Declaration of Competing Interests

The authors acknowledge there are no conflicts of interest recorded.
A method to generate initial fault stresses for physics-based ground motion prediction consistent with regional seismicity

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Key Points:
• We introduce a practical approximation of initial fault stresses resulting from past background seismicity.
• Stress heterogeneity due to past seismicity leaves traceable signatures on rupture and ground motion.
• A stratified fault medium amplifies seismic radiation at high frequencies.

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Abstract

Near-field ground motion is the major blind spot of seismic hazard studies, mainly because of the challenges in accounting for source effects. Initial stress heterogeneity is an important component of physics-based approaches to ground motion prediction that represent source effects through dynamic earthquake rupture modeling. We hypothesize that stress heterogeneity on a fault primarily originates from past background seismicity. We develop a new method to generate stochastic stress distributions as a superposition of residual stresses left by previous ruptures that are consistent with regional distributions of earthquake size and hypocentral depth. We validate our method on $M_w$ 7 earthquake models suitable for California, by obtaining a satisfactory agreement with empirical earthquake scaling laws and ground motion prediction equations. To avoid the excessive seismic radiation produced by dynamic models with abrupt arrest at preset rupture borders, we achieve spontaneous rupture arrest by incorporating a scale-dependent fracture energy adjusted with fracture mechanics theory. Our analyses of rupture and ground motion reveal particular signatures of the initial stress heterogeneity: rupture can locally propagate at supershear speed near the highly-stressed areas; the position of high-stress and low-stress areas due to initial stress heterogeneity determines how the peak ground motion amplitudes and polarization spatially vary along the fault, as low-stress areas slows down the rupture, decrease stress drop, and change the radiation distribution before the rupture arrest. We also find that the medium stratification amplifies the moment rate spectrum at frequencies above 2 Hz, which requires understanding the interaction between site effects and rupture dynamics; therefore, we highlight the need to consider a realistic fault medium on future studies of rupture dynamics. Our approach advances our understanding of the relations between dynamic features of earthquake ruptures and the statistics of regional seismicity, and our capability to model source effects for near-field ground motion prediction studies.

1 Introduction

Ground motion prediction is key for earthquake engineering, but faces challenges in the vicinity of faults because of outstanding knowledge gaps in modeling earthquake rupture processes. The conventional approach in earthquake engineering is to estimate ground motion intensity measures at a certain distance from an active fault, for a future event with a given magnitude, by using empirical ground motion prediction equations (GMPEs) compiled through statistical analyses of available recordings of past earthquakes. Yet, these equations assume a lognormal distribution of ground motion estimates that is only described by a standard deviation, and do not provide any information about the physical limits. Additionally, the uncertainty of GMPEs can be high when the empirical data is scarce. Even in well-monitored areas, available data can be limited for large, rare, events, such as $M_7+$ earthquakes in California. Close to the rupture, the uncertainty is even higher because the strong spatial variability of source properties along a rupture — such as rupture direction, rupture speed and slip distribution — can have a large impact on ground motion, as inferred from worldwide observations (e.g., Ozacar & Beck, 2004; Courboulex et al., 2013; Bao et al., 2019) and supported by theoretical and numerical studies of rupture dynamics and consequent ground motions (e.g., Ben-Menahem & Harkrider, 1964; Archuleta & Hartzell, 1981; Somerville et al., 1997; Aagaard et al., 2004; Dunham & Bhat, 2008). Therefore, physics-based approaches, involving dynamic rupture modeling that can produce realistic variability of source properties, are a promising avenue to circumvent the data gap and to advance the predictability of near-field ground motions.

Dynamic rupture modeling requires setting initial fault stresses according to physical assumptions and prior information, and stochastic models for the generation of initial stresses have been previously proposed. Initial stress heterogeneity has been necessary to reproduce the ground motion recordings of past earthquakes (Ide & Takeo, 1997;
Andrews, 1980; Guatteri & Spudich, 2000; Nielsen & Olsen, 2000). Possible origins of stress heterogeneity include the residual stresses left by past seismicity, variations in fault zone pore fluid pressure and fault strength, and irregular fault geometry (Harris, 2004). Here, as a starting point, we focus on the former. Considering that each earthquake perturbs stresses in the vicinity of its rupture area, and that earthquake sizes follow the Gutenberg-Richter (GR) power-law distribution, Andrews (1980) reasoned that seismicity should maintain a heterogeneity of crustal stresses across all length scales, and introduced the concept of stochastic initial stresses in earthquake modeling (Andrews & Barall, 2011).

Mai & Beroza (2002) constrained the parameters of random initial stress fields using finite-fault slip inversion models of past earthquakes worldwide. Recent geological and geodetic observations of surface deformation during many earthquakes further support the possibility of such heterogeneous distribution (e.g., Manighetti et al., 2015; Milliner et al., 2015).

