Time-domain source parameter estimation of $M_w 3–7$ earthquakes in Japan

from a large database of moment-rate functions

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Summary

Time-domain analyses of seismic waveforms have revealed diverse source complexity in large earthquakes ($M_w$>7). However, source characteristics of small earthquakes have been studied by assuming a simple rupture pattern in the frequency domain. This study utilized high-quality seismic network data from Japan to systematically address the source complexities and radiated energies of $M_w$ 3–7 earthquakes in the time domain. We first determined the apparent moment-rate functions (AMRFs) of the earthquakes using the empirical Green's functions. Some of the AMRFs showed multiple peaks, suggesting complex ruptures at multiple patches. We then estimated the radiated energies ($E_R$) of 1736 events having more than ten reliable AMRFs. The scaled energy ($e_R=E_R/M_0$) did not strongly depend on the seismic moment ($M_0$), focal mechanisms, or depth. The median value of $e_R$ was $3.7 \times 10^{-5}$, which is comparable to those of previous studies; however, $e_R$ varied by approximately one order of magnitude among earthquakes. Additionally, we measured the source complexity based on the radiated energy enhancement factor ($REEF$). The values of $REEF$ differed among earthquakes, implying diverse source complexity. The values of $REEF$ did not show strong scale dependence for $M_w$ 3–7 earthquakes, suggesting that the source diversity of smaller earthquakes is similar to that of larger earthquakes at their representative spatial scales. Applying a simple spectral model (e.g., the $\omega^2$-source model) to complex ruptures may produce substantial estimation errors of source parameters.
1. Introduction

Studies on the spatiotemporal evolution of large ($M_w > 7$) earthquakes have significantly contributed to our understanding of earthquake rupture physics and diversity in different tectonic environments. Most of these studies have been made by time-domain analyses of seismic waveforms.

For small earthquakes ($M_w < 5$), however, it is often difficult to resolve details of the rupture pattern; thus characterizing an earthquake source using several parameters is common, such as the seismic moment ($M_0$), radiated energy ($E_R$), stress drop ($\Delta \sigma$), rupture duration ($T$), spectral corner frequency ($f_c$), directivity, and complexity. In principle, these parameters can be estimated from the analysis of seismic waveforms on either the time domain or frequency domain. Since Brune (1970) developed a simple frequency-domain method, many studies have been made to establish various scaling relations (e.g., Allmann & Shearer 2009), as recently reviewed by Abercrombie (2021).

The corner frequency of the amplitude displacement spectrum, $f_c$, is used to estimate the rupture duration, $T$, and stress drop, $\Delta \sigma$. In most cases, the spatial dimension is only indirectly estimated from the corner frequency based on the assumption that the corner frequency is inversely proportional to the spatial dimension. This assumption is valid when the source time function has a simple pulse-like waveform. However, if the rupture pattern is complex, the relationship between the corner frequency and the source dimension is non-unique, and its relationship to the stress drop is ambiguous (e.g., Beresnev 2001). If the phase spectrum is used together with the amplitude spectrum, this ambiguity may be eliminated. However, few studies include the phase spectrum in their analysis because it is generally cumbersome.

To overcome this difficulty, we apply a time-domain method to high-quality seismic network
data from Japan, with the hope that we can systematically characterize the source properties of $M_w$ 3 to 7 earthquakes. The issues in analysing small earthquakes in the time domain arises from the difficulty in obtaining reliable Green's functions at high frequencies. However, with the good seismic network data in Japan, we can estimate reliable apparent moment-rate functions (AMRFs) for small earthquakes ($M_w$>3) using empirical Green's functions (eGF) (Yoshida 2019). Here, we first created a database of AMRFs of many (~1700) earthquakes, and then systematically examined the relationships between various source parameters.
2. Data and Method

2.1. Earthquakes and waveform data

We applied a time-domain analysis to the earthquakes for which the moment tensors are listed in the F-net moment tensor catalogue (Fukuyama et al. 1998). We targeted crustal events on land from 2003 to 2021 that were surrounded by seismic networks (Fig. 1). Their moment magnitude $M_w$ ranged from 3.0 to 7.0. We obtained velocity waveform data from the stations of national universities, Japan Meteorological Agency (JMA), and Hi-net (NIED, 2019a), F-net (NIED, 2019b), and V-net (NIED, 2019c) of the National Research Institute for Earth Science and Disaster Resilience (NIED) (Fig. 1b). Additionally, we obtained acceleration waveform data from the downhole acceleration sensors of the NIED KiK-net (NIED, 2019d), which are collocated with the Hi-net velocity sensors. Velocity waveform data were used to analyse $M_w$ $<$ 5.5 earthquakes, whereas acceleration waveform data were used to analyse $M_w$ $\geq$ 4.5 earthquakes. Earthquakes with $M_w$ 4.5–5.5 were analysed with both data sets separately. Yoshida (2019) estimated the AMRFs in this region in a preliminary study of $M_w$ 3–5 earthquakes from 2004 to 2019. In this study, we expanded the data period, magnitude range, and analysed frequency range and implemented new quality control for AMRFs.
Figure 1. Earthquake data and seismic stations. (a) Earthquakes ($M_w$ 3-7 in the crust of land areas) from the F-net moment tensor catalogue from March 2003 to May 2021. The earthquakes for which more than ten AMRFs were derived are coloured according to their faulting style. (b) Seismic stations. Blue crosses denote the stations with only velocity seismometers, and red triangles denote the stations with both velocity seismometers and accelerometers (KiK-net and Hi-net stations).

