Differential rotation in Jupiter’s interior revealed by simultaneous inversion for the magnetic field and zonal flux velocity

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Key Points:

\begin{itemize}
  \item Magnetometer data from the Juno spacecraft exhibit clear indications of secular variation of Jupiter’s magnetic field.
  \item Although a large part of the apparent secular variation can be explained by a solid body prograde rotation of approximately 0.1°/yr, there is evidence of latitude-dependent zonal drift of the magnetic field.
  \item The recent secular variation we find agrees well with that determined over much longer timescales.
\end{itemize}

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Abstract

A key objective of the current Juno mission (Bolton et al., 2017) is the direct determination of the secular variation (time dependency) of Jupiter’s internal magnetic field in order to further understand the dynamics of Jupiter’s interior. Here, we find that the residuals to a static, baseline model of the magnetic field are consistent with the effects of secular variation, specifically secular variation arising from zonal drift of the field. We present a technique for simultaneously inverting for the main magnetic field and secular variation due to zonal drift of the field. We explore the required drift systematically and argue that although the drift is dominated by a prograde super-rotation, corresponding to approximately 1 part in $10^6$ relative to System IIIa (1965), there is also evidence for differential drift of the field. We compare the resultant secular variation with that determined by Moore et al. (2019) and Connerney et al. (2022) and find good agreement. This suggests that the drift rate of Jupiter’s magnetic field is steady over time periods of several decades, though short period secular variation (such as that resulting from torsional oscillations) superimposed on this steady secular variation is still possible.

Plain Language Summary

Magnetometer data from the Juno spacecraft in orbit around Jupiter show clear signs of time-dependency in Jupiter’s magnetic field. Although a large part of the time-dependency can be explained by a small change in the rotation rate of the reference frame, there remains a signal of time-dependency after solving for such a correction. This remaining time-dependency represents true secular variation of the magnetic field, instead of an apparent secular variation resulting from a slightly incorrect choice of rotation rate. The true secular variation results from a simple latitude-dependent zonal drift of the field, and includes an equatorial jet that is symmetric about the equator.

1 Introduction

Observations of a planet’s magnetic field provide insight into dynamical processes deep in the planet’s interior. Mapping the secular variation of the field, extends those insights well beyond what is provided by a single snapshot of the field in time. Until recently, the only planet for which it has been possible to map secular variation was Earth. That changed to some extent with the work of Ridley and Holme (2016) and more definitively with the work of Moore et al. (2019), who compared the magnetic field calculated using a Jupiter reference model based on Juno data (Connerney et al., 2018) with observations from much earlier spacecraft missions extending back almost fifty years, and found compelling evidence of secular variation. They detected peak secular variation of approximately $15\mu T/yr$ at the planet’s surface, about 100 times larger than the surface secular variation of Earth’s magnetic field. Further, they found it to be consistent with zonal drift of the planet’s magnetic field, with the pattern of drift itself consistent with a downward projection (parallel to the rotation axis) of the planet’s observed surface winds (though with a velocity four orders of magnitude smaller).

While that study found evidence of secular variation in Jupiter’s magnetic field, a primary objective of the Juno mission is the direct mapping of secular variation from...
An important question is whether secular variation averaged over almost 50 years is representative of more recent secular variation over a shorter time interval. We note that the strong secular variation inferred over the last several decades implies that Jupiter’s surface field may have changed by as much as 75 $\mu$T (i.e. ~1% of the surface field strength) over the first five years of the Juno mission. Thus, it should be possible, but remains an open question, to detect secular variation directly from Juno magnetometer data. Recent support for the ability to detect secular variation directly from Juno data comes from (Connerney et al., 2022). They difference the JRM09 reference model based on PJ01 through PJ09 (where PJ number refers to the perijove number) and the more recent JRM33 reference model based on PJ01 through PJ33 and find that the difference in the field models is chiefly in a region proximal to the Great Blue Spot (GBS). (Throughout this paper we use the naming of features in Jupiter’s magnetic field introduced by Moore et al. (2018)). Given that the two models have a different central time a natural interpretation is that the difference is due to secular variation. Furthermore, the model difference is not only concentrated around the GBS but is similar in pattern to the secular variation that would result from an eastward drift of the spot, both of which are consistent with the findings of Moore et al. (2019).

