

Semileptonic Λ_b decay to excited Λ_c baryons at order Λ_{QCD}/m_Q

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Exclusive semileptonic Λ_b decays to excited charmed Λ_c baryons are investigated at order Λ_{QCD}/m_Q in the heavy quark effective theory. The differential decay rates are analyzed for the $J^\pi=1/2^-$ $\Lambda_c(2593)$ and the $J^\pi=3/2^-$ $\Lambda_c(2625)$. They receive $1/m_{c,b}$ corrections at zero recoil that are determined by mass splittings and the leading order Isgur-Wise function. With some assumptions, we find that the branching fraction for Λ_b decays to these states is 2.5–3.3%. The decay rate to the helicity $\pm 3/2$ states, which vanishes for $m_Q \rightarrow \infty$, remains small at order Λ_{QCD}/m_Q since $1/m_c$ corrections do not contribute. Matrix elements of weak currents between a Λ_b and other excited Λ_c states are analyzed at zero-recoil to order Λ_{QCD}/m_Q . Applications to baryonic heavy quark sum rules are explored.

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I. INTRODUCTION

The use of heavy quark symmetry [1] resulted in a dramatic improvement in our understanding of exclusive semileptonic decays of hadrons containing a single heavy quark. In the infinite mass limit, the spin and parity of the heavy quark Q and the strongly interacting light degrees of freedom are separately conserved, and can be used to classify the particle spectrum. Light degrees of freedom with spin-parity $s_l^{\pi_l}$ yield a doublet with total angular momentum $J=s_l \pm \frac{1}{2}$ and parity $P=\pi_l$ (or a singlet if $s_l=0$). This classification can be applied to the Λ_Q baryons where $Q=c,b$. For the charmed baryons some of the spin multiplets are summarized in Table I, with masses given for the observed particles [2]. Here $\Lambda_c^{1/2}$ and $\Lambda_c^{3/2}$ are the observed $\Lambda_c(2593)$ and $\Lambda_c(2625)$ with total spin 1/2 and 3/2 respectively.

For $m_Q \rightarrow \infty$ the semileptonic decay of a Λ_b into either Λ_c in a heavy doublet are described by one universal form factor, the leading order Isgur-Wise function [3]. This function will vanish identically if the parity of the final state doublet is unnatural [4–6]. A semileptonic baryonic transition is unnatural if $(\Delta\pi_l)(-1)^{\Delta s_l} = -1$, where Δs_l is the change in the spin of the light degrees of freedom, and $\Delta\pi_l = -1$ if the sign of π_l changes, and $+1$ if it does not. This rule follows from parity considerations along with the fact that for $m_Q \rightarrow \infty$ the angular momentum of the light degrees of freedom

along the decay axis is conserved [4]. For natural decays the hadronic matrix elements do not vanish identically as $m_Q \rightarrow \infty$, and at zero recoil these matrix elements have a value which is fixed by heavy quark symmetry. For initial and final state doublets with the same light degrees of freedom this determines the normalization of the leading order Isgur-Wise function. If the light degrees of freedom for the two states differ, then the matrix elements vanish at zero recoil, and the normalization of the leading order Isgur-Wise function is not determined.

In general for Λ_b decays, these infinite mass limit predictions are corrected at order Λ_{QCD}/m_Q . An unnatural transition can have a non-zero decay rate at this order. For the natural transition to the ground state Λ_c ($s_l^{\pi_l}=0^+$), the Λ_{QCD}/m_Q corrections vanish at zero recoil [7]. However, for a natural transition to an excited Λ_c the zero recoil hadronic matrix elements need not be zero at this order. These corrections can substantially effect the decay rate into excited states since they dominate at zero recoil and the available phase space is quite small. In the heavy quark effective theory (HQET), it is useful to write form factors as functions of $w=v \cdot v'$, where v is the four-velocity of the Λ_b baryon and v' is that of the recoiling charmed baryon. Zero recoil then corresponds to $v=v'$, where $w=1$.

For a spin symmetry doublet of hadrons H_\pm with total spin $J_\pm = s_l \pm \frac{1}{2}$ the HQET mass formula is

$$m_{H_\pm} = m_Q + \bar{\Lambda}^H - \frac{\lambda_1^H}{2m_Q} \pm \frac{n_\pm \lambda_2^H}{2m_Q} + \mathcal{O}(1/m_Q^2). \quad (1.1)$$

Here $n_\pm = 2J_\pm + 1$ is the number of spin states in the hadron H_\pm and $\bar{\Lambda}^H$ denotes the energy of the light degrees of freedom in the $m_Q \rightarrow \infty$ limit. $\lambda_{1,2}$ are the usual kinetic and chromomagnetic matrix elements

TABLE I. Isospin zero charmed baryon spin multiplets with $s_l^{\pi_l} < 2$. Masses are given for the observed particles [2].

| $s_l^{\pi_l}$ | Particles | J^π | $m(\text{GeV})$ |
|---------------|--------------------------------------|--------------------------------|-----------------|
| 0^+ | Λ_c | $\frac{1}{2}^+$ | 2.284 |
| 1^- | $\Lambda_c^{1/2}, \Lambda_c^{3/2}$ | $\frac{1}{2}^-, \frac{3}{2}^-$ | 2.594, 2.627 |
| 0^- | Λ_c^* | $\frac{1}{2}^-$ | |
| 1^+ | $\Lambda_c^{1/2*}, \Lambda_c^{3/2*}$ | $\frac{1}{2}^+, \frac{3}{2}^+$ | |

$$\lambda_1^H = \frac{1}{2 m_{H_\pm}} \langle H_\pm(v) | \bar{h}_v^{(Q)} (iD)^2 h_v^{(Q)} | H_\pm(v) \rangle,$$

$$\lambda_2^H = \frac{\mp 1}{2 m_{H_\pm} n_\mp} \langle H_\pm(v) | \bar{h}_v^{(Q)} \frac{g_s}{2} \sigma_{\alpha\beta} G^{\alpha\beta} h_v^{(Q)} | H_\pm(v) \rangle,$$
(1.2)

written in terms of $h_v^{(Q)}$, the heavy quark field in HQET, using a relativistic normalization for the states, $\langle H(p') | H(p) \rangle = (2\pi)^3 2E_H \delta^3(\vec{p}' - \vec{p})$.

The excited charmed baryons $\Lambda_c^{1/2}$ and $\Lambda_c^{3/2}$, which belong to the doublet with $s_l^{\pi_l} = 1^-$, have been observed. We will use $\bar{\Lambda}$ for the ground state Λ_Q , and $\bar{\Lambda}'$ for the $s_l^{\pi_l} = 1^-$ doublet.¹ For semileptonic Λ_b decays to excited Λ_c 's the members of the charmed $s_l^{\pi_l} = 1^-$ doublet are special. At zero recoil and order Λ_{QCD}/m_Q their hadronic matrix elements are determined by the leading order Isgur-Wise function and the difference $\bar{\Lambda}' - \bar{\Lambda}$ (as will be seen explicitly in Sec. II). This is analogous to the case of semileptonic B decays to excited charmed mesons with $s_l^{\pi_l} = 1/2^+, 3/2^+$ [8,9].

The difference $\bar{\Lambda}' - \bar{\Lambda}$ can be expressed in terms of measurable baryon masses. From Eq. (1.1) λ_2 can be eliminated by taking the helicity weighted average mass for the doublet

$$\bar{m}_H = \frac{n_- m_{H_-} + n_+ m_{H_+}}{n_+ + n_-}. \quad (1.3)$$

If \bar{m}_H is known in both the b and c sectors then $\bar{\Lambda}_H$ can be calculated in terms of $m_{c,b}$ by eliminating λ_1^H . With $m_{\Lambda_b} = 5.623$ GeV [2], $m_{\Lambda_c} = 2.284$ GeV [2], and $m_b - m_c = 3.4$ GeV [10], taking $m_c = 1.4$ GeV gives $\bar{\Lambda} = 0.8$ GeV. While this value of $\bar{\Lambda}$ depends sensitively on the value of m_c , the difference

$$\bar{\Lambda}' - \bar{\Lambda} = \frac{m_b(\bar{m}'_{\Lambda_b} - m_{\Lambda_b}) - m_c(\bar{m}'_{\Lambda_c} - m_{\Lambda_c})}{m_b - m_c} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^3}{m_Q^2}\right), \quad (1.4)$$

is less sensitive to m_c . Baryons with $s_l^{\pi_l} = 1^-$ in the bottom sector have not yet been observed, so the mass splitting $\Delta m_{\Lambda_b} = \bar{m}'_{\Lambda_b} - m_{\Lambda_b}$ is not known. In the limit $N_c \rightarrow \infty$ this mass splitting is predicted to be $\Delta m_{\Lambda_b} = 0.29$ GeV, as shown in the Appendix. We will see that sum rules imply that $\Delta m_{\Lambda_b} < 0.24$ GeV (for $m_c = 1.4$ GeV). Taking $\Delta m_{\Lambda_b} \approx 0.24$ gives $\bar{\Lambda}' - \bar{\Lambda} \approx 0.20$ GeV as a rough estimate. Since $\bar{\Lambda}'/(2m_c) \approx 0.36$ the Λ_{QCD}/m_Q corrections may be large and the effective theory might not be a good description for these excited states. However, near zero recoil only the dif-

ference, $\bar{\Lambda}' - \bar{\Lambda}$, occurs and furthermore some form factors do not receive Λ_{QCD}/m_c corrections.

In this paper decays of Λ_b to excited Λ_c 's are investigated to order Λ_{QCD}/m_Q in the heavy quark effective theory.² In Sec. II we examine the differential decay rates for $\Lambda_b \rightarrow \Lambda_c^{1/2} e \bar{\nu}_e$ and $\Lambda_b \rightarrow \Lambda_c^{3/2} e \bar{\nu}_e$ to order Λ_{QCD}/m_Q . There is large model dependence away from zero recoil due to unknown Λ_{QCD}/m_Q corrections, but there is less uncertainty when the rates to these two states are combined. Note that when baryonic decays are considered in the limit $N_c \rightarrow \infty$ it is possible to predict the leading order Isgur-Wise function [11,12] as well as some of the sub-dominant Isgur-Wise functions. The large N_c results which are relevant for the decays considered in Sec. II are summarized in the Appendix. In Sec. III the Λ_{QCD}/m_Q corrections to zero recoil matrix elements for weak currents between a Λ_b state and all other excited Λ_c states are investigated. The effect of these excited states on baryonic heavy quark sum rules is discussed in Sec. IV. In Sec. V we summarize our results. This extends the analysis of semileptonic B decay into excited charmed mesons in Refs. [8,9] to the analogous baryonic decays.