We used SH waves observed at stations within 100 km of the target earthquakes. The waveforms beginning 2.0 seconds before the arrival of the S-wave and lasting 10 s to 120 s, depending on the magnitude, were used. We used the arrival time of the S-wave listed in the JMA unified catalogue, if available. Otherwise, we computed the arrival time assuming the one-dimensional velocity structure of Ueno et al. (2002), which is routinely used to determine the hypocentres in the JMA unified catalogue.

We estimated the AMRFs of the target events using the waveforms of nearby smaller earthquakes (eGF events) to correct for the site and path effects (Hartzell 1978). These eGF events
satisfied the following two criteria: (1) the hypocentral distance from the target event was < 3.0 km according to the JMA unified catalogue, and (2) the magnitude was 1–2 magnitude units less than the target earthquake. Each target earthquake could have multiple eGF events.

2.2. Estimation of AMRFs

We determined each AMRF by deconvolving the observed waveforms with the eGFs at the same station, using the deconvolution algorithm developed by Ligorria & Ammon (1999) that employs the method of Kikuchi & Kanamori (1982). The moment-rate function was constrained to be positive in this inversion. All the basic analyses were performed on the time domain; however, we decided the frequency band and corrected for the estimated $E_R$ with the widely-used $\omega^2$-source model.

To perform stable deconvolution, we first applied a low-pass filter to the seismograms. The cut-off frequency of this filter, $f_l$, depends on the signal-to-noise (SN) ratio of the seismograms. For a given seismogram, $f_l$ should be as high as possible to obtain an accurate $E_R$ while maintaining good deconvolution stability. To treat earthquakes with different magnitude consistently, we set $f_l$ as proportional to the corner frequency of the event. Using Brune’s (1970) model, the corner frequency $f_c$ of an event with moment $M_0$ is given by $f_c(M_0) = k\beta \left( \frac{16}{7} \Delta\sigma \right)^{1/3} M_0^{-1/3}$, where $\beta$ is the S-wave velocity (3.3 km/s), $k = 0.37$ (SI unit), and $\Delta\sigma$ is the scaling stress parameter. Fig. 2 shows this relation for three values of $\Delta\sigma$. Because $\Delta\sigma$ typically ranges from 0.3 to 50 MPa (e.g., Allmann & Shearer 2009), we chose the $f_l$ given by this relation for $\Delta\sigma = 100$ MPa. Because of the low-pass filter, our estimate of $E_R$ is band-limited and missing some energy at high frequencies. Thus, we need to correct for the missing energy, as we will discuss in Subsection 2.3.
**Figure 2.** Relationship between the moment magnitude ($M_w$) and the low-pass cut-off frequency ($f_l$) (solid line) and spectral corner frequencies (dashed lines). The dashed lines show the source corner frequencies of the Brune (1970) model for the stress parameters $\Delta \sigma$ of 1 MPa, 3 MPa, and 10 MPa.

Fig. 3 shows five typical examples of the target-event waveform, $eGF$, and AMRF. We
computed the synthetic waveform of each target event by convolving the AMRF with the eGF. We included the event in our final list only if the synthetic waveform reproduced more than 80% of the power of the observed waveform. Furthermore, even if the derived AMRF met the above condition, we removed it from the final list if it appeared very noisy (about 15% of the total; Text S1).

**Figure 3.** Examples of waveform deconvolution for five earthquakes (a–e). From left to right, the first and second columns show the target seismic waveforms and the eGFs, respectively, and third column shows the AMRF obtained by deconvolution. White, black, and blue triangles show the initiation time $t_1$, the termination time $t_2$, and the centroid time $t_c$, respectively. Red horizontal
lines show the measured duration $T = t_2 - t_1$. The fourth column shows the spectra of the AMRFs.

Solid red curves denote the spectra. The black dashed curve shows the best-fit omega-square spectrum, with the black inverted triangle indicating the corner frequency ($f_c$).

When multiple eGF events were available for one target earthquake, the eGF event that produced AMRFs at the largest number of stations was used. If the number of AMRFs from an eGF event was less than ten, we discarded the AMRFs. We measured the durations of AMRFs by using peak detection (Text S2). We automatically determined the initiation time of the first peak $t_1$ and the termination time of the last peak $t_2$, and then calculated the duration by $T = t_2 - t_1$. We cut out the range from $t = t_1 - T/2$ to $t = t_2 + T/2$ of the AMRFs and normalized them so that their time integrals were equal to the seismic moments listed in the F-net catalogue.

Fig. 3a shows a simple triangular AMRF, whereas Figs. 3b–e show examples of AMRFs with multiple pulses, implying complex ruptures at multiple patches. Additionally, we computed the spectra of the AMRFs using the fast Fourier transform (FFT) algorithm. The spectrum of the simple triangular AMRF shows a monotonic decay in amplitude with frequency above a well-defined corner frequency (Fig. 3a). However, the spectra of the complex AMRFs have distinctive troughs (Figs. 3b-e), reflecting the multiple pulses. Because of this complex spectral shape, fitting the spectrum with the commonly-used $\omega^2$-model is difficult, and no clear corner frequency can be defined.

