on the results from laterally mounted stress gauges in plate impact shock experiments, which revealed a decrease in shear strength behind the shock front. Two possible mechanisms were proposed for the observed decrease in shear strength: (1) so-called "failure waves" initiated at the shock front as suggested for other brittle materials [13], and (2) plasticity due to the generation and motion of dislocations behind the shock front.

Significant work has been performed on WC using the pressure-shear plate impact (PSPI) technique [14-16] at high strain rates and pressures. In a sandwich configuration, the experiment features combined normal and transverse loading of a thin specimen sandwiched between two

WC anvil plates. Frutschy et al. [11,17] conducted symmetric PSPI experiments on pure WC at different temperatures and reported only a small deviation from the elastic response up to pressures of ~ 7 GPa. Following Frutschy's work [17], Zuanetti et al. [18] conducted PSPI experiments at different temperatures with pressures up to 5 GPa. Experiments conducted at higher shear stresses, using larger skew angles, revealed fluctuations of the transverse (shear) signal, suggesting a possible failure in tension due to the interaction of the longitudinal wave reflected from the rear surface of the anvil plate, and the oncoming shear wave.

Clifton and Jiao [19] report results from a PSPI experiment conducted at normal stress approaching 25 GPa, which showed significant shear strength with no evidence of failure. They incorporated a Voce-Palm strain-hardening model in their numerical simulations to account for the plastic shock structure of the normal velocity profile. A material strength of 7.5-11.0 GPa was used to obtain the best match of the measured normal velocity profile, but that model overestimated the measured transverse resistance observed in the experiment.

PSPI experiments have been conducted on a wide variety of materials, using anvil plates selected to ensure their elastic response [14-17]. The primary motivation of the present work is to extend PSPI experiments to much higher pressures. The measured free surface velocities in a PSPI experiment could be affected by wave attenuation and dispersion of the inelastically deforming anvils. Determining the inelastic response of WC at high pressures, and addressing its strength and possible damage mechanisms under these conditions, is crucial to characterizing the strength of the material of interest at high pressures.

In this work, we present and examine the results from pressure-shear experiments conducted on WC in the range of 9-37 GPa, using the gas gun facilities at Brown University [14] and a powder gun facility at Caltech [20,21]. Material models have been developed to account for the inelastic behavior of WC both for strength and damage beyond its HEL. Two modeling approaches are considered: (1) an elasto-viscoplastic response, accounting for strain and strain rate hardening, and (2) a brittle like response using the Johnson-Holmquist (JH2) model, which incorporates pressure hardening and damaged states of the material. Material models are calibrated through numerical simulations adjusted to achieve the best match of both the time-resolved normal and transverse free surface velocity signals. In Section 2, the PSPI experiment is described along with details of the high-pressure experiments. Results are presented from experiments conducted on WC. Numerical simulations, using both ductile and brittle material models, are described in Section 3. Results of the simulations are compared with measured PSPI normal and transverse particle velocity records, and with additional published results in the literature. Section 4 follows with a discussion of the behavior of WC under extreme loading conditions. Section 5 presents the summary and conclusions.

2. PSPI experiments on WC

The experimental work used to calibrate and validate a material model for tungsten carbide includes two types of experiments: (1) symmetric PSPI experiments, and (2) sandwich experiments in which a thin layer of soda lime glass (SLG) is sandwiched between two WC anvils. The latter case, while introducing an unknown response of the SLG is nevertheless helpful because the normal velocity in these experiments is only weakly dependent on the

presence of the thin SLG layer. The experimental gas guns at Brown University have been described previously [e.g., 14-17]. The recently established powder gun set up at Caltech for PSPI at high-pressure, and the tools for measuring normal and transverse motion, are described elsewhere [21, 22]. Figure 1 shows the two types of target configurations. In the symmetric configuration, the flyer and target are thick WC plates. Both normal and transverse particle velocities, measured at the free surface of the target plate, probe WC's response. This coupled measurement, as will be shown later, provides a strong constraint when calibrating a material model – necessitating a simultaneous match of measurements and simulations for both transverse and normal motions. The target in the sandwich configuration consists of front and back WC anvil plates, which sandwich a thin layer of the sample material. This configuration induces very high shear strains and shear strain rates in the sample. The sample is a 5 μm -thick soda-lime glass (SLG) layer that is significantly weaker than the WC anvils. The measured normal velocity is determined primarily by the behavior of WC, which undergoes significant normal strains under the compressive loading. Normal motions, measured for a wide range of pressures, were used to validate and extend the calibrated material model to a broad range of pressures.

