Coronagraph design with the electric field conjugation algorithm

Jorge Llop-Sayson, A. J. Eldorado Riggs, Dimitri Mawet, Dwight C. Moody, Brian Kern, and Erkin Sidick

California Institute of Technology, Pasadena, California, United States
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, United States

Abstract. The requirements for a coronagraph instrument to image and obtain spectra of rocky planets around bright stars from space are tight. Indeed, the goal of imaging an Earth-like planet requires a starlight suppression system that cancels light to a level of $10^{-10}$ with sufficient stability and robustness to errors. Furthermore, the key science questions necessitate an adequate sample size; consequently, the throughput of the coronagraph drives the achievable yield of a given mission. The trade among achievable raw contrast, sensitivity to wavefront errors, and throughput poses a challenging problem in coronagraph design. The complexity of this problem drives us toward the simultaneous solving of all optical elements. We present a set of methods to optimize the design of a coronagraph. We implement these for the case of the hybrid Lyot coronagraph in the context of the Nancy Grace Roman Space Telescope Coronagraph Instrument. We discuss our findings in terms of coronagraph instrument design, and optical subsystems, and performance interplay. © 2022 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.JATIS.8.1.015003]

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1 Introduction

The numerous techniques of detection and characterization have allowed the discovery of thousands of exoplanets of which we are just starting to grasp the diversity in planetary characteristics. Indirect techniques, such as transiting and radial velocity techniques, have been widely successful in detecting lower separation planets orbiting relatively bright and mature stars. On the other hand, direct imaging can probe wider separation planets around younger stars. Direct imaging and atmospheric characterization of exoplanets is also one of the driving science cases of NASA mission concepts Habitable Exoplanet Observatory (HabEx) and Large UV Optical Infrared Surveyor (LUVOIR). Consequently, a key technology for these observatories is the coronagraph instrument design.

Several coronagraph architectures have been demonstrated to suppress unwanted starlight for this science case. For instance, the hybrid Lyot coronagraph (HLC) consists of two deformable mirrors (DMs): a focal plane mask (FPM) and a Lyot stop. At the FPM, a metal occulter diffracts most of the light to the outer edge in the pupil plane, where a Lyot stop filters it, cancelling most of the starlight at the final image plane. Shaped dielectric is deposited on top of the metal occulter to control the phase of the wavefront thus adding extra degrees of freedom in the design. The HLC has the best performing record in terms of raw contrast in a vacuum testbed environment, and, although it has been generally believed to suffer from sensitivity to low-order errors, this has been mitigated through research for LUVOIR-A HLC designs. The requirements for LUVOIR and HabEx coronagraph instruments on raw contrast and sensitivity to low-order errors are remarkably strict. Furthermore, instruments must allow for as high
planet throughput as possible to increase the science yield of a mission.\textsuperscript{14} This is especially challenging in the presence of discontinuities in the pupil, e.g., from the telescope segmentation and from the secondary mirror. Particularly adverse to coronagraph performance is the central obscuration from the secondary mirror in an on-axis telescope.\textsuperscript{14,15} For these reasons, coronagraph design optimization remains a high-priority research pathway for future direct imaging missions.

Optimizing a coronagraph design is a complex task. Many efforts in the last few years have been focused on dealing with the pupil discontinuities.\textsuperscript{16–19} For instance, adding an apodizer to the vortex coronagraph concept can help deal with segmentation discontinuities,\textsuperscript{20,21} or, combining the shaped pupil coronagraph concept with the apodizing phase plate coronagraph can deal with big central obscurations.\textsuperscript{22} Coronagraph apodizers can be designed using a convex linearized version of the problem that achieves globally optimal solutions.\textsuperscript{23,24} Fogarty et al.\textsuperscript{25} used a linear programming approach to find an optimal apodizer for the case of a vortex coronagraph. Mazoyer et al.\textsuperscript{26} presented a method to deal with the diffraction originated from the pupil discontinuities with two DMs by solving the nonlinear problem. A design process for an HLC was presented in Ref.\textsuperscript{11}: the DMs and FPM mask are optimized through a multistep process that iteratively modifies the optical parameters. Indeed, optimizing for multiple optical surfaces makes the design process highly complex.

Ultimately, there exists a fundamental trade between the achievable raw contrast and sensitivity to low-order errors and inner working angle, and, given the current state of the art, there is still much room for improvement toward the theoretical limit.\textsuperscript{27–29} Furthermore, much progress is still needed to understand the limitations of coronagraph systems in a testbed environment. Indeed, the physical processes taking place at the optical elements and their interplay at very high contrast are poorly understood, e.g., the chromatic effects of the DM surface errors,\textsuperscript{13} the DM actuator quantization effects,\textsuperscript{30,31} or polarization-induced aberrations.\textsuperscript{12,13,32} Modeling efforts at the Jet Propulsion Laboratory have been tackling the gap between simulation and testbed performance.\textsuperscript{12,33,34}