This paper is organized as follows: we begin in Section 2 by considering evidence of the effects of secular variation on Juno magnetometer data by examining the along-track residuals to a static baseline model. Then, in Section 3, we describe a new method for solving simultaneously for Jupiter’s main magnetic field and secular variation due to zonal drift of the field. We first use this method to determine the best fitting solid body rotation and then proceed to consider more complex models of the zonal drift. Finally, we interpret our results.

2 A baseline model of Jupiter’s magnetic field

We first solve for a static baseline model of Jupiter’s magnetic field which we use to examine the extent to which a static (i.e. non time-dependent) model fits the Juno observations. We use magnetometer data (Connerney et al., 2017) from PJ01 in August 2016 through PJ33 in April 2021 (with the exception of PJ02 for which no data was collected), resulting in a mean time for the dataset of 2019.0: we refer to this as the baseline epoch. We then select data within 2.0 $R_J$ of the jovi-center and down-sample the data in time to 1s resolution. In addition, we remove a model of Jupiter’s external magnetodisc field from the data (Connerney et al., 1981; Edwards et al., 2001; Connerney et al., 2020). The magnetodisc parameters we adopt are $[R_0, R_1, \mu_0 I_0/2, D] = [5, 50, 225, 2.5]$ where $R_0$ and $R_1$ are the disk inner and outer radii respectively, $D$ is the half-thickness of the disk, and $\mu_0 I_0/2$ is the current constant. The unit for $R_0$, $R_1$, and $D$ is $R_J$, while that for the current constant is nT. The magnetodisc is assumed to be aligned with the magnetic dipole-axis defined by JRM09 (Connerney et al., 2017). We assign equal weights to the data given that the misfit is likely to be dominated by effects other than uncorrelated Gaussian noise in the data, including, and most perniciously, unmodelled fields that contribute to the data, such as small-scale structure in the field. For this reason we do not present a formal model covariance matrix for our results, as these would likely be

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grossly-underestimate the true uncertainties. In all the plots of residuals that follow in this paper the residuals are clearly not independent.

Figure 1: Baseline model: Radial component, $B_r$, of the magnetic field at 0.9 $R_J$ at 2019.0. The projection is a global Hammer equal-area with the central meridian at 180° in System III coordinates. The spacing of the lines of latitude is 45° and of longitude 60°. The colorscale is linear between the indicated limits.

In Figure 1 we show the baseline model at 0.9 $R_J$ from which we can see that it is extremely similar to models based on data from PJ01 to PJ09 (Connerney et al., 2018; Moore et al., 2018), in fact, superficially almost indistinguishable from the regularized solution of Moore et al. (2018) (cf. their Figure 1e), which was produced using the same regularized inversion method as this model. The similarity might be taken to suggest that the secular variation is weak, but a change in the field of 1% is hard to distinguish by comparing maps. Furthermore, when comparing maps other factors such as the differing regularization (or truncation) may have a greater effect on the comparison than secular variation. Either way, our baseline model should be considered a very conservative solution — despite being based on 32 orbits it is very similar to a solution based on the initial 8 orbits.

The rms misfit of this model to the data is 768 nT, which is considerably larger than the estimated uncertainty in the data of 0.01% (Connerney et al., 2017), suggesting that there is additional signal in the data that the baseline model does not adequately capture; the number of degrees of freedom in this solution is 323. To investigate this remaining signal further, in Figure 2 we overlay the baseline model with the along-track residuals to the radial component of the magnetic field. In the uppermost panel we show the first ten tracks used in the model, in the middle panel the next eleven tracks, and in the lowermost panel the last eleven tracks used. It is immediately apparent that the resid-
uals are largest near the Northern Hemisphere Flux Band (NHFB) and near the Great Blue Spot (GBS). The association of large residuals with the NHFB or the GBS has three possible, non-mutually exclusive explanations: one is simply that the mean residuals are largest where the field is strongest; another is that there is small-scale structure in the field that is not well-represented by the baseline model; and, finally, that the misfit is due to secular variation.