II. DECAY RATES FOR $\Lambda_b \rightarrow \Lambda_c^{1/2} e \bar{\nu}_e$ AND $\Lambda_b \rightarrow \Lambda_c^{3/2} e \bar{\nu}_e$

The matrix elements of the vector and axial currents ($V^\mu = \bar{c} \gamma^\mu b$ and $A^\mu = \bar{c} \gamma^\mu \gamma_5 b$) between the Λ_b and $\Lambda_c^{1/2}$ or $\Lambda_c^{3/2}$ baryon states can be parametrized as

$$\frac{\langle \Lambda_c^{1/2}(v', s') | V^\mu | \Lambda_b(v, s) \rangle}{\sqrt{4m_{\Lambda_c^{1/2}} m_{\Lambda_b}}} = \bar{u}(v', s') [d_{V_1} \gamma^\mu + d_{V_2} v^\mu + d_{V_3} v'^\mu] \gamma_5 u(v, s),$$

$$\frac{\langle \Lambda_c^{1/2}(v', s') | A^\mu | \Lambda_b(v, s) \rangle}{\sqrt{4m_{\Lambda_c^{1/2}} m_{\Lambda_b}}} = \bar{u}(v', s') [d_{A_1} \gamma^\mu + d_{A_2} v^\mu + d_{A_3} v'^\mu] u(v, s), \quad (2.1)$$

$$\frac{\langle \Lambda_c^{3/2}(v', s') | V^\mu | \Lambda_b(v, s) \rangle}{\sqrt{4m_{\Lambda_c^{3/2}} m_{\Lambda_b}}} = \bar{u}_\alpha(v', s') [v^\alpha (l_{V_1} \gamma^\mu + l_{V_2} v^\mu + l_{V_3} v'^\mu) + l_{V_4} g^{\alpha\mu}] u(v, s),$$

$$\frac{\langle \Lambda_c^{3/2}(v', s') | A^\mu | \Lambda_b(v, s) \rangle}{\sqrt{4m_{\Lambda_c^{3/2}} m_{\Lambda_b}}} = \bar{u}_\alpha(v', s') [v^\alpha (l_{A_1} \gamma^\mu + l_{A_2} v^\mu + l_{A_3} v'^\mu) + l_{A_4} g^{\alpha\mu}] \gamma_5 u(v, s), \quad (2.2)$$

¹The notation $\bar{\Lambda}$ is commonly used in the mass formula for the mesons $B^{(*)}$ and $D^{(*)}$, however in this paper $\bar{\Lambda}$ will be used exclusively for the baryons.

²Corrections of order Λ_{QCD}/m_c were previously considered in [6]. We disagree with the statement made there that the Λ_{QCD}/m_c current and chromomagnetic corrections to the matrix elements vanish at the zero recoil point for decays to all but the ground state Λ_c .

where s and s' are for spin, and d_i and l_i are dimensionless functions of w . The spinor $u(v, s)$ and Rarita-Swinger spinor $u_\alpha(v', s')$ are normalized so that $\bar{u}(v, s)u(v, s) = 1$ and $\bar{u}_\alpha(v', s')u^\alpha(v', s') = -1$, and satisfy $\not{v}u = u$, $\not{v}'u_\alpha = u_\alpha$, $v'_\alpha u^\alpha = 0$, and $\gamma_\alpha u^\alpha = 0$. At zero recoil ($v = v'$) these properties, along with $\bar{u}_\alpha \gamma_5 u = \bar{u} \gamma_5 u = 0$, imply that only d_{V_1} , $d_{A_1} + d_{A_2} + d_{A_3}$, and l_{V_4} can contribute to the matrix elements in Eqs. (2.1) and (2.2).

In the infinite mass limit decays to excited Λ_c 's with helicity $\lambda = \pm 3/2$ are forbidden by heavy quark spin symmetry since the light helicity, λ_l , is conserved in the transition [4]. For the Λ_b , $s_l^{\pi_l} = 0^+$ so $\lambda_l = 0$, and the final state excited Λ_c can only have $\lambda = \pm 1/2$. It is useful to consider separately decay rates to the different helicities to see what effect corrections of order Λ_{QCD}/m_Q have on this infinite mass limit prediction. For a massive particle with 4-velocity v the polarization sums over individual helicity levels can be done by introducing an auxiliary four vector $n_\alpha(v)$ such that $n \cdot v = 0$ and $n^2 = -1$. For the spin 3/2 Rarita-Swinger spinors $u^\mu(s)$ the spin sums are then³

$$\begin{aligned} \sum_{|s|=1/2} u^\alpha(v, s) \bar{u}^\beta(v, s) &= \frac{(1 + \not{v})}{12} [-g^{\alpha\beta} + v^\alpha v^\beta + 3n^\alpha n^\beta \\ &\quad - i\gamma_5 \epsilon^{\alpha\beta\sigma\tau} v_\sigma (2\gamma_\tau + 3n_\tau \not{h})], \\ \sum_{|s|=3/2} u^\alpha(v, s) \bar{u}^\beta(v, s) &= \frac{(1 + \not{v})}{4} [-g^{\alpha\beta} + v^\alpha v^\beta - n^\alpha n^\beta \\ &\quad + i\gamma_5 \epsilon^{\alpha\beta\sigma\tau} v_\sigma n_\tau \not{h}], \end{aligned} \quad (2.3)$$

where $\epsilon^{0123} = 1$. In the rest frame of the Λ_b the auxiliary vector $n(v') = (|\vec{v}'|, v'_0 \hat{v}') = (\sqrt{w^2 - 1}, w \hat{v}')$, where $\hat{v}' = \vec{v}'/|\vec{v}'|$.

The differential decay rates are expressible in terms of the form factors in Eqs. (2.1) and (2.2), and the kinematic variables $w = v \cdot v'$ and θ . Here θ is the angle between the charged lepton and the charmed baryon in the rest frame of the virtual W boson, i.e., in the center of momentum frame of the lepton pair. For $\Lambda_b \rightarrow \Lambda_c^{1/2} l \bar{\nu}$ the differential decay rate is

$$\begin{aligned} \frac{d^2 \Gamma_{\Lambda_{1/2}}}{dw d \cos \theta} &= 6\Gamma_0 r_1^3 \sqrt{w^2 - 1} (\sin^2 \theta \{ (w+1)[(r_1 - 1)d_{V_1} + (w-1)(d_{V_3} + r_1 d_{V_2})]^2 + (w-1)[(r_1 + 1)d_{A_1} \\ &\quad + (w+1)(d_{A_3} + r_1 d_{A_2})]^2 \} + (1 - 2r_1 w + r_1^2) \{ (1 + \cos^2 \theta)[(w-1)d_{A_1}^2 + (w+1)d_{V_1}^2] - 4 \cos \theta \sqrt{w^2 - 1} d_{A_1} d_{V_1} \}), \end{aligned} \quad (2.4)$$

while for $\Lambda_b \rightarrow \Lambda_c^{3/2} l \bar{\nu}$ the rates are

$$\begin{aligned} \frac{d^2 \Gamma_{\Lambda_{3/2}}^{(|\lambda|=1/2)}}{dw d \cos \theta} &= \Gamma_0 r_3^3 \sqrt{w^2 - 1} [(-4 \cos \theta \sqrt{w^2 - 1} [l_{A_4} - 2(w+1)l_{A_1}][l_{V_4} - 2(w-1)l_{V_1}] + (1 + \cos^2 \theta) \{ (w-1)[l_{A_4} - 2(w+1)l_{A_1}]^2 \\ &\quad + (w+1)[l_{V_4} - 2(w-1)l_{V_1}]^2 \} (1 - 2r_3 w + r_3^2) + 4 \sin^2 \theta \{ (w+1)[(w-1)(r_3 + 1)l_{V_1} + (w^2 - 1) \\ &\quad \times (l_{V_3} + r_3 l_{V_2}) + (w - r_3)l_{V_4}]^2 + (w-1)[(w+1)(r_3 - 1)l_{A_1} + (w^2 - 1)(l_{A_3} + r_3 l_{A_2}) + (w - r_3)l_{A_4}]^2 \}], \\ \frac{d^2 \Gamma_{\Lambda_{3/2}}^{(|\lambda|=3/2)}}{dw d \cos \theta} &= 3\Gamma_0 r_3^3 \sqrt{w^2 - 1} (1 - 2r_3 w + r_3^2) \{ (1 + \cos^2 \theta)[(w+1)l_{V_4}^2 + (w-1)l_{A_4}^2] + 4 \cos \theta \sqrt{w^2 - 1} l_{V_4} l_{A_4} \}. \end{aligned} \quad (2.5)$$

Here $\Gamma_0 = G_F^2 |V_{cb}|^2 m_{\Lambda_b}^5 / (192\pi^3)$, $r_1 = m_{\Lambda_c^{1/2}} / m_{\Lambda_b}$, and $r_3 = m_{\Lambda_c^{3/2}} / m_{\Lambda_b}$. $d\Gamma/dw$ is found by integrating over $d \cos \theta$, which amounts to the replacements $\sin^2 \theta \rightarrow 4/3$, $(1 + \cos^2 \theta) \rightarrow 8/3$, and $\cos \theta \rightarrow 0$. Note that near zero recoil ($w = 1$) the form factors d_{V_1} and l_{V_4} determine the rates in Eqs. (2.4) and (2.5). The electron energy spectrum may be found by changing the variable $\cos \theta$ to $E_e = (m_{\Lambda_b}/2)(1 - r w - r \sqrt{w^2 - 1} \cos \theta)$.

In HQET the form factors d_i and l_i are parametrized in terms of one universal Isgur-Wise function in the infinite mass limit and additional sub-leading Isgur-Wise functions which arise at each order in Λ_{QCD}/m_Q . The form of this parametrization is most easily found by introducing interpolating fields which transform in a simple way under heavy quark symmetry [14]. The ground state spinor field, Λ_v , destroys the Λ baryon with $s_l^{\pi_l} = 0^+$ and four-velocity v , and furthermore satisfies $\not{v} \Lambda_v = \Lambda_v$. For the $s_l^{\pi_l} = 1^-$ doublet, the fields with four-velocity v are in

$$\psi_v^\mu = \psi_v^{3/2 \mu} + \frac{1}{\sqrt{3}} (\gamma^\mu + v^\mu) \gamma_5 \psi_v^{1/2}, \quad (2.6)$$

³This agrees with Ref. [13], although there is a sign mistake in Eq. (24) of that paper (the fourth plus sign should be a minus).

where the spinor field $\psi_v^{1/2}$ and Rarita-Schwinger field $\psi_v^{3/2\mu}$ destroy the spin 1/2 and spin 3/2 members of this doublet respectively. The field defined in Eq. (2.6) satisfies $\not{v}\psi_v^\mu = \psi_v^\mu$, and $v_\mu\psi_v^\mu = 0$. Note also that $\gamma_\mu\psi_v^{3/2\mu} = 0$.