Figure 1. PSPI experimental configurations: (a) symmetric and (b) sandwiched target.

Tungsten carbide flyers and anvils with 3% cobalt binder were procured from Basic Carbide Corporation (Lowber, PA). The material properties are listed in Table 1.

Table 1. Selected properties of tungsten carbide with 3% cobalt binder.

Density, ρ [kg/m ³]	Longitudinal wave speed, C_L [mm/ μs]	Shear wave speed, C_S [mm/ μs]	Bulk modulus K [GPa]	Shear modulus G [GPa]
15,600	6.91	4.18	379	273

SEM analysis of the material revealed negligible porosity. Scattered pores of triangular shaped voids were found in sizes ranging from 300 to 400 nm.

2.1. Results

Table 2 summarizes experimental parameters for PSPI experiments conducted at Brown University (identified by TJ) and Caltech (identified by CK). The WC material used at both

experimental facilities were produced by the same manufacturer with identical specifications. The peak normal stress in the experiments, σ_{peak} , is calculated using the WC inelastic material model discussed in Section 2.

Table 2. Experimental parameters for PSPI experiments on WC conducted at Brown University (TJ) and Caltech (CK).

Experiment	Type	V_{impact} [m/s]	Skew Angle	Thickness [mm]			Normal stress σ_{peak} [GPa]	Tilt [mrad]
				SLG Sample	Flyer	Target*		
TJ-1603	Sandwich	188	18°	0.005	6.39	3.97/5.92	9.0	0.40
CK-1806	Symmetric	319	18°	-	4.97	3.93	14.1	0.30
TJ-1802	Sandwich	483	16°	0.005	6.43	3.95/5.88	21.2	1.90
CK-1803	Sandwich	606	18°	0.005	4.97	1.96/3.78	25.8	0.80
CK-1804	Symmetric	617	18°	-	4.99	3.91	26.5	0.50
CK-1701	Sandwich	736	18°	0.005	4.98	2.01/3.77	31.4	N/A
CK-1801**	Symmetric	863	18°	-	4.97	3.85	36.9	3.4

*Target thicknesses correspond to the front (thin) and rear (thick) WC anvils

**For this symmetric shot, only the normal signal was measured successfully because of large tilt.

2.1.1. Symmetric PSPI experiments on WC

Figure 2 shows measured normal and transverse free surface particle velocities for two symmetric WC shots: CK-1806 and CK-1804. A large amplitude transverse signal is recorded for the CK-1806 shot, indicating high shear strength of WC. For shot CK-1804, at an impact velocity of 617 m/s, a very low transverse velocity was recorded. This response is believed to be due to slip at the impact face; consequently, this transverse motion is not included in the subsequent modeling of material response.

Figure 2. Free surface velocities for symmetric PSPI experiments CK-1806 and CK-1804.

2.1.2. Normal response of sandwich targets

The results for five additional free surface normal velocity profiles recorded over a broad range of impact velocities and pressures are shown in Fig. 3. All normal velocity profiles except for CK-1801 (at $V=863$ m/s) were measured for the sandwich configuration. CK-1801 corresponds to a symmetric WC experiment; due to large tilt, only the normal response was measured successfully. For comparison purposes, the signals were shifted to have the same arrival times. The structure of the plastic wave and its evolution at different pressures is valuable in examining and validating the material model over a large range of pressures. The plastic wave in the high-velocity experiment ($V=863$ m/s) shows the pronounced steepening of the development of a shock wave.

Figure 3. Additional free surface normal velocities at various impact velocities (see Table 2).