There are outstanding opportunities to advance physics-based ground motion modeling based on stochastic initial stresses. Many studies adopt the stochastic initial stress model of Andrews & Barall (2011), for rupture and ground motion analyses (e.g., Oglesby & Day, 2002; Ripperger et al., 2007, 2008; Aagaard & Heaton, 2008; Mena et al., 2012; Lozos et al., 2015; Andrews & Ma, 2016; Liu & Duan, 2018), for GMPE development in induced seismicity areas (Bydlon et al., 2019), for tsunami modeling (e.g., Geist et al., 2019). These studies show that initial stress heterogeneity has a direct effect on the main characteristics of dynamic rupture and ground motion. A detailed comparison of synthetics with GMPEs by Ripperger et al. (2008) did not reproduce spectral accelerations above 1 Hz. Guatteri & Spudich (2000), setting stochastic stress heterogeneity constrained by the earthquake slip database of Mai & Beroza (2002), report a good agreement between GMPEs and long period synthetics (> 3 s), but at shorter periods their synthetics underestimate the GMPEs. In Baumann & Dalguer (2014) where the stress heterogeneity follows a power law similar to Andrews (1980), the fit of short period synthetics to GMPEs is improved up to 1 Hz, but the simulated ground motions at distances below 10 km underestimate the GMPEs. Andrews & Ma (2016) find a good fit of their synthetics to GMPEs at 10 km distances, and highlight that the compatibility with GMPEs is controlled by the amplitude of the stress heterogeneity. Indeed, sensitivity analyses based on kinematic rupture models of Crempien & Archuleta (2018) show that a larger slip correlation length increases the ground motion variability. Also in dynamic rupture models, as underlined by Ripperger et al. (2008), the standard deviation of initial stresses substantially controls the final rupture size, slip fluctuations and rupture speed variations. Their result reveals the necessity to better constrain the parameters of stochastic stress models, in order to quantify the uncertainties associated with future earthquake ground motions. Enforcing physical constraints on a purely stochastic model, on the other hand, is challenging. Instead, here we consider a physics-based generation mechanism for stress heterogeneity driven by past seismicity.

We propose a new method to approximate the heterogeneous stresses induced by past seismicity, which allows us to introduce constraints based on regional seismicity data. A self-consistent approach to generate heterogeneous fault stresses would be to model earthquake cycles that produce realistic GR seismicity. Seismic cycle models on planar faults — with uniform friction properties — generate GR seismicity when the seismogenic zone is much larger than the nucleation length, i.e., the size of the smallest fault patch that can slip seismically (Barbot, 2019; Cattania, 2019). Yet, achieving such conditions in 3D earthquake cycle simulations remains a formidable computational challenge.

We overcome this challenge by constructing the stress field as the sum of residual stresses left by a stochastic distribution of past ruptures that is consistent with the statistical properties of regional seismicity. Our work is the 3D extension of a 2D methodology that was developed by Ruiz et al. (2008), which showed promise by its versatility to produce enhanced high-frequency radiation.
In the following, we first introduce our methodology. Then, we present our main findings on the effect of the initial stress heterogeneity on rupture and ground motion in the context of M 7 earthquakes in California. Next, we discuss the current limitations and potential extensions of our method. Finally, we present the conclusions of our work.

2 Methodology

2.1 Generation of initial fault stress heterogeneity

Our method to generate initial stresses on a given fault suitable for dynamic rupture modeling is built around a target earthquake with a given magnitude. For illustration purposes, our target is a $M_{w} 7$ earthquake on a generic (and idealized) vertical planar strike-slip fault in southern California. An example for our conceptualization is given in Fig. 1, for the seismicity of the San Jacinto fault since 1960, with each past earthquake within 10 km of the fault shown as a circular rupture. For our target model, we consider a fault with a length of 75 km and a depth of 25 km, which we discretize with sub-faults of 100 m size.

We first generate a set of past earthquakes that are representative of the background seismicity of the fault zone, consistently with the available regional seismicity data. We set the number of past earthquakes per magnitude such that, when adding the target earthquake, the magnitude distribution follows the GR law. The $b$-values in southern California lie around 1 (e.g., Hutton et al., 2010; Tormann et al., 2010; Field et al., 2014) and we set $b=0.95$ as obtained by Page & Felzer (2015) based on the instrumental data of M 4-6 earthquakes. Figure 2a shows an example of magnitude distribution for a $M_{w} 7$ target event.

