Apart from the special quantum-mechanical results based on the knowledge of the function $\psi(q, p)$ of Eq. (5), all the general conceptions and theorems of quantum theory, those concerning discrete sets of mutually orthogonal states with fractional likeness between states in general, the probability interpretation of the likeness fractions q, the splitting effect with conservation of statistical weight, i.e., quantum statistics, and last but not least, the mathematical, i.e., logical, necessity of introducing probability amplitudes ψ linked together by the matrix multiplication law (4)—all these results are immediate consequences of the postulate of thermodynamic continuity: "the Gibbs paradox does not occur." One may speculate on what would have been the development of physics if Gibbs in the 1890's had consistently pursued his objection to the discontinuity inherent in the classical theory of diffusion.

An Absolute Ampere Current Balance for Laboratory Use

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The ampere current balance herein described has been in use for four years as a regular experiment of a sophomore physics course. It has the following merits: (1) It is simple to construct; (2) it is absolute in that the current may be calculated from the dimensions and balancing weight only; (3) it is subject to a high degree of accuracy, especially for the larger currents; (4) it may be used for alternating as well as direct currents; (5) when current is passed through the movable conductor only, the balance may be used to measure the horizontal component of the earth's magnetic field.

1. INTRODUCTION

THE ampere is now defined in terms of the force between conductors.[1] This being so, it becomes necessary to construct balances having current-carrying conductors whose dimensions are such that accurate calculations may be made so that the values of the currents may be computed in terms of these dimensions and the balancing weights. Such balances exist, for example, at the Bureau of Standards in Washington.[2]

Elaborate pieces of equipment such as these are of course not suited for ordinary laboratory use, but none the less it is highly desirable to be able to make such measurements because of the importance of the principles involved. Ampere balances suitable for measuring *relative* currents are in common use. Described below is an *absolute* balance of simple construction that is used in a sophomore physics laboratory.

[1] F. B. Silsbee, "Establishment and Maintenance of the Electrical Units," Natl. Bur. Standards Circular 475 (1949).

[2] Curtis, Driscoll, and Critchfield, J. Research Natl. Bur. Standards **28**, 133 (1942), RP 1449.

2. GENERAL DESCRIPTION

For students of the sophomore level to comprehend properly, it is desirable to be able to use the simple equation derived from the law of Biot and Savart, $F/l=\mu_0 i_1 i_2/2\pi a$, (for the force F between parallel conductors of lengths l, carrying currents i_1, i_2 when separated by a distance a in free space of permeability μ_0), as the basic equation in calculating the current. In order that this should strictly apply, the radius of curvature, for example, of any conductors must be large compared with the distances between conductors. The simplest geometrical shape which at the same time can be made sensitive to small torques, is a uniform circular conductor which is able to rotate about a diameter. Another concentric conductor with the same radius of curvature, whose plane is parallel to but displaced a small distance from this one, will produce a net torque provided current flows parallel in the two adjacent conductors around one-half the loop and antiparallel around the other half.

Aside from the difficulties of making currents flow like this, a further practical disadvantage comes in not being able to measure conveniently the distance between centers of the conductors. These two difficulties are solved by placing at an equal distance on the other side of the movable conductor another concentric split ring as shown in Fig. 1, and so connected that the currents flow in the proper directions. The torques now add up between the stationary conductors and all parts of the pivoted, central conductor and the same current, in magnitude, flows in each.

The important spacing now becomes that between the two outer conductors. Consider the three parallel conductors with currents flowing, as shown in Fig. 2, giving a force on the central conductor as indicated. Let the center conductor be displaced an amount Δa from its central position. As the movable conductor comes closer to the upper one, the force will increase, but there will be a decrease in force due to the current in the lower conductor. The force per unit length on the center one will be

$$\frac{F}{l}=\frac{\mu_0 i^2}{2\pi}\left[\frac{1}{a+\Delta a}+\frac{1}{a-\Delta a}\right]$$

$$=\frac{\mu_0 i^2}{2\pi a}\left[1-\frac{\Delta a}{a}+\left(\frac{\Delta a}{a}\right)^2 - \cdots +1+\frac{\Delta a}{a}+\left(\frac{\Delta a}{a}\right)^2+\cdots\right];$$

and if $\Delta a \ll a$,

$$\frac{F}{l}=\frac{\mu_0 i^2}{\pi a}\left[1+\left(\frac{\Delta a}{a}\right)^2\right] \text{ to second-order terms.} \quad (1)$$

