[1] Th. Lyman, *Astrophys. J.*, **60**, No. 1, July, 1924.

[2] H. B. Dorgelo and T. H. Abbink, *Zeitschr. Physik*, **37**, 667, 1926.

[3] A. Fowler, *Proc. Roy. Soc.*, **A91**, 208, 1915; W. E. Curtis and R. G. Long, *Proc. Roy. Soc.*, **A108**, 513, 1925.

[4] R. S. Mulliken, *Proc. Nat. Acad. Sci.*, **12**, No. 3, 158, March, 1926.

[5] The electron jumps in the CuH-spectrum show, for instance, that in a molecule transitions $^1S-{}^1S$ are possible under ordinary conditions. Although such transitions are unknown in the visible He_2 spectrum, it cannot be concluded that the transition 1^1S-2^1S is also absent. For it may be (comp. R. S. Mulliken, *Phys. Rev.*, **28**, No. 6, Dec., 1926, p. 1208, etc.) that in the lowest state 1^1S the levels $j = 1/2, 2\,1/2, 4\,1/2$, etc., are not missing, but are present with partial, or even full "weight," and that in the state 2^1S they also appear with small "weight."

WHITE-LIGHT INTERFERENCE FRINGES WITH A THICK GLASS PLATE IN ONE PATH. PART II

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In Part I of this paper[1] the writer developed a theory of the white-light fringes observed in the Michelson interferometer when a thick plate of glass, or other refractive substance, is placed in one of the paths. Part I treated of the axial beam, whose composition determines the color of the central spot in the fringe pattern. An explanation was given of the very great number of colored fringes, running up to several thousand, which is observed under these conditions. In this part we discuss certain phenomena which depend for their explanation upon the analysis of a beam oblique to the optical axis. To observe these phenomena we obtain, as described in Part I, circular fringes in sodium light, insert the thick plate in one of the paths, shorten that path until in the neighborhood of the most brilliant sodium maximum, in which position colored fringes will be seen with white light. If we observe these fringes through a spectroscope with the slit placed across the center of the system, the slit being long enough to cut across the whole field, the spectrum is seen to be crossed by a system of concentric dark rings, looking very much like a shadow of the fringe system itself. If now one of the paths is slowly shortened or lengthened, these dark rings expand or contract, as the case may be, just as in the fringe system. In addition, the center of the system on the spectrum travels very slowly from one end of the spectrum to the other. These phenomena may be observed easily and distinctly if a rather wide slit, cut in a card, is placed in the path of the light before it enters the interferometer, and the fringes are then viewed through a direct-vision spectroscope, without slit.

Derivation of the Fundamental Equation.—We shall proceed to derive an expression for the phase-difference on re-uniting of the two beams into which an oblique beam is split by the interferometer, in terms of the wave-length, the angle made with the optical axis and other factors. The source is supposed to give a set of beams of plane wave-front coming from all directions. The course of one of these beams, inclined at an angle i to the optical axis, the angle lying in the base plane of the instrument, is drawn in figure 4. Although the derivation is limited to beams whose angles of inclination lie in the base-plane the result would be the same, on account of the symmetry about the optical axis, if the angle lay in any other plane. In the diagram M_1 and M_2 are the interferometer mirrors, OQ is the half-silvered surface. The thick plate is placed in contact with M_2 for convenience in making the derivation, though this is, of course, not essential. P_2 gives the direction of the optical axis before reflection from the half-silvered surface, P_1 the direction after reflection. The compensating plate and the half-silvered plate itself, excepting the surface OQ, are omitted, as the portions of the paths through them cancel out in every case, and have no influence on the result. Only two rays of the beam are drawn, a pair that will meet at the same point on the half-silvered surface on leaving the instrument. It should be kept in mind that the beam drawn in the diagram forms only one point of the fringe pattern, the whole pattern being formed by beams coming from an infinite number of directions. The part of the path marked GHJ is in the thick plate, and the angle between the optical axis and the beam in this part of the path is r. P_1 and P_2 are the distances from B, on the half-silvered surface, to M_1 and M_2, respectively. As the difference only, $P_1 - P_2$, appears in the result, it does not matter what point on the half-silvered surface is used in defining them. The thickness and the index of refraction of the plate are denoted by t and μ, respectively. The wave-length of the light in air is λ, and the difference of phase of the two parts of the beam on re-uniting is ϕ.