We then randomly distribute the hypocenters of the past events on a fault grid. While real seismicity is distributed over a fault zone volume (Powers & Jordan, 2010) or a fault network, we consider that heterogeneous stresses on the target fault are dominated by stresses induced by those past events that are located at distances to the fault shorter than or comparable to their rupture sizes, and approximate those stresses as if the past ruptures were located on the target fault. We distribute the hypocentral depths according to the empirical depth distribution developed by Hauksson & Meier (2019) based on a recent catalog of southern California. The empirical distribution is provided on depth bins of 2 km (Figure 2c). We generate random depth values by sampling this empirical distribution (Figure 2b). The lateral position of the hypocenters is set randomly according to a uniform distribution. We represent each past event by a circular crack with uniform stress drop. The crack radius ($R$) is derived from the seismic moment ($M_0$) and an assumed stress drop ($\Delta \sigma$) by $M_0 = 16/7 \Delta \sigma R^3$. We set $\Delta \sigma = 3$ MPa, the average stress drop value for active tectonic regimes such as California (Kanamori & Anderson, 1975). Figure 2d shows the spatial distribution of past ruptures in the example set.

Next, we randomly generate an order of occurrence for the set of past earthquakes. This step introduces an additional stochastic layer in our method. Ruiz et al. (2008) constrained the order of occurrence by forcing the hypocenter position of the next event to lie in zones of high stress, above 50% of the background stress. For simplicity, here we entirely randomize the occurrence order.

We then compute the cumulative stress changes induced by the past events. For each event, we compute the changes in shear stress at each point of a preset fault grid based on the circular crack approximation, following the procedure detailed in Appendix A. This procedure gives the shear stress changes normalized by a constant value of normal stress. It assumes uniform values of static and dynamic friction coefficients, set here to 0.7 and 0.1, respectively. Figures 2e,f show the normalized along-strike and along-dip shear stresses, respectively, for our working example. The stress change is the lowest and most uniform inside the ruptures that occurred later in the sequence and that overlap...
little with subsequent ruptures. Hereafter, we refer to these areas of low stress as barriers. The largest stress values occur around barriers. The areas with less seismicity, mostly at depth, have relatively larger stress than the barriers.

We generated three different realizations of stochastic stresses for this study. Model I is a reference case that we calibrate for a $M_w 7$ event in Section 3.1. In Model II, we drew a new random set of hypocenter locations. We set the minimum magnitude of past events as 4.75 in these two models for numerical simplicity. In Model III, we lowered it to 3.5 for a sensitivity analysis and drew a new random set of hypocenter locations. The minimum crack radius is 1.35 km in Models I and II, and 0.32 km in Model III, such that the largest node spacing of both stress generation (0.1 km) and rupture modeling (0.585 km) grids are smaller than the minimum crack size (0.64 km). The total number of earthquakes equals 123 in Models I and II, and 1846 in Model III. Figure 3 shows the magnitude distribution, depth distribution, and normalized stress changes for the three models. Table 1 summarizes the ranges of magnitude and rupture size of past events in each model. The depth distributions of the past events in the three models are, by design, very similar to the reference distribution (Figure 2c). The largest and smallest stress spots differ between the models, owing to their different hypocentral distributions. In addition, Model III has smaller high-stress zones due to its larger number of small events that contribute extra barriers.

Finally, we compute absolute initial stresses, by multiplying the normalized stress changes with an assumed depth-dependent effective normal stress, as detailed in Appendix B. We adopt the effective normal stress profile of Shebalin & Narteau (2017), as shown in Figure S1, which accounts for non-hydrostatic pore fluid pressure that was discussed in Rice (1992).

### 2.2 3D modeling of rupture and wave propagation with initial stress heterogeneity

The heterogeneous initial stresses constructed in the previous section serve as input for the dynamic rupture modeling of the target event. We prepare a 3D mesh of the propagation medium containing the fault. We linearly interpolate the stress ratios from the stress-generation grid onto the dynamic rupture modeling fault grid. We assume that the fault behaviour is governed by linear slip-weakening friction.

We initiate the rupture in a highly stressed spot by an artificial time-weakening procedure. The hypocenter location of each model is shown in Fig. 3 and provided in Table 1. We located the hypocenter far from the fault lateral borders, with a random depth in agreement with the reference depth distribution. We enforce nucleation by a time-dependent reduction of the friction coefficient (linearly from static to dynamic) inside a circular ring that expands from the hypocenter up to a radius of 2 km, at a constant speed of 3 km/s, and with a width of 300 m.

To devise a procedure to control the arrest of the rupture, we refer to Griffith (1921)’s criterion: rupture stops if the static energy release rate, $G_0$, is smaller than the fracture energy, $G_c$. For a circular crack-like rupture of radius $R$ and stress drop $\Delta \sigma$, $G_0 \propto \Delta \sigma^2 R$.