### 2.3. Estimation of radiated energy and source complexity

The radiated energy, $E_R$, is estimated by the following (e.g., Vassiliou & Kanamori 1982):

$$E_R = \left(\frac{1}{15\pi\rho\alpha^5} + \frac{1}{10\pi\rho\beta^5}\right) \int M^2(t) dt$$ (1)
where \( \rho \) is the density, \( \alpha \) is the P-wave velocity, \( \beta \) is the S-wave velocity at the source, and \( M(t) \) is the moment-rate function. (In Appendix A, we list the symbols and relationships frequently used in this paper.) We used the medium parameters employed in the F-net moment tensor inversion algorithm (Fig. S1). For \( M(t) \), we used the AMRFs obtained in Subsection 2.2. Because the AMRFs were estimated by applying a low-pass filter (Fig. 2), the radiated energy computed with Eq. (1) represents a band-limited value. Hereafter, we refer to this band-limited radiated energy as \( \hat{E}_R \).

To estimate the missing high-frequency energy, we assume that the amplitude of the displacement spectrum decays with \( f^2 \) for \( f > f_l \) in consistency with the \( \omega^2 \)-model (Aki 1967; Brune 1970). In this case, the missing high-frequency energy is calculated as follows:

\[
E_R^{\text{missing}} = \frac{4\pi a_l^3 f_l^3}{5\rho \beta^5}
\]  

(2),

where \( a_l \) is the spectral amplitude at \( f = f_l \). We used the FFT algorithm to compute \( a_l \), and obtained the radiated energy as \( E_R = \hat{E}_R + E_R^{\text{missing}} \). Fig. 4a shows the scaled energy \( (\hat{E}_R/M_0) \), before the correction for missing high-frequency energy. Fig. 4b compares the scaled energy before and after the correction. For earthquakes with a small \( \hat{E}_R/M_0 \), the effect of the correction is negligible, whereas for earthquakes with a large \( \hat{E}_R/M_0 \), the correction is large. This is because earthquakes with high scaled energy before the correction have a large amount of high-frequency energy, which is removed by the low-pass filter.
Figure 4. Estimated band-limited scaled energy $\hat{E}_R/M_0$. (a) Frequency distribution of $\hat{E}_R/M_0$. The red colour shows the results from velocity seismograms for $M_w<5.5$ events, and the blue colour shows the results from strong-motion seismograms for $M_w>4.5$ events. (b) Comparison of scaled energy before and after the correction for the missing high-frequency energy ($\hat{E}_R/M_0$ and $E_R/M_0$, respectively).

As a measure of source complexity, we used the radiated energy enhancement factor ($REEF$) proposed by Ye et al. (2018).

$$REEF = \frac{E_R}{E_{Rmin}} = \frac{5\pi\rho\beta^5}{6} \left( \frac{E_R}{M_0} \right) \left( \frac{T^3}{M_0} \right),$$

where $E_{Rmin}$ is the minimum radiated energy for a given $M_0$ and source duration $T$ (Kanamori & Rivera 2004). The minimum energy radiation occurs when the moment-rate function is a parabolic (i.e., quadratic) function and $REEF = 1$. As the shape of the moment-rate function becomes more complex, $REEF$ increases. Examples of $REEF$ for some typical moment-rate functions are shown in Fig. S2.

Figs. 5–7 show examples of the azimuthal variation in the AMRF, the scaled energy
\( e_R = \frac{E_R}{M_0}, REEF, \) and \( M_0/T^3 \) for 12 events. They show the systematic directional variations caused by rupture propagation. Some earthquakes show simple directivity (Ben-Menahem 1961), where the pulse width and amplitude vary systematically with direction. Fig. 7d shows the 2016 \( M_w=7.0 \) Kumamoto earthquake, for which the AMRFs are very complex and long in the southwest direction but relatively simple and short in the northeast direction. This directivity is consistent with the previous studies based on different analyses using similar data (Asano & Iwata 2006; Kanamori et al. 2020). Additionally, the directional dependence of the AMRFs of the 2017 Akita-Daisen earthquake (Fig. 7b) is consistent with that in previous study using a similar method (Yoshida et al. 2020). We selected the station with the median radiated energy for each event and used that AMRF as representative of the event. The representative values of \( E_R, REEF, \) and \( T \) for that event were calculated from the representative AMRF.
Figure 5. Examples of directional dependence in AMRFs, $e_R = E_R/M_0$, REEF, and $M_0/T^3$ for four earthquakes of (a) $M_w$ 3.4, (b) $M_w$ 3.6, (c) $M_w$ 3.8, and (d) $M_w$ 3.9. The horizontal lines in the upper three diagrams in each panel represent the median values of $e_R$, REEF, and $M_0/T^3$. The horizontal dashed line represents the 95% confidence interval.
Figure 6. Examples of directional dependence in AMRFs, $e_R = E_R/M_0$, REEF, and $M_0/T^3$ for four earthquakes of (a) $M_w$ 4.0, (b) $M_w$ 4.0, (c) $M_w$ 4.3, and (d) $M_w$ 4.4. The horizontal lines in the upper three diagrams in each panel represent the median values of $e_R$, REEF, and $M_0/T^3$. The horizontal dashed line represents the 95% confidence interval.
Figure 7. Examples of directional dependence in AMRFs, $e_R = E_R/M_0$, REEF, and $M_0/T^3$ for four earthquakes of (a) $M_w$ 4.8, (b) $M_w$ 4.9, (c) $M_w$ 6.9, and (d) $M_w$ 7.0. The horizontal lines in the upper three diagrams in each panel represent the median values of $e_R$, REEF, and $M_0/T^3$. The horizontal dashed line represents the 95% confidence interval.
There were 5445 cases where the AMRFs were obtained with more than ten different eGF events for the same earthquake-station pair. To see the stability of these results, we calculated the median (Med) and median absolute deviation (MAD) of the results obtained by using the different eGF events for each pair (Figs. S3a–c). We then computed \( r\text{MAD} = \text{MAD}/\text{Med} \) as a rough indicator of the percentage spread of the \( T_e R_e \) and \( \text{REEF} \) estimates. The distributions of \( r\text{MAD} \) are shown in Figs. S3d–f. The medians \( r\text{MAD} \) values were 0.11, 0.27, and 0.25 for \( T_e R_e \) and \( \text{REEF} \), respectively.
3. Results

We estimated the scaled energy $e_R = E_R / M_0$, $REEF$, and duration $T$ for 1736 earthquakes. Of these, 1700 were of $M_w < 5.5$, for which we used velocity waveforms, and 36 were of $M_w \geq 4.5$, for which we used acceleration waveforms.