The work presented here was done in the context of optimizing the design process for the Nancy Grace Roman Space Telescope (Roman) Coronagraph Instrument’s HLC. The Roman coronagraph\textsuperscript{35} is an advanced technology demonstrator that will pave the way for future direct imaging missions by demonstrating key coronagraph technologies in space. The HLC in the Roman coronagraph is designed by carefully solving for the DMs shapes and the dielectric layer shape simultaneously in a small-step linearized approach. It is beyond the scope of this paper to provide the specifics of the optimal solution for the HLC design for the Roman coronagraph; our goal is instead to publish the optimization tools we developed. These consist of the changes to the electric field conjugation (EFC) algorithm cost function and the several methods to drive EFC into the optimal design solution space. We consider in particular contrast and throughput to planet light; the methods presented here are all aiming at directly or indirectly driving the coronagraph design toward an optimal contrast and throughput. The sensitivity to low-order errors is not directly addressed here; we refer, however, to Ref.\textsuperscript{36}, where an improvement in tip and tilt sensitivity of two orders of magnitude at small separations was achieved utilizing EFC. The methods presented in this work can be used in combination with the technique from Ref.\textsuperscript{36}.

This paper is organized as follows: in Sec.\textsuperscript{2}, we lay out the EFC algorithm and its conventional cost function. In Sec.\textsuperscript{3}, we present the modifications to the EFC cost function. In Sec.\textsuperscript{4}, we present some methods on how we control the EFC design runs to seek for the optimal design points and show the results from the modifications to the cost function. We discuss these results in Sec.\textsuperscript{5}. For all EFC design run optimizations, we use the Fast Linearized Coronagraph Optimizer\textsuperscript{37} software package (https://github.com/ajeldorado/falco-matlab, https://github.com/ajeldorado/falco-python).

\textbf{2 Electric Field Conjugation Algorithm}

In this section, we review the mathematical formalism of EFC. EFC as described in Ref.\textsuperscript{38} iteratively reduces the intensity in the dark hole (DH) of image plane of a coronagraphic system.

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\textsuperscript{J. Astron. Telesc. Instrum. Syst. 015003-2 Jan–Mar 2022  Vol. 8(1)
Originally conceived as a wavefront control algorithm, EFC is usually implemented to optimize one or two DM shapes: the algorithm finds a vector $\Delta u$ containing the change in actuator heights needed to null the $E$-field in the image plane.

The approach we follow to obtain the EFC equations, which we derive starting with the EFC cost function, is similar to the one followed in Ref. 39. We use a notation similar to Ref. 40, where vectors are represented in bold and lower-case letters, matrices in bold and upper-case letters, and scalars in unbold letters.

With $e_{DH}$ being the vector of complex-valued electric fields in the pixels in the region of interest, or DH, after control iteration $k+1$, the cost function of EFC in its most fundamental mode can be expressed as

$$J = e_{DH,k+1}^H e_{DH,k+1},$$  \hspace{1cm} (1)

where $H$ represents the Hermitian operator. At the core of the EFC algorithm is the linearization assumption, in which we assume that at any given control iteration the effect of the DMs is small enough that they can be approximated to first order. We can thus write the electric field at the image plane at iteration $k+1$ as the electric field in the previous iteration plus the linearized effect of the DM:

$$e_{DH,k+1} \approx e_{DH,k} + \Delta e_u = e_{DH,k} + G \Delta u,$$  \hspace{1cm} (2)

where $G$ is the Jacobian of the system with respect to $u$, $G = \partial e_{DH,k} / \partial u$. The Jacobian contains the effect of each actuator over the electric field in the DH and is computed by poking each actuator independently and recording the effect in a complex-valued, $n_{\text{pixel}} \times n_{\text{actuator}}$ matrix. Equation (2) gives us the linearized relationship between the quantity of interest $e_{DH,k+1}$ and $u$. We can use this expression with Eq. (1) to obtain the explicit expression of $J$ as a function of $\Delta u$, from which we can derive and apply $\partial J / \partial \Delta u = 0$:

$$G^H G \Delta u = -R \{ Ge_{DH,k} \}. \hspace{1cm} (3)$$

We solve for $\Delta u$ with these equations via least squares. In practice, to ensure stability given that the problem is ill-posed, a common modification is to add Tikhonov regularization:

$$J = e_{DH,k+1}^H e_{DH,k+1} + \alpha \Delta u^T \Delta u,$$  \hspace{1cm} (4)

where $\alpha$ is usually expressed as $\alpha = \alpha_0 \times 10^\beta$, $\alpha_0$ being a scaling factor and $\beta \in \mathbb{R}$ an exponential scaling term. Including the regularization, Eq. (3) becomes

$$(G^H G + \alpha I) \Delta u = R \{ Ge_{DH,k} \}, \hspace{1cm} (5)$$

where $I$ is the identity matrix.