When evaluated between a $s_l^{\pi_l} = 1^-$ excited Λ_c state and the Λ_b ground state the $b \rightarrow c$ flavor changing current is

$$\bar{c}\Gamma b = \bar{h}_v^{(c)}\Gamma h_v^{(b)} = \sigma(w)v_\alpha\bar{\psi}_v^\alpha\Gamma\Lambda_v, \quad (2.7)$$

at leading order in Λ_{QCD}/m_Q and α_s . Here $\sigma(w)$ is the dimensionless leading Isgur-Wise function for the transition to this excited doublet. The matrix element in Eq. (2.7) vanishes at zero recoil, and leads to the infinite mass predictions of Ref. [5].

At order Λ_{QCD}/m_Q , there are corrections originating from the matching of the $b \rightarrow c$ flavor changing current onto the effective theory and from order Λ_{QCD}/m_Q corrections to the effective Lagrangian. The current corrections modify the first equality in Eq. (2.7) to

$$\bar{c}\Gamma b = \bar{h}_v^{(c)}\left(\Gamma - \frac{i}{2m_c}\not{D}\Gamma + \frac{i}{2m_b}\Gamma\not{D}\right)h_v^{(b)}. \quad (2.8)$$

For matrix elements between a $s_l^{\pi_l} = 1^-$ excited Λ_c state and the Λ_b ground state, the order Λ_{QCD}/m_Q operators in Eq. (2.8) are

$$\begin{aligned} \bar{h}_v^{(c)}i\bar{D}_\lambda\Gamma h_v^{(b)} &= b_{\alpha\lambda}^{(c)}\bar{\psi}_v^\alpha\Gamma\Lambda_v, \\ \bar{h}_v^{(c)}\Gamma i\bar{D}_\lambda h_v^{(b)} &= b_{\alpha\lambda}^{(b)}\bar{\psi}_v^\alpha\Gamma\Lambda_v. \end{aligned} \quad (2.9)$$

The most general sub-leading current form factors that can be introduced are

$$b_{\alpha\lambda}^{(Q)} = \sigma_1^{(Q)}v_\alpha v_\lambda + \sigma_2^{(Q)}v_\alpha v'_\lambda + \sigma_3^{(Q)}g_{\alpha\lambda}, \quad (2.10)$$

where the $\sigma_i^{(Q)}$ are functions of w and have mass dimension 1. Using the heavy quark equation of motion, $(v \cdot D)h_v^{(Q)} = 0$, gives two relations among these form factors

$$\begin{aligned} w\sigma_1^{(c)} + \sigma_2^{(c)} &= 0, \\ \sigma_1^{(b)} + w\sigma_2^{(b)} + \sigma_3^{(b)} &= 0. \end{aligned} \quad (2.11)$$

When evaluated between the states destroyed by ψ_v^μ and Λ_v translational invariance gives

$$i\partial_\nu(\bar{h}_v^{(c)}\Gamma h_v^{(b)}) = (\bar{\Lambda}v_\nu - \bar{\Lambda}'v'_\nu)\bar{h}_v^{(c)}\Gamma h_v^{(b)}, \quad (2.12)$$

which implies that

$$b_{\alpha\lambda}^{(c)} + b_{\alpha\lambda}^{(b)} = (\bar{\Lambda}v_\lambda - \bar{\Lambda}'v'_\lambda)v_\alpha\sigma. \quad (2.13)$$

Equation (2.13) gives three relations between the current form factors in Eq. (2.10),

$$\begin{aligned} \sigma_1^{(c)} + \sigma_1^{(b)} &= \bar{\Lambda}\sigma, \\ \sigma_2^{(c)} + \sigma_2^{(b)} &= -\bar{\Lambda}'\sigma, \\ \sigma_3^{(c)} + \sigma_3^{(b)} &= 0, \end{aligned} \quad (2.14)$$

which enables us to eliminate the $\sigma_i^{(b)}$. Combining Eq. (2.14) with Eq. (2.11) allows two more form factors to be eliminated,

$$\begin{aligned} \sigma_2^{(c)} &= -w\sigma_1^{(c)}, \\ \sigma_3^{(c)} &= (\bar{\Lambda} - w\bar{\Lambda}')\sigma + (w^2 - 1)\sigma_1^{(c)}, \end{aligned} \quad (2.15)$$

leaving only one unknown current form factor, $\sigma_1 \equiv \sigma_1^{(c)}$, at order $\Lambda_{\text{QCD}}/m_{c,b}$. At zero recoil we see from Eqs. (2.9) and (2.10) that only $\sigma_3^{(Q)}$ can contribute, and from Eq. (2.14) and Eq. (2.15) that $\sigma_3^{(b)}(1) = -\sigma_3^{(c)}(1) = (\bar{\Lambda}' - \bar{\Lambda})\sigma(1)$.

There are also corrections from the order Λ_{QCD}/m_Q effective Lagrangian, $\delta\mathcal{L}^{(Q)} = (O_{\text{kin},v}^{(Q)} + O_{\text{mag},v}^{(Q)})/(2m_Q)$. Here $O_{\text{kin},v}^{(Q)} = \bar{h}_v^{(Q)}(iD)^2 h_v^{(Q)}$ is the heavy quark kinetic energy and $O_{\text{mag},v}^{(Q)} = \bar{h}_v^{(Q)}(g_s/2)\sigma_{\alpha\beta}G^{\alpha\beta}h_v^{(Q)}$ is the chromomagnetic term. The kinetic energy operators modify the infinite mass states giving corrections to the matrix elements of Eq. (2.7) of the form

$$\begin{aligned} i\int d^4x T\{O_{\text{kin},v}^{(c)}(x)[\bar{h}_v^{(c)}\Gamma h_v^{(b)}](0)\} &= \phi_{\text{kin}}^{(c)}v_\alpha\bar{\psi}_v^\alpha\Gamma\Lambda_v, \\ i\int d^4x T\{O_{\text{kin},v}^{(b)}(x)[\bar{h}_v^{(c)}\Gamma h_v^{(b)}](0)\} &= \phi_{\text{kin}}^{(b)}v_\alpha\bar{\psi}_v^\alpha\Gamma\Lambda_v. \end{aligned} \quad (2.16)$$

These corrections do not violate spin symmetry, so their contributions enter the same way as the $m_Q \rightarrow \infty$ Isgur-Wise function σ and vanish at zero recoil. The chromomagnetic operator, which violates spin symmetry, gives contributions of the form

$$\begin{aligned} i\int d^4x T\{O_{\text{mag},v}^{(c)}(x)[\bar{h}_v^{(c)}\Gamma h_v^{(b)}](0)\} & \\ &= (\phi_{\text{mag}}^{(c)}g_{\mu\alpha}v_\nu)\bar{\psi}_v^\alpha i\sigma^{\mu\nu} \frac{1+\not{v}}{2}\Gamma\Lambda_v, \\ i\int d^4x T\{O_{\text{mag},v}^{(b)}(x)[\bar{h}_v^{(c)}\Gamma h_v^{(b)}](0)\} & \\ &= (\phi_{\text{mag}}^{(b)}g_{\mu\alpha}v'_\nu)\bar{\psi}_v^\alpha \Gamma \frac{1+\not{v}}{2}i\sigma^{\mu\nu}\Lambda_v. \end{aligned} \quad (2.17)$$

At zero recoil these chromomagnetic corrections vanish since $v_\alpha(1+\not{v})\sigma^{\alpha\beta}(1+\not{v}) = 0$. Thus the only Λ_{QCD}/m_Q corrections that contribute at zero recoil are determined by measurable baryon mass splittings and the value of the leading order Isgur-Wise function at zero recoil.

Using Eqs. (2.10)–(2.17), it is straightforward to express the form factors d_i and l_i parametrizing these semileptonic decays in terms of Isgur-Wise functions σ , σ_1 , $\phi_{\text{kin}}^{(Q)}$, and $\phi_{\text{mag}}^{(Q)}$. Let $\varepsilon_Q = 1/(2m_Q)$. For decays to $\Lambda_c^{1/2}$ we have

$$\begin{aligned}
\sqrt{3}d_{A_1} &= (w+1)\sigma + \varepsilon_c[3(w\bar{\Lambda}' - \bar{\Lambda})\sigma - 2(w^2-1)\sigma_1 \\
&\quad + (w+1)(\phi_{\text{kin}}^{(c)} - 2\phi_{\text{mag}}^{(c)})] - \varepsilon_b[(\bar{\Lambda}' - w\bar{\Lambda})\sigma \\
&\quad - (w+1)\phi_{\text{kin}}^{(b)}], \\
\sqrt{3}d_{A_2} &= -2\sigma - 2\varepsilon_c(\phi_{\text{kin}}^{(c)} - 2\phi_{\text{mag}}^{(c)}) + 2\varepsilon_b[(\bar{\Lambda}' - \bar{\Lambda})\sigma \\
&\quad - (w-1)\sigma_1 - \phi_{\text{kin}}^{(b)} + \phi_{\text{mag}}^{(b)}], \\
\sqrt{3}d_{A_3} &= 2\varepsilon_b[(\bar{\Lambda}' - \bar{\Lambda})\sigma - (w-1)\sigma_1 - \phi_{\text{mag}}^{(b)}], \\
\sqrt{3}d_{V_1} &= (w-1)\sigma + \varepsilon_c[3(w\bar{\Lambda}' - \bar{\Lambda})\sigma - 2(w^2-1)\sigma_1 \\
&\quad + (w-1)(\phi_{\text{kin}}^{(c)} - 2\phi_{\text{mag}}^{(c)})] - \varepsilon_b[(\bar{\Lambda}' - w\bar{\Lambda})\sigma \\
&\quad - (w-1)\phi_{\text{kin}}^{(b)}], \\
\sqrt{3}d_{V_2} &= -2\sigma - 2\varepsilon_c(\phi_{\text{kin}}^{(c)} - 2\phi_{\text{mag}}^{(c)}) - 2\varepsilon_b[(\bar{\Lambda}' + \bar{\Lambda})\sigma \\
&\quad - (w+1)\sigma_1 + \phi_{\text{kin}}^{(b)} + \phi_{\text{mag}}^{(b)}], \\
\sqrt{3}d_{V_3} &= 2\varepsilon_b[(\bar{\Lambda}' + \bar{\Lambda})\sigma - (w+1)\sigma_1 - \phi_{\text{mag}}^{(b)}].
\end{aligned} \tag{2.18}$$