Two additional aspects of the residuals are apparent: first, there are large amplitude (up to nearly 10000 nT) short wavelength (approximately 15° in latitude) oscillations; second there is a mean residual (i.e. a systematic offset) when averaging over two or three of the short wavelength oscillations. We discuss the short wavelength oscillations later; for now we focus on the mean residuals. In the lowermost panel of Figure 2, comparing PJ24 and PJ26 near the GBS, we see a mean positive residual for PJ24, which passes slightly to the west of the GBS, and a mean negative residual for PJ26, which passes to the east. In other words, the PJ24 radial field data are more positive (less negative) with respect to the baseline model and the PJ26 radial field data are more negative with respect to the baseline model. This pattern is consistent with eastward drift of the (negatively-signed) GBS so that the radial field to its west is growing less negative with time and the radial field to its east more negative with time. Furthermore, this pattern is inconsistent with magnetometer drift, given that PJ24 and PJ26 show oppositely signed residuals relative to the baseline model while sampling radial field of the same sign. Looking at the uppermost panel, PJ01 is also consistent with this picture, noting that PJ01 precedes the baseline epoch and hence its residual with respect to the baseline model is of opposite sign. In the middle panel, PJ14 passes slightly east of the center of the GBS though the amplitude is larger than might be expected given that it is close (five months) to the baseline epoch. However, where PJ14 passes over the GBS where the azimuthal field gradient is very large and so a small time difference might nonetheless result in substantial secular variation.

A similar picture emerges from the NHFB. In the uppermost panel, PJ09, PJ11 and PJ05 each to the west of the flux concentration within the NHFB have positive residuals, while PJ06 to its east has a negative residual. This is consistent with eastward drift of the flux concentration within the NHFB so that the tracks to the west recorded a more positive radial field than that at the baseline epoch while PJ06 saw a weaker radial field. In the lowermost panel, PJ30 and PJ23 to the west and PJ25 and PJ31 to the east are again consistent with this picture. PJ29, in between the western and eastern PJs, is more complicated.

This analysis of the residuals is consistent with the inference of zonal drift of the field (Moore et al., 2019; Connerney et al., 2022), and suggests the need to consider the effects of secular variation. It also suggests that if secular variation were ignored, then the effects of secular variation could be mapped into spurious azimuthal field structure. For example, spatially-close but temporally-separated tracks such as PJ11 and PJ30 have oppositely-signed residuals, which if fit by a static model would result in small-scale azimuthal structure on the scale of the separation of these tracks. Here, we wish to address whether the residuals are consistent with a simple solid-body rotation of the field (in other words demanding a different rotation rate from that of System III) or whether a more complicated pattern of zonal drift (involving differential drift) is required.
Figure 2: Residuals of the radial component of the magnetic field data along track after subtracting the baseline model. The residuals, here calculated every 15s, are plotted along track, with positive residuals plotted to the west of the track and negative residuals to the east of the track as the spacecraft passes through periapsis from north to south. The baseline model is shown in the background. The projection is cylindrical with central meridian at 180° in System III coordinates. The colorscale is as in Figure 1. The bar below the colorscale depicts the residual scale. The uppermost panel shows the first ten orbits used in the baseline model, the middle panel the next eleven orbits, and in the lowermost panel the last eleven orbits. Thus, all the tracks in the upper panel are before the baseline epoch and all those in the lower panel are after the baseline epoch.
3 Modeling the secular variation of Jupiter’s magnetic field