In dynamic rupture models governed by fault friction, $G_c$ is constant if the frictional properties are uniform. However, $G_0$ increases as the rupture grows, thus it becomes increasingly difficult to stop the rupture. In natural fault zones, energy is dissipated not only by fault friction but also by off-fault inelastic deformation, which contributes to increase $G_c$. In dynamic rupture models with off-fault inelasticity, $G_c$ increases with rupture size (Andrews, 2005; Gabriel et al., 2013), which promotes rupture arrest if it overcomes the growth of $G_0$ with rupture size. Here we do not explicitly model off-fault plasticity but mimic its effect on $G_c$ by setting a spatial distribution of the slip-weakening distance $D_c$. 


that grows as a function of horizontal distance to the hypocenter \((x)\) as
\[
D_c(x) = D_{c,\text{min}} + (D_{c,\text{max}} - D_{c,\text{min}})(x/L)^\beta
\]  
(1)

where \(D_{c,\text{min}}\) and \(D_{c,\text{max}}\) are lower and upper values of \(D_c\), \(\beta\) is a power-law exponent that we set to 3, and \(L\) is the rupture distance at which \(D_{\text{max}}\) is reached. In our current modeling we only consider the horizontal distance \(x\); incorporating the vertical component and setting a smooth transition to velocity-strengthening behaviour at depth is part of our planned future work.

The distance-dependent \(D_c\) helps control rupture arrest at a target distance on average, but the eventual rupture path and final size of a given realization of our stochastic model are mainly controlled by its specific stress heterogeneity. We set the parameters controlling the \(D_c\) distribution and the average stress drop in Model I by trial-and-error in order to satisfy empirical constraints from scaling laws and GMPEs. The rupture lengths of earthquakes with \(M_w=7\) range from 28 to 84 km (Wells & Coppersmith, 1994). Following the benchmark procedures in the SCEC Dynamic Rupture Validation project (Withers et al., 2021), we considered a GMPE developed in Next Generation Attenuation-West2 (NGA-West2; Ancheta et al., 2014). Ground motion amplitude is known to correlate with stress drop (e.g. Cotton et al., 2013). As a starting point, we prepare initial stresses, as detailed in Appendix B. Next, we search for \(D_{\text{min}}\) and \(D_{\text{max}}\) values that give a final rupture length consistent with scaling laws. Then, we compare the simulated ground motions to GMPEs and, if necessary, we adjust the average stress drop by re-scaling the initial stresses. We specifically change \(\mu_d\) and \(\mu_s\) values, while keeping the same values of \(\mu_d\) and \(S\) as in Appendix A, where \(\mu_d\) and \(\mu_s\) stand for the background initial shear to normal stress ratio, static friction coefficient, and the strength excess to stress drop ratio (or strength parameter) defined by Das & Aki (1977), respectively. Since seismic moment scales with rupture area and stress drop, each time we change the stress drop to better fit the GMPEs, we also need to adjust the rupture area not to compromise the event magnitude. To do so, we tune \(D_{\text{min}}\) and \(D_{\text{max}}\) values, referring to the above-discussed balance between elastic and fracture energy. The parameter set that we determined in this procedure to get a \(M_w\) 7 event for Model I: \(D_{\text{min}}=0.10556\) m, \(D_{\text{max}}=0.72384\) m, \(L=21\) km. For other heterogeneous models, we use the same \(D_c\) function.

We model 3D wave propagation with a proper resolution up to 3 Hz. We use the 3D spectral element code SPECFEM3D for wave propagation and rupture dynamics (Tromp et al., 2008; Kaneko et al., 2008; Galvez et al., 2014). We set 5 GLL nodes for each spectral element (i.e., polynomial degree of 4). In post-processing, we Butterworth low-pass filtered all synthetics at 3 Hz. The propagation medium is vertically stratified: S-wave speed equals 760 m/s at the surface and gradually increases with depth (Figure S1), as prescribed in an ongoing SCEC community benchmark effort (Withers et al., 2021) following (Andrews & Ma, 2016). We set a fault length of 75 km which is sufficiently large for a \(M\) 7 event to allow for a smooth arrest at fault sides, and hence avoiding any artificial amplification of radiation by abrupt rupture arrest. We limited the fault depth to 18 km, consistently with the depth distribution of seismicity in California (Hauksson & Meier, 2019).