The analogous formulae for $\Lambda_c^{3/2}$ are

$$\begin{aligned}
l_{A_1} &= \sigma + \varepsilon_c[(w-1)\sigma_1 + \phi_{\text{kin}}^{(c)} + \phi_{\text{mag}}^{(c)}] - \varepsilon_b[(\bar{\Lambda}' - \bar{\Lambda})\sigma \\
&\quad - (w-1)\sigma_1 - \phi_{\text{kin}}^{(b)} + \phi_{\text{mag}}^{(b)}], \\
l_{A_2} &= -2\varepsilon_c\sigma_1, \\
l_{A_3} &= 2\varepsilon_b(\bar{\Lambda}'\sigma - w\sigma_1 + \phi_{\text{mag}}^{(b)}), \\
l_{A_4} &= -2\varepsilon_b[(w\bar{\Lambda}' - \bar{\Lambda})\sigma - (w^2-1)\sigma_1 + (w+1)\phi_{\text{mag}}^{(b)}], \\
l_{V_1} &= \sigma + \varepsilon_c[(w+1)\sigma_1 + \phi_{\text{kin}}^{(c)} + \phi_{\text{mag}}^{(c)}] + \varepsilon_b[(\bar{\Lambda}' + \bar{\Lambda})\sigma \\
&\quad - (w+1)\sigma_1 + \phi_{\text{kin}}^{(b)} + \phi_{\text{mag}}^{(b)}], \\
l_{V_2} &= -2\varepsilon_c\sigma_1, \\
l_{V_3} &= -2\varepsilon_b(\bar{\Lambda}'\sigma - w\sigma_1 + \phi_{\text{mag}}^{(b)}), \\
l_{V_4} &= 2\varepsilon_b[(w\bar{\Lambda}' - \bar{\Lambda})\sigma - (w^2-1)\sigma_1 + (w-1)\phi_{\text{mag}}^{(b)}].
\end{aligned} \tag{2.19}$$

The form factors which occur for the helicity $|\lambda|=3/2$ rate in Eq. (2.5), l_{A_4} and l_{V_4} , only receive corrections proportional to ε_b , so this rate remains small at order Λ_{QCD}/m_Q . The form factors d_{V_1} and l_{V_4} which determine the rates near zero recoil have the values

$$\begin{aligned}
\sqrt{3}d_{V_1}(1) &= (3\varepsilon_c - \varepsilon_b)(\bar{\Lambda}' - \bar{\Lambda})\sigma(1), \\
l_{V_4}(1) &= 2\varepsilon_b(\bar{\Lambda}' - \bar{\Lambda})\sigma(1).
\end{aligned} \tag{2.20}$$

The Isgur-Wise functions that appear in Eqs. (2.18) and (2.19) have unknown functional forms, so to predict the decay rates some assumptions must be made. The functions $\phi_{\text{kin}}^{(Q)}$ can be absorbed by replacing σ with

$$\tilde{\sigma} = \sigma + \varepsilon_c\phi_{\text{kin}}^{(c)} + \varepsilon_b\phi_{\text{kin}}^{(b)}. \tag{2.21}$$

This introduces higher order terms of the form $\phi_{\text{kin}}(\bar{\Lambda}' - \bar{\Lambda})\mathcal{O}(\varepsilon_Q^2)$. These terms are small for the spin 3/2 form factors since they are always suppressed by at least one ε_b , but could be large for the spin 1/2 form factors since ε_c^2 occurs. However, in the limit $N_c \rightarrow \infty$ we have $\phi_{\text{kin}}^{(c)}(1) = 0$ (as discussed in the Appendix) so the latter contributions are also small. Hereafter, unless explicitly stated otherwise, we will use $\tilde{\sigma}$. The chromomagnetic functions, $\phi_{\text{mag}}^{(Q)}$, are expected to be small relative to Λ_{QCD} and will therefore be neglected. This is supported by the small $s_l^{\pi'} = 1^-$ doublet mass splitting, the fact that at order Λ_{QCD}/m_Q spin-symmetry violating effects are sub-dominant in the $N_c \rightarrow \infty$ limit [12], and that the members of this doublet are P-wave excitations in the quark model. Following Ref. [8] we note that since the available phase space is small ($1 < w \leq 1.3$), it is useful to consider the differential rates treating $(w-1)$ as order Λ_{QCD}/m_Q and expanding in these parameters. This has the advantage of showing explicitly at what order various unknown factors appear. Expanding the differential rates in powers of $(w-1)$ gives

$$\begin{aligned}
\frac{d^2\Gamma_{\Lambda_{1/2}}}{dw d\cos\theta} &= 4\Gamma_0\tilde{\sigma}^2(1)r_1^3\sqrt{w^2-1}\sum_n(w-1)^n\{\sin^2\theta s_1^{(n)} \\
&\quad + (1-2r_1w+r_1^2)[(1+\cos^2\theta)t_1^{(n)} \\
&\quad - 4\cos\theta\sqrt{w^2-1}u_1^{(n)}\}], \\
\frac{d^2\Gamma_{\Lambda_{3/2}^{|\lambda|=1/2}}}{dw d\cos\theta} &= 8\Gamma_0\tilde{\sigma}^2(1)r_3^3\sqrt{w^2-1}\sum_n(w-1)^n\{\sin^2\theta s_3^{(n)} \\
&\quad + (1-2r_3w+r_3^2)[(1+\cos^2\theta)t_3^{(n)} \\
&\quad - 4\cos\theta\sqrt{w^2-1}u_3^{(n)}\}],
\end{aligned} \tag{2.22}$$

where $s_i^{(n)}$, $t_i^{(n)}$, and $u_i^{(n)}$ are expansion coefficients. The entire rate for spin-3/2 $|\lambda|=3/2$ is suppressed by a ε_b^2 so it is not useful to consider the $w-1$ expansion. Corrections of order ε_c^2 to the form factors l_{V_4} and l_{A_4} in Eq. (2.19) have not been considered and may give terms of similar order in this rate. Even so, a conservative estimate puts the contribution from the $|\lambda|=3/2$ states to the total $\Lambda_c^{3/2}$ rate as at least 30 times smaller⁴ than that of the $|\lambda|=1/2$ states.

Treating $(w-1)$ as order ε_Q we keep the coefficients $s^{(n)}$ and $t^{(n)}$ to order $\varepsilon_Q^{(2-n)}$. Since the coefficients $u^{(n)}$ are multiplied by an additional $\sqrt{w^2-1}$ we keep them to order

⁴This estimate is made using Eq. (2.25b) and the method described below. Varying $\hat{\sigma}_1$ over the range $-1 \text{ GeV} < \hat{\sigma}_1 < 1 \text{ GeV}$ gives $3 \times 10^{-4} < \Gamma_{\Lambda_{3/2}^{|\lambda|=3/2}}/\Gamma_{\Lambda_{3/2}^{|\lambda|=1/2}} < 0.02$. The bound is taken to be 1/30 rather than 1/50 to be conservative.

$\varepsilon_Q^{(1-n)}$. Recall that these latter coefficients do not contribute to the single differential $d\Gamma/dw$ rates. It is straightforward to derive these coefficients using Eqs. (2.4), (2.5), (2.18), and (2.19) so only a few will be displayed here for illustrative purposes. The coefficients $s^{(0)}$ and $t^{(0)}$ are order $\varepsilon_Q^2(\bar{\Lambda}' - \bar{\Lambda})^2$

$$\begin{aligned} s_1^{(0)} &= (1-r_1)^2(3\varepsilon_c - \varepsilon_b)^2(\bar{\Lambda}' - \bar{\Lambda})^2, \\ t_1^{(0)} &= (3\varepsilon_c - \varepsilon_b)^2(\bar{\Lambda}' - \bar{\Lambda})^2, \\ s_3^{(0)} &= 4(1-r_3)^2\varepsilon_b^2(\bar{\Lambda}' - \bar{\Lambda})^2, \\ t_3^{(0)} &= \varepsilon_b^2(\bar{\Lambda}' - \bar{\Lambda})^2, \end{aligned} \quad (2.23)$$

while the $u^{(0)}$ coefficients are order $\varepsilon_Q(\bar{\Lambda}' - \bar{\Lambda})$. The coefficients $s^{(1)}$ and $t^{(1)}$ have terms with ε_Q^0 and with ε_Q^1 . The ε_Q^1 contributions do not involve σ_1 , and for the spin 3/2 coefficients there are no ε_c^1 contributions. For example, we have

$$\begin{aligned} t_1^{(1)} &= 2 + 4(3\varepsilon_c - \varepsilon_b)(\bar{\Lambda}' - \bar{\Lambda}), \\ t_3^{(1)} &= 2 - 4\varepsilon_b(\bar{\Lambda}' - \bar{\Lambda}). \end{aligned} \quad (2.24)$$

Finally, the coefficients $s^{(2)}$, $t^{(2)}$, and $u^{(1)}$ are kept to order ε_Q^0 , and depend on $\hat{\sigma}' = \tilde{\sigma}'(1)/\tilde{\sigma}(1)$ (a caret will be used to denote normalization with respect to $\tilde{\sigma}$). With these assumptions the coefficients are determined at this order in terms of $\bar{\Lambda}' - \bar{\Lambda}$ and $\hat{\sigma}'$, while terms with σ_1 and more derivatives of $\tilde{\sigma}$ come in at higher orders in the double expansion. The value of $\hat{\sigma}'_1(1)$ [where $\hat{\sigma}'_1(w) \equiv \sigma_1(w)/\tilde{\sigma}(w)$] gives smaller uncertainties than might naively be expected for this reason.

It is also possible to estimate the rates without a w expansion by inserting the form factors in Eqs. (2.18) and (2.19) directly into Eqs. (2.4) and (2.5). To determine the differential rates we take the large N_c predictions

$$\sigma(w) = 1.2[1 - 1.4(w-1)], \quad (2.25a)$$

$$\tilde{\sigma}(w) = 1.2[1 - 1.6(w-1)], \quad (2.25b)$$

using the former in the infinite mass limit and the latter when Λ_{QCD}/m_Q effects are included. The derivation of Eqs. (2.25a) and (2.25b) are given in the Appendix. The $\phi_{\text{mag}}^{(Q)}$ will be neglected for the reasons given above, leaving $\hat{\sigma}'_1$ as the remaining unknown form factor needed to predict the differential rates at order Λ_{QCD}/m_Q .

