Supplemental Document

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Quasi-static optical parametric amplification: supplement

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Quasi-static Optical Parametric Amplification: supplemental document

This supplemental document presents a more detailed analysis of quasi-static OPA by considering the frequency-domain evolution of the generated signal field in the presence of an undepleted pump. This approach highlights the roles played by various dispersion orders of the nonlinear waveguide. Both group-velocity mismatch ($\Delta k'$, GVM) and the group velocity dispersion (GVD) of the second harmonic ($k''_{2\omega}$) restrict the input pump bandwidth that may effectively contribute to parametric gain. Similarly, the group velocity dispersion of the fundamental (k''_{ω}) and fourth order dispersion of the fundamental ($k^{(4)}_{\omega}$) limit the bandwidth that may undergo parametric gain. In the absence of dispersion, the model presented here simplifies to a quasi-continuous wave (quasi-CW) treatment for the generated signal and idler, with the peak gain of the input pump pulses setting the characteristic gain lengths. We then briefly discuss the extension of this model to optical parametric generation (OPG). Finally, we conclude this document by showing the evolution of the dominant dispersion orders, namely $\Delta k'$ and k''_{ω}, as a function of waveguide geometry. A suitable choice of waveguide geometry may eliminate these dispersion orders, thereby achieving quasi-static interactions.

1. FREQUENCY DOMAIN THEORY OF QUASI-STATIC OPA

The coupled-wave equations (CWEs) for OPA in the time-domain are given by

$$\partial_z A_\omega(z,t) = -i\kappa A_{2\omega}(z,t)A^*_\omega(z,t)\exp(-i\Delta k z) + \hat{D}_\omega A_\omega(z,t), \tag{S1a}$$

$$\partial_z A_{2\omega}(z,t) = -i\kappa A^2_\omega(z,t)\exp(i\Delta k z) - \Delta k'\partial_t A_{2\omega}(z,t) + \hat{D}_{2\omega}A_{2\omega}(z,t), \tag{S1b}$$

where the quantities here are defined in the main text. In the absence of pump depletion we may obtain a more complete solution to Eqns. S1a-S1b that accounts for dispersion to arbitrary order by considering the evolution of generated signal in the frequency domain.

In the undepleted limit, the evolution of a signal generated at frequency $\omega + \Omega'$ is given by

$$\partial_z \hat{A}_\omega(z,\Omega') = -i\kappa \int \hat{A}_{2\omega}(0,\Omega)\hat{A}^*_\omega(z,\Omega-\Omega')\exp(-i\Delta k(\Omega,\Omega')z)d\Omega, \tag{S2}$$

where Ω parameterizes the frequency detuning within the pump envelope, and the corresponding idler is generated at frequency $\omega + \Omega - \Omega'$. The energy spectral density of the generated signal pulse is given by $|\hat{A}_\omega(z,\Omega')|^2$. The bandwidth of the nonlinear polarization that can drive the generated signal is determined by the phase-mismatch, $\Delta k(\Omega,\Omega') = k(2\omega+\Omega) - k(\omega+\Omega') - k(\omega+\Omega-\Omega')$. We note here that given the finite bandwidth associated with the pump pulses, the limits of integration in Eqn. S2 can be truncated at a characteristic frequencies $\pm\Delta\Omega_p$.

We may obtain insight into the role each dispersion order plays by series expanding $\Delta k(\Omega,\Omega')$ to second order in Ω and Ω',

$$\Delta k(\Omega,\Omega') \approx \Delta k_0 + \Delta k'\Omega + \frac{k''_{2\omega} - k''_{\omega}}{2}\Omega^2 - k''_{\omega}(\Omega'^2 - \Omega\Omega'). \tag{S3}$$

We again find that the phase-mismatch is dominated to leading order in Ω and Ω' by $\Delta k'$ and k''_{ω}, respectively. For broadband OPA, the $\Omega\Omega'$ term in S3 can be neglected since the range of generated Ω' is typically much larger than $\Delta\Omega_P$.