3. Numerical simulations

The finite element solver LS-DYNA (Version R9.0.1) [23], with explicit time integration, was used for the simulations. A planar geometry was modeled by taking a thin slice of the 3D geometry representative of the middle cross-section of the plates. All nodes were constrained to ensure plane wave propagation. Constant stress 3D solid elements with a Flanagan-Belytschko hourglass stabilization scheme were used. A mesh size of $10\ \mu\text{m}$ in the shock direction was used as a converged mesh size to account for the high-velocity gradients in the normal elastic precursor [24]. All surfaces between plates were joined at the nodes to avoid additional numerical issues associated with modeling contact. Initial conditions were given by prescribing initial flyer plate velocities in normal and transverse directions. Actual sample dimensions were used to account for release waves emanating from lateral boundaries. Two material models for WC were examined in this work: (i) a plasticity model encompassing its metal-like characteristics and (ii) a brittle, damage model accounting for its character as a sintered ceramic-

like material. The two models are described next, each followed by the comparison of simulations with experimental data.

3.1. Plasticity Model

Results from the literature described in the introduction, and the structure of the measured plastic waves in the current experiments, suggest the inclusion of strain and strain-rate hardening effects in the constitutive relation for WC. The model incorporates a power-law strain hardening description for the flow stress [6]:

$$S(\varepsilon^{pl}) = Y_0 + B(\varepsilon^{pl})^n \quad (1)$$

where ε^{pl} is the effective plastic strain, Y_0 represents the yield strength of the material, B , is a hardening parameter and n is the strain hardening exponent. The strain-rate hardening is modeled through a Cowper-Symonds law of the form [25]:

$$R(\dot{\varepsilon}^{pl}) = \left(1 + \left(\frac{\dot{\varepsilon}_{eff}^{pl}}{C \dot{\varepsilon}_0^{pl}} \right)^{1/p} \right) \quad (2)$$

where C is a rate parameter, p is the power of strain-rate hardening, $\dot{\varepsilon}_{eff}^{pl}$ and $\dot{\varepsilon}_0^{pl}$ are the effective plastic strain rate and a reference plastic strain rate, respectively. Swegle and Grady [26] observed that the strain rate in steady shock experiments for many different materials followed a simple universal relation: $\dot{\varepsilon} = A(\Delta\sigma)^4$, where A is a material constant and $\Delta\sigma$ is the jump in stress associated with the plastic shock front. It can be shown that the Cowper Symonds relation when adopting a power of $p \sim 2$, corresponds to the Swegle and Grady fourth-power, strain-rate law [27]. The complete material model for strength (flow stress) is then achieved by the multiplication of the two terms for strain hardening, $S(\varepsilon^{pl})$, (Eq. (1)) and strain-rate hardening, $R(\dot{\varepsilon}^{pl})$, (Eq. (2)):

$$Y(\varepsilon^{pl}, \dot{\varepsilon}^{pl}) = (Y_0 + B(\varepsilon^{pl})^n) \cdot \left(1 + \left(\frac{\dot{\varepsilon}_{eff}^{pl}}{C \dot{\varepsilon}_0^{pl}} \right)^{1/p} \right) \quad (3)$$

The values of the parameters in the model were calibrated by simultaneously fitting both the normal and transverse particle velocity signals. Then, the fitted model was examined in light of the experiments at lower and higher pressures.

The pressure-volume relation is modeled by a Gruneisen mechanical equation of state (EOS) [28]: that neglects thermo-mechanical coupling:

$$P = \frac{\rho_0 c_0 \mu [1 + (1 - \gamma_0 / 2) \mu]}{(1 - (S_1 - 1) \mu)^2} \quad (4)$$

where P is the pressure, $\mu = \rho / \rho_0 - 1$, is the volumetric strain, i.e., the relative change in density (ρ), ρ_0 is the reference density, c_0 is the bulk wave speed, γ_0 is the Gruneisen parameter and S_I is the first-order coefficient of the dependence of shock velocity on particle velocity (U_s - u_p) relation, $U_s = c_0 + S_I u_p$. The U_s - u_p relation presented in [9] for WC was used in this work.