With $r_1 = 0.461$, $r_3 = 0.467$, $\bar{\Lambda}' - \bar{\Lambda} = 0.2$ GeV and $\bar{\Lambda} = 0.8$ GeV, our results for the $d\Gamma/dw$ spectrums are shown in Fig. 1. Plotted are the infinite mass limit predictions without expansion (dotted lines), the predictions with $1/m_Q$ effects using the expansions in Eq. (2.22) (dashed lines), and the predictions including $1/m_Q$ effects without expansion and taking $\hat{\sigma}'_1 = 0$ (solid lines). A factor of $\Gamma_0 \tilde{\sigma}(1)^2 \sqrt{w^2 - 1}$ has been scaled out of the decay rates making the displayed curves independent of the normalization. Therefore, the only large N_c input for these curves is the value of the slope

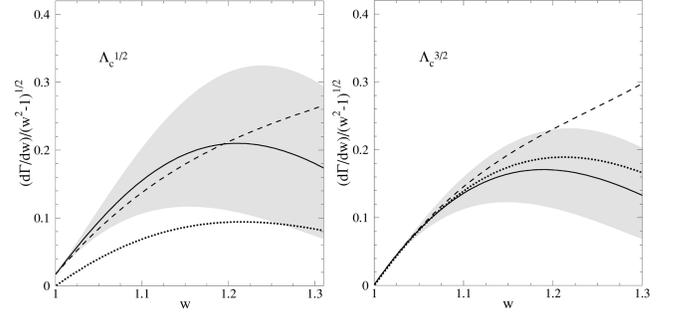


FIG. 1. The spectrum for $\Lambda_b \rightarrow \Lambda_c^{1/2} e \bar{\nu}_e$, in Fig. 1a, and the spectrum for $\Lambda_b \rightarrow \Lambda_c^{3/2} e \bar{\nu}_e$, in Fig. 1b, are shown in units of $\Gamma_0 \tilde{\sigma}(1)^2$. The dashed curves are the prediction of the expansions in Eq. (2.22) with $\hat{\sigma}' = -1.6$ and include $1/m_Q$ effects. The dotted curves are the $m_Q \rightarrow \infty$ predictions with no expansion and with $\sigma'(1)/\sigma(1) = -1.4$. The solid curves are the results with no expansion using $\hat{\sigma}' = -1.6$ and include $1/m_Q$ effects with $\hat{\sigma}'_1 = 0$. The shaded regions show the range the solid curves cover when $\hat{\sigma}'_1$ is varied through the range $-1 \text{ GeV} < \hat{\sigma}'_1 < 1 \text{ GeV}$.

parameter $\hat{\sigma}'$ [or $\sigma'(1)/\sigma(1)$ for $m_Q \rightarrow \infty$]. The contribution from the helicity $\pm 3/2$ states to the $\Lambda_c^{3/2}$ rate in Fig. 1b is invisible on the scale shown.

The spectra in Fig. 1 have uncertainty associated with the values of $\bar{\Lambda}' - \bar{\Lambda}$ and $\bar{\Lambda}$. Changing the value of $\bar{\Lambda}' - \bar{\Lambda}$ by ± 0.1 GeV has a large effect for the $\Lambda_c^{1/2}$ ($\lesssim 30\%$ for a given point on the curve in Fig. 1a) but a small effect for $\Lambda_c^{3/2}$ ($\lesssim 3\%$). A measurement of the mass of a $s_1^{n_l=1^-}$ bottom baryon will substantially reduce this uncertainty. Changing the value of $\bar{\Lambda}$ has a small effect for both $\Lambda_c^{1/2}$ and $\Lambda_c^{3/2}$ ($\lesssim 5\%$ and $\lesssim 1\%$ respectively). To estimate the uncertainty in predicting the rates associated with the value of $\hat{\sigma}'_1$ we take it to be w independent and vary it over the range $-1 \text{ GeV} < \hat{\sigma}'_1 < 1 \text{ GeV}$. This gives the shaded regions shown in Fig. 1. It is important to note that the lower bound comes from $\hat{\sigma}'_1 = 1 \text{ GeV}$ for the $\Lambda_c^{1/2}$, but from $\hat{\sigma}'_1 = -1 \text{ GeV}$ for the $\Lambda_c^{3/2}$. Thus the sum of these rates is less sensitive to $\hat{\sigma}'_1$ than the $\Lambda_c^{1/2}$ rate alone.

The Λ_b^0 lifetime $\tau = 1.11$ ps and 10% branching fraction for $\Lambda_b \rightarrow \Lambda_c e \bar{\nu}_e X$ [2] give an inclusive rate of $0.29 \Gamma_0$. We can estimate what percentage of this rate is made up of decays to $\Lambda_{1/2}$ and $\Lambda_{3/2}$ by taking the large N_c normalization, $\tilde{\sigma}(1) = 1.2$, and integrating the differential rates in Eqs. (2.4) and (2.5) over the ranges $1 < w < 1.31$ and $1 < w < 1.30$ respectively. Varying $\hat{\sigma}'_1$ in the range $-1 \text{ GeV} < \hat{\sigma}'_1 < 1 \text{ GeV}$ then gives

$$\begin{aligned} 0.024 &< \frac{\Gamma_{\Lambda_{1/2}}}{\Gamma_0} < 0.072, \\ 0.023 &< \frac{\Gamma_{\Lambda_{3/2}}}{\Gamma_0} < 0.048. \end{aligned} \quad (2.26)$$

The $\Gamma_{\Lambda_{1/2}}$ rate is enhanced compared to the infinite mass prediction $\Gamma_{\Lambda_{1/2}}/\Gamma_0=0.020$. Adding the rates in Eq. (2.26) and comparing with the inclusive rate $0.29\Gamma_0$, we find that decays to these states contribute between 25% to 33% of the semileptonic Λ_b branching fraction. This range corresponds to $-1 \text{ GeV} < \hat{\sigma}_1 < 1 \text{ GeV}$ and has less uncertainty than that in Eq. (2.26) since varying $\hat{\sigma}_1$ changes the two rates in opposite ways. To test the dependence of this prediction on the shape of $\hat{\sigma}(w)$ we take $\hat{\sigma}_1(1)=0$ and vary $\hat{\sigma}'_1(1)$ over the range $-1 \text{ GeV} < \hat{\sigma}'_1(1) < 1 \text{ GeV}$. This has a small effect on the prediction giving a range from 26% to 28%.

Factorization should be a good approximation for Λ_b decay into charmed baryons and a charged pion. Contributions that violate factorization are suppressed by Λ_{QCD} divided by the energy of the pion in the B rest frame [15] or by $\alpha_s(m_Q)$. Furthermore, for these decays, factorization holds in the large N_c limit. Neglecting the pion mass, the two-body decay rate, Γ_π , is related to the differential decay rate $d\Gamma_{\text{sl}}/dw$ at maximal recoil for the analogous semileptonic decay (with the π replaced by the $e\bar{\nu}_e$ pair). This relation is independent of which charmed baryon appears in the final state:

$$\Gamma_\pi = \frac{3\pi^2 |V_{ud}|^2 C^2 f_\pi^2}{m_{\Lambda_b}^2 r} \left(\frac{d\Gamma_{\text{sl}}}{dw} \right)_{w_{\text{max}}}. \quad (2.27)$$

Here r is the mass of the charmed baryon divided by m_{Λ_b} , $w_{\text{max}}=(1+r^2)/(2r)$, and $f_\pi \simeq 132 \text{ MeV}$ is the pion decay constant. C is a combination of Wilson coefficients of four-quark operators [16], and numerically $C|V_{ud}|$ is very close to unity.

Using the large N_c prediction for the Isgur-Wise function, Eq. (2.25b), and evaluating Eqs. (2.4) and (2.5) at $w=1.31$ and 1.30 respectively, it is possible to obtain predictions for these nonleptonic decays. Since these predictions depend on $d\Gamma_{\text{sl}}/dw$ at w_{max} there is a large uncertainty due to $\hat{\sigma}_1$. Varying $\hat{\sigma}_1$ in the range $-1 \text{ GeV} < \hat{\sigma}_1 < 1 \text{ GeV}$ gives

$$0.003 < \frac{\Gamma_{\Lambda_{1/2}}^\pi}{\Gamma_0} < 0.014, \\ 0.003 < \frac{\Gamma_{\Lambda_{3/2}}^\pi}{\Gamma_0} < 0.009. \quad (2.28)$$

Adding these rates and using $\tau=1.11 \text{ ps}$ for the Λ_b^0 lifetime gives 0.4–0.6 % for the branching fraction for these decays. Here again the uncertainty in the total branching fraction is smaller than the individual rates. Varying the slope of $\hat{\sigma}_1$ again makes only a small difference for this prediction.

In this section the decays $\Lambda_b \rightarrow \Lambda_c^{1/2} e \bar{\nu}_e$ and $\Lambda_b \rightarrow \Lambda_c^{3/2} e \bar{\nu}_e$ were considered. Predictions were given for the differential decay distributions, and the total decay rates. Factorization was also used to make a prediction for the nonleptonic $\Lambda_b \rightarrow \Lambda_c^{1/2} \pi$ and $\Lambda_b \rightarrow \Lambda_c^{3/2} \pi$ decay rates. The determination of the Isgur-Wise function in the $N_c \rightarrow \infty$ limit was used to make these predictions. At order Λ_{QCD}/m_Q , all these

TABLE II. Contributions to the zero recoil matrix elements in Eq. (3.1) to order Λ_{QCD}/m_Q . An asterisk denotes that the corresponding contribution to the matrix element is identically zero for any value of w . Here 0^+ refers only to the radially excited $s_l^{\pi l} = 0^+$ states.

| $s_l^{\pi l}$ | 0^+ | 1^- | 0^- | 1^+ |
|--------------------------|---------|---|-------|---------|
| $m_Q \rightarrow \infty$ | 0 | 0 | 0* | 0* |
| $1/m_Q$ currents | 0 | $\propto (\bar{\Lambda}' - \bar{\Lambda})\sigma(1)$ | 0* | 0 |
| $1/m_Q$ kin T-products | nonzero | 0 | 0* | 0* |
| $1/m_Q$ mag T-products | 0* | 0 | 0 | nonzero |

predictions depend on the unknown $\hat{\sigma}_1$. A measurement of any of these quantities will constrain the normalization of this function.