3.1. Radiated energy

The frequency distributions of the scaled energy $e_R = E_R / M_0$, $REEF$ after the correction, and $M_0 / T^3$ are shown in Fig. 8a–c. The correction (Fig. 4b) increased the median of the scaled energy from $2.4 \times 10^{-5}$ to $3.7 \times 10^{-5}$. For each event, we computed the 95% confidence intervals of the median values of $e_R$ and $REEF$ using 1000 different combinations of stations based on bootstrap resamplings. We calculated $e_R^{\text{err}} = e_R^{\text{max}} / e_R^{\text{min}}$ and $REEF^{\text{err}} = REEF^{\text{max}} / REEF^{\text{min}}$ to express the uncertainty of the estimated results, where the superscripts max and min represent the upper and lower limits, respectively. The frequency distribution of $e_R^{\text{err}}$ and $REEF^{\text{err}}$ is shown in Fig. S4; the median $e_R^{\text{err}}$ is 2.0 and the median $REEF^{\text{err}}$ is 1.8. However, the overall diversity of $e_R$ and $REEF$ among different earthquakes varied approximately several- to ten-fold (Figs. 8a-b), significantly larger than the estimation uncertainties for individual results.
Figure 8. Estimated scaled energy $e_R = E_R/M_0$, $REEF$, and $M_0/T^3$. (a)-(c) Frequency distributions for the three variables, respectively. Red colour shows the results from velocity seismograms for $Mw<5.5$ events, while blue shows the results from acceleration seismograms for $Mw>4.5$ events. The numbers in parentheses indicate the 95% confidence interval of the median. Dependence of (d) $e_R$, (e) $REEF$, and (f) $M_0/T^3$ on the seismic moment $M_0$. Circles show the results from velocity seismograms, while squares show the results from acceleration seismograms. $REEF$ is shown according to the colour scale given in the figure. Large black symbols show the median values, and the horizontal line denotes the range of the data used. Vertical lines indicate the 95% confidence interval of the median values. The dashed horizontal line in (f) shows the global $M_0/T^3$.
trend (Duputel et al. 2013).

The present results include 13 earthquakes in common with Kanamori et al. (2020) in which $E_R$ is estimated with a very different method. Table 1 compares the $E_R$ values in this study with the $E_R_{\text{final}}$ in Kanamori et al. (2020). For 12 events, the difference is within a factor of three. Even for the remaining one event, the difference is approximately a factor of four. Moreover, the results of this study are in good agreement with those of other studies using the empirical Green's function method, as shown in Table 1.

**Table 1.** Comparison with results from Kanamori et al. (2020) and other studies. (1) Izutani (2005), (2) Izutani (2008), (3) Baltay et al. (2011), and (4) Ross et al. (2018).

<table>
<thead>
<tr>
<th>Event</th>
<th>JMA ID</th>
<th>$M_w$ (F-net)</th>
<th>$E_R$ (This study)</th>
<th>$E_R_{final}$ (Kanamori et al. 2020)</th>
<th>$E_R$ (Other studies)</th>
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</thead>
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<tr>
<td>2004 Chuetsu</td>
<td>2004102317560030</td>
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<td>2.11 E+14 J</td>
<td>3.92 E+14 J</td>
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<td>2004102318340569</td>
<td>6.24</td>
<td>1.44 E+14 J</td>
<td>6.76 E+13 J</td>
<td>2.9 E+14 J (3)</td>
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<td>6.69</td>
<td>3.12 E+14 J</td>
<td>8.69 E+14 J</td>
<td>6.8 E+14 J (2)</td>
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<td>2008 Iwate-Miyagi</td>
<td>2008061408434536</td>
<td>6.89</td>
<td>1.44 E+15 J</td>
<td>7.91 E+14 J</td>
<td>1.8 E+15 J (3)</td>
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<td>4.95 E+14 J</td>
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<td>6.46 E+13 J</td>
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2016 Kumamoto 2016041601250547 7.03 1.03 E+15 J 2.09 E+15 J
2016 Tottori 2016102114072257 6.17 4.51 E+13 J 4.95 E+13 J 5.8 E+13 J (4)
2016 Ibaraki 2016122821384904 5.90 2.03 E+13 J 9.85 E+12 J
2018 Shimane 2018040901323081 5.62 6.65 E+12 J 4.91 E+12 J
2018 Osaka 2018061807583414 5.51 8.72 E+12 J 1.12 E+13 J

3.2. Absolute value and scale dependence of the scaled energy, REEF, and $M_0/T^3$

The median values of the scaled energy $e_R = E_R/M_0$, $REEF$, and $M_0/T^3$ were $3.7 \times 10^{-5}$, 3.8, and $3.5 \times 10^{16}$, respectively. We found no clear difference between the estimates for the $M_w < 5.5$ earthquakes using velocity waveform data and those for the $M_w \geq 4.5$ earthquakes using acceleration waveform data.