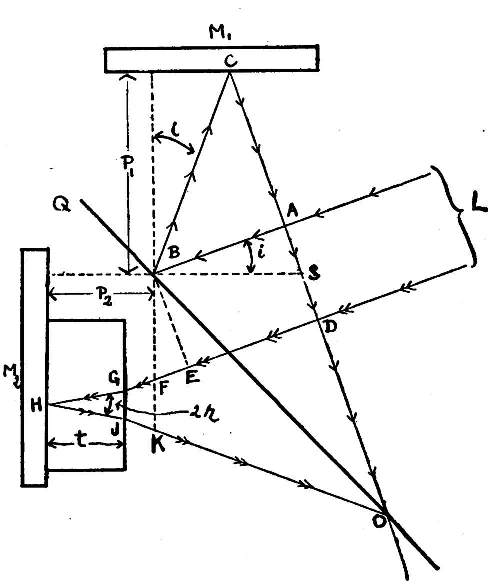

FIGURE 4

We wish to form an expression for $\phi/(2\pi)$, the difference of the numbers of light waves in the two paths. The paths may be considered to start from any two simultaneous points on the wave-front, such as A and D in the diagram. Then the paths of the two rays are $ABCSO$ and $DEFGHJKO$, respectively. In forming the difference, SO cancels KO, and AB cancels DE, leaving $2BC$ for one path, and $EF + 2FG + 2GH$ for the other. $BC = P_1 \sec i$; $FG = (P_2 - t) \sec i$; $GH = t \sec r$. To find the number of light waves in each part, divide the length of path by λ, multiplying the length of the path through glass by μ before doing so. In this way we get

$$\phi/(2\pi) = \frac{1}{\lambda}(2BC - 2FG - 2\mu GH - EF)$$

or

$$\phi/(2\pi) = \frac{1}{\lambda}[2P_1 \sec i - 2(P_2 - t) \sec i - 2\mu t \sec r - EF].$$

To find EF, we have $BK = BS = 2P_1 \tan i$; $FK = 2t \tan r + 2(P_2 - t) \tan i$. Therefore, $BF = 2(P_1 - P_2) \tan i + 2t(\tan i - \tan r)$ and $EF = BF \sin i = 2(P_1 - P_2) \sin^2 i \sec i + 2t(\sin^2 i \sec i - \tan r \sin i)$ since $\sin i = \mu \sin r$, this becomes

$$EF = 2(P_1 - P_2) \sin^2 i \sec i + 2t \sin^2 i \sec i - 2\mu t \sin^2 r \sec r.$$

Hence

$$\frac{\phi\lambda}{(4\pi)} = (P_1 - P_2) \sec i(1 - \sin^2 i) + t \sec i(1 - \sin^2 i) - \mu t \sec r(1 - \sin^2 r).$$

Finally, putting $P_1 - P_2 = D$, we get the desired expression:

$$\frac{\phi\lambda}{(4\pi)} = (D + t) \cos i - \mu t \cos r \qquad (12)$$

Discussion of the Equation.—Equation (12) can be used to explain the spectrum rings in the following way: On the two-dimensional field into which the image of the slit is drawn out by the spectroscope, the direction down the length of the spectrum represents λ, the direction across the width of the spectrum gives i, with $i = 0$ down the middle line of the width. On account of the small range of i which is used, not more than from $-3°$ to $+3°$, $\sin i$, $\tan i$ and i itself in radian measure, may be considered interchangeable in drawing a diagram. If we think of i and λ as the coördinates, and plot the curve represented by equation (12), holding the other quantities constant, we shall have a curve at every point of which the phase-difference has the same value, or in other words, one of the spectrum rings. Repeating the process, changing ϕ by multiples of 2π we can draw the whole system.

Wave-Length and Phase-Difference at the Center of the Spectrum Rings.—If we put $i = 0$, and consequently $r = 0$, in (12) we shall be restricted to the line of zero inclination, down the middle of the spectrum diagram. Assigning a value to ϕ, and solving the resulting equation for λ, we shall find the points at which a particular spectrum ring crosses the middle line. This work cannot be carried out without assuming some form for the relation between μ and λ. If we use Cauchy's formula, $\mu = A + B/\lambda^2$, in (12), after putting $i = 0$, we get the following cubic in λ:

$$\left(\frac{\phi}{(2\pi)}\right)\lambda^3 - 2(D + t - At)\lambda^2 + 2Bt = 0. \qquad (13)$$

It can be shown that this equation has either two or no real positive roots. It always has one negative root, with which we are not concerned. If there are two positive roots we have a real ring crossing the zero axis at the two points indicated. If the two roots coincide, we have the value of λ for the central spot of the spectrum ring system. The condition for equal roots, obtained by putting the minimum value of the left-hand side of (13) equal to zero, is

$$\frac{\phi_c}{(2\pi)} = \frac{4}{3}\sqrt{\frac{(D + t - At)^3}{3Bt}} \qquad (14)$$

This equation gives the phase-difference at the central spot, and shows that it increases with D. Placing this value of ϕ in (13), and solving for λ, we obtain

$$\lambda_c = \sqrt{\frac{3Bt}{D + t - At}}, \qquad (15)$$

the wave-length of the central spot. As this expression decreases when D is increased, it gives the explanation of the observed fact that the center of the system of rings moves down the spectrum as the position of the moveable mirror is changed. If we place in this equation in succession the values of λ at the extremes of the visible spectrum, λ_1 and λ_2, and the corresponding values, D_1 and D_2, of the path-difference, and subtract after solving each equation for D, we get

$$D_2 - D_1 = 3Bt\left(\frac{1}{\lambda_2^2} - \frac{1}{\lambda_1^2}\right) \qquad (16)$$

which is the same as equation (10) of the previous paper, thus showing that the range of D for white-light fringes is the same as the range during which the central spot is traveling through the spectrum. The phase-difference at the central spot is evidently the greatest or least phase-difference for all the spectrum-rings. That it is the greatest follows from figure 2 of the previous paper. Since ϕ decreases as we go outward from the center, and ϕ_c increases with D, it follows that the rings expand as the center moves toward the blue end, and *vice versa*. These conclusions agree with the observed facts.