III. ZERO RECOIL MATRIX ELEMENTS FOR EXCITED TRANSITIONS

In this section matrix elements for semileptonic Λ_b transitions to other excited Λ_c states are investigated. In particular we are interested in matrix elements of the form

$$\langle \Lambda_c(s_l^{\pi l}, v') | J^\mu | \Lambda_b(v) \rangle \Big|_{v' \rightarrow v} \quad (3.1)$$

at order Λ_{QCD}/m_Q . (Some statements about the form of these matrix elements away from zero recoil will also be made.) In Eq. (3.1) J^μ refers to the vector or axial-vector part of a weak current. At zero recoil it is sufficient to consider excited states with $s_l^{\pi l} = 0^\pm, 1^\pm$ (the states summarized in Table I), since for $s_l^{\pi l} \geq 2$ the matrix element in Eq. (3.1) vanishes at order Λ_{QCD}/m_Q . With $J \geq 5/2$ the matrix elements vanish by conservation of angular momentum. For transitions to $J=3/2$ where $s_l=2$ they vanish at zero recoil and order Λ_{QCD}/m_Q since the effective fields are transverse to v (we agree with the proof of this fact given in [6], but only for $s_l \geq 2$). For each $s_l^{\pi l}$ there is a tower of particle excitations with increasing mass. The states in this tower will be referred to as radial excitations, and the n th such state will be denoted with a superscript (n). In general the properties of the Λ_b transition to a radially excited charmed state can be directly inferred from those of the lowest excited state with the same $s_l^{\pi l}$. The exception is radial excitations of the ground state, $s_l^{\pi l} = 0^+$, where a separate analysis is required.

A summary of how the various states receive order Λ_{QCD}/m_Q corrections at zero recoil is given in Table II. The results in the previous section for $s_l^{\pi l} = 1^-$ are included for easy reference. For the $m_Q \rightarrow \infty$ matrix elements, recall that the leading order Isgur-Wise function for decays to radial excitations with $s_l^{\pi l} = 0^+$ vanishes at zero recoil, while for the unnatural parity transitions these matrix elements vanish identically. For the unnatural transitions to $s_l^{\pi l} = 0^-$ and 1^+ one can use the same effective fields, Λ_v and ψ_v^μ , introduced in Sec. II, but the form factors must be pseudoscalar and therefore involve an epsilon tensor [17]. For the leading order current in Eq. (2.7) there are not enough vectors available

to contract with the indices of the epsilon tensor so these unnatural parity matrix elements vanish [6].

The matrix elements of the $1/m_Q$ current corrections in Eq. (2.8) vanish at zero recoil for excitations with $s_l^{\pi_l} = 0^+, 0^-, 1^+$. Between a $s_l = 0$ excited Λ_c state and a Λ_b state the corrections in Eq. (2.9) are

$$\begin{aligned}\bar{h}_v^{(c)} i \vec{D}_\lambda \Gamma h_v^{(b)} &= b_\lambda^{(c)} \bar{\Lambda}_v \Gamma \Lambda_v, \\ \bar{h}_v^{(c)} \Gamma i \vec{D}_\lambda h_v^{(b)} &= b_\lambda^{(b)} \bar{\Lambda}_v \Gamma \Lambda_v.\end{aligned}\quad (3.2)$$

For $s_l^{\pi_l} = 0^+$, the most general form is $b_\lambda^{(Q)} = a_1^{(Q)} v_\lambda + a_2^{(Q)} v'_\lambda$. The equations of motion, $(v \cdot D) h_v^{(Q)} = 0$, imply $w a_1^{(c)} + a_2^{(c)} = 0$ and $a_1^{(b)} + w a_2^{(b)} = 0$; so the current corrections vanish at zero recoil. Using in addition Eq. (2.12) one can easily show that $a_{1,2}^{(Q)}$ are determined in terms of $\bar{\Lambda}^{(n)}$, $\bar{\Lambda}$, and the leading order Isgur-Wise function for the transition. For $s_l^{\pi_l} = 0^-$, $b_\lambda^{(Q)}$ must include an epsilon tensor, but there are not enough vectors to contract with the indices, so $b_\lambda^{(Q)} \equiv 0$. For $s_l^{\pi_l} = 1^+$ the current corrections are given by Eq. (2.9) with $b_{\alpha\lambda}^{(Q)} = \sigma_{1*}^{(Q)} \epsilon_{\alpha\lambda\sigma\tau} v^\sigma v'^\tau$ and therefore vanish at zero recoil. Note that from Eq. (2.12) it follows that $\sigma_{1*}^{(b)} = -\sigma_{1*}^{(c)}$.

Next consider the Λ_{QCD}/m_Q contributions to the matrix elements coming from time ordered products of the corrections to the Lagrangian, $\delta\mathcal{L}^{(Q)} = (O_{\text{kin}}^{(Q)} + O_{\text{mag}}^{(Q)})/(2m_Q)$, with the leading order current, $\bar{h}_v^{(c)} \Gamma h_v^{(b)}$. For the unnatural transitions ($s_l^{\pi_l} = 0^-, 1^+$) corrections from the kinetic energy operator do not break the spin symmetry and therefore vanish for the same reason that the leading form factor vanished (i.e., $\Delta\lambda_l = 0$ and parity). For $s_l = 0$ the time ordered products involving the chromomagnetic operator are

$$\begin{aligned}i \int d^4x T \{ O_{\text{mag},v}^{(c)}(x) [\bar{h}_v^{(c)} \Gamma h_v^{(b)}](0) \} \\ = R_{\mu\nu}^{(c)} \bar{\Lambda}_v i \sigma^{\mu\nu} \frac{1 + \not{v}'}{2} \Gamma \Lambda_v, \\ i \int d^4x T \{ O_{\text{mag},v}^{(b)}(x) [\bar{h}_v^{(c)} \Gamma h_v^{(b)}](0) \} \\ = R_{\mu\nu}^{(b)} \bar{\Lambda}_v \Gamma \frac{1 + \not{v}}{2} i \sigma^{\mu\nu} \Lambda_v,\end{aligned}\quad (3.3)$$

where the indices μ and ν are anti-symmetric. For $s_l^{\pi_l} = 0^+$, $R_{\mu\nu}^{(Q)} = c_1^{(Q)} (v_\mu v'_\nu - v_\nu v'_\mu)$, and $\not{v} \Lambda_v = \Lambda_v$, so these time ordered products vanish identically since $v_\mu (1 + \not{v}) \sigma^{\mu\nu} (1 + \not{v}) = 0$. For $s_l^{\pi_l} = 0^-$, $R_{\mu\nu}^{(Q)} = c_2^{(Q)} \epsilon_{\mu\nu\sigma\tau} v^\sigma v'^\tau$, so the time ordered products in Eq. (3.3) vanish at zero recoil. For $s_l^{\pi_l} = 1^+$ chromomagnetic Lagrangian corrections have a form similar to Eq. (2.17), but we must have a tensor involving epsilon multiplying possible form factors. At zero recoil we find a nonzero contribution from the tensor $\epsilon_{\mu\nu\alpha\beta} v^\beta$ as indicated in Table II.

The kinetic Lagrangian correction for $s_l^{\pi_l} = 0^+$ and the chromomagnetic Lagrangian correction for $s_l^{\pi_l} = 1^+$ do not

vanish at zero recoil. These corrections can be written in terms of local matrix elements by inserting a complete set of states between the leading order $m_Q \rightarrow \infty$ currents and the operators $O_{\text{kin}}^{(Q)}$ or $O_{\text{mag}}^{(Q)}$. Working in the rest frame $v = v' = (1, \vec{0})$ and performing the space-time integral gives

$$\begin{aligned}\frac{\langle \Lambda_c^f | J | \Lambda_b \rangle}{\sqrt{m_{\Lambda_c^f} m_{\Lambda_b}}} &= \sum_I \left(\frac{\langle \Lambda_c^f | \delta\mathcal{L}_v^{(c)} | \Lambda_c^I \rangle_\infty \langle \Lambda_c^I | J | \Lambda_b \rangle_\infty}{2(\bar{\Lambda}_I - \bar{\Lambda}_c^f)} \right. \\ &\quad \left. + \frac{\langle \Lambda_c^f | J | \Lambda_b^I \rangle_\infty \langle \Lambda_b^I | \delta\mathcal{L}_v^{(b)} | \Lambda_b \rangle_\infty}{2(\bar{\Lambda}_I - \bar{\Lambda})} \right),\end{aligned}\quad (3.4)$$

where $J = \bar{h}_v^{(c)} \Gamma h_v^{(b)}$. The subscript ∞ is used to denote states in the effective theory, which are normalized so $\langle \infty(H(p') | H(p)) \rangle_\infty = (2\pi)^3 2v^0 \delta^3(\vec{p}' - \vec{p})$ for $p = m_H v$. Since the zero recoil weak currents are charge densities of heavy quark spin-flavor symmetry, only one state from this sum contributes. For the radially excited $s_l^{\pi_l} = 0^+$ states we find the following non-vanishing matrix elements

$$\begin{aligned}\frac{\langle \Lambda_c^{(n)}(s) | \vec{A} | \Lambda_b(s) \rangle}{\sqrt{m_{\Lambda_c^{(n)}} m_{\Lambda_b}}} &= \frac{-\vec{s}}{(\bar{\Lambda}^{(n)} - \bar{\Lambda})} \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \\ &\quad \times \langle \infty \Lambda_c^{(n)}(s) | O_{\text{kin}}^{(c)}(0) | \Lambda_c(s) \rangle_\infty, \\ \frac{\langle \Lambda_c^{(n)}(s) | V^0 | \Lambda_b(s) \rangle}{\sqrt{m_{\Lambda_c^{(n)}} m_{\Lambda_b}}} &= \frac{-1}{(\bar{\Lambda}^{(n)} - \bar{\Lambda})} \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \\ &\quad \times \langle \infty \Lambda_c^{(n)}(s) | O_{\text{kin}}^{(c)}(0) | \Lambda_c(s) \rangle_\infty,\end{aligned}\quad (3.5)$$

where $\vec{s} = \vec{u}(s) \vec{\gamma} \gamma_5 u(s)$. For the spin 1/2 member of the $s_l^{\pi_l} = 1^+$ doublet we have

$$\begin{aligned}\frac{\langle \Lambda_c^{1/2*(n)}(s) | \vec{A} | \Lambda_b(s) \rangle}{\sqrt{m_{\Lambda_c^{1/2*(n)}} m_{\Lambda_b}}} \\ = \frac{-\vec{s}}{(\bar{\Lambda}'^{*(n)} - \bar{\Lambda})} \left(\frac{1}{2m_c} + \frac{1}{6m_b} \right) \\ \times \langle \infty \Lambda_c^{1/2*(n)}(s) | O_{\text{mag}}^{(c)}(0) | \Lambda_c(s) \rangle_\infty, \\ \frac{\langle \Lambda_c^{1/2*(n)}(s) | V^0 | \Lambda_b(s) \rangle}{\sqrt{m_{\Lambda_c^{1/2*(n)}} m_{\Lambda_b}}} \\ = \frac{-1}{(\bar{\Lambda}'^{*(n)} - \bar{\Lambda})} \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) \\ \times \langle \infty \Lambda_c^{1/2*(n)}(s) | O_{\text{mag}}^{(c)}(0) | \Lambda_c(s) \rangle_\infty.\end{aligned}\quad (3.6)$$