Fig. 8d–f compares the scaled energy $e_R = E_R/M_0$, $REEF$, and $M_0/T^3$ with $M_0$. $E_R$ increases with $M_0$ (Fig. S5); nevertheless, no strong correlations were found between $e_R$ and $M_0$ for $M_w$ 3–7 (Fig. 8d). However, the $e_R$ for smaller earthquakes ($M_w < 4$) appears to have a weak, decreasing trend with $M_0$. Similarly, the median $M_0/T^3$ decreases slightly as $M_w$ increases from 3 to 4 (Fig. 8f). No such trends are evident for earthquakes with $M_w$ 4–7. Hence, the weak decreasing trends for $M_w < 4$ events may be related to low signal-to-noise ratios of the eGFs for smaller earthquakes.

The median values of $REEF$ do not show a significant dependence on $M_0$ (Fig. 8e).

The apparent stress $\sigma_a = \mu(E_R/M_0)$ (Wyss & Brune 1968; Wyss 1970) is a useful stress parameter that can be related to energy radiation. Because the static stress drop $\Delta \sigma$ is subject to large uncertainties, the scaled energy $e_R = E_R/M_0$ or $\sigma_a$ are alternative stress parameters for studying the difference in earthquake source characteristics (Kanamori et al. 1993; Kanamori & Heaton 2000; Kanamori & Rivera 2006). However, comparing the scaled energies of earthquakes with different magnitudes is challenging because the comparison requires a wide range of
frequencies, and the scale dependence remains under debate. Some previous studies showed that scaled energy or apparent stress increases with earthquake size (e.g., Abercrombie 1995; Mayeda & Walter 1996; Jost et al. 1998; Izutani & Kanamori 2001; Prejean & Ellsworth 2001; Mori et al. 2003; Izutani 2005, 2008; Takahashi et al. 2005; Mayeda et al. 2005; Malagnini et al. 2008; Malagnini et al. 2014; Nishitsuji & Mori 2014), whereas others suggested that the scaled energy is independent of earthquake size (e.g., Ide & Beroza 2001; Pérez-Campos & Beroza 2001; Baltay et al. 2010; Baltay et al. 2011; Baltay et al. 2014; Zollo et al. 2014; Denolle & Shearer 2016; Ye et al. 2016a; Chounet et al. 2018). The $e_R$ in our dataset does not show a strong size dependence from $M_w$ 3–7.

The scaled energy $e_R$ obtained in this study varied from $5 \times 10^{-6}$ to $4 \times 10^{-4}$ (Fig. 8a). This range is much narrower than the range of $10^{-7}$ to $10^{-3}$ obtained in previous studies (Vasiliou & Kanamori 1982; Singh & Ordaz 1994; Abercrombie 1995; Newman & Okal 1998; Ide & Beroza 2001; Kanamori & Rivera 2004; Venkataraman & Kanamori 2004; Jin & Fukuyama 2005; Choy et al. 2006; Convers & Newman 2011; Baltay et al. 2014; Kanamori et al. 2020). The value of $e_R$ varies among tectonic environments (Choy & Boatwright 1995; Newman & Okal 1998; Venkataraman & Kanamori 2004; Ji & Fukuyama 2005; Ye et al. 2013; Ye et al. 2016b; Kanamori & Ross 2019). Crustal earthquakes often have a larger $e_R$ than interplate earthquakes (Venkataraman & Kanamori 2004). Venkataraman & Kanamori (2004) estimated the $e_R$ of six $M_w$>6.7 crustal earthquakes at $2 \times 10^{-5}$ to $3 \times 10^{-4}$. Kanamori et al. (2020) estimated the $e_R$ of 29 $M_w$>5.5 crustal earthquakes in Japan at $6 \times 10^{-6}$ to $1 \times 10^{-4}$. These ranges are comparable to the results of this study.

The median $REEF$ obtained in this study was 3.8, with individual values ranging from 2 to 30. Ye et al. (2018) estimated the $REEF$ of $M_w$>7 interplate earthquakes, and found a range of 10 to 100, which is larger than the present results. $REEF$ is proportional to $T^3$ (where $T$ is duration,
Eq. 3); therefore, this difference could result from differences in the way $T$ was measured. The median $M_0/T^3$ obtained in this study for $M_w$ 3–7 earthquakes was $3.5 \times 10^{16}$, which was approximately five times larger than that of the global $M_w$>6.5 events, at $7.1 \times 10^{15}$, as estimated from the centroid time delay (Duputel et al. 2013) (Fig. 8f). Thus, given the differences in the definition of $T$, we consider that the relative, rather than absolute, variation in $REEF$ in the individual data is relevant. Our results (Fig. 8e) show that $REEF$ has no obvious size dependence at $M_w$ 3–7.