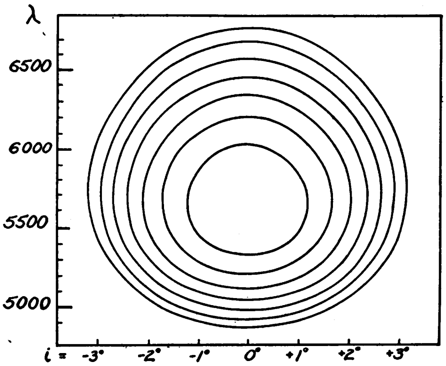

FIGURE 5

To find the wave-lengths at which the successive rings cross the line of zero inclination, down the spectrum, we use (13), putting $\frac{\phi}{(2\pi)} = \frac{\phi_c}{(2\pi)} - n$, and using the value for $\frac{\phi_c}{(2\pi)}$ given by (14). This gives

$$\left(\frac{4}{3}\sqrt{\frac{(D+t-At)^3}{3Bt}} - n\right)\lambda^3 - 2(D+t-At)\lambda^2 + 2Bt = 0 \ldots\ldots \quad (17)$$

With $n = 0$, this equation has a double root whose value is given by (15). Putting $n = 1, 2, 3 \ldots$ in succession, and solving for the two positive roots, we get the wave-lengths at which the successive rings cross the spectrum.

Simplified Form of Equation (12)[2] *for Small Values of i.*—In order to graph the spectrum fringes we shall make some approximations in equation (12) which are justifiable for small values of i. It is permissible to take $\cos i = \sqrt{1 - \sin^2 i}$ as approximately $1 - \frac{i^2}{2}$, and $\cos r = \sqrt{1 - \frac{\sin^2 i}{\mu^2}}$ as $1 - \frac{i^2}{(2\mu^2)}$. Making these substitutions in (12) and collecting the coefficient of i^2 gives

$$\frac{\phi\lambda}{(4\pi)} = (D + t - \mu t) - \frac{i^2}{2}\left(D + t - \frac{t}{\mu}\right).$$

Placing $\mu = A + B/\lambda^2$ gives

$$\frac{\phi\lambda}{(4\pi)} = \left(D + t - At - \frac{Bt}{\lambda^2}\right) - \frac{i^2}{2}\left(D + t - \frac{t\lambda^2}{(A\lambda^2 + B)}\right).$$

Since B is only about 2% of the denominator in the last term, we may neglect it, giving

$$\frac{\phi\lambda}{(2\pi)} = 2(D + t - At) - i^2\left(D + t - \frac{t}{A}\right) - \frac{2Bt}{\lambda^2} \quad \ldots\ldots\ldots \quad (18)$$

or

$$\left(D + t - \frac{t}{A}\right)i^2 = 2(D + t - At) - \left(\frac{\phi}{(2\pi)}\right)\lambda - \frac{2Bt}{\lambda^2} \quad \ldots\ldots \quad (19)$$

From this equation, by assigning the proper value to $\frac{\phi}{(2\pi)}$, any spectrum ring may be graphed. When $i = 0$, (18) reduces to (13).

Numerical Illustration.—By assuming a set of values of the various constants, a graph of the spectrum rings, figure 5, was made. The following values were taken: $D = 1.65$; $t = 3$; $\mu = A + B/\lambda^2$; $A = 1.5$; $B = 53 \times 10^{-10}$. With these values (14) gives $\frac{\phi_c}{(2\pi)} = 354.66$; (15) gives $\lambda_c = 0.0005639$. Equation (17) becomes

$$(354.66 - n)\lambda^3 - 0.30\lambda^2 + 318 \times 10^{-10} = 0.$$

From this equation the following table of the points where the first 13 rings cross the zero line was made:

n	1	2	3	4	5	6	7	8	9	10	11	12	13
$\lambda_1 \times 10^7$	5406	5316	5249	5195	5147	5106	5068	5033	5002	4972	4944	4918	4893
$\lambda_2 \times 10^7$	5897	6012	6104	6183	6256	6323	6386	6447	6505	6560	6615	6667	6719

Equation (19) becomes

$$i^2 = 0.11321 - 0.37736\left(\frac{\phi}{(2\pi)}\right)\lambda - 120.00 \times 10^{-10}\left(\frac{1}{\lambda^2}\right).$$

Assigning to $\phi/(2\pi)$ the values 353.66, 352.66, etc., in succession, the successive rings were graphed. For the sake of clearness, the even-numbered rings only are drawn in the diagram.

The writer again takes great pleasure in acknowledging the helpful and constructive interest of Dr. Paul S. Epstein during the preparation of this paper.

[1] *Proc. Nat. Acad. Sci.*, **10**, No. 11, pp. 452–457, November, 1924.

[2] This modification of equation (12) is due to Dr. Epstein.