For the spin 3/2 member of the $s_l^{\pi_l} = 1^+$ doublet only the axial current gives a nonzero matrix element

$$\begin{aligned}
& \frac{\langle \Lambda_c^{3/2*(n)}(s) | A^i | \Lambda_b(s) \rangle}{\sqrt{m_{\Lambda_c^{3/2*(n)}} m_{\Lambda_b}}} \\
&= \frac{\bar{u}^i u}{(\bar{\Lambda}'^{*(n)} - \bar{\Lambda})} \frac{1}{\sqrt{3} m_b} \\
& \times {}_\infty \langle \Lambda_c^{1/2*(n)}(s) | O_{\text{mag}}^{(c)}(0) | \Lambda_c(s) \rangle_\infty, \quad (3.7)
\end{aligned}$$

In Eqs. (3.6) and (3.7) heavy quark spin-flavor symmetry was used to write the effects of $O_{\text{kin}}^{(b)}$ and $O_{\text{mag}}^{(b)}$ in terms of matrix elements of $O_{\text{kin}}^{(c)}$ and $O_{\text{mag}}^{(c)}$. This neglects the weak logarithmic dependence on the heavy quark mass in the matrix elements of O_{mag} . At zero recoil and order Λ_{QCD}/m_Q this completes the classification of all nonzero hadronic matrix elements for semileptonic Λ_b to excited Λ_c decays.

IV. SUM RULES

In this section we consider baryon sum rules that relate the inclusive decays $\Lambda_b \rightarrow X_c e \bar{\nu}_e$ to a sum of exclusive channels [18]. The starting point is a time ordered product of the form

$$\begin{aligned}
T &= \frac{i}{4m_{\Lambda_b}} a^{\mu\nu} \int d^4x e^{-iq \cdot x} \\
& \times \sum_s \langle \Lambda_b(v, s) | T \{ J_\mu^\dagger(x), J_\nu(0) \} | \Lambda_b(v, s) \rangle, \quad (4.1)
\end{aligned}$$

where the current $J_\mu = \bar{c} \Gamma b$, and $a^{\mu\nu}$ is chosen to project out the desired part of the current correlator [19]. (The extra factor of 1/2 compared to the $|B\rangle$ case is for the average over initial spin.) Suitable moments of $T_{\mu\nu}$ may then be compared making use of an OPE on the inclusive side [20] and inserting a complete set of Λ_c states on the exclusive side. Usually a hard cutoff is introduced so that only hadronic resonances up to an excitation energy $\Delta \sim 1$ GeV are included in these moments.

In [5,21] a Bjorken sum rule was considered which bounds the slope $-\rho^2$ of the ground state Isgur-Wise function $\zeta(w) = 1 - (w-1)\rho^2 + \dots$. It was determined that only excited states with $s_l^{\pi_l} = 1^-$ can contribute to the exclusive side of this sum rule and that

$$\rho^2 = \sum_n |\sigma^{(n)}(1)|^2 + \dots \quad (4.2)$$

(neglecting perturbative QCD corrections). The sum is over $s_l^{\pi_l} = 1^-$ radial excitations with excitation energies up to the scale Δ and the ellipses here and below refer to non-resonant contributions. In the large N_c limit ρ^2 is determined [11] and this sum rule is saturated by $|\sigma(1)|^2$ alone.

A similar statement about which excited states contribute can be made for the Voloshin type [22] ‘‘optical’’ sum rule for $\bar{\Lambda}$. Taking the first moment of the vector-vector ($J_\mu = V_\mu = \bar{c} \gamma_\mu b$) sum rule and $a^{\mu\nu} = -g^{\mu\nu} + v^\mu v^\nu$ we find

$$\begin{aligned}
\frac{3(w-1)^2}{2w^2} \bar{\Lambda} &= \frac{(-g^{\mu\nu} + v^\mu v^\nu)}{2} \sum_{s, s'} \sum_{X_c \neq \Lambda_c} (E_{X_c} - E_{\Lambda_c}) \\
& \times \frac{\langle \Lambda_b(v, s) | V_\mu^\dagger | X_c \rangle \langle X_c | V_\nu | \Lambda_b(v, s) \rangle}{4w m_{X_c} m_{\Lambda_b}}. \quad (4.3)
\end{aligned}$$

Here the excited charmed states $|X_c\rangle$ have four-velocity v' and spin s' . Spin symmetry will enable us to determine which baryonic states contribute to this $\bar{\Lambda}$ sum rule since only matrix elements which vanish as $(w-1)^2$ as $w \rightarrow 1$ give a nonzero contribution. States with unnatural parity cannot contribute since their matrix elements vanish identically in the infinite mass limit. For radial excitations of the ground state, the Isgur-Wise function must vanish at zero recoil and using spin symmetry we find that summed over spins $a^{\mu\nu} \langle \Lambda_b | V_\mu^\dagger | \Lambda_c^{(n)} \rangle \langle \Lambda_c^{(n)} | V_\nu | \Lambda_b \rangle \sim (w-1)^3$. We also find that states with $s_l \geq 2$ go at least as $(w-1)^3$, so only the $s_l^{\pi_l} = 1^-$ states can contribute. Using the matrix elements and form factors from Eqs. (2.1), (2.2), (2.18) and (2.19) we find that Eq. (4.3) gives

$$\bar{\Lambda} = 2 \sum_n (\bar{\Lambda}'^{(n)} - \bar{\Lambda}) |\sigma^{(n)}(1)|^2 + \dots \quad (4.4)$$

This agrees with the result which was found in Refs. [12, 23] using different methods.

A sum rule that bounds λ_1 can be derived by considering the vector current at zero recoil and working to order $\Lambda_{\text{QCD}}^2/m_Q^2$ on both the inclusive [24] and exclusive sides. For this case, following [18] we take a vector current and sum over the spatial components using $a_{\mu\nu} = -g^{\mu\nu} + v^\mu v^\nu$. Recalling that for the ground state baryons $\lambda_2 = 0$ we have

$$\begin{aligned}
-\lambda_1 &= \frac{1}{4} \left(\frac{1}{m_c^2} + \frac{1}{m_b^2} - \frac{2}{3m_c m_b} \right) \\
&= \frac{1}{6} \sum_{X_c} \sum_{s, s'} \frac{|\langle X_c(v, s') | V_i | \Lambda_b(v, s) \rangle|^2}{4m_{X_c} m_{\Lambda_b}}. \quad (4.5)
\end{aligned}$$

For any state with $s_l^{\pi_l} = 0^+$ the spatial component of the vector matrix element vanishes at zero recoil in the Λ_b rest frame. The same is true for states with $s_l^{\pi_l} = 1^+$. In Sec. III we pointed out that for states with $s_l^{\pi_l} = 0^-$ or $s_l^{\pi_l} \geq 2$ the matrix elements vanish at order Λ_{QCD}/m_Q . Therefore, again only states with $s_l^{\pi_l} = 1^-$ can contribute and we find

$$-\lambda_1 = 3 \sum_n (\bar{\Lambda}'^{(n)} - \bar{\Lambda})^2 |\sigma^{(n)}(1)|^2 + \dots \quad (4.6)$$

This agrees with the result of Ref. [25], even though the derivation there relied on orbital angular momentum being a good quantum number (which is true for large N_c) [26].

These sum rules can be used to place an interesting bound on $\bar{\Lambda}'$ and hence on the mass of the unobserved $s_l^{\pi_l} = 1^-$ excited baryon multiplet, \bar{m}'_{Λ_b} . Since the mass of the light

degrees of freedom $\bar{\Lambda}'^{(n)}$ increases with n Eqs. (4.4) and (4.6) can be combined to give

$$-\lambda_1 \geq \frac{3}{2} \bar{\Lambda} (\bar{\Lambda}' - \bar{\Lambda}). \quad (4.7)$$

This assumes there is a negligible contribution from non-resonant states with excitation energies less than $\bar{\Lambda}' - \bar{\Lambda}$. An upper bound on $\bar{\Lambda}'$ can then be obtained by using the mass formula, Eq. (1.1), and $m_b = m_c + 3.4$ GeV [10] to write λ_1 and $\bar{\Lambda}$ in terms of measured masses and m_c . For $m_c = 1.4$ GeV we have $\bar{\Lambda}' < 1$ GeV. Using Eq. (1.4) this translates into an upper bound on \bar{m}'_{Λ_b}

$$\bar{m}'_{\Lambda_b} < 5.86 \text{ GeV}, \quad (4.8)$$

which corresponds to a splitting $\Delta m'_{\Lambda_b} < 0.24$ GeV above the ground state Λ_b mass. These bounds are very sensitive to the value of m_c . Taking $m_c = 1.1$ GeV strengthens the bound giving $\bar{m}'_{\Lambda_b} < 5.79$ GeV while taking $m_c = 1.7$ GeV weakens the bound to $\bar{m}'_{\Lambda_b} < 6.01$ GeV. Note that perturbative corrections to the sum rules [27] have not been included here and could also give a sizeable correction to these bounds.

V. CONCLUSIONS

At zero recoil, the weak vector and axial-vector currents for Λ_b decay to a charmed baryon correspond to charges of the heavy quark spin-flavor symmetry. Therefore, in the $m_Q \rightarrow \infty$ limit, the zero recoil matrix elements of the weak current between a Λ_b and any excited charmed baryon vanish. At order Λ_{QCD}/m_Q , however, these matrix elements need not be zero. These Λ_{QCD}/m_Q corrections can play an important role, since most of the phase space is near zero recoil for these decays.