The calibrated parameters for the plasticity model that provided the best fit of the experimental results are listed in Table 3:

Table 3. Calibrated Parameters for the WC plasticity model.

Y_0 [GPa]	B [GPa]	n	C	ε_0^{pl} [μs^{-1}]	p	c_0 [m/s]	γ_0	S_I
3.2	11.0	0.25	500	1.0	2.0	4930	1.62	1.309

The simulated normal and transverse free surface velocity-time profiles are compared with those measured in experiments CK-1806 and CK-1804 in Fig. 4. As mentioned earlier, the transverse signal of the CK-1804 was affected by slip, restricting the comparison for that experiment to the normal free surface velocity. The simulated normal velocity profile reproduces the HEL and evolution of the plastic wave. The simulation underestimates the peak velocity observed in the experiment by approximately 4%. Following a steep initial rise (<30 m/s), the simulated transverse velocity follows a similarly dispersed profile. The steep rise is a numerical artifact resulting from the nodes joined on the interface of the two impacting plates, causing an instantaneous build-up of the shear forces, which in the experiment is observed as a gradual build-up resulting from frictional processes. Above a transverse velocity of 80 m/s, the simulated response slightly overpredicts the final velocity. Furthermore, the simulated smooth evolution to a constant final transverse velocity does not fully capture the more wavy and slightly upward sloping response observed in the experiment. This wavy response will be discussed in more detail in the next section, relating to possible tensile damage resulting from the interaction of the rarefaction wave reflected from the free surface.

Figure 4. Comparison of simulated normal and transverse velocity-time profiles, obtained with the plasticity model, with the experimental velocimetry measurements for experiments CK-1804 and CK-1806.

For the higher velocity shot, CK-1804, the plasticity model does not capture well the shape of the normal velocity profile, especially the structure of the plastic shock. The strong “knee” observed in the experiment is smoothed out in the simulations. For CK-1806 the modeling of the shearing resistance is initially too stiff and is more realistic as the plateau is approached.

Figure 5 compares the normal free surface velocity signals shown in Fig. 2 and Fig. 3 with plasticity model predictions over a wide range of impact velocities. Up to 20 GPa — the model shows reasonable agreement in capturing the plastic wave structure with a small deviation of 3-4% in predicting the final velocity of the signals. For the higher pressure experiments, all signals seem to underpredict the steepness of the primary plastic wave. This under-prediction of steepness could correspond to an under-prediction of stress relaxation times or, correspondingly, to over-prediction of strain rate sensitivity.

Figure 5. Normal velocity profiles in PSPI measurements: comparison between experimental results and numerical simulations with the calibrated plasticity model (Eq. (3), Table 3).

3.2. Brittle Material Model

Another approach to model the behavior of WC at high-pressures and strain rates is to consider its brittle character as a sintered ceramic material. A phenomenological model, which is widely used in simulating impact of brittle solids, is a modified Johnson-Holmquist model, commonly known as the JH2 model [28]. The material strength relates to two models: one for the intact material and one for the failed material. Both intact and failed states are described by a power-law pressure hardening function and a logarithmic strain-rate hardening function. The normalized intact strength, σ_i^* , and the normalized failed strength, σ_f^* , are defined as:

$$\sigma_i^*(P, \dot{\epsilon}) = A_1 (P^* + T^*)^k (1 + A_2 \ln(\dot{\epsilon})) \quad (5)$$

$$\sigma_f^*(P, \dot{\epsilon}) = A_3 (P^*)^m (1 + A_2 \ln(\dot{\epsilon})) \quad (6)$$

where A_1 , A_2 , A_3 , k and m , are dimensionless parameters, and P^* and T^* are the normalized pressure and normalized maximum tensile hydrostatic pressure, which serves as a tensile stress cut off. The equivalent stresses are normalized with respect to the Hugoniot elastic limit (HEL); the pressure and tensile cut off are normalized with respect to P_{HEL} , which is the hydrostatic pressure at the HEL. The normalized fracture strength could be limited by a maximum cut off value: $\sigma_f^* < SFMAX$.