3 Results

3.1 A family of plausible rupture models for California

The model introduced here produces ground motions in agreement with the GMPE at periods between 0.5 and 3 s. We calculated the spectral accelerations at distances up to \(\sim 45\) km from the surface rupture (Joyner-Boore distance), at periods of 0.5, 1, and 3 s, and evaluated the orientation-independent geometric mean of horizontal response spectra, GMRotD50, defined by Boore et al. (2006). Figure 4 shows the comparison of
Model I synthetics with GMPEs. Considering the mean values, our synthetics are in satisfactory agreement with the GMPE. Additional comparisons for shorter periods (Fig. S2), show that the GMPE fit is also satisfactory at 0.4 s. The ground motion amplitude at equidistant stations can be different (Figure S3); as Ripperger et al. (2008) suggested, such intra-event variability is governed by the radiation pattern and the rupture directivity. When changing the stochastic distribution of stresses, we can expect changes in these rupture details, and the ground motion amplitude at a given station, i.e., the inter-event variability. Indeed, the standard deviation of spectral accelerations at a given distance can vary in Models II and III (Figures S4 and S5). Ripperger et al. (2008) suggest that the inter-event variability is mainly controlled by the hypocenter-station configuration rather than the changes in stress heterogeneity. Here rupture is bilateral and the hypocenter-station configuration is similar for our three heterogeneous models. The differences between the three models are slight and mostly visible below 10 km Joyner-Boore distance for 3 s period; the decay trend with respect to GMPE remains comparable.

By generating several stochastic realizations of the spatial distribution of past seismicity, we generate a set of earthquakes that are consistent with the regional statistics. After calibrating Model I, we kept the parameters controlling the distribution of $D_e$ the same in Models II and III. Figure 5 shows the evolution of slip in the three models. The rupture path and slip are strongly controlled by the locations of barriers and high-stress areas. We summarized the resulting magnitude, stress drop, and surface rupture length in Table 2. In Model II, the surface rupture length is slightly larger than in Model I, and the average stress drop is the same, 2.7 MPa, such that the resulting magnitude equals 7.05. In Model III, the rupture area and the average stress drop, 2.9 MPa, are larger, yielding a magnitude of 7.12. We verified that surface rupture lengths in each model are coherent with the empirical values documented by Wells & Coppersmith (1994). In summary, the source properties of the three models, generated with a single set of parameter values, are consistent with the empirical expectations about source scaling.

The source spectra (the Fourier amplitude of the moment-rate function) of the three models have a similar shape (Figure 6). All models exhibit high-frequency radiation induced by stress heterogeneity. Details of the rupture evolution in the three models are shown in supplementary figures: the evolution of slip rate in Figure S6, and the spatial distribution of peak slip velocity in Figures S7-S9. In particular, below $\sim$1 Hz, Model I is closer to the $\omega^{-2}$ model (Aki, 1967; Brune, 1970). We calculated the double corner frequencies as a function of $M_w$ after Ji & Archuleta (2021). We set the corner frequency of the $\omega^{-2}$ model as the geometric mean of the double corner frequencies. On the other hand, we see a significant amplification with respect to the $\omega^{-2}$ model above 2 Hz in all the models. We show in the Discussions section that this amplification is partially caused by the stratified medium.

The spatial spectra of our slip and initial stress distributions are within the expectations of previous studies. Mai & Beroza (2002) showed that the spatial spectra of earthquake slip inferred by finite source inversion can be approximated by Von Karman correlation functions. Following their approach, we fitted a Von Karman function to the Fourier spectra of slip of our three models. We obtained values of correlation length and Hurst exponent that are consistent with the values reported by Mai & Beroza (2002) for M 7 events (Figures S10-S12). Regarding the 2D Fourier spectrum of initial stresses, Andrews (1980) suggests a spectrum decay following a power law, with exponent $\nu < 1$. Given the linear relationship between slip gradients and stress implied by elasticity, the exponent of slip spectrum is $< \nu + 1$, hence $< 2$. Later, Lavallée & Archuleta (2003) calculate these values for a set of past earthquakes and report $0.78 \leq \nu \leq 1.71$. In our models, the exponent of slip and initial stress spectra follow a power law with approximate exponents of 0.7 and 1.7, respectively (Fig. S13), and therefore agrees with the previous studies. Moreover, Lavallée et al. (2006) evaluate the probability density func-
tion of slip in a set of past earthquakes and conclude that a non-Gaussian distribution better represents the observations, as opposed to the Gaussian distribution assumed in Andrews (1980). Here we found that the slip and initial stress distributions of our models are both non-Gaussian (provided in Fig. S14). The decay of initial stress distribution is smoother and has partial increase because of local stress concentrations, differently than a Gaussian distribution. A non-Gaussian function, such as Levy or Cauchy, works better for our models, as concluded in Lavallée & Archuleta (2003).

3.2 Local supershear at initially high stress patches

Initial stress heterogeneity causes variations in rupture speed, including supershear speeds. Figure 7 (middle) shows the spatial distribution of rupture speed in Model I. The rupture front accelerates in areas of high initial stress, such as the deeper parts with fewer barriers, and in barrier edges, and can reach supershear speeds. Yet, the resulting supershear ruptures are not sustained but transient. The average speed is 0.6 $V_s$, similar to the pseudo-dynamic models by Mena et al. (2012) that concluded the possibility of a local supershear because of stress heterogeneity. We obtain similar results in the other two models (Figures S15 and S16).