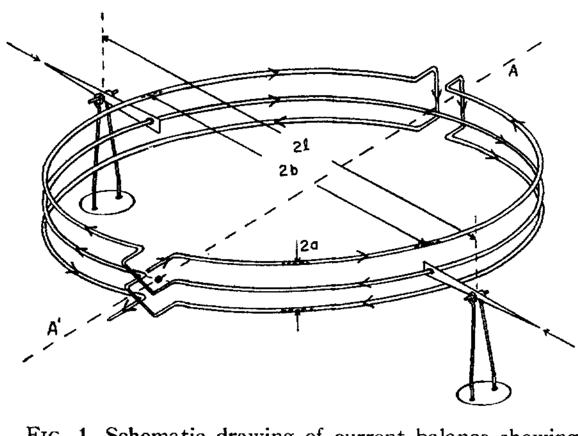

FIG. 1. Schematic drawing of current balance showing one solution of connections for currents to give torques on movable, central conductor, in the same direction.

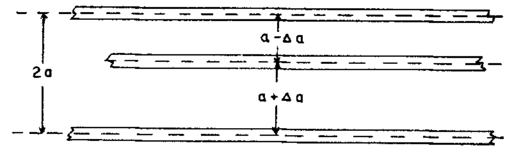

FIG. 2. The force on the middle movable conductor is to be calculated. The movable conductor lies between two fixed conductors a distance $2a$ apart. Equation (1) shows that the position of the middle conductor is unimportant unless Δa is greater than a few percent of a.

Thus, the position of the center conductor is unimportant until Δa becomes of the order of 5 or 10 percent of a. The important spacing is the distance between centers of the fixed, outer conductors and this distance can be measured accurately with micrometer calipers.

3. DETAILED DESCRIPTION

a. The Fixed and Movable Conductors

In the instrument as constructed, the diameter of all the circular conductors is 50.0 cm, and they are made of $\frac{3}{16}$-inch copper tubing. The distance between outer conductors is approximately 2.7 cm, or just slightly too great for the usual one-inch micrometer caliper to fit over. The distance between the outer conductors is accurately determined by eight Lucite spacers, as shown in Figs. 3 and 4. These are all machined while clamped together. The micrometer calipers do fit across the parallel faces on the Lucite spacers, and these are much less easily distorted by the calipers than are the circular conductors. The distance between centers of the outer conductors is then determined by adding to this dimension the radius of each conductor. Connections to the four semicircular stationary loops are brought out to large binding posts mounted on two Bakelite panels. These terminals, including those from the movable conductor, are equally spaced so that slotted strap conductors may be used to provide the proper connections as shown in Fig. 3. An important part of the experiment for the student is to work out the proper arrangement of these straps at both terminal panels so that the torques are all in the same direction on the movable conductor.

The movable conductor is made in two halves with a portion of a single-edge razor blade serving as the knife-edge at each end of a diameter where the two halves join. The details of that junction, where the current is brought in and goes out, are shown in Fig. 4. The knife-edge itself is just slightly above the center of the conductors forming the central loop. The knife-edge, necessary mica insulation, and conductors that lead to shallow mercury cups, are all clamped together by the four small bolts seen in the middle of Fig. 4.

The knife-edges rest in V-shaped grooves. That one at the junction, where the current enters and leaves, is midway between the two uprights that contain the mercury contacts. The line joining the knife-edges must be slightly above the center of mass of the movable system if the system is to be in stable equilibrium. It has been found that when the period of oscillation is about five seconds, the disturbances caused by the usual air currents in the laboratory are not noticeable.

The conductors that lead to and from the ends of the movable conductor to the mercury cups are made of copper and are heavily nickel plated. Their lower ends that dip into the mercury are in the same straight line as the knife-edges. This alignment tends to minimize any torques due to the mercury surface.

The mercury is contained in shallow cups made by bending thin nickel sheet over a rectangular form, thus making folded corners. The cups are soldered into the ends of brass uprights. The conductors that dip into the mercury reach within about one millimeter of the bottom of the cups, thus keeping the path of the current through the mercury to a minimum. All metal parts are heavily nickel plated before final assembly.

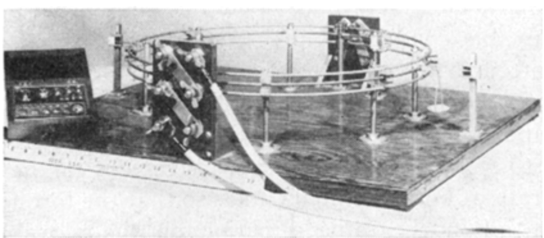

FIG. 3. Slotted, strap connectors are provided for each terminal board. The student is expected to find a combination of connections that will give a resultant torque on the central conductor, due to each half-loop, in the same direction.

