In order to calculate the relative positron activities the experimentally determined[4] ratio $(K_{64}/\beta_{64}^+)/(K_{61}/\beta_{61}^+)=5.5$ was used. This gives a value of $\beta_{64}^+/\beta_{61}^+=0.00141$ from our observed value $K_{64}/K_{61}=0.00775$.

The longest period of cyclotron bombardment which has been used for production of Cu^{61} has been 4 hours. On the basis of the ratio of activities after $2\frac{1}{2}$ hours of bombardment time one finds that after 4 hours of bombardment time the relative positron activities $\beta_{64}^+/\beta_{61}^+$ would be 0.00157. No positron source has been studied for a period lasting more than three half-lives of Cu^{61} following the removal of the nickel target from the cyclotron. Using the data above one finds that even after three half-lives the sample which was bombarded for four hours would have a positron activity ratio $\beta_{64}^+/\beta_{61}^+=0.00733$. Quite a bit less than one percent of the activity can thus be attributed to Cu^{64}.

This study verifies the fact that only negligible quantities of the Cu^{64} positron activity existed in the samples which have been studied to determine the shape of the positron spectrum of Cu^{61}.

This work has been supported by the joint program of the ONR and AEC.

[1] C. S. Cook and L. M. Langer, Phys. Rev. **74**, 227 (1948).
[2] G. E. Owen and C. S. Cook, Phys. Rev. **76**, 1536 (1949).
[3] The Auger electrons were assumed for this experiment to represent a ratio of intensities for the K-capture processes in the two isotopes under consideration. In reality a small amount of the intensity of the Auger line for Cu^{61} would be caused by the internal conversion electrons. However, their intensity is much less than the K-capture intensity and could not cause a detectable change in the experimentally observed value. [See Boehm, Blaser, Marmier, and Preiswerk, Phys. Rev. **77**, 295 (1950) and Owen, Cook, and Owen (article to be submitted to Phys. Rev.).]
[4] Bradt, Gugelot, Huber, Medicus, Preiswerk, and Scherrer, Helv. Phys. Acta **18**, 252 (1945).

The Latitude and Longitude Effects in Cosmic Rays Over the United States and Canada at 30,000 Feet*

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The change with geomagnetic latitude of the ionizing particles at 310 g/cm^{-2} atmospheric pressure (30,000 ft.) that could penetrate various thicknesses of absorber was measured by Biehl, Neher, and Roesch[1] over a range of latitude of 64° geomagnetic north to the geomagnetic equator along longitude 80°W. These flights show that most of the latitude effect is over at 50° as one proceeds north and this was independent of the absorber used or of whether the total radiation at that altitude was measured or only that near the vertical. On the other hand Swann, Morris and Seymour[2] find a 10 percent increase for a 10° increase in geomagnetic latitude at 30,000 feet even beyond the "knee" of the curve. Although no mention of an absorber was made, presumably their measurements were made with a lead absorber.

A number of flights having a further bearing on this point have recently been made, covering the latitude range particularly from 40° to 64° geomagnetic north with counter telescopes and ionization chambers, the latter both unshielded and shielded with 10 cm of lead.

In Fig. 1 the measurements made with two of our ionization chambers,[3] which were sealed off 12 years ago, are shown, the one with no lead shield, the other with a 10 cm lead shield. Curve A is that taken[1] in June 1948 along longitude 80° W while B was taken in October, 1949 along 115° W using the same instrument as for A. Curve C was taken with a companion instrument along 115° W but surrounded with 10 cm of lead. B is the result of 3 flights made within a period of five days. Where these flights overlapped they differed by ±2.0 percent, which was well outside the experimental errors and are hence assumed to represent real changes in the primary radiation during this period. Curve B is therefore drawn to represent the mean of these flights.

In Fig. 2 are shown the corresponding results with counter telescopes over the same range of latitudes at the same two longitudes, namely 80° and 115° W. Curve A′ and C′ have been published previously[1] but are here given in greater detail. Curve B′ was taken in October, 1949. In addition curve D′ is given which represents the mean of six flights made during the summer of 1948 along longitude 117° W and at 33,000 ft. pressure altitude.

Comparing corresponding curves in Figs. 1 and 2 the following conclusions are evident: (a) There is a small increase both with counter telescopes and ionization chambers of 2.5 to 3 percent from latitude 50° to 64° N and is approximately the same with or without a 10 cm lead absorber. (b) In going south with latitude 50° N the curves taken along longitude 80° W fall off more slowly than along longitude 115° W.

Table I gives a summary of the experimental situation. (Numbers given in the table are in percent change of radiation per degree change in geomagnetic latitude.)