The normalized equivalent strength is defined in relation to a damage parameter, D , interpolating between the two strength functions at intact and failed states:

$$\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_f^*) \quad (7)$$

The damage for fracture is accumulated based on the plastic strains:

$$D = \Sigma \Delta \varepsilon^p / \varepsilon_f^p \quad (8)$$

where $\Delta \varepsilon^p$ is the increment of plastic strain in each cycle of integration, and ε_f^p is the plastic strain to fracture; ε_f^p is a function of pressure, defined by:

$$\varepsilon_f^p = D_1 (P^* + T^*)^{D_2} \quad (9)$$

where D_1 and D_2 are model parameters. The material cannot undergo any plastic strain when $P^* = -T^*$.

The hydrostatic (volumetric) behavior is defined before fracture begins ($D=0$) as a third-order polynomial function:

$$P = K_1 \mu + K_2 \mu^2 + K_3 \mu^3 \quad (10)$$

where K_1 , K_2 , and K_3 are material constants and K_1 is the bulk modulus. In tension, only the first term as a linear function of the volumetric strain (relative change in density) is used.

An additional significant part of the model relates to how the pressure changes as damage evolves. An additional incremental pressure change is added to the right-hand side of Eq. (10), accounting for the bulking behavior when failure occurs. The additional pressure is calculated from energy considerations as described in [28]. A parameter β is used to describe the fraction of elastic energy that is converted to potential hydrostatic energy. The numerical implementation of the JH2 model for the LS-DYNA software used in this work is described in [29].

The model adopted for tungsten carbide in this work is a simplified version of the JH2 model presented above. The calibration did not account for a damaged or failed state by plastic strain evolution, meaning that the same pressure and strain rate hardening was used for both states, with $A_1 = A_3$ and $k=m$. The evolution of damage function with strain was “bypassed” by using a very large value for D_1 , see Eq. (9). Yet, damage could evolve when the normalized tensile cut-off, T^* , is reached under a state of tension. Specifically, no softening is expected as a result of reaching the tensile cutoff. At this stage, this formulation was focused on the pressure-dependent strength modeling rather than on the use of damage. The calibrated parameters for the JH2 model are summarized in Table 4. For the hydrostatic behavior, only the first term (K_1) in Eq. (10), equal to the bulk modulus, was used.

Table 4. Model parameters of WC for JH2 model

A_1	A_3	k	m	$\varepsilon_0^{pl} [\mu s^{-1}]$	C	HEL [GPa]	P_{HEL} [GPa]	T^*	SFMAX
0.80	0.80	0.50	0.50	1.0	0.03	6.0	3.0	0.40	0.10
β	K_1 [GPa]					K_2 [GPa]	K_3 [GPa]	D_1	D_2
0.0	379.0					0.0	0.0	10.0	0

Figure 6 shows a comparison between the simulated and experimental free surface normal velocity profiles using the calibrated JH2 model and the experimental measurements for the symmetric experiments. For experiment CK-1806, the brittle material model captures the measured free surface velocities quite faithfully and provides insight into specific features in the wave profiles. The feature labeled ‘A’ shows the close match with the normal velocity profile, as well as the gradual pressure increase as a result of the interaction with the shear wave arrival. The second feature, labeled ‘B’ refers to the “dip” in the transverse velocity. This dip is related to the maximum tensile stress and its value was calibrated in relation to this dip. The calibrated value corresponds to 10 μm mesh size resolution and was found to be consistent with other reported results, at different conditions, as shown in Section 3.3. This important feature is discussed in detail in the following section. The simulated normal velocity for experiment CK-1804 compares well with the measured plastic wave profile and the final plateau value. However, the simulation under-predicts the normal velocity in the transition region between the primary plastic wave and the velocity plateau.

Figure 6. Comparison of normal and transverse velocity profiles obtained from velocimetry measurements and numerical simulations with the calibrated JH2 model.

Figure 7. Normal velocity profiles in PSPI measurements: comparison between experimentally measured profiles and numerical simulations with the calibrated JH2 model.