The approach to mapping the secular variation that is most commonly used for studies of the Earth’s magnetic field is to simultaneously invert for the planet’s main field and its secular variation (Cain et al., 1965). In such an inversion, the secular variation would be represented by, for example, a Taylor series expansion of the main field coefficients in time or by an expansion of the spatial coefficients with a set of temporal basis functions (e.g. B-splines). Then, the field and its secular variation can be used to invert for models of the flow at the top of the dynamo region using the frozen-flux form of the magnetic induction equation (Roberts & Scott, 1965; Backus, 1968). This approach has been tried for Jupiter (Ridley & Holme, 2016), though prior to the availability of Juno data. However, such simultaneous inversion for the main field and secular variation greatly increases the number of coefficients that must be determined. For example, the CHAOS-7 model (Finlay et al., 2020) of the Earth’s magnetic field, which spans 33 years, expands each spatial coefficient in a series consisting of over 200 terms. Even the inclusion of a simple linear secular variation model would double the number of coefficients. As a result, incorporating secular variation directly would require data that are extremely well-distributed in both space and time. Juno magnetometer data are heterogeneously distributed, and so such an approach is likely infeasible, at least at this stage in the Juno mission.

Here we propose a different approach. Given that both the secular variation found by Moore et al. (2019) and by Connerney et al. (2022) and the pattern of residuals described above are consistent with a simple zonal drift of the field, we adopt a model in which the secular variation is assumed to be due to latitude-dependent zonal drift of the field. We invert simultaneously for a static spatial model of Jupiter’s magnetic field and for the zonal drift. This represents a very parsimonious approach to inverting for the secular variation since only a small number of additional parameters are required in order to parameterize the drift. While the drift is entirely zonal, the resultant magnetic secular variation is purely non-axisymmetric, and so we obtain no information on the secular variation of the axisymmetric part of the magnetic field, including the axial dipole (but see Ridley and Holme (2016) for a solution that includes that part of the field).

The drift of Jupiter’s magnetic field may arise, at least in part, in a region in which the frozen flux approximation (Roberts & Scott, 1965; Backus, 1968) is unlikely to hold. In contrast to the situation in the Earth where the electrical conductivity jumps abruptly at the core-mantle boundary, the electrical conductivity in Jupiter increases smoothly, though rapidly in a certain radius band, with depth. The secular variation likely arises (Moore et al., 2019) within a semi-conducting region (French et al., 2012) in which flux is not frozen on the multi-year timescales of interest here. Our aim here, though, is simply to parameterize the drift of the field rather than to invert for the fluid flow that gives rise, along with diffusion, to the drift.

The time dependency of the magnetic field is described by the magnetic induction equation

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) + \nabla \times (\eta \nabla \times B)$$ (1)
where $B$ and $u$ are the magnetic field and fluid velocity respectively, and $\eta$ is the magnetic diffusivity. The effects of advection of the field are represented by the first term on the right hand side and the effects of diffusion by the second term. In the frozen flux approximation the effects of diffusion are neglected. Here, though, we have argued that they are likely non-negligible. Following Wilmot-Smith et al. (2005), we introduce a flux velocity $w$ given by

$$\nabla \times (w \times B) = \nabla \times (u \times B) + \nabla \times (\eta \nabla \times B).$$

(2)

The flux velocity represents the combined effects of induction and the effects of diffusion. Wilmot-Smith et al. (2005) discuss sufficient conditions for the existence of solutions to Equation 2. Our aim here is simply to construct solutions to this equation which are necessarily approximate, while recognizing that not all diffusive fields are necessarily consistent with Equation 2 (Wilmot-Smith et al., 2005), for example diffusive decay in the absence of fluid flow. In the situation at hand, namely purely zonal flow, the effects of diffusion most likely take the form of a slippage of field lines, so that the flux velocity is likely less than the flow velocity (Moore et al., 2019). In addition, it seems implausible that zonal flux velocity could result purely from diffusion. Our inversion will yield the zonal pattern of flux line motion that best fits the combined effects of fluid flow and diffusion acting on the field. We have

$$\frac{\partial B_r}{\partial t} = -\frac{1}{r \sin \theta} w_\phi(\theta) \frac{\partial B_r}{\partial \phi}(\theta, \phi)$$

(3)

where $w_\phi(\theta)$ is the zonal flux velocity (assumed steady in time), and $(r, \theta, \phi)$ are spherical polar coordinates. Note that strong secular variation is generated where the field has strong azimuthal gradients.