Quasi-static operation occurs when these two leading contributions to the phase-mismatch are made to be negligible. In this limit, a number of assumptions can be made to simplify Eqn. S2. First, we assume that the bandwidth of the generated signal is sufficiently large compared to $\Delta\Omega_p$ that the contribution of the generated idler to the phase-mismatch can be approximated as $k(\omega+\Omega-\Omega') \approx k(\omega-\Omega')$, which renders

$$\exp(-i\Delta k(\Omega,\Omega')z) \approx \exp(-i\Delta k_{2\omega}(\Omega)z)\exp(-i\Delta k_\omega(\Omega')z), \tag{S4}$$

a separable function of Ω and Ω'. Here $\Delta k_\omega(\Omega') = \Delta k(\Omega = 0, \Omega')$ contains contributions to the phase-mismatch by the generated signal and idler bandwidth, and is an even function of Ω',

$$\Delta k_\omega(\Omega') \approx \Delta k_\omega(0) + k''_\omega (\Omega')^2 + \frac{k^{(4)}_\omega}{12} (\Omega')^4 + \mathcal{O}((\Omega')^6). \tag{S5}$$

Similarly, $k_{2\omega}(\Omega)$ describes contributions of the pump bandwidth to phase-mismatch,

$$\Delta k_{2\omega}(\Omega) \approx \Delta k'\Omega + \frac{k''_{2\omega}}{2}\Omega^2 + \frac{k'''_{2\omega}}{3!}\Omega^3 + \mathcal{O}(\Omega^4). \tag{S6}$$

With these simplifications, Eqn. S2 becomes

$$\partial_z \hat{A}_\omega(z,\Omega') = -i\kappa \exp(-i\Delta k_\omega(\Omega')z) \int \hat{A}_{2\omega}(z,\Omega)\hat{A}^*_\omega(z,\Omega-\Omega')d\Omega, \tag{S7}$$

where the field envelope of the linearly propagating pump in a reference frame co-moving with the group velocity of the signal is given by

$$\hat{A}_{2\omega}(z,\Omega) = \hat{A}_{2\omega}(0,\Omega)\exp(-i\Delta k_{2\omega}(\Omega)z). \tag{S8}$$

For the quasi-static devices considered here temporal walk-off and higher-order dispersion of the pump is negligible, $\hat{A}_{2\omega}(z,\Omega) \approx \hat{A}_{2\omega}(0,\Omega)$. We note here that the integral in Eqn. S7 is the Fourier transform of $A_{2\omega}(z,t)A^*_\omega(z,t)$ and can be readily calculated using the time-domain field envelopes. For convenience, we define the effective gain coefficient,

$$\gamma_{\text{eff}}(z,\Omega') = -i\kappa \int \hat{A}_{2\omega}(z,\Omega)\frac{\hat{A}^*_\omega(z,\Omega-\Omega')}{\hat{A}^*_\omega(z,-\Omega')}d\Omega, \tag{S9}$$

which simplifies Eqn. S7 to

$$\partial_z \hat{A}_\omega(z,\Omega') = \gamma_{\text{eff}}(z,\Omega')\exp(-i\Delta k_\omega(\Omega')z)\hat{A}^*_\omega(z,-\Omega'). \tag{S10}$$

Eqn. S10 has the same form as the CWEs for OPA in the presence of a continuous-wave pump; the key difference is an effective gain coefficient $\gamma_{\text{eff}}(z,\Omega')$ that depends on the field envelopes for both the pump and signal, and therefore evolves during propagation.

The second key simplifying assumption we make for quasi-static devices is that the shape of the amplified signal spectrum is preserved during propagation $\hat{A}_\omega(z,\Omega') = A(z)f(\Omega')$. With this assumption, and our previous assumption that $\hat{A}_{2\omega}(z,\Omega) \approx \hat{A}_{2\omega}(0,\Omega)$, the effective gain coefficient becomes invariant with respect to z, $\gamma_{\text{eff}}(z,\Omega') = \gamma_{\text{eff}}(0,\Omega')$. When the generated signal bandwidth is much larger than $\Delta\Omega_p$ the idler envelope $\hat{A}_\omega(z,\Omega-\Omega')$ varies slowly with respect to pump detuning $\hat{A}_\omega(z,\Omega-\Omega') \approx \hat{A}_\omega(z,-\Omega')$, and the effective gain coefficient simplifies to the peak gain associated with the peak of the time-domain pump envelope $\gamma_{\text{eff}} = -i\kappa\int \hat{A}_{2\omega}(0,\Omega)d\Omega = -i\kappa A_{2\omega}(z=0,t=0) = \gamma_{\text{pk}}$.