### 2.1 Using EFC as a Coronagraph Design Tool

Although EFC was conceived as a wavefront control algorithm, its formalism applies for any surface in the optical train as long as the difference in surface heights at each iteration is small, and the effect in the electric field in the DH can be linearized. Therefore, this algorithm can be used to design a coronagraph: the control vector $u$ can be generalized to contain any degrees of freedom given by the designer to affect the optical propagation through the system. For instance, the cumulative control vector $u$ can contain the actuators of two DMs and the parameterization of an FPM’s complex transmission:
Thus the features of a hypothetical FPM can be iteratively adjusted in the same way as the DM actuator heights as shown above. It is used in particular for the design of HLCs, which consist of a Lyot coronagraph plus a complex-valued FPM. We follow this approach in this paper.

3 Modifying the Cost Function

The standard EFC cost function in Eq. (4) is concerned only with the intensity in the DH and ignores other key quantities that directly drive the performance of a coronagraph, namely, the throughput of an off-axis source in the DH and the sensitivity to low-order aberrations. In this section, we present some modifications to the EFC cost function to account for the throughput. As for the low-order error sensitivity, a possible solution is to add channels to the electric field vector $e_{DH}$ in the same way we would add wavelengths for a broadband problem, but instead we add Zernike polynomials to the entrance pupil. Since this method is thoroughly discussed in Ref. 36 and was demonstrated to achieve a significant improvement, we do not attempt to drive the sensitivity to low-order errors directly.

3.1 Adding the Total DM Stroke

A typical EFC design run for an HLC takes hundreds of iterations to attain the desirable DM and FPM solution in terms of raw contrast in the DH. Usually, the first iterations tackle the easy control modes, which require lower spatial frequencies and smaller DM stroke. At later iterations, the hard modes are addressed. To identify easy and hard modes, we do a singular value decomposition of the Jacobian matrix: the modes associated with higher energy singular values are easier to control, thus denoted easy. The modes linked lower singular values are harder or even inaccessible. The easy modes are tackled first, heavily reducing the raw contrast. When the harder modes are addressed, the DM is required to add higher frequencies in its pattern. At each iteration, the raw contrast decreases at a slower rate while rapidly increasing the RMS stroke of the DMs. The DM stroke is crudely correlated to a loss in throughput and higher sensitivity to tip and tilt errors. However, two-DM solutions often cancel each others’ phase effects leaving amplitude effects, yielding less reduction in Strehl than the RMS actuator heights might otherwise produce.

A straightforward way of controlling the DM stroke is by adding a term in the EFC cost function that explicitly punishes adding excessive stroke:

$$J = e_{DH}^H e_{k+1} + \alpha \Delta u^\dagger \Delta u + \gamma u^\dagger u.$$  

(7)

The real-valued scalar $\gamma$ regulates the strength of this new term. The total stroke being added explicitly in $u^\dagger u$ allows us to control an important parameter heavily linked to the coronagraph performance. This approach, however, has its limitations, as will be shown in Sec. 4.4. The corresponding expression for $\Delta u$ in the $k+1$ iteration is

$$\Delta u = (G^H G + (\alpha + \gamma) I)^{-1} \text{Re}\{G^H e_k + \gamma u\}.$$  

(8)

It is worth noting that the wavefront control technique stroke minimization and this modification to EFC are different. Stroke minimization seeks to minimize the DM stroke under the constraint that a target contrast is achieved. However, stroke minimization has the same mathematical solution as EFC in Eq. (4), i.e., when using the Tikhonov regularization.
3.2 Adding the Throughput Explicitly

Another more sophisticated way of dealing with the loss of throughput is to include the change in throughput $\Delta t$ of the $k$'th iteration explicitly:

$$J = e_k^H e_{k+1} + \alpha \Delta u^* \Delta u + \gamma u^* u - \omega \Delta t^* \Delta t. \tag{9}$$

The term $-\omega \Delta t^* \Delta t$ penalizes the loss of throughput, where $\omega$ is a real-valued coefficient to regulate the strength of this term; $\Delta u$ is a one-element vector in order to keep the same notation. In our optics model of the system, $\Delta t$ can be computed as a function of $\Delta u$. Indeed, in the same way, we did in Eq. (2) where the change of the electric field vector was expressed in terms of $\Delta u$, we can do the same for the change in throughput:

$$\Delta t = G_{cp} \Delta u, \tag{10}$$

where $G_{cp}$ is the Jacobian of the system for the intensity of the central pixel of an unobscured PSF; $cp$ in the subindex stands for central pixel. The peak intensity of an unobscured PSF is directly proportional to the throughput of the system. $G_{cp}$ is computed in a similar way as the original Jacobian, as explained in Sec. 2, this time evaluating the peak electric field of the unobscured PSF.