In this paper we studied the predictions of HQET for the $\Lambda_b \rightarrow \Lambda_c^{1/2} e \bar{\nu}_e$ and $\Lambda_b \rightarrow \Lambda_c^{3/2} e \bar{\nu}_e$ decays including order Λ_{QCD}/m_Q corrections to the matrix elements of the weak currents. Here $\Lambda_c^{1/2}$ and $\Lambda_c^{3/2}$ are excited charmed baryons with $s_l^{\pi_l} = 1^-$. At zero recoil these corrections can be written in terms of the leading, $m_Q \rightarrow \infty$, Isgur-Wise function, and measured baryon masses. In the large N_c limit of QCD, it is possible to calculate the Isgur-Wise function for heavy to heavy baryon decays, using the bound state soliton picture. Using this calculation, the shape of the differential w spectra, shown in Fig. 1, and the total decay rates were predicted at order Λ_{QCD}/m_Q . The contribution from the helicity $\pm 3/2$ states to the $\Lambda_c^{3/2}$ rate remains negligible at this order. We found that the total branching fraction for Λ_b decays to these states is 2.5–3.3%. Also, factorization was used to predict the decay rates for $\Lambda_b \rightarrow \Lambda_c^{1/2} \pi$ and $\Lambda_b \rightarrow \Lambda_c^{3/2} \pi$ giving a total branching fraction of 0.4–0.6%. The uncertainty from the unknown Λ_{QCD}/m_Q form factor σ_1 was found to be smaller in total branching fractions to the $s_l^{\pi_l} = 1^-$ states than in the individual rates to $\Lambda_c^{1/2}$ and $\Lambda_c^{3/2}$.

We considered the zero recoil matrix elements of weak currents between a Λ_b baryon and other excited charmed

baryons at order Λ_{QCD}/m_Q . Our results are summarized in Table II. For excitations where $s_l^{\pi_l} = 0^+, 1^+$ these matrix elements are nonzero. Only corrections to the states contribute, and these corrections were expressed in terms of matrix elements of local operators.

Heavy quark sum rules for Λ_b decays have contributions from excited charmed baryons. The Bjorken sum rule as well as sum rules for $\bar{\Lambda}$ and λ_1 have contributions only from excited states with $s_l^{\pi_l} = 1^-$. Combining sum rules for $\bar{\Lambda}$ and λ_1 , and using the HQET mass formula for heavy baryons, an upper bound on the spin-averaged mass for the $s_l^{\pi_l} = 1^-$ doublet of beautiful baryons was obtained in Eq. (4.8).

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APPENDIX: $\Lambda_b \rightarrow \Lambda_c^{1/2} e \bar{\nu}_e$ AND $\Lambda_b \rightarrow \Lambda_c^{3/2} e \bar{\nu}_e$ FOR $N_c \rightarrow \infty$

In this appendix we review the simplified description that occurs for Λ_Q baryons in the $N_c \rightarrow \infty$ limit [11,12], focusing on the part relevant for the decays $\Lambda_b \rightarrow \Lambda_c^{1/2} e \bar{\nu}_e$ and $\Lambda_b \rightarrow \Lambda_c^{3/2} e \bar{\nu}_e$. Using as input the observed mass splitting, $\Delta m_{\Lambda_c} = \bar{m}'_{\Lambda_c} - m_{\Lambda_c}$, it is possible to determine the corresponding splitting in the bottom sector, as well as the functions $\sigma(w)$ and $\hat{\phi}_{\text{kin}}^{(Q)}(w)$ discussed in the text. In the large N_c limit the $\Lambda_{c,b}$ states are described as bound states of a nucleon N (viewed as a soliton of the nonlinear chiral Lagrangian) and a heavy meson $D^{(*)}$ or $B^{(*)}$. The bound state dynamics are governed by the harmonic oscillator potential

$$V(\vec{x}) = V_0 + \frac{1}{2} \kappa \vec{x}^2, \quad (A1)$$

and the reduced mass $\mu_Q = (m_H^{-1} + m_N^{-1})^{-1}$ where $H = B$ or D . The parameters κ and μ_Q then determine the mass spectrum, with splittings $\Delta m = \sqrt{\kappa/\mu_Q}$ between excited multiplets. Using the experimental values $\Delta m_{\Lambda_c} = 0.33$ GeV, $m_D = \bar{m}_D = 1.971$ GeV and $m_N = 0.939$ GeV [2] determines $\kappa = (0.411 \text{ GeV})^3$. With $m_B = \bar{m}_B = 5.313$ GeV the prediction for the mass splitting in the bottom sector is then $\Delta m_{\Lambda_b} = 0.29$ GeV.

As the wave functions for the system are determined, form factors for the weak heavy-heavy baryon transition can be found by calculating the hadronic matrix element as an overlap integral. For instance, in the rest frame of the Λ_b and for excited Λ_c velocity \vec{v}' such that $\vec{v}'^2 \lesssim N_c^{-3/4}$ we have [12]

$$\begin{aligned} & \frac{\langle \Lambda_c^{1/2}(\vec{v}', m_s) | \bar{h}_v^{(c)} \gamma_0 h_v^{(b)} | \Lambda_b(m_s) \rangle}{\sqrt{4m_{\Lambda_c^{1/2}m_{\Lambda_b}}}} \\ &= -i \left(1, 0; \frac{1}{2}, m_s \middle| \frac{1}{2}, m_s \right) \int d^3q \varphi_c^*(\vec{q}) \varphi_b(\vec{q} - m_N \vec{v}'), \end{aligned} \quad (\text{A2})$$

where m_s is the magnetic spin quantum number with projection on the axis defined by \vec{v}' which we take to be the z axis. Here φ_b is the ground state harmonic oscillator wave function in momentum space,

$$\varphi_b(\vec{q}) = \pi^{-3/4} (\mu_b \kappa)^{-3/8} \exp(-\sqrt{\mu_b \kappa} \vec{q}^2 / 2), \quad (\text{A3})$$

and φ_c is the wave function for the $l=1$ orbitally excited state with z projection $m_l = m'_s - m_s = 0$:

$$\varphi_c(\vec{q}) = -i\sqrt{2} \pi^{-3/4} (\mu_c \kappa)^{-5/8} q_z \exp(-\sqrt{\mu_c \kappa} \vec{q}^2 / 2). \quad (\text{A4})$$

Doing the integral in Eq. (A2) gives

$$\begin{aligned} & \frac{\langle \Lambda_c^{1/2}(\vec{v}', m_s) | \bar{h}_v^{(c)} \gamma_0 h_v^{(b)} | \Lambda_b(m_s) \rangle}{\sqrt{4m_{\Lambda_c^{1/2}m_{\Lambda_b}}}} \\ &= -4 \left(1, 0; \frac{1}{2}, m_s \middle| \frac{1}{2}, m_s \right) v' \kappa^{-1/4} m_N \\ & \quad \times \frac{\mu_c^{5/8} \mu_b^{3/8}}{(\sqrt{\mu_c} + \sqrt{\mu_b})^{5/2}} \exp \left[\frac{-m_N^2 \kappa^{-1/2}}{(\sqrt{\mu_c} + \sqrt{\mu_b})} \frac{\vec{v}'^2}{2} \right]. \end{aligned} \quad (\text{A5})$$

We wish to consider corrections at order Λ_{QCD}/m_Q so we take the leading term in the mass formula in Eq. (1.1), $m_H = m_Q$. Furthermore, a heavy baryon has $N_c - 1$ light quarks, which generate the dominant contribution to the color field felt by the light degrees of freedom as $N_c \rightarrow \infty$. Therefore replacing the heavy quark by a light quark has a negligible effect on the light degrees of freedom [12], so we take $m_N = \bar{\Lambda}$. In the large N_c limit Λ_{QCD}/m_Q corrections from the current and from the part of the effective Lagrangian, $\delta\mathcal{L}$,

that breaks spin symmetry are sub-leading in N_c [11]. In Eq. (A2) the m_Q dependence in the wave functions does not break the spin symmetry, and the part going as Λ_{QCD}/m_Q therefore corresponds to $\phi_{\text{kin}}^{(Q)}$. Expanding the expression in Eq. (A5) about the infinite mass limit and taking $\vec{v}'^2 = w^2 - 1$ gives the $m_Q \rightarrow \infty$ result of Ref. [12]:⁵

$$\sigma(w) = \left(\frac{\bar{\Lambda}^3}{\kappa} \right)^{1/4} \frac{1}{\sqrt{w+1}} \exp \left[-\frac{1}{4} \sqrt{\frac{\bar{\Lambda}^3}{\kappa}} (w^2 - 1) \right]. \quad (\text{A6})$$

Plotting this function over the phase space, $1 < w < 1.3$, we see that the shape differs from that of the straight line,

$$\sigma(w) = 1.165 - 1.682(w - 1), \quad (\text{A7})$$

by less than 3%. At order Λ_{QCD}/m_Q we find

$$\phi_{\text{kin}}^{(c)}(w) = -\frac{\bar{\Lambda}}{8} \sqrt{\frac{\bar{\Lambda}^3}{\kappa}} (w^2 - 1) \sigma(w),$$

$$\phi_{\text{kin}}^{(b)}(w) = \left[\frac{\bar{\Lambda}}{2} - \frac{\bar{\Lambda}}{8} \sqrt{\frac{\bar{\Lambda}^3}{\kappa}} (w^2 - 1) \right] \sigma(w). \quad (\text{A8})$$

This allows a determination of the rescaled Isgur-Wise function $\tilde{\sigma}(w) = \sigma + \varepsilon_c \phi_{\text{kin}}^{(c)} + \varepsilon_b \phi_{\text{kin}}^{(b)}$. For $1 < w < 1.3$ the shape of $\tilde{\sigma}(w)$ differs from that of the straight line,

$$\tilde{\sigma}(w) = 1.214 - 1.971(w - 1), \quad (\text{A9})$$

by about 2%, except near $w = 1.3$ where it differs by 4%. The N_c power counting of Ref. [11] restricts the range of validity of equations Eqs. (A6) and (A8) to $w^2 \leq 1 + N_c^{-3/2}$. Despite this we will use Eqs. (A7) and (A9) for the entire phase space with the qualification that we expect less predictive power in the region further from zero recoil in any case.

⁵Unlike [12] in writing the expression for $\sigma(w)$ we have not used approximations that are appropriate near zero recoil such as $\vec{v}'^2 \approx 2(w - 1)$.

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