Figure 7, compares simulated and experimental normal velocity profiles over a broad range of pressures. The simulations show reasonably good agreement over the pressure range reported here. At higher pressures, the model seems to be slightly stiffer and predicts higher particle velocities at the plastic shock front than observed in the experiments.

In an effort to further examine the accuracy of the calibrated models, additional simulations were performed to compare with published results on WC.

3.3. Comparison with data in the literature

In a recent work [18], the results for WC are reported from PSPI experiments conducted at pressures below the HEL. Figure 8 depicts a numerical simulation using the calibrated JH2 model of shot #107 in [18], conducted at an impact velocity of 92 m/s. Distinct features are observed in the experimentally measured velocity profiles. In particular, a sharp dip is noted in the transverse velocity profile, centered at $t=2 \mu\text{s}$, and a corresponding increase registered in the normal velocity profile. Both features are captured using the calibrated JH2 model. In [18] the authors suggest this is an occurrence of *failure*, yet it is seen that the transverse velocity recovers later to higher velocities and that it does not further soften, as indicative of failure. Furthermore, at much higher impact velocities, (e.g., CK-1806), for which the initial compressive stresses are higher, the numerical simulation demonstrates that the observed behavior is related to damage due to tension, while maintaining the shear strength. The shear wave propagates into a region near the free surface where the normal stress has been reduced nominally to zero. However, principal stresses at approximately 45 degrees to the normal have values of $\pm\sigma$ and the tensile stress ($+\sigma$) can induce damage when its value exceeds the material's tensile strength. The noted features correspond to the reflection of the shear wave from the damaged material observed at

the free surface. The increased velocities on the normal signal is a result of bulking of the damage area, a well-known phenomenon in granular solids [30].

Figure 8. Normal and transverse velocity profiles obtained from velocimetry measurements for experiment #107 in [18] and numerical simulation with the calibrated JH2 model.

Figure 9 shows a comparison with a symmetric WC PSPI shot conducted at 167 m/s, as reported in [17]. The WC alloy used in this experiment exhibits a lower HEL and is different from the WC material used in the current experiments. A dip in the transverse velocity profile was also observed in this experiment. A reduction in the normalized tensile strength was used (T^* from 0.4 to 0.3) corresponding to the reduction in the HEL between the two materials. The simulation presented in Fig. 9, shows that this dip could be captured accordingly. A good overall match is achieved for both the normal and the transverse velocity profiles.

Figure 9. Normal and transverse velocity-time profiles obtained from velocimetry measurements for experiment KF-9407 in [17], compared with numerical simulation using the calibrated JH2 model.

The results from a numerical simulation of a symmetric PSPI experiment [19] conducted at an impact velocity at 456 m/s are shown in Fig.10. The material of the target and anvil is pure WC. A significantly higher HEL is evident from the experimentally measured normal velocity profile. The simulated normal velocity is generally lower than the measured profile since the model is calibrated for a WC with a lower HEL and thus shows a more inelastic response. The transverse velocity is predicted relatively well.

Figure 10. Normal and transverse velocity profiles obtained from velocimetry measurements for a symmetric PSPI experiment reported in [19] and numerical simulation with the calibrated JH2 model for WC

4. Discussion

4.1. Damage due to tension

In Figs. 6 and 8, a dip (e.g. at B in Fig. 6) in the transverse signal is observed with this feature appearing more dominant in the experiment conducted at a lower pressure (Fig. 8). The noted dip was thought to be related to possible damage due to the tensile stress (i.e., in excess of the tension cut-off, T^* in the JH2 model), that arises in a symmetric PSPI experiment because of the reflection of the compressive wave at the traction-free boundary (rear surface). By this line of reasoning, one may expect damage to increase at higher impact stresses, resulting in tensile failure at some stage of the loading. However, experimental data shows that at higher pressures, this phenomenon actually diminishes, and eventually vanishes, with no sign of tension damage observed at high impact stresses. Recall the experimental records reported in [18] and shown in Fig. 8, resulted from an impact velocity of 92 m/s (~ 5 GPa). As previously noted, the transverse velocity exhibits a pronounced “dip” indicating a significant damage effect relative to the initial stress measured for WC.