In seeking an explanation for why the curves along 80° W longitude fall off at a different rate from along 115° W, we measured the film taken with the unshielded ionization chamber along a constant geomagnetic latitude line on a flight at 30,000 ft. from California to Florida on August 1, 1949. The route varied less than 0.5° from geomagnetic latitude 40.8° N and a slight correction taken from Figs. 1 and 2 of approximately 1 percent per degree

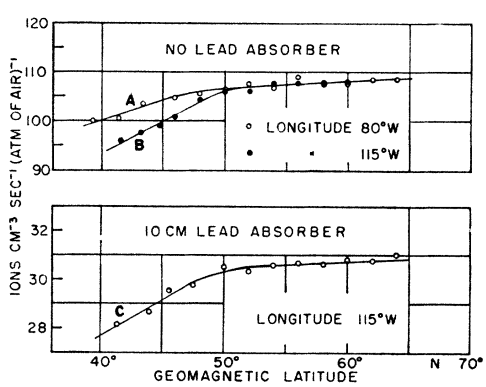

Fig. 1. These curves show the ionization as a function of latitude at two different longitudes at an altitude corresponding to 310 g/cm^{-2} air pressure. The probable errors in the points, as determined by the scatter of the individual discharges on the film, are given approximately by the diameter of the points.

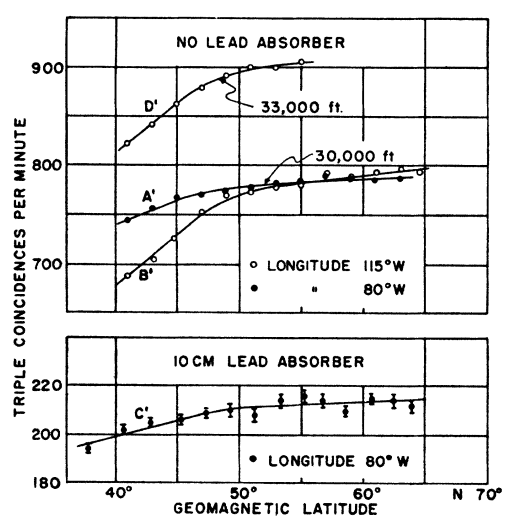

Fig. 2. Similar curves to those in Fig. 1 but taken with counter telescopes pointing vertically.

TABLE I. Percentage increase per degree change in latitude.

λ_{mag}	No added absorber Total ionization	No added absorber Vertical radiation	10 cm lead Total ionization	10 cm lead Vertical radiation
	A. Along longitude 80° W			
45°N	1.3	1.3	1.9	—
50°–64°N	0.16	0.18	0.25	—
	B. Along longitude 115° W			
45°N	0.7	0.6	—	0.6
50°–64°N	0.2	0.1	—	0.18

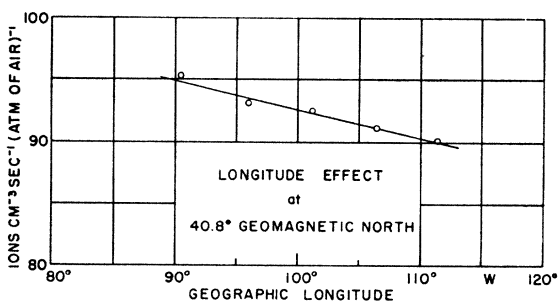

FIG. 3. The longitude effect at 310 g/cm^{-2} air pressure as determined with the unshielded ionization chamber.

was applied when necessary. The result is plotted in Fig. 3. It is seen that there is in fact a longitude effect amounting to -0.24 percent per degree change in longitude. Extrapolated to 80° W and 115° W this curve would indicate on 8.2 percent decrease in going from 80° W to 115° W at this geomagnetic latitude and altitude. From Figs. 1 and 2 we find a corresponding difference of 6.2 percent for the ionization chamber and 8.2 percent for the counter telescope, each unshielded.

With the help of the theory of the behavior of charged particles in the magnetic field of a dipole as worked out by Vallarta *et al.* we have made an estimate of the longitude effect to be expected at this latitude due only to the eccentricity of the equivalent magnetic dipole of the earth. Thus one can calculate approximately the change in minimum momentum of the incident particles for a given change in longitude in terms of the corresponding change in latitude. Using the experimental data for the latitude effect as given in Fig. 1 the calculated value of the longitude effect at 40.8° geomagnetic north and 100° geographic west is -0.29, a value consistent with the measured value of -0.24 percent per degree change in longitude.

We wish to express our gratitude to the ONR and the United States Air Forces for making these flights possible. We also wish to thank Major W. A. Gustafson, Captain Z. Shawhan, Captain George Freyer and other members of the crew of the B-29 for their cooperation and expert handling of the plane.

* Supported in part by the joint program of the ONR and AEC.