3.3 Spatial variability of ground motion amplitude and polarization

The spatial variability of ground motion amplitude is sensitive to the initial stress heterogeneity. We analyzed the peak ground velocity (PGV) in our models (Figure 8b for Model I) and in an additional homogeneous stress model (detailed in Supplemental Material) (Figure 8a). In the latter, the magnitude is $M_w$ 7, the surface rupture extends between -20 and 20 km along strike, and the largest PGV values lie along the surface rupture. At an approximate epicentral distance of 7 km, rupture accelerates due to free-surface effects (Figures S17 and S18) and radiates waves carrying large amplitudes to further off-fault distances. Upon rupture arrest at abrupt barriers, additional fronts produce large-amplitude ground motion at stations ahead of the two rupture directions. In the heterogeneous Model I, the spatial distribution of PGV is also heterogeneous and the largest values, exceeding 1 m/s, reach distances up to $\sim$4 km perpendicular to the fault. Such largest PGV areas are associated with the rupture segment in between the two largest barriers (Figure 7). Differently from the homogeneous model, the fronts carrying large PGV to farther off-fault distances start close to barriers distributed all along the rupture, instead of the fault ends. In Ripperger et al. (2008), the areas perpendicular to the fault plane and rupture direction in the band inside $x=[-20,20]$ km are found to be unaffected by directivity and exhibit most the inter-event differences in ground motion caused by stress heterogeneity. Indeed, here we can associate the changes in peak ground motion amplitudes with the event-specific stress heterogeneity. A different heterogeneous stress distribution (Models II and III) changes the PGV distribution, and in particular the location of the largest PGV values near the fault (shown in Figures S15 and S16), similar to the results by Oglesby & Day (2002) and Ripperger et al. (2008).

The orientation of the maximum amplitude of ground velocity, both in the near and far field, can change because of initial stress heterogeneity. Figures 8c,d show the spatial distribution of the peak ground motion polarization for homogeneous and heterogeneous models. Near the epicenter, the polarization is in the fault-parallel direction at stations that are located perpendicular to fault strike in both models. In the homogeneous model, the polarization switches to fault-normal direction where the slip rate increases upon surface rupture after 3 s (Fig. S18) as indicated by arrows in Fig. 8a. In the heterogeneous model, we notice a similar transition from fault-parallel to fault-normal component. The fault-normal polarization zone is larger in the heterogeneous case, and the difference between the two cases persists to Joyner-Boore distances larger than 20 km. The pattern of PGV polarization is event-specific, depending on the spatial distribution of barriers, supporting the findings of Oglesby & Day (2002).
4 Discussion

One limitation of our current study is the lack of unilateral ruptures and additional
source complexities. Rupture is bilateral and the hypocenter depth is comparable (5-7
km) in our models. Verification of our results for different hypocenter locations is one
of the primary perspectives. To focus on initial stress heterogeneity, we also excluded
other potential factors such as multi-segmented faults, fault roughness, presence of fault-
damage zones, and non-uniform frictional properties. Understanding how the coupling
of these additional factors impact the rupture process and ground motions requires fur-
ther research.

To guide future studies that require generating a large set of earthquakes of a given
magnitude, we performed a sensitivity analysis by changing various features of the model
settings: hypocenter location, $L$ parameter in equation 1, strength parameter $S$, and the
order of occurrence of past events. The resulting magnitudes range from 6.9 to 7.3. Only
for the case of a larger $S$ value, rupture fails to propagate upon nucleation (Figs. S19-
S24). A more systematic study is warranted to evaluate the model performance within
a comprehensive set of rupture models considering the above factors.

Medium stratification amplifies high-frequency radiation and ground motion; a bet-
ter description of the propagation medium in terms of site geometry and stratification
can help to constrain high-frequency radiation. Our heterogeneous models generate en-
hanced high-frequency radiation, but compared to $\omega^{-2}$ decay, all models have larger spec-
tral amplitudes above 2 Hz. To better understand the possible origins of such excessive
high-frequency radiation, besides the stress heterogeneity, we created two homogeneous
stress cases that only differ by the presence of medium stratification. Comparing their
spectra (Figure 9), we identified that medium stratification alone can amplify the source
spectrum at high frequencies, by more than a factor of 10, and consequent ground mo-
tions (Figures S25 and S26). The model with stratification exceeds the estimations of
the empirical models $\omega^{-2}$ and Ji & Archuleta (2021) particularly above 2 Hz. Our ad-
ditional analyses (Figures S27-S30) show that the peak slip rate in the stratified model
is larger than in the homogeneous model, and such relative amplification increases to-
wards the surface, up to a factor of 10 at the surface. Despite the weaker rigidity in the
stratified model, the increase in slip rate at shallow depths amplifies the moment rate
above 2 Hz. To clarify whether we can improve the synthetic source spectra with respect
to the references when using a realistic medium, without compromising the GMPE fit,
we note the necessity to consider a realistic fault zone and wave propagation medium.
The effect of crustal velocity structure on dynamic rupture and source properties is in-
creasingly recognized (e.g., Prada et al., 2021; Huang, 2021). This effect is exacerbated
in our examples, because we did not include the competing effects of cohesion and fric-
tional strengthening at shallow depth. For southern California, subsurface velocity mod-
els are available (SCEDC, 2013), as well as recent refined models for the Ridgecrest area
as those derived from dense array data by White et al. (2021) and from distributed acous-
tic sensing data by Yang et al. (2022). Our planned future work includes the assessment
of the impact of medium heterogeneity on rupture dynamics and seismic radiation in south-
ern California.