Figure 11. Maximum principal stresses at varying distances from the free surface in the CK-1806 experiment (319 m/s, ~ 15 GPa), modeled with the calibrated JH2 model and with an elastic model (dashed lines) for comparison.

The calibrated tension cap in the model is 1.0 GPa, which is close to the reported spall strength measured for pressures above 3 GPa [10]. Results from experiment CK-1806 shown in Fig. 6, conducted at an impact velocity of 319 m/s, shows a relatively small dip at (B) whereas no dip is observed at higher pressures (i.e. see Fig. 10 from [19]). To explain this behavior, results from the numerical simulations for two cases were examined as a function of distance from the free surface. The analysis was carried out for experiments CK-1806 (319 m/s, ~ 15 GPa) and CK-1804 (617 m/s, ~ 25 GPa). Results for the time evolution of the maximum principal stress at

different locations are shown in Figs. 11 and 12. The figures compare simulations performed with the JH2 model to a purely elastic model (for some selected locations). In Fig. 12, it is seen that for the elastic model, a significant tension above 3 GPa would be expected, with a steep rise time. On the other hand, for the inelastic case (JH2), release times are significantly longer and during this dispersed release, a much smaller tension stress develops. For the 319 m/s experiment (CK-1806) this value is just slightly above 1 GPa. For the 617 m/s experiment CK-1804, the release occurs from higher stress values. Time for release is longer and the actual tension stresses are lower than 1 GPa. It is also seen that tension occurs in the vicinity of the free surface and tensile stresses are almost non-existent beyond a depth of 0.5 mm from the free surface.

Figure 12. Maximum principal stresses at different distances from the free surface in CK-1804 experiment (617 m/s, ~25 GPa), modeled with the calibrated JH2 model and with an elastic model (dashed lines) for comparison.

5. Summary and conclusion

For PSPI experiments at high pressures, the inelastic behavior of WC anvils needs to be accounted for in order to extract the strength of the material of interest from these experiments. In this work, two material strength models for WC were calibrated. Pressure-shear experiments at high pressures on WC, in the range of 9-37 GPa, formed the basis for calibrating and validating the material strength models. Two approaches were examined: an elasto-viscoplastic response, accounting for strain and strain-rate hardening, and a brittle-like response using the JH2 model, incorporating damaged states of WC. The material models were calibrated using numerical simulations to reconstruct the measured free surface velocity measurements, both in the normal and transverse directions. Both models show quite good agreement up to pressures of 20 GPa. Above 20 GPa, the JH2 model seems to fit the measured velocity-time profiles better. The JH2 model also shows promise for interpreting tensile failure associated with longitudinal

wave reflection from the traction-free rear surface. This tensile failure occurs in the vicinity of the free surface.

The following conclusions are drawn from this study.

1. Both plasticity and brittle modeling approaches show possibility for modeling the inelastic behavior of WC at high pressures. Strain rate effects appear to play a minor role in the observed response. Above 20 GPa, the brittle modeling of WC seems to fit the measured velocity-time profiles better, and the dominant hardening mechanism, responsible for the inelastic response of tungsten carbide (WC) at high pressures appears to be pressure hardening.
2. Symmetric PSPI experiments, conducted with relatively thick plates ($> 3\text{mm}$), provide valuable data for developing a material model for WC. Matching both normal and transverse velocity-time profiles provides a strong constraint on the acceptability of potential material models.
3. Simulations of the experiments suggest possible damage in WC at higher pressures. A suggested possible damage mechanism is that of pore collapse [17] at the higher pressures. Such damage could also explain reported decreases in spall strength with increasing pressure [10].
4. Damage due to tension arising from the release wave reflected from the traction-free rear surface of the target plate was captured by numerical simulations at different pressures. Counterintuitively, such damage seems to be suppressed at increasing shock pressures. This behavior could be a result of the release wave being spread out by the inelastic response of void growth. As a result, the ramp-like propagation of the release wave can suppress or delay development of large tensile stresses.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. I hear by declare that this statement has been approved by all authors

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