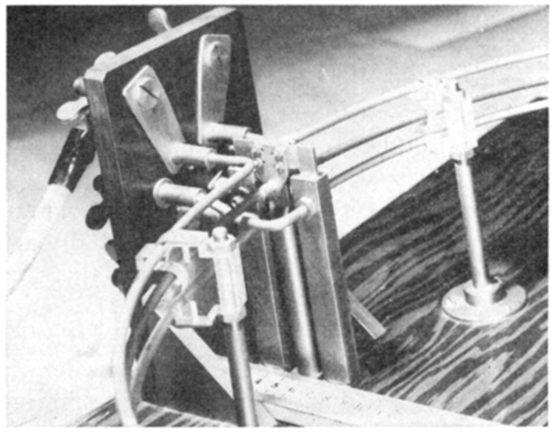

FIG. 4. Enlarged view of the rear of the main terminal board.

The above construction has proven very satisfactory. No frictional effects in the mercury contacts have been noticed; also very little heating occurs even when currents of 50 amperes are carried for some time. Further, no apparent amalgamation of the mercury with other metal parts has taken place in spite of the fact that the mercury has been in the cups continuously for four years.

b. Weight Pans and Pointers

To provide a proper torque to balance the torque due to the current, two identical weight pans and pointers with mirrors and scales are provided as shown in Figs. 1 and 3. The weight pans are supported by hardened steel triangular knife-edges soldered into the pointers.

The dual pointers allow two observers to watch simultaneously. The two weight pans are necessary to find the average lever arm. Thus, in practice, a mass is placed in one pan, and the current is increased until a balance is obtained. Then the current in the movable loop is reversed, the mass is changed to the other pan and a new balancing current is read. Then if (l_1+l_2) is the distance between knife-edges, $\frac{1}{2}(l_1+l_2)=l$ will be the average lever arm; and if $l_1\approx l_2$, the true current i can be shown to be the average of the currents i_1 and i_2. For the two pieces of apparatus constructed, the two lever arms are so nearly the same that the two currents are the same to within less than one part in 500. (See examples below.)

c. Auxiliary Apparatus

The experiment, as performed, regards the ampere balance as a standard and an ammeter is calibrated in terms of the values calculated from measurements with the balance. A 0–50 ammeter is quite satisfactory. In addition, a resistor, such as a carbon pile that has a low resistance and gives a smooth, continuous adjustment, is desirable. As a source of current, a 2-volt lead storage cell is quite sufficient. For weights, those used with the usual analytical balance are suitable. It is also necessary to provide connectors that will carry the necessary currents without having too large iR drops.

4. CALCULATIONS

To calculate the torque on the movable conductor, use is made of the law of Biot and Savart, combined with the expression for the force per unit length on a conductor carrying a current in a magnetic field. Using the rationalized mks system of units, the torque on the movable conductor about the diameter AA', owing to the stationary conductors when all the currents are numerically the same and the torques are all in the same direction, is easily shown to be

$$L=4\mu_0 i^2 b^2/\pi a, \tag{2}$$

where μ_0 has the value $4\pi\times10^{-7}$ webers per ampere-meter, i is the common current, b the common radius of all loops, and $2a$ the distance between the centers of the outer loops.

When this torque is balanced by the torque due to the added weights mgl, the current will then be given by

$$i=\left(\frac{\pi}{4}\frac{mgla}{\mu_0 b^2}\right)^{\frac{1}{2}}=km^{\frac{1}{2}}. \tag{3}$$

The above calculation using the law of Biot and Savart is based on the assumption that the conductors are straight and very long compared with the distances under consideration. For a more accurate calculation, one may refer to Smythe's book.[3] For two concentric circular loops of the same radius b, distance between planes a, each carrying a current i, the mutual force is

$$F=\frac{\mu_0 i^2 a}{(4b^2+a^2)^{\frac{1}{2}}}\left[-K+\frac{(2b^2+a^2)E}{a^2}\right],$$

where K and E are complete elliptic integrals of the first and second kind, respectively, with modulus $k^2=4b^2/(4b^2+a^2)$. Using the values $b=50$ cm and $a=1.45$ cm, the force thus calculated is one-half percent less than that calculated by using the simple law of Biot and Savart.

It should be emphasized that both calculations neglect the fact that in the actual apparatus the loops are not complete, and that there are other conductors going to the terminal boards. There are apparently compensating effects, for the actual current compared with that calculated from the simple formula is well within the error that might be expected due to the curvature of the conductors as will be seen in the examples cited below.