With this definition of the effective gain, Eqn. S10 reduces to the CWEs for non-degenerate OPA with a continuous-wave pump,

$$\begin{bmatrix} \tilde{A}_\omega(z,\Omega') \\ \tilde{A}^*_\omega(z,-\Omega') \end{bmatrix} = \begin{bmatrix} \mu(\Omega') & \nu(\Omega') \\ \nu^*(-\Omega') & \mu^*(-\Omega') \end{bmatrix} \begin{bmatrix} A_\omega(0,\Omega') \\ A^*_\omega(0,-\Omega') \end{bmatrix}, \tag{S11a}$$

$$\mu(z,\Omega') = \cosh(\gamma(\Omega')z) + \frac{i\Delta k_\omega(\Omega')}{2\gamma(\Omega')}\sinh(\gamma(\Omega')z) \tag{S11b}$$

$$\nu(z,\Omega') = \frac{\gamma_{\text{eff}}(\Omega')}{\gamma(\Omega')}\sinh(\gamma(\Omega')z), \tag{S11c}$$

where $\tilde{A}_\omega(z,\Omega') = A_\omega(z,\Omega')\exp(i\Delta k_\omega(\Omega')z/2)$, and the frequency dependent field gain coefficient is given by $\gamma(\Omega') = \sqrt{\gamma_{\text{eff}}(\Omega')\gamma^*_{\text{eff}}(-\Omega') - (\Delta k_\omega(\Omega')/2)^2}$. It can be shown that the maximum attainable power gain is given by [1]

$$G = \left(\sqrt{\mu(\Omega')\mu^*(-\Omega')} + \sqrt{\nu(\Omega')\nu^*(-\Omega')}\right)^2, \tag{S12}$$

or $G \approx \exp(2\gamma(\Omega')z)$ in the limit of large gain ($\gamma_{\text{eff}}(\Omega')\gamma^*_{\text{eff}}(-\Omega') \gg (\Delta k/2)^2$). Equation S12 is plotted in Figure 1e. as a function of generated signal wavelength and phase-mismatch $\Delta k(\Omega = 0, \Omega' = 0)$, with $\gamma_{\text{eff}} \approx \gamma_{\text{pk}}$ for the effective gain. We note here that for a given phase-mismatch, $G(\Omega')$ is essentially flat across the range of generated signal frequencies. We therefore expect the generated pulse energy to grow as $G = \exp(b\sqrt{U_{\text{in}}}) \approx \exp(2\gamma(0)z)$, with the power gain coefficient b in dB/$\sqrt{\text{pJ}}$ determined to good approximation by the peak gain associated with the pump γ_{pk} and the phase-mismatch of a degenerate signal at ω.

For OPG, the seed is given by vacuum fluctuations. The treatment used here is readily extended to the field creation and annihilation operators $\hat{a}_\omega(\Omega')^\dagger$ and $\hat{a}_\omega(\Omega')$ for a mode at frequency $\omega + \Omega'$, following a procedure similar to [2]. The evolution of the signal and idler is given by

$$\begin{bmatrix} \hat{a}_\omega(z, \Omega') \\ \hat{a}^\dagger_\omega(z, -\Omega') \end{bmatrix} = \begin{bmatrix} \mu(\Omega') & \nu(\Omega') \\ \nu^*(-\Omega') & \mu^*(-\Omega') \end{bmatrix} \begin{bmatrix} \hat{a}_\omega(0, \Omega') \\ \hat{a}^\dagger_\omega(0, -\Omega') \end{bmatrix}, \tag{S13}$$

where μ and ν have the same form as in Eqns S11b-S11c. Here, the photon number density at $\omega + \Omega'$ is given by

$$S(\Omega') = \int \langle 0|\hat{a}_\omega(\Omega')^\dagger \hat{a}_\omega(\Omega'')|0\rangle \exp\left[i(\Omega'' - \Omega')t\right] d\Omega'' = |\nu(\Omega'_j)|^2, \tag{S14}$$

where the latter expression comes from the commutator $\left[\hat{a}(0, \Omega'), \hat{a}^\dagger(0, \Omega'')\right] = \delta\left[(\Omega' - \Omega'')/2\pi\right]$. The total photon number generated by the pump pulse is given by $\int_{-\infty}^{\infty} S(\Omega')\Omega'$. For a pulsed source with repetition frequency f_R, the power contributed by frequencies around $\omega_j = \omega + \Omega'_j = \omega + 2\pi j f_R$ is therefore given by

$$P(z, \omega_j) = \hbar\omega_j f_R (\gamma_{\text{pk}} z)^2 \left| \frac{\sinh(\gamma(\Omega'_j)z)}{\gamma(\Omega'_j)z} \right|^2. \tag{S15}$$