3.3. Dependence on the depth and the faulting style

Fig. 9 compares the depth with $e_r$, $REEF$, and $M_0/T^3$. Although few events deeper than 15 km were examined, no significant variation with depth is discernible. Our finding that $e_r$ does not strongly depend on depth is consistent with the globally compiled results (Bilek et al. 2004; Venkataraman & Kanamori 2004; Ye et al. 2016b, Denolle & Shearer 2016), except for shallow tsunami earthquakes.
Figure 9. Dependence of (a) scaled energy $e_R = E_R/M_0$, (b) apparent stress, (c) REEF, and (d) $M_0/T^3$ on depth. REEF is shown according to the colour scale in the figure. Large black circles show the median values, and the horizontal line denotes the range of the data. Vertical lines indicate the 95% confidence interval of the median values.
Some controversy has arisen over the dependence of $e_R$ on the focal mechanism type (Choy & Boatwright 1995; Newman & Okal 1998; Pérez-Campos & Beroza 2001). In particular, some studies reported that $e_R$ is systematically larger for strike-slip fault earthquakes (Choy & Boatwright 1995; Pérez-Campos & Beroza 2001; Choy & Kirby 2004; Choy et al. 2006; Convers & Newman 2011; Batlay et al. 2014); however, others found no significant differences (Abercrombie 1995; Mayeda & Walter 1996; Newman & Okal 2004; Kanamori & Ross 2019; Kanamori et al. 2020). Others have suggested that the difficulty of correcting the radiation pattern may have caused the apparent variations (Newman & Okal 1998).

Fig. 10 compares $e_R$, $REEF$, and $M_0/T^3$ with the type of faulting. We classified the focal mechanisms listed in the F-net catalogue by the scalar parameter $C_m = \frac{\phi}{|\phi|} \frac{180 - |\phi|}{90}$ according to Shearer et al. (2006). Here, $\phi$ is the rake angle (ranging from -180° to 180°) of the nodal plane with the smaller absolute value of rake angle. $C_m$ is related to the slip direction and ranges from -1 to 1, taking the value of -1 for pure normal-fault motion, 0 for pure strike-slip-fault motion, and 1 for pure reverse-fault motion. We found no obvious dependence of $e_R$, $REEF$, and $M_0/T^3$ on faulting type.
Figure 10. Dependence of scaled energy \( e_R = E_R/M_0 \), \( REEF \), and \( M_0/T^3 \) on faulting style. Black circles show the median values, with the horizontal line denoting the data range. Vertical lines indicate the 95% confidence interval of the median values.

4. Discussion

4.1. Relation between source duration \( T \) and corner frequency \( f_c \)

In our time-domain analysis, we used the duration, \( T \), rather than the commonly used corner
frequency, $f_c$, to define the temporal scale of the events. To clarify the relationship between $T$ and $f_c$, we compared them for the events used in this study. We measured the corner frequencies in the frequency domain using the standard procedure based on the $\omega^2$-model. We first computed the spectrum of each AMRF using the FFT method. We did not smooth the spectra, because smoothing may distort the spectral shape. Fig. 11 shows the spectra of AMRFs at each station of the $M_w=4.0$ event shown in Fig. 6a. The low-frequency level is normalized at 1. Other examples are shown in Figs. S6-9. The spectra of the AMRFs with high $REEF$ tend to have complex spectral shapes, significantly different from their $\omega^2$-spectrum (azimuth from 180° to 300°).
Figure 11. Examples of AMRF spectra for an $M_w = 4.0$ event. The corresponding AMRFs are shown in Fig. 6a. The red curves on the left side of each panel show the AMRF spectra. The black dashed curves show the best-fit omega-square spectra, and the black inverted triangles indicate the corner frequency, $f_c$. The vertical line represents the cut-off frequency. The number in parentheses indicates the azimuths of the respective seismic station. The blue curve on the right side of each panel shows the AMRF.
We fit the spectrum of all the events to the $\omega^2$–spectrum and minimized the variance using

$$J = \sum_{k=1}^{n_{\text{freq}}} \left\{ \log(A(f_k)) - \log\left(\frac{1}{1+(f_k/f_c)^2}\right) \right\}^2$$

(4),

where $n_{\text{freq}}$ is the number of frequency points, $f_k$ is the frequency point ($f_k < f_l$), and $A(f)$ is the normalized AMRF spectral amplitude. The corner frequency, $f_c$, is determined by minimizing $J$. In equation (4), both the frequency and the amplitude are sampled at equal intervals on a logarithmic scale (this sampling scheme is denoted by loglog in this paper). Fig. 12a shows the variance as a function of $REEF$; it is minimized at $REEF=3$. This result can be interpreted as follows. The moment-rate function that has the $\omega^2$-spectrum is given by $t \exp(-2\pi f_c t)$ ($t$: time) (Brune 1970). This function reaches a maximum at $t = t_p = 1/2\pi f_c$. Although the duration of this function is not finite, at $t = 5t_p$ it decreases to less than 10% of the maximum; hence, we define the duration by $T = 5/2\pi f_c$. In our time-domain analysis, we measured the duration of the observed AMRF with approximately 10% amplitude threshold. Fig. S2c shows that $REEF$ is approximately 3 for this truncated AMRF function. Thus, Fig. 12a indicates that, if the AMRF is a simple pulse-like function with $REEF \approx 3$, then the $\omega^2$-model is a good model; additionally, $T$ and $f_c$ can be directly related by $T = 5/2\pi f_c$. For complex AMRFs with large $REEF$s, simple relationship does not exist between $T$ and $1/f_c$. Figs. 12b and c show the relationships we obtained for our data set. Although we used the loglog sampling scheme above, we can similarly define "linlin", "linlog" and "loglin" sampling schemes and show the same trend (Fig. S10).
Figure 12. Relationships with corner frequency estimated using spectral fitting and duration. (a) Relationship between REEF and the residual of spectral fitting $J$. (b) Comparison of the durations measured in the time domain ($T$) and those in the frequency domain ($T'$) from the corner frequency. $T'$ was calculated assuming the triangular moment-rate function (Eq. A2). REEF is shown according to the colour scale. (c) Comparison of REEF and $T'/T$.