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[3] R. A. Millikan and H. V. Neher, Phys. Rev. **50**, 15 (1936).

Energy Band Structures in Semiconductors

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A METHOD of attack is proposed which for semiconductors has the potentiality of actually determining the shapes of the energy surfaces in the Brillouin zone in detail by proper comparison with experiment. The essential feature is that the most complicated family of energy surfaces arising from a degenerate energy band in a cubic crystal can be described by three parameters. Similar results for the restricted case of tight binding have been discussed by Sommerfeld and Bethe.[1] We shall illustrate the more general case in terms of the wave functions near the bottom of an energy band which has its lowest energy H_0 at the center of the Brillouin zone and is triply degenerate with p-type wave functions,[2] type δ, which have the periodicity of the lattice. These are denoted by

$$\psi_x = xu_x(x^2, y^2, z^2) = xu_x(x^2, z^2, y^2), \quad \text{etc.}$$

A wave function near the lowest energy of the band may be written as

$$\Phi = [a_x\psi_x + a_y\psi_y + a_z\psi_z + \Sigma_{\alpha i}\, b_{\alpha i}\varphi_{\alpha i}] \exp(i\mathbf{P}\cdot\mathbf{r}/\hbar),$$

where the $\varphi_{\alpha i}$ are also periodic and satisfy $H\varphi_{\alpha i} = H_\alpha \varphi_{\alpha i}$. In order to solve the eigenvalue problem $(H-W)\Phi = 0$, a set of equations for the coefficients is found by multiplying by $\exp(-i\mathbf{P}\cdot\mathbf{r}/\hbar)\psi_x^*$, etc. and the resulting system treated by a method similar to that of Van Vleck[3] so as to eliminate the b's. This gives

$$[AP_x^2 + B(P_y^2 + P_z^2) - K]a_x + C[P_xP_ya_y + P_xP_za_z] = 0, \quad (1)$$

and similar equations with permuted indices (x, y, z), where $K = W - H_0$ is essentially the kinetic energy of motion and

$$A - \tfrac{1}{2}m = \Sigma_{\alpha i}(x|p_x|\alpha i)(\alpha i|p_x|x)/m^2(H_0 - H_\alpha),$$
$$B - \tfrac{1}{2}m = \Sigma_{\alpha i}(x|p_y|\alpha i)(\alpha i|p_y|x)/m^2(H_0 - H_\alpha),$$
$$C = \Sigma_{\alpha i}[(x|p_x|\alpha i)(\alpha i|p_y|y) + (x|p_y|\alpha i)(\alpha i|p_x|y)]/m^2(H_0 - H_\alpha).$$

These equations are formally identical with equations for the frequency and polarization of an acoustical wave of displacement

$$\delta\mathbf{r} = \mathbf{G}\exp i(\mathbf{k}\cdot\mathbf{r} + \omega t)$$

which gives

$$[c_{11}k_x^2 + c_{44}(k_y^2 + k_z^2) - \rho\omega^2]G_x + (c_{12} - c_{44})(k_xk_yG_y + k_xk_zG_z) = 0.$$

The secular equations for both the quantum and the mechanical cases give surfaces of three sheets in $\mathbf{k}$ space for a given eigenvalue, the "polarization vector" (a_x, a_y, a_z) of the wave function being analogous to $\mathbf{G}$. The present treatment was suggested by the analogy of the surfaces Fig. 1 computed[4] for the $3p$ band in NaCl to acoustical surfaces. The symmetry of (1) follows directly, of course, from the fact that it is quadratic in P and that the crystal is cubic.

A similar procedure for wave functions of the form[2] $x^2 - y^2$, $y^2 - z^2$ (type γ) leads to secular equation of two constants:

$$K^2 + 2FK + GP^4 + 3(F^2 - L(P_x^4 + P_y^4 + P_z^4) = 0,$$

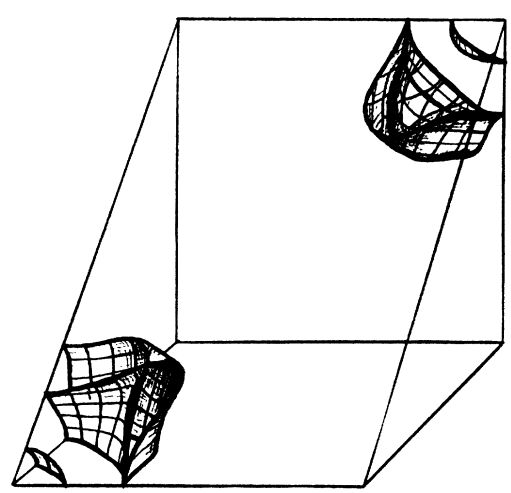

FIG. 1. Energy surface for degenerate band for $\mathbf{k} \approx 0$.