From Eq. (9), we can express the solution for $\Delta u$ for the $k + 1$ iteration:

$$\Delta u = (G^H G + (\alpha + \gamma) I - \omega G_{cp}^H G_{cp})^{-1} \text{Re}\{G^H e_k + \gamma u\}. \tag{11}$$

3.3 Peak-Normalized Intensity

Another way of adding the throughput constraint to the EFC cost function is by normalizing the DH intensity to the unocculted peak intensity. This value is closer to a key design parameter—the ratio of starlight detected to planet light detected squared $\eta_s/\eta_p^2$, which is the main driver of the detectable planet yield of a mission. The peak unobscured PSF intensity is directly proportional to the system’s throughput. We can write the cost function to minimize as

$$J = (e_{k+1}/p_{k+1})^H (e_{k+1}/p_{k+1}). \tag{12}$$

where $p$ is a complex-valued scalar representing the peak electric field of the unobscured PSF. The Jacobian for this case is denoted $G_{pn} = \partial(e_k/p_k)/\partial u$. We adopt an analogous linear assumption to that of conventional EFC, as in Eq. (2). Therefore, since the cost function $J$ has the same form as seen in Eq. (1), we can rewrite Eq. (5) as

$$\Delta u = (G_{pn}^H G_{pn} + \alpha I)^{-1} \text{Re}\{G_{pn}^H e_k/p_k\}. \tag{13}$$

To compute the new Jacobian $G_{pn}$, we need to apply the chain rule:

$$G_{pn} = \frac{\partial e_k}{\partial u}/p_k - e_k/p_k^2 \frac{\partial p_k}{\partial u}. \tag{14}$$

The term $\frac{\partial e_k}{\partial u}$ is simply $G$, as seen previously. To derive the term $\frac{\partial p_k}{\partial u}$, we do as in Sec. 3.2 when we derived a separate Jacobian $G_{cp}$ for the changes in the peak, or central pixel, of the unocculted PSF.

4 Design Optimization

4.1 Our Case Study: the Roman Coronagraph HLC

The work presented in this paper was done in the context of optimizing the design for the Roman coronagraph HLC. Using this coronagraph case to test the techniques described in Sec. 3 serves as a way to illustrate how difficult the problem of optimizing a coronagraph is in practice.
The Roman Space Telescope has certain characteristics that make it particularly challenging for high-contrast imaging. Specifically, the central obscuration is relatively large (∼30% of pupil diameter), and the six struts are several times wider (with widths of 3.2% of pupil diameter) than for any proposed mission concepts for high-contrast imaging. Discontinuities in the pupil and especially central obstructions heavily affect the performance of the system. The pupil version that we use is not the final, slightly asymmetric one. Instead, we use an earlier, on-axis version with threefold azimuthal symmetry and a perfectly circular central obscuration. Another limitation from the Roman coronagraph that we do not consider in our study is the DM surface stroke RMS requirement; this is a rather constraining requirement (based on considerations regarding allocating DM stroke for wavefront control) that adds unnecessary complexity to the problem. We do consider the DM limitations in terms of actuator voltage limits, i.e., how far the actuators are able to move.

The specifications and other coronagraph parameters are shown in Table 1. Given the current mission specifications, our target contrast is $10^{-9}$ normalized intensity (NI). The mission specifications and predicted performance are available online (Ref. 42). The choice of polymethylglutarimide (PMGI) as the dielectric is driven by the MDL's current HLC manufacturing capabilities; PMGI outside of the occulter mask results in incoherent light, so we limit the inclusion of dielectric to the occulter mask only. Dielectric deposition on the mask and shaping technologies limit what dielectric material is to be used; other materials have been considered for this application. We, however, limit ourselves to the Roman coronagraph material. The resolution at the pupil planes drives the computing time; we compute the minimum resolution to give a reliable result at high contrast to be ∼200 pixels across the pupil and use 250 pixels for margin. We use a thin-film model for the dielectric material and the metal spot in the FPM since using a complex transmission matrix and then converting the resulting matrix to a real material model is not possible in practice without losing the resultant FPM performance. We measure the contrast level with the NI, here defined as the mean intensity in the DH normalized to the peak intensity of the unobstructed PSF, i.e., with the FPM out of the beam, and the DMs in their current configuration. We define the throughput as the total energy within half-max isophote for a PSF offset $6 \lambda / D$ from on-axis divided by the total energy at the telescope pupil; losses from the reflectivity and transmissivity of optical surfaces are not included.

### Table 1 Roman coronagraph HLC design parameters.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target NI</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>10%</td>
</tr>
<tr>
<td>Number of wavelengths during WFC</td>
<td>4</td>
</tr>
<tr>
<td>Evaluation of throughput angle</td>
<td>$6 \lambda / D$</td>
</tr>
<tr>
<td>Resolution in pupil planes</td>
<td>250 pixels</td>
</tr>
<tr>
<td>Resolution in FPM plane</td>
<td>3 pixels per $\lambda / D$</td>
</tr>
<tr>
<td>Correction inner and outer radius</td>
<td>2.8 and 10 $\lambda / D$</td>
</tr>
<tr>
<td>DMs</td>
<td>Xinetics, 48 × 48 actuators</td>
</tr>
<tr>
<td>Metal material on FPM</td>
<td>Nickel</td>
</tr>
<tr>
<td>Dielectric material on FPM</td>
<td>PMGI</td>
</tr>
</tbody>
</table>

4.2 **Artificial Gain for the Dielectric Actuator Vector**

As explained in Sec. 2.1, the actuator control vector contains the information about the surface shape of the dielectric layer on the FPM. The FPM degrees of freedom in this vector have a
significantly smaller effect on the contrast than the DM actuators. If left unweighted, the DMs are loaded with most of the work; at each iteration the controller puts most of the work on the DMs since, due to the linearization assumption, at the vicinity of the current actuator vector, only the DMs have any control over the contrast. The controller is thus blind to what the dielectric can actually achieve. To fix this we add an artificial gain to the actuator vector part associated with the dielectric layer, which forces the controller to offload some of the work from the DMs to the FPM. This gain is defined as a multiplying factor (real value and positive) that artificially augments the effect of the dielectric scaling the control solution as well as to the Jacobian part associated with the dielectric degrees of freedom. Although the natural path seems to be better at the immediate iterations, we achieve a better contrast-throughput trade-off overall when using this gain.