We represent the magnetic field using a standard spherical harmonic expansion with coefficients $g$, and we represent the flux velocity by

$$w_\phi = -\sum_\ell v_\ell \frac{dP_\ell^0(\cos \theta)}{d\theta}$$

(4)

where the $P_\ell^0(\cos \theta)$ are associated Legendre polynomials with coefficients $v = \{v_\ell\}$. Equation 3 can then be written in two forms

$$\partial_t g = G(g) v$$

(5)

$$= V(v) g$$

(6)

The elements of the matrices $G(g)$ and $V(v)$ depend on Elsasser integrals; see, for example, Jackson and Bloxham (1991).

Integrating in time we have

$$g(t) = g(t_0) + \left( \int_{t_0}^t G(g) \, dt \right) v$$

(7)
for the first form (Equation 5) and

\[ g(t) = g(t_0) + \int_{t_0}^{t} V(v) g \, dt \] (8)

\[ g(t) = g(t_0) + V(v) \int_{t_0}^{t} g \, dt \] (9)

for the second form (Equation 6).

If the data are represented by a vector \( y \), then for a particular datum \( y_i \) at coordinates \((r_i, \theta_i, \phi_i, t_i)\), we have

\[ y_i = d(r_i, \theta_i, \phi_i)^T g(t_i) \] (10)

where the data kernels \( d(r_i, \theta_i, \phi_i) \) consist of a product of functions of radius (for example \((1/r)^{l+2}\)) and associated Legendre polynomials (and their derivatives). For data from a particular perijove, we set \( t_i = t_{PJ} \) where \( t_{PJ} \) is a representative time for that perijove. This is a reasonable approximation since the time between perijoves (roughly 53 days) is much greater than the time span from which we select data within a particular perijove (roughly 2 hours).

Our goal is to solve the nonlinear inverse problem simultaneously for the field coefficients \( g_0 = g(t_0) \) and for the flux velocity coefficients \( v \). To linearize the problem, we need to find the derivatives of \( y_i \) with respect to \( g_0 \) and \( v \), which requires finding the derivatives of \( g(t) \). These derivatives are given by

\[ \frac{\partial g_i}{\partial v} = \left( \frac{\partial}{\partial v} \int_{t_0}^{t} G(g(t)) \, dt \right) + \int_{t_0}^{t_i} G(g(t)) \, dt \] (11)

and

\[ \frac{\partial g_i}{\partial g_0} = \left[ 1 + V(v) \frac{\partial}{\partial g_0} \int_{t_0}^{t_i} g(t) \, dt \right] \] (12)

Keeping terms that are first-order in \( \Delta t = t_i - t_0 \), we obtain

\[ \frac{\partial g_i}{\partial v} = G(g_0)(t_i - t_0) \] (13)

and

\[ \frac{\partial g_i}{\partial g_0} = [1 + V(v)(t_i - t_0)]. \] (14)

The solution is found by iteration with the update at the jth iteration given by

\[ m_{j+1} = m_j + (A^T C_e^{-1} A + \Lambda)^{-1} (A^T C_e^{-1} r_j - \Lambda m_j) \] (15)

where \( m = (g_0; v) \) and \( r \) is the residual vector, which is calculated using Equation 7.

Here \( C_e \) is the data error covariance matrix and \( \Lambda \) is the regularization matrix, in this case based on the \( L2 \)-norm of the second spatial derivatives of both \( B_r(g_0) \) and \( w_\phi(v) \).