A natural extension of our work is to study the effect of low velocity, damaged fault
zones on rupture speed. Damaged fault zones can facilitate sustained supershear rup-
tures (Huang et al., 2016; Oral et al., 2020). California is one of the regions where such
damage zones are well documented (e.g., Cochran et al., 2009; Lewis & Ben-Zion, 2010).
Very recently, co-seismic and pre-seismic damage zones with notable along-strike and off-
fault variability were identified on the fault system that hosted the 2019 M 7.1 Ridge-
crest earthquake (Qiu et al., 2021; Rodriguez Padilla et al., 2022). Accounting for the
documented damage zones in California, a primary question to address in a future study
is whether the local transitions of speed due to initial stress heterogeneity can evolve to
sustained supershear ruptures.
Applying our method to other regions with different Gutenberg-Richter b-values or seismotectonic settings, as a further verification of the applicability of our method, is another perspective. We focused on \( M_w 7 \) strike-slip earthquakes in California and obtained promising results constrained by properties of the regional seismicity. Extending our study to other regions with the available data of ground motion and past seismicity stands as a further test to validate the applicability of our method.

5 Conclusions

We develop a new method to generate heterogeneous initial fault stresses for dynamic rupture modeling consistent with statistical properties of regional seismicity. Our modeling gives promising results for \( M_w 7-7.1 \) strike-slip earthquakes in California, producing ground motions and rupture characteristics consistent with empirical relations.

We determined certain signatures of the initial stress heterogeneity on the rupture process and consequent ground motion. Variations in rupture speed can locally reach supershear speed, where the stress is relatively large, around barriers or in areas with less barriers. The largest peak ground motion amplitudes expand to larger distance from the fault than in a homogeneous stress model. The spatial patterns of PGV amplitude and polarization are sensitive to the location of stress barriers, and hence event-specific.

We also find a feedback between site effects and source effects: the medium stratification can amplify the source spectrum at high frequencies (above 2 Hz). To better understand the medium effect on rupture and consequent ground motion, we suggest for future dynamic rupture studies to adopt realistic descriptions of the propagation medium.

6 Data and resources

SPECFEM3D is available at https://github.com/geodynamics/specfem3d/tree/devel. The modifications that we made can be found https://github.com/elifo/specfem3d upon the publication of our manuscript. The supplemental material includes one section of homogeneous models and thirty figures.

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<table>
<thead>
<tr>
<th>Model</th>
<th>$M_w$</th>
<th>Radius (km)</th>
<th>Hypocenter (km, km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>4.75-6.5</td>
<td>1.35-10.11</td>
<td>(1.45, -7.0)</td>
</tr>
<tr>
<td>II</td>
<td>4.75-6.5</td>
<td>1.35-10.11</td>
<td>(-2.70, -5.4)</td>
</tr>
<tr>
<td>III</td>
<td>3.5-6.5</td>
<td>0.32-10.11</td>
<td>(1, -7)</td>
</tr>
</tbody>
</table>

Table 2: Rupture properties of the three heterogeneous models.

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_w$</th>
<th>Ave. stress drop (MPa)</th>
<th>Surface rupture (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>7.00</td>
<td>2.7</td>
<td>49.7</td>
</tr>
<tr>
<td>II</td>
<td>7.05</td>
<td>2.7</td>
<td>52.8</td>
</tr>
<tr>
<td>III</td>
<td>7.12</td>
<td>2.9</td>
<td>63.5</td>
</tr>
</tbody>
</table>
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Appendix A  Stress change induced by each background earthquake

We treat each past earthquake as a circular rupture with uniform stress drop. The slip $\Delta u$ at a distance $r < a$ from the center of a rupture of radius $a$ is (Keilis-Borok & Monin, 1959):

$$\Delta u(r) = u_{\text{max}}(1 - r^2/a^2)^{1/2} \quad (A1)$$

where $u_{\text{max}}$ is the maximum slip.

