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COMMENTS

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Comments on "General analysis of the stability of superposed fluids"

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According to the result (39) in this paper,[1] the compressibility effects completely disappear from the Rayleigh–Taylor instability problem if the speeds of sound in the two fluids are equal. This contradicts the findings of P. Vandervoort[2] on the same problem.

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Reply to comments on "General analysis of the stability of superposed fluids"

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Previous results by Plesset and Hsieh on the effects of compressibility for Rayleigh–Taylor instability are shown to be valid, and an alternative brief deduction is given.

As Shivamoggi[1] points out, the analysis by Vandervoort[2] contradicts the results of Ref. 3. We have examined the Vandervoort calculations and find a significant inconsistency in his transition from his Eq. (22) to his Eq. (26). The error arises because he assumes in this step that *both* the density ρ and the speed of sound

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V are constants. If one takes V to be a constant, then ρ has to be a linear function of pressure and consequently a function of the vertical coordinate. If this variation is appropriately accounted for, Vandervoort's Eq. (22) becomes identical with Eq. (20) or (21) of Ref. 3 when one takes $\gamma = 1$ in the equation of state $p = A + B\rho^\gamma$. This value of γ corresponds, of course, to the assumed constancy of V. The finding of Ref. 3 is valid that the effects of compressibility in Rayleigh–Taylor instability disappear to the order of V^{-2} when the speed of sound in the two fluids are equal.

It may be of interest to note that the effect of compressibility on the Rayleigh–Taylor instability may be treated in a very compact and straightforward way. As in the customary configuration, one has two semi-infinite fluids of density ρ_1, ρ_2 separated by a horizontal plane interface in the unperturbed state of zero motion. In each fluid the unperturbed pressure gradient is $\nabla p = -\rho g \mathbf{e}$, where $\mathbf{e}$ is a unit vector in the vertical direction and g is the magnitude of the force per unit volume which is supposed to be in the $-\mathbf{e}$ direction. Each fluid has its own equation of state $\rho = \rho(p)$. Unperturbed quantities are unprimed, and the corresponding perturbed quantities when the interface is disturbed are denoted by a prime. Upon linearization about the unperturbed state the momentum equation gives

$$\rho \frac{\partial \mathbf{u}'}{\partial t} = -\nabla p' - g V^{-2} p' \mathbf{e}, \qquad (1)$$

where $\mathbf{u}'$ is the perturbed velocity for either fluid, p' is the perturbed pressure, and $V^2 = dp/d\rho \simeq p'/\rho'$. Similarly the equation of continuity gives

$$V^{-2} \frac{\partial p'}{\partial t} - \rho g V^{-2} \mathbf{u}' \cdot \mathbf{e} + \rho \nabla \cdot \mathbf{u}' = 0. \qquad (2)$$

A useful relationship in putting the equation of continuity in this form is

$$\nabla \rho = -g \rho V^{-2} \mathbf{e}. \qquad (3)$$

Next one takes the derivative with respect to time of Eq. (2) and then eliminates $\partial \mathbf{u}'/\partial t$ from the result by means of Eq. (1). The result is

$$V^{-2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' - g V^{-2} \mathbf{e} \cdot \nabla p' + g^2 \rho \frac{d^2 \rho}{dp^2} p' = 0, \qquad (4)$$

where use is made of (3).

Equation (4) describes the behavior of pressure perturbations in either fluid for any $\rho = \rho(p)$. In particular, if one assumes with Vandervoort that the speed of sound is constant so that ρ is a linear function of p, the last term in Eq. (4) disappears. This term will be omitted in the following.

A normal mode analysis in which the time dependence of the perturbed quantities is e^{int} can be made in the usual way[4] to give the following dispersion relation

$$n^2 = n_0^2 \frac{(\rho_1 + \rho_2)\lambda_1 \lambda_2}{\lambda_1 \rho_2 + \lambda_2 \rho_1}, \qquad (5)$$

where n_0^2 is the familiar incompressible Rayleigh–Taylor results

$$n_0^2 = k[(\rho_2 - \rho_1) g + \sigma k^2]/(\rho_1 + \rho_2),$$

and

$$\lambda_1 = \left(1 - \frac{n^2}{k^2 V_1^2} + \frac{g^2}{4k^2 V_1^4}\right)^{1/2} - \frac{g}{2k V_1^2},$$

$$\lambda_2 = \left(1 - \frac{n^2}{k^2 V_2^2} + \frac{g^2}{4k^2 V_2^4}\right)^{1/2} + \frac{g}{2k V_2^2}.$$

In these relations k is the wavenumber of the interfacial wave and σ is the surface tension coefficient. In deriving (5) we have taken the subscript 1 to refer to the upper fluid and 2 to the lower fluid. As $V_1, V_2 \to \infty$, λ_1 and λ_2 approach unity and the incompressible result is recovered. The next order approximation in V^{-2} gives

$$n^2 = n_0^2 \left\{ 1 + \frac{1}{k(\rho_1 + \rho_2)^2} \left[\left(\frac{1}{V_2^2} - \frac{1}{V_1^2} \right) \rho_1 \rho_2 g - \frac{1}{2} \sigma k^2 \left(\frac{\rho_1}{V_1^2} + \frac{\rho_2}{V_2^2} \right) \right] \right\},$$

which coincides with Eq. (40) of Ref. 3 aside from the surface tension term which was not included in that analysis.

It may be remarked that this result, as was shown in Ref. 3, is valid for a general $p(\rho)$. In the particular case $V_1 = V_2$, if surface tension is neglected, the effects of compressibility on the Rayleigh–Taylor instability disappear at this order as Shivamoggi[1] has noted.

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