The azimuthal resolution of Juno data is much less than the meridional resolution as the field is sampled far more densely along track than between tracks, although this is alleviated to some extent by the altitude of the spacecraft (as quantified by the relevant Green's function), but the effect is still large. We mitigate this by applying an anisotropic regularization with stronger damping at orders \( m > 16 \). This allows finding solutions
with higher meridional resolution without introducing, or at least limiting, azimuthal artifacts. As discussed earlier in this paper, we take \( C_e = 1 \).

As a diagnostic of the solutions, we compute the resolution matrix \( R \)

\[
R = (A^T C_e^{-1} A + \Lambda)^{-1}(A^T C_e^{-1} A)
\]  

(16)

The trace of the resolution matrix, \( \text{tr}(R) \), is a measure of the number of degrees of freedom in the model, which we report for the various solutions below.

4 Results

We begin by considering a solid body rotation of the field, in other words a flux velocity represented by \( w_\phi = -v_1 \sin \theta \). We compute the solution at \( r = 0.9R_J \): this choice represents a trade-off between on the one hand being deep enough to be in, or close to, the region in which the secular variation arises while not being so deep that downwardly continuing the potential magnetic field becomes unreliable owing to the increasing electrical conductivity. We use the baseline model as a starting point for the inversion and find that after five iterations the model is well converged. We call this model SBFR-1 (where SBFR stands for solid body flux rotation). We show the along-track residuals in Figure 3. The mean residuals are substantially reduced, an observation that is born out by the reduction in rms misfit from 768 nT to 571 nT (or equivalently a variance reduction of 45%), achieved by adding a single degree of freedom to the model; for this solution \( \text{tr}(R) = 324 \). The misfit is now more clearly due to the short wavelength oscillations rather than the mean residuals, in other words a solid-body flux velocity has explained a large part of the mean residuals. The solid body flux velocity for the solution is \( 0.14^\circ/\text{yr} \).

Is a solid body rotation sufficient to explain the inferred secular variation? It is quite clear that a solid body flux rotation is necessary, and may simply be thought of as an adjustment of the rotation rate of the coordinate system, but is there evidence of a differential flux rotation in the observations? A complication in addressing this by further examination of the residuals is the presence of the short-wavelength oscillatory component, which is much larger in amplitude than the likely signal of differential flux velocity. In order to examine the residuals further, we seek first to more fully explain the oscillatory residuals so that any signal of differential flux velocity will be more clearly expressed. One possible origin of the short wavelength oscillations is that they result from a Gibbs-like phenomenon given that they appear correlated with the meridional field gradient and the meridional field gradients may have been under fit. We can explore this possibility by examining a spatially rougher solution computed by relaxing the spatial regularization of the magnetic field part of the solution.
Figure 3: Residuals of the radial component of the magnetic field data along track after subtracting SBFR-1. The figure is otherwise the same as Figure 2, except the background field is plotted at $t_0 = 2016.5$ using SBFR-1. Of note are the oscillatory residuals associated with PJ19: these result from a spacecraft manoeuvre that was performed shortly before perijove in order to align the MWR instrument to make cross-track sweeps, leaving insufficient time for the spacecraft nutation dampers to stabilize the spacecraft. As the motion is oscillatory and of short period, the data from this orbit are retained.

We again consider a solid body flux rotation solution but now with a spatially more complex magnetic field solution. We do not produce a more spatially complex baseline solution because such a solution would alias the effects of the solid body flux rotation into spurious azimuthal structure: orbits that are spatially adjacent but temporally sep-
arated would demand different magnetic field structure. We show the residuals to this solid body solution, which we call SBFR-2, in Figure 4. Adding spatial complexity, especially in the meridional direction, has reduced the along-track oscillatory residuals, suggesting that they did result from under-fitting the data, though possibly at the cost of introducing some unnecessary azimuthal structure. The overall misfit of this solution is $442 \text{ nT}$, and $\text{tr}(\mathbf{R}) = 567$.

Figure 4: Residuals of the radial component of the magnetic field data along track after subtracting SBFR-2. The figure is otherwise the same as Figure 3, though with SBFR-2 as the background model.
We use this solution as a comparison for one with a differential zonal