We find that tuning this artificial gain is a rather delicate task. For better results, the gain must be higher at early iterations, and then it must go back to the natural state, i.e., equal to one, so that the FPM effect does not impede going down in contrast. Indeed, the effect of the FPM is mostly to maintain throughput (see Sec. 4.2), so the effect of the dielectric must be moderated to avoid getting stuck in a low-contrast solution. A variable gain is found to be the best solution, but it must be finely tuned to each case for every control variable set. It is beyond the scope of this paper to fine tune to the optimal solution, so we use a constant gain.

In Fig. 1, we show two EFC runs with its conventional cost function, one without the dielectric and one with. We use a gain of 10; without the gain the curve would closely follow the case of flat FPM. The other two curves in Fig. 1 are discussed in the following sections.

### 4.3 $\beta$-Scheduling

A common practice when using EFC for wavefront control is to have a scheduled regularization scheme in order to help with achieving a better contrast floor. This technique, known as...
β-scheduling, is explained in detail Ref. 40. In Eq. (4), \( \alpha \) is split into \( \alpha = \alpha_0 \times 10^\beta \), where \( \alpha_0 \) is the scaling factor and \( \beta \) is the exponent called out in the regularization scheduling.

The linearization performed to obtain the linear relationship between the electric field and the actuator vector (see Sec. 1) entails that the Jacobian of the system in this formalism intrinsically misses information. This is a form of model mismatch that also affects simulations. Indeed, although model mismatch is more severe in the case of a real system, it also affects simulations due to the linearization assumption and results in a worse contrast floor with respect to the actual achievable floor. A way of dealing with this is by doing β-scheduling.

To understand how β-scheduling helps with model mismatch, we resort to a singular value spectrum analysis of the Jacobian \( G \) for the problem laid out in Eq. (5). Solving for \( \Delta u \), the Tikhonov regularization is to be understood as a high-pass filter that limits the controller to attack a certain number of singular modes. The smaller (i.e., the more negative) the \( \beta \) is, the more modes are controlled. We usually pick a \( \beta \) that yields the best contrast (or best contrast to throughput ratio) through a grid search at each iteration, in which case the controller attacks bigger singular value modes, or easy modes (see Sec. 3.1), and when these are mostly done, at later iterations, the controller deals with the hard modes. A way of doing β-scheduling is by forcing the \( \beta \) value to be smaller than what the grid search would otherwise choose, which makes the intensity increase in the DH. However, when resuming the grid search, the contrast is significantly improved, and the achievable contrast floor is lowered; this is illustrated in Fig. 6 of Ref. 40. In the singular value domain, when \( \beta \) is forced to small values, the intensity of the easy modes is increased, but the controller starts to deal with hard modes that were previously inaccessible. So when the controller goes back to the grid-search of \( \beta \), the easy modes are quickly dealt with and new hard ones are now accessible. The β-scheduling enables the carving out of harder singular modes and thus improves the final contrast floor.

In Fig. 2, we illustrate the effect of performing β-scheduling by showing several EFC design runs with different schemes. By picking the right scheme, we obtain a better achievable contrast, while improving the final throughput. Finding a convenient scheme is done by trial and error: we identify where the design run reaches a plateau in terms of contrast, we then schedule a smaller \( \beta \) for a number of iterations. The value of \( \beta \) and the number of iterations are picked such that the final solution is more optimal.

The design runs utilizing conventional EFC achieve an acceptable NI relatively easy. However, when modifying the cost function as seen in previous sections, a reasonable achievable
contrast seems to be harder to attain. Therefore, this technique will be used when running the EFC controller for the modified cost functions.

The Tikhonov regularization is applied to the part of the Jacobian matrix associated to the DMs; there is no reason why it should not be applied to the part associated to the dielectric shape in the FPM. In Eq. (5), or wherever $aI$ is present, we substitute this term by $A$, a diagonal matrix where each diagonal element is either $a_{0,\text{DM}} 10^{\text{rms}}$ or $a_{0,\text{FPM}} 10^{\text{rms}}$. This allows us to control each regularization independently. Given the nature of the problem at hand, since the dielectric has a significantly smaller effect on the electric field in the DH with respect to the DMs, such regularization is not as critical. However, since we artificially weight the dielectric effect on the DH, we rely on this new regularization as another design tool to search for the optimal coronagraph solution.