The stress change on the fault at $(x,y) = (r \cos \alpha, r \sin \alpha)$ is (Sato, 1972; Singh, 1977):

$$\Delta \sigma = 1/2 \cdot (\pi \cdot a/2)^{1/2} \mu u_{\text{max}} [K_0 I_0(r/a) - K_1 \cos(2\alpha) I_2(r/a)] \quad (A2)$$

where $K_0 = \gamma + 1/2$, $K_1 = \gamma - 1/2$, $\gamma = (\lambda + \mu)/(\lambda + 2\mu)$, $\lambda$ and $\mu$ are Lame constants, $I_0$ and $I_2$ are integrals provided by Singh (1977) for all values of $r/a$.

The integrals are singular at $r = a$. To avoid this singularity, we convolve the shear stresses with a unitary area boxcar function of width $da$. The convolution integral is applied along the $r$ variable. The resulting regularized shear stresses used in the numerical computation are:

$$\tau_{12} = -C \left\{ \begin{array}{ll} K_0 \left( \frac{\pi}{2} \right)^{1/2} & r < a \\
K_0 \left( \frac{\pi}{2} \right)^{1/2} \left[ I_0(r) - I_2(r) \right] + \frac{f(r-a)}{da} & a \leq r < a + da \\
\frac{1}{da} & r \geq a + da \end{array} \right. \quad (A3)$$

$$\tau_{23} = -C \left\{ \begin{array}{ll} 0 & r < a \\
K_1 \left( \frac{2a}{a} \right)^{1/2} \sin(2\alpha) \left[ I_2(r) - I_2(r-da) \right] \frac{1}{da} & a \leq r < a + da \\
K_1 \left( \frac{2a}{a} \right)^{1/2} \sin(2\alpha) \left[ I_2(r) - I_2(r-da) \right] \frac{1}{da} & r \geq a + da \end{array} \right. \quad (A4)$$

where $\tau_{12}$ and $\tau_{23}$ are shear stress along strike and dip, respectively; $C$ scalar and the functions $I_1(r)$, $I_2(r)$, and $f(r)$ are set as follows:

$$C = \frac{1}{2} \left( \frac{\pi a}{2} \right)^{1/2} \mu u_{\text{max}} \quad (A5)$$

$$f(r) = K_0 \left( \frac{2}{\pi a} \right)^{1/2} I_1(r) - K_1 \left( \frac{2a}{a} \right)^{1/2} \cos(2\alpha) I_2(r) \quad (A6)$$

$$I_2(r) = \sqrt{r^2 - a^2} / ar \quad (A7)$$

$$I_1(r) = -r \arctan \left( \frac{a}{\sqrt{r^2 - a^2}} \right) \quad (A8)$$

After calculating the stress changes for a past event, we scale the stress values by a factor $r_{\text{scal}}$ to set the maximum stress change equal to a cut-off value, $\tau_{\text{max}}$:

$$r_{\text{scal}} = (\tau_{\text{max}}/\max(\tau_{12}) - 1)(\tau_{12} + \mu)/(\max(\tau_{12}) + \mu) \quad (A9)$$

We set $\tau_{\text{max}} = \mu S$ where $S = (\mu_s - \mu_b)/(\mu_b - \mu_d)$. Here, $S = 2.33$ ($\mu_s = 0.7$, $\mu_d = 0.1$ and $\mu_b = 0.28$).
After each event, we set the cumulative stress equal to $\tau_{12}$ and $\tau_{23}$ inside its rupture. Outside the rupture, we add the changes $\tau_{12}$ and $\tau_{23}$ to the pre-existing shear stresses and set the upper limit for $\tau_{12}$ as $\tau_{\text{max}}$.

### Appendix B Adjusting stress ratios to the 3D model

The procedure in Appendix A is based on the assumption of a spatially uniform medium. We adjust the shear stresses to our depth-dependent model, such that we derive stress ratios $r_{12,23} = \tau_{12,23}/\mu$. We determine the initial stresses, $\tau_0$, at each point of the fault grid in the 3D mesh by the following equation, such that the maximum stress equals the static strength, $\mu_s\sigma'$:

$$
\tau_{0,12,23} = \sigma' \cdot [\mu_b + r_{12,23}(\mu_b - \mu_d)]
$$

where $\sigma'$ is the effective stress. We set the upper limit of shear stress slightly below the static strength as $\text{max}(\tau) = \tau_s - 0.1(\tau_s - \tau_d)$, where $\tau_s$ and $\tau_d = \mu_d\sigma'$ are static and dynamic shear strength, respectively. We do this by cutting down the values that exceed this threshold.