In the limit of low gain, Eqn. S15 reduces to the power per mode produced by spontaneous parametric down-coversion $P(z, \omega_j) = \hbar(\omega_j) f_R \gamma_{\text{pk}}^2 z^2 \text{sinc}^2(\Delta k z/2)$. In the limit of large gain, and for a phase-mismatch dominated by fourth-order dispersion, $\gamma(\Omega'_j)$ is essentially flat until Ω' reaches the characteristic bandwidth for OPA $(\Delta\Omega'_{\text{OPA}})^4 \approx 48\gamma_{\text{pk}}/k^{(4)}_\omega$. The generated bandwidth is therefore a weak function of pump power, and the total power is given by

$$P(z) = \sum_j P(z, \omega_j) \approx \hbar\omega f_R |\sinh^2(\gamma(0)z)|^2 N_{\text{OPA}}, \tag{S16}$$

where the number of OPA modes is given by $N_{\text{OPA}} = \Delta\Omega_{\text{OPA}}/(2\pi f_R)$. The scaling of the detected signal power as a function of pulse energy input to the waveguide allows the gain coefficient $\gamma(0) \approx \gamma_{\text{pk}}$ to be determined directly from P_{out} using a simultaneous fit of $\gamma(0)$ and N_{OPA}.

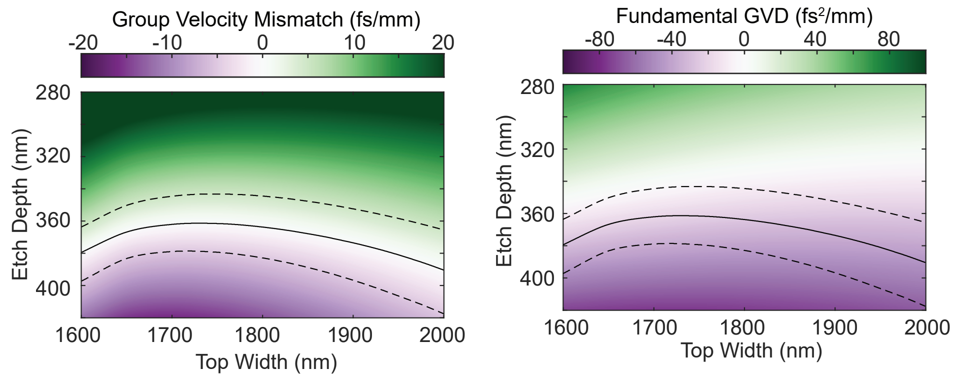

Fig. S1. Simulated waveguide dispersion. a) The group velocity mismatch ($\Delta k'$), and b) the fundamental GVD (k''_ω), as a function of waveguide geometry for a 2060-nm fundamental.

2. WAVEGUIDE DESIGN

The theory presented in Sec. 1 establishes the roles played by each dispersion order and highlights two dominant dispersion orders that effectively restrict the parametric gain and the generated signal bandwidth, $\Delta k'$ and k''_{ω}, respectively. We close this document by showing an example waveguide design that eliminates these dispersion orders to achieve quasi-static operation. $\Delta k'$ and k''_{ω} are shown as a function of waveguide width and etch depth in Fig. S1(a) and Fig. S1(b), respectively, assuming a fundamental wavelength of 2060-nm and a film thickness of 700 nm. Here, the solid black contour line corresponds to $\Delta k' = 0$ fs/mm, and the dashed black contour lines correspond to $\Delta k' = \pm 5$ fs/mm. For the numbers shown here, quasi-static operation occurs around a top width of 1750 nm and an etch depth of 360 nm, where $\Delta k' = 5$ fs/mm and $k''_{\omega} \approx 0$ fs^2/mm. For the 1045-nm pump used in the main text, corresponding to a 2090-nm fundamental, these curves shift slightly and quasi-static operation now occurs for top widths around 1850 nm and etch depths around 340 nm. We note here that the film thickness may additionally be used to fine tune the waveguide dispersion to simultaneously render both $\Delta k' = 0$ and $k''_{\omega} = 0$.

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