4.4 Adding the Total DM Stroke Term

As seen in Sec. 3.1, adding the $\gamma$ term helps with restraining the total DM stroke, which in turn prevents an excessive loss of throughput and poor sensitivity to low-order aberrations. The effect of $\gamma$ is illustrated in Fig. 3. For a given value of $\gamma$ the effect at early iterations is relatively small, but after a certain number of iterations the run deviates from the conventional EFC curve. This is when the new term introduced in the cost function is comparable in value with the conventional EFC cost function value. In Table 2, we list the resultant performance metrics for different $\gamma$ values. We find that $\gamma$ offers a direct trade-off between achievable final contrast, and the DM stroke RMS; there appears to be an associated contrast floor that decreases monotonically with the value of $\gamma$. In all runs in Fig. 3 where $\gamma$ is nonzero, there is a change in curve direction in terms of DM surface RMS; it decreases this value when it would naturally increase. Indeed, there seems to be a point in which the $\gamma$-term initiates its effect, depending on the value of $\gamma$, up to the point of changing the curvature of the NI-DM surface RMS curve. A brief analysis on the units of $\gamma$ can help build some intuition. Looking at Eq. (7), $\gamma$ has units of (NI/nm$^2$); when the NI in a design run reaches $\sim 10^{-5}$ with a DM surface RMS of $\sim 10$ nm, it takes a $\gamma$ of $10^{-2}$ to halt the

![Fig. 3](https://www.spiedigitallibrary.org/journals/Journal-of-Astronomical-Telescopes,-Instruments,-and-Systems)  

Fig. 3 Adding the term associated with the total stroke of the DMs, the $\gamma$ term, helps halt the increase in surface RMS, which in its turn helps with the throughput and sensitivity to low-order aberrations (see Table 2).
controller from digging further. For the case of the conventional EFC cost function, we claim that there is an optimal value of $\gamma$ for a given raw contrast design specification.

With other modifications of the cost function, as the ones described in Secs. 3.2 and 3.3, the EFC runs severely increase in complexity in terms of accessing different areas of the throughput-contrast space. However, the $\gamma$ term effect maintains its effect of directly bounding the DM stroke RMS, which makes it a useful tool for obtaining reasonable EFC solutions. For the following sections in which we introduce more sophisticated modifications of the cost function, we work with a small value of $\gamma$ that guarantees an acceptable DM stroke RMS and allows the controller to go beyond $10^{-9}$ in NI.

### 4.5 Adding the Explicit Throughput Term

Implementing the cost function in Eq. (9) is a direct way of bounding the loss of throughput in a similar way as done with the total stroke term (see Sec. 4.4). If the weight factor for this term $\omega$ is left at a fixed value, the throughput is maintained at the cost of not reaching an acceptable contrast. In practice, we leave $\omega$ as a free parameter for which a value is chosen via a grid search to give the best $\text{NI}/T^2$ at each iteration.

We find that this term opens a new array of accessible points in the contrast-throughput space (see Fig. 1). Depending on what the controller is allowed to do in the first iterations, the grid search of the regularization (including the regularization for the dielectric actuators), and $\omega$, and combined with the effect of the $\gamma$ term, the EFC run yields a different result. The design run presented in Fig. 1 achieves a ~35% better throughput with respect to using conventional EFC while reaching a similar NI. The sensitivity to tip and tilt was also improved (see Table 3) probably due to the indirect effect of constraining the loss of throughput on the sensitivity to low-order errors.

To access the best points in the contrast-throughput parameter space, we require over 1000 iterations. This makes optimizing the EFC run by means of tuning several design parameters an

### Table 2 Effect of $\gamma$ in performance.

| $\gamma$ | NI     | Throughput (%) | Surf. DM1 (nmRMS) | Surf. DM2 (nmRMS) | $|\Delta E|^2$ for 1 nm RMS T/T |
|----------|--------|----------------|--------------------|--------------------|-------------------------------|
| 0        | 1.31 $\times 10^{-9}$ | 4.82          | 26.55              | 29.36              | 2.2925 $\times 10^{-8}$     |
| 10$^{-6}$ | 1.53 $\times 10^{-9}$ | 5.18          | 20.76              | 22.13              | 2.2926 $\times 10^{-8}$     |
| 10$^{-5}$ | 7.19 $\times 10^{-9}$ | 5.45          | 18.13              | 18.98              | 2.2929 $\times 10^{-8}$     |
| 10$^{-4}$ | 7.05 $\times 10^{-8}$ | 6.22          | 14.53              | 14.85              | 2.2963 $\times 10^{-8}$     |
| 10$^{-3}$ | 4.01 $\times 10^{-7}$ | 7.02          | 8.91               | 8.93               | 2.3410 $\times 10^{-8}$     |
| 10$^{-2}$ | 9.15 $\times 10^{-7}$ | 6.97          | 7.56               | 7.53               | 2.6081 $\times 10^{-8}$     |