5. PROCEDURES AND EXAMPLES

The student is given the balance without the connecting straps on the terminal boards being in place. From his knowledge of the direction of forces on conductors in magnetic fields he is then expected to arrive at solutions which give (1) the same absolute value of current in all conductors, (2) all the torques on the movable conductor in the same direction, and (3) the reverse direction of torque on the movable conductor. This last is necessary to find the average length of lever arm. One solution is that shown in Fig. 2. The torque can be reversed by changing the positions of the connecting straps on only one terminal panel.

In use, the axis of rotation of the movable loop is pointed magnetic north and south to avoid the torque due to the earth's magnetic field. The diameter of the loops is measured at several places by placing a meter stick on its edge on the top stationary loop and noting the points of contact with the $\frac{3}{16}$-inch conductors. This distance can thus be measured to ±0.2 mm. The distance between knife-edges that hold the weight pans is also determined when the meter stick is on edge across the top stationary loop. The usual meter stick has mm markings along the two sides, thus allowing the observer to sight along similar markings to the knife edges of the weight pans. This distance may thus be measured to ±0.2 mm.

[3] Smythe, *Static and Dynamic Electricity* (McGraw-Hill Book Company, Inc., New York, 1939), p. 277.

Examples as a Current Measuring Instrument

The current balance was compared against a Weston Model 5, standard ammeter with a 12.5-inch scale. This instrument had been recently calibrated with a potentiometer and standard cell. Constants of the ampere balance:

twice the lever arm $=2l=0.5559\pm0.0002$m,
twice the radius of the loops $=2b=0.4994\pm0.0002$m,
distance between centers of outer conductors $=2a=0.02898\pm0.00002$m.

The constant k, given by Eq. (3), is then

$$k=\left(\frac{\pi agl}{4\mu_0 b^2}\right)^{\frac{1}{2}}=628.8\pm0.4 \text{ amp kg}^{-\frac{1}{2}},$$

if m is in kg, or

$$k=19.885\pm0.012 \text{ amp g}^{-\frac{1}{2}},$$

if m is in grams.

	Trial No. 1	Trial No. 2
Mass in pans (g)	4.000	9.000
Reading of st'd ammeter (amp)	39.725	59.595
Reading of st'd ammeter with current reversed (amp)	39.725	59.695
Average current (amp)	39.725	59.645
Current computed from constant of current balance, $I=km^{\frac{1}{2}}$ (amp)	39.770	59.655
Percent difference	0.14	0.016

At currents from 40 to 60 amperes, the period of oscillation noticeably increases. At the larger value, instability has nearly set in, resulting in a large displacement of the movable conductor for changes of current that are not noticeable on the ammeter.

The current balance is accurate to about ±1 percent for currents of 8 to 10 amperes, while below 2 amperes the accuracy is not better than ±5 percent due to the small torques involved. (A balancing weight of 10 mg corresponds to 2 amperes.) For currents from 20 to 60 amperes the balance is accurate to ±0.2 percent.

The instrument may also be used to measure the horizontal component of the earth's magnetic field. When so used, the axis of rotation of the movable conductor is pointed magnetic east and west and current is sent through it only. Stray magnetic fields must now be guarded against. The following procedures have been found sufficient to eliminate the effects of stray fields due to currents flowing in connecting conductors: (1) use either a concentric line or a twisted pair to the movable conductor; (2) reverse the current and take the average value. With these precautions an extended magnetic field of 0.25 gauss may be measured to ±2 percent.

SUMMARY

The current balance described above has proven to be most satisfactory as a precision instrument for measuring currents between 15 and 60 amperes. After four years' use by the sophomore class, the two instruments have required no adjustments, and their present constants are very close to their original values. No precision shopwork is required on any of the parts in their fabrication, except on the Lucite spacers. These should be made with some care to insure uniformity and to provide parallel faces for micrometer measurements.

Pedagogically the instrument has been found useful in helping the student (1) apply his knowledge of forces on current-carrying conductors in magnetic fields, (2) gain a feeling for the magnitude of the forces thus involved, (3) test his ability to make accurate measurements.

In conclusion the author wishes to thank Mr. T. Harvey for taking the photographs which are reproduced in Figs. 3 and 4.

Among the admirable American expressions which enter our language from time to time to lend it new vitality, there is one which I would have every audience, and every lecturer, use at the end of every lecture or course: So what? *What does all this amount to?*—T. R. Henn, The Apple and The Spectroscope (Methuen, 1951) by permission.