4.2. Comparison of the results with time-domain and frequency-domain methods

In the time-domain method, we have three independent parameters, $M_0$, $E_R$, and $T$. To compare our time-domain results with the frequency domain results, Fig. 13 shows the relationship given by equation (3), using 2 non-dimensional parameters $e_R = \frac{E_R}{M_0}$ and $\left(\frac{M_0}{\mu \beta^3 T^3}\right)$ with REEF as a parameter. For the $\omega^2$-model, $REEF \approx 3$ as shown above. The black solid line on Fig. 13 shows the relationship for the $\omega^2$-model where $T = 5/2 \pi f_c$ is used.
Figure 13. The relationship between \( e_R = E_R/M_0 \), \( REEF \), and \( M_0/\mu \beta^3 T^3 \). \( REEF \) is shown according to the colour scale. The thick black line shows the relationship for the \( \omega^2 \)-model (Brune 1970).

About 60% of our events (in blue) closely follow this line. Thus, if the events are relatively simple with a small \( REEF \) (< 5), our results are consistent with those for the \( \omega^2 \)-model. However, the results for events with a large \( REEF \) deviate significantly from this trend, as shown in Fig. 13.

Note that there is no parameter in the \( \omega^2 \)-model that directly measures the spatial length.
in the model (e.g., radius $a$ of a circular fault). The corner frequency $f_c$ is a kinematic parameter determined from the wave-field. Thus, in principle, the static stress drop, $\Delta \sigma$, cannot be determined from this type of analysis. Brune (1970) linked the $\omega^2$-model to a circular crack model with a radius $a$ using a simple relationship ($f_c = c \beta / 2 \pi a$, where $c$=2.34), which is based on qualitative energy partitioning and is somewhat heuristic, but it is a reasonable relation for relatively smooth ruptures, because if $T = 5 / 2 \pi f_c$, the corresponding rupture speed, $a/T$, is a significant fraction of the shear-wave speed, $0.468 \beta$. With this relationship, we can relate $f_c$ to the static stress drop $\Delta \sigma$ of a circular fault using Eshelby's (1957) relation as follows:

$$\Delta \sigma = \left( \frac{7 \pi^3}{2 c^3} \right) \mu \left( \frac{M_0 f_c^3}{\mu \beta^3} \right) = \left( \frac{875}{16 c^3} \right) \mu \left( \frac{M_0}{\mu \beta^3 T^3} \right).$$

(5)

However, because no directly measured spatial parameter is used, this should be regarded as an approximate stress drop inferred from $f_c$, which may be more appropriately called a stress parameter (Boore 1983; Atkinson & Beresnev 1997). Nevertheless, for a smooth simple rupture, it is a conceptually useful measure of the static stress drop.

For the $\omega^2$-model, the radiated energy $E_R$ is given by $E_R = \frac{\pi^2}{5 \mu \beta^3} M_0^2 f_c^3$. Using this equation and equation (5), the radiation efficiency (the ratio of $E_R$ to the total static energy released by a fault if the stress drop is from $\Delta \sigma$ to 0) for this model is given by the following:

$$\eta_R = 2 \left( \frac{\mu}{\Delta \sigma} \right) \left( \frac{E_R}{M_0} \right) = \frac{4 c^3}{35 \pi}$$

(6)

For Brune's (1970) model, $c$ is fixed at 2.34, and $\eta_R = 0.47$ regardless of $\Delta \sigma$ (e.g., Madariaga &
Ruiz 2016). Note that only if a circular fault with $c = 2.34$ is used, $\Delta \sigma$ can be determined and $\eta_R = 0.47$. The radiation efficiency $\eta_R = 0.47$ appears to be a reasonable rough average for ordinary brittle events in the crust, but it may vary considerably among individual events.

4.3. Earthquakes with large REEF

The large REEF events, such as those shown in Fig. 3b–e, significantly deviate from the trend of the $\omega^2$-model shown in Fig. 13. No obvious way exists to relate $T$ to the spatial scale for complex events; hence, $\Delta \sigma$ and $\eta_R$ cannot be determined for these events. However, despite its ambiguity, because $\Delta \sigma$ is a stress parameter widely used in seismology, here we attempt to relate our results to the $\Delta \sigma$ and $\eta_R$ obtained with the $\omega^2$-model.

From the definition of $\eta_R$, we can write $\eta_R$ as

$$\eta_R = \frac{2\mu}{\Delta \sigma} \left( \frac{E_{R_{\text{min}}}}{M_0} \right) \text{REEF}$$

in terms of REEF. Then, for an event with $\text{REEF} = R$ and duration $T$, $\Delta \sigma$ can be written as

$$\Delta \sigma = \left( \frac{\eta_{R_{\omega}}}{\eta_R} \right) \left( \frac{R}{R_{\omega}} \right) \Delta \sigma_{\omega}$$

where the subscript $\omega$ indicates the parameters for the $\omega^2$-model with the same event duration $T$.

In the above, $R_{\omega} = 3$ for the $\omega^2$-model. This relation can be regarded as an extension of the widely used method for estimating $\Delta \sigma$ from seismic observations.

Because equation (6) involves both REEF and $\eta_R$, $\Delta \sigma$ cannot be uniquely related to
\[ \Delta \sigma = \left( R / R_{\infty} \right) \Delta \sigma_{\infty} \] 

\[ \text{if we assume that simple and complex events in similar tectonic environments have} \]

approximately the same average \( \eta_R \), then \[ \Delta \sigma = \left( R / R_{\infty} \right) \Delta \sigma_{\infty}. \] 

If \( R = 15 \), then \( \Delta \sigma \) is 5 times larger than that for the \( \omega^2 \)-model. It has been shown that the stress drop can be considerably larger for a complex rupture (e.g., Rudnicki & Kanamori 1981; Das 1988), although exactly how large depends on the rupture characteristics. Thus, this result is qualitatively consistent with larger \( \Delta \sigma \) values for events with a larger \( \text{REEF} \).