### Table 3 Performance comparison between different changes in the cost function.

| Design run type             | NI     | Throughput (%) | DM1 surf. (nmRMS) | DM2 surf. (nmRMS) | $|\Delta E|^2$ for 1 nm RMS T/T |
|-----------------------------|--------|----------------|--------------------|--------------------|-------------------------------|
| Conventional EFC 2 DMs      | 1.79 $\times 10^{-9}$ | 4.97          | 25.88              | 28.56              | 2.29 $\times 10^{-8}$       |
| Conventional EFC 2 DMs + dielectric | 2.16 $\times 10^{-9}$ | 5.26          | 24.75              | 27.04              | 2.16 $\times 10^{-8}$       |
| Explicit throughput term ($\omega$ term) | 1.25 $\times 10^{-9}$ | 6.61          | 46.16              | 47.00              | 1.34 $\times 10^{-8}$       |
| Peak-NI                     | 1.58 $\times 10^{-9}$ | 6.68          | 44.90              | 46.10              | 2.67 $\times 10^{-8}$       |
overwhelming task. The addition of a new Jacobian for the throughput term, and the grid search over the two regularization coefficients and the $\omega$ value, makes these runs take days to complete on a powerful desktop computer.

4.6 Peak-Normalized Results

The result of implementing the cost function presented in Sec. 3.3 is shown in Fig. 1. For our particular problem, the controller seems to get stuck at $10^{-8}$ NI. We thus use the $\beta$-scheduling technique to reach better contrast results. We achieve a similar coronagraph performance compared to using the explicit throughput term (see Fig. 1); we reach similar contrast and improved throughput $\sim 35\%$ with respect to using conventional EFC. The sensitivity to tip and tilt errors was not improved (see Table 3), which would require using the method presented in Ref. 36 in combination with this approach to directly drive the sensitivity to low-order errors.

Compared to the cost function presented in Sec. 3.2, for which the parameter $\omega$ served as a way to tweak the behavior of the controller at every iteration, with the current cost function, the user’s control is more limited. We can control the final point in the throughput-contrast space by modifying the weight of the dielectric actuator vector, combined with the $\beta$-schedule and regularization for the dielectric vector, and $\gamma$ value. However, these parameters seem to have a limited effect on the final reach in terms of throughput-contrast. The lack of a parameter equivalent to $\omega$ or $\gamma$ makes this method harder to tune; for instance, these terms can be used to emphasize the throughput early or later. However, this method is easier to implement since it does not require a multidimensional tuning. Therefore, this method is preferred in the case the designer wants to avoid the tuning of multiple parameters, at the expense of losing the opportunity to probe more optimal areas in the parameter space.

5 Discussion

5.1 Problems with the Linear Approach

As hinted by the results presented in Sec. 4, the complexity of the problem at hand is titanic. Some of the reasons we identify are: (1) the high number of free variables; the DMs consist of two times $48 \times 48$ free variables, and the dielectric actuator vector although of arbitrary size, adds on the order of 200 variables. (2) The amount of controller variables, e.g., $\beta$ at each iteration (see Sec. 4.3), $\gamma$ (see Sec. 4.4), the weight gain on the dielectric vector (see Sec. 4.2), etc. (3) The competing nature of contrast and throughput; for the most part the gain on one is at the loss of the other. Furthermore, we are required to achieve an acceptable level of sensitivity pointing jitter. (4) The DM influence function: its tail effect, or cross talk, results in a loss of orthogonality of the DM effect. (5) The material properties of the dielectric and metal layers at the FPM, where amplitude and phase control are entangled.

Another complication comes from the limitations associated with the algorithm used, the most important of which is the linear approximation. The assumption that the step size in the DM commands is small is good enough \textit{a priori} since, although some information is lost in second-order effects that are not accounted for, these are fixed in further iterations, or even when doing the $\beta$-scheduling that corrects for model mismatch. However, particularly during the first iterations, when the step size is the highest given the energy displacement required, the controller places itself in a point that it did not intend given the linear assumption. This influences the following steps and, ultimately, where the controller ends in the throughput-contrast space. In particular, we find that the first iteration heavily affects where the controller follows in the throughput-contrast parameter space.

Hence, there are a large array of possibilities attainable in terms of solutions that provide an acceptable design. To illustrate this, we show in Fig. 4 the shape solutions for the DMs and dielectric surfaces for the optimization runs of Fig. 1. The controller arrives at very different solutions depending on the cost function. We find that even when the cost function is the same, when tweaking certain parameters, e.g., the weighting of the dielectric actuator vector or the $\beta$-scheduling, the resulting solutions are different as well.
All of these factors contribute to making finding an optimal design a tremendous task. A solution to mitigate the problems associated with the linear approach is to use a nonlinear algorithm. We have been exploring the L-BFGS-B algorithm, which has been implemented for phase retrieval providing a framework for the problem of high-contrast imaging. Recently, Will et al. introduced this framework in the context of wavefront control as an alternative to EFC.

### 5.2 Impact of Complex Transmission in the FPM on Performance

In Fig. 1, we plot two conventional EFC runs: one with the actuator dielectric vector and the other without. By choosing the right artificial weight for the dielectric actuator vector (see Sec. 4.2), the runs with the dielectric perform consistently better. In the worst case, when the weight is not correctly tuned, the performance is similar to the 2 DMs only but never worse. We find that the dielectric shape has relatively little effect on the final achievable contrast: when leaving the natural effect of the dielectric actuator vector, i.e., weight equals 1, it effectively has little effect. However, the weight helps the controller find different routes in the throughput-contrast space that end up making the dielectric do some of the work, in particular it helps with the interaction between the DMs and the Lyot stop, assisting in reshaping the diffracted light from the struts and central obscuration into the Lyot stop. Intuitively, an additional shaped surface at the FPM helps alleviate some of the work done by the DMs. This results in a smaller stroke RMS from which a better throughput and sensitivity to jitter follow.

When accounting for the throughput in the cost function, the role of the dielectric is enhanced. The FPM now rearranges the electric field shape at the center of the PSF attempting to retain the intensity at the center. We speculate that the dielectric, in a similar way to how it helps with the interaction of the DMs and the Lyot stop, helps reshape the PSF to interact with the part of the Lyot stop corresponding to the central obscuration in such a way that it would result in a better sensitivity.

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**Fig. 4** The DM shapes and dielectric layer shape solutions depend significantly on the design run and method used. The shapes displayed here correspond to the curves shown in Fig. 1. The throughput-conserving terms cause more pupil remapping to occur as can be seen in the DM shapes.

All of these factors contribute to making finding an optimal design a tremendous task.
The disparate possibilities for the dielectric shape solutions seen in Fig. 4 indicate that there are complex dependencies with design parameters as discussed in Sec. 5.1 and intricate interactions with the DMs. However, there seems to always be certain features on a threefold azimuthal symmetry associated with the strut obscurations. Ultimately, there is still much to learn on how shaping the FPM can improve performance, and how it may be used to improve the performance of future instruments.

In this work, we have limited the dielectric to the current manufacturing process limits; with sharper features than currently allowed there is more room for improvement. The manufacturing process also prevents the shaping of the metal layer and using a different dielectric other than PMGI.

6 Conclusion

We have presented a set of tools to perform coronagraph design with the EFC algorithm. The modifications to the EFC cost function directly assist in trading off contrast, throughput, and low-order aberration sensitivities, and yield better results compared to a conventional use of EFC. We showed how, with these modifications, the controller can access the more optimal areas in the performance parameter space with careful treatment of the design run parameters. The improvements shown here amount to $\sim 35\%$ gain in throughput for the same NI; however, we believe there is significant room for improvement with a more thorough tuning of parameters. Some of the main findings of this work can be summarized as follows.

- The modifications to the EFC cost function help probe more optimal areas of the throughput-NI space.
- The explicit addition of a throughput term to the cost function (see Sec. 3.2) provides significant improvement in terms of achievable NI and throughput. It is, however, non-trivial to tune.
- The peak-NI modification to the cost function (see Sec. 3.3) is easier to tune and also provides similar improvement in terms of NI and throughput. However, the lack of parameters to tune results in a more limited adjustment potential by the designer.
- We present the DM stroke term, or $\gamma$ term, that helps with contain the throughput loss in a design run. We review the $\beta$-scheduling method that deals with model uncertainty.
- We discuss the potential and limitations of adding a designable complex transmission to the FPM. This coronagraph element can reshape the PSF to optimize the throughput with nontrivial interactions with the DMs and the Lyot stop.

One of the main limitations to these methods is the linear approximation, which hinders the controller of finding the optimal path given its limited capacity of probing the right areas in the design parameter space. A nonlinear approach could address this issue and will be the subject of future work.

7 Appendix A: Lyot Stop Size Effects on Contrast and Throughput

In the Roman coronagraph HLC, and in general in any coronagraph, the interplay between optical elements and final performance is very complex. For instance, the Lyot stop shape plays a big role in how easy is for the DMs to cancel the unwanted starlight and has a big impact on the throughput. Although intuitively a bigger Lyot stop, i.e., a Lyot stop that blocks more light, would help driving the contrast down, the opposite effect eventually occurs: with a big Lyot stop there is not enough light in certain areas to destructively interfere at the final plane. To illustrate the complex interactions that take place, we show how the Lyot stop size affects the contrast and throughput for the Roman coronagraph case in Figs. 5 and 6. To obtain this, we performed a survey of EFC runs, with a conventional cost function, for several combinations of the LSID and LSOD. Some runs did not finish for unknown reasons are displayed as minus infinite in these figures.
Fig. 5 Raw contrast solution for FPM metal layer thickness versus FPM dielectric layer thickness for different Lyot stop inner diameter (LSID) and Lyot stop outer diameter (LSOD). Each individual plot shows the raw contrast distribution for a fixed LSID and LSOD. This complex interplay between optical elements and its effect on the final raw contrast solution illustrates the challenges in coronagraph design. In particular, when considering the other design metrics, such as the throughput (see Fig. 6), finding a global optimal solution can be a daunting task.

Fig. 6 Throughput to an off-axis source solution for FPM metal layer thickness versus FPM dielectric layer thickness for different LSID and LSOD. Each individual plot shows the throughput distribution for a fixed LSID and LSOD.
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Biographies of the authors are not available.