5. Conclusion

We systematically estimated the radiated energy, \( E_R \), and the source complexity of \( M_w 3–7 \) earthquakes in the island crust of Japan using a time-domain analysis method. We first created a large database of apparent moment-rate function (AMRF) for 1,736 events using high-quality seismic network data from Japan. We estimated the \( E_R \) and the source duration, \( T \) from the AMRFs and used them to quantify the source complexity using the radiated energy enhancement factor \( \text{REEF} \); Ye et al. 2018). The \( e_R \) ranged from \( 5 \times 10^{-6} \) to \( 4 \times 10^{-4} \) with a median of \( 3.7 \times 10^{-5} \), and \( \text{REEF} \) ranged from 1 to 200 with a median of 3.8. The \( e_R \) and \( \text{REEF} \) did not strongly depend on the seismic moment for \( M_w 3–7 \); moreover, \( e_R \) and \( \text{REEF} \) did not depend on the focal mechanism or the event depth. Therefore, the ruptures of small earthquakes have a similar degree of complexity to those of large earthquakes at their representative spatial scales.

Approximately one third of the total events had complex AMRFs that were distinct from a single-pulse AMRF. For simple events, the time-domain and the frequency-domain methods yielded similar source parameter results. However, the spectral shape of complex events deviated from the \( \omega^2 \)-model; thus, the significance of the corner frequency was ambiguous. In our time-
domain method, the use of $T$, determined with empirical Green’s functions, allows for
determination of a more detailed energy release pattern. Moreover, through the use of the
complexity parameter, $REEF$, we can systemically present the scaling relation between $e_R$ and $T$
in a systematic way.

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References


NIED. (2019a) NIED Hi-net, National Research Institute for Earth Science and Disaster Resilience. doi:10.17598/NIED.0003

NIED. (2019b) NIED F-net, National Research Institute for Earth Science and Disaster Resilience. doi:10.17598/NIED.0005
NIED. (2019c) NIED V-net, National Research Institute for Earth Science and Disaster Resilience. doi:10.17598/NIED.0006

NIED. (2019d) NIED K-NET, KiK-net, National Research Institute for Earth Science and Disaster Resilience. doi:10.17598/NIED.0004


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Appendix A- Frequently used symbols and relationships

Symbols

\( M_0 \) seismic moment

\( E_R \) radiated energy

\( e_R \) scaled energy

\( \rho, \alpha, \beta, \mu \) medium parameters. Density, P-wave velocity, S-wave velocity, shear modulus.

\( f_c \) corner frequency of the \( \omega^2 \)-model

\( T \) duration of the moment rate function

\[
T = \frac{2}{\pi f_c} \text{ (triangle)}, \quad T = \frac{5}{2\pi f_c} \text{ (truncated } t \exp(-t/2\pi f_c) \text{ )}
\]

\( REE(R) \) radiated energy enhancement factor

\( \Delta \sigma \) static stress drop, stress parameter

\( \eta_R \) radiation efficiency

\( a \) radius of a circular fault

\( c \) constant relating \( f_c \) to the radius of a circular fault

Relationships

(A1) \[
\frac{e_R}{M_0} = \frac{\pi^2}{5} \left( \frac{M_0 f_c^3}{\rho \beta^3} \right) \quad \text{for the } \omega^2 \text{-model (P-wave energy ignored)}
\]

(A2) \[
T = \frac{2}{\pi f_c} \text{ (for an isosceles triangle)}, \quad T = \frac{5}{2\pi f_c} \text{ (for the truncated } t \exp(-t/2\pi f_c) \text{ )}
\]

(A3) \[
\frac{e_R}{M_0} = \frac{25}{8\pi} \left( \frac{M_0}{\mu \beta^3 T^3} \right) \quad \text{similar to (A1) but with } T \text{ instead of } f_c \text{ (for the truncated exponential MRF)}
\]
(A4) \( \text{REEF} = \frac{5\pi}{6} \left( \frac{E_g}{M_0} \right) \left( \frac{\mu \beta^3 T^3}{M_0} \right) \) definition of \( \text{REEF} \)

(A5) \( M_0 = \frac{16}{7} a^3 \Delta \sigma \) for a circular fault, static relation

(A6) \( f_c = \frac{c \beta}{2\pi a} \)

Brune’s (1970) heuristic relation partially based on energy conservation \( (c=2.34) \).

(A7) \( f_c = \frac{c}{2\pi} \beta \left( \frac{16}{7} \frac{\Delta \sigma}{M_0} \right)^{1/3} \) from (A5) and (A6)

(A8) \( \Delta \sigma = \frac{7\pi^3}{2c^3} \mu \left( \frac{M_0 f_c^3}{\mu \beta^3} \right) \) relation for the \( \omega^2 \) model with the Brune’s constant \( c \)

(A8’) \( \Delta \sigma = \frac{875}{16c^3} \mu \left( \frac{M_0}{\mu \beta^3 T^3} \right) \) same as above for a truncated exponential MRF

Data availability

This study used hypocentres and S-wave arrival time data reported in the JMA unified catalogue.

The seismograms were collected and stored by the JMA, national universities, and NIED

(http://www.hinet.bosai.go.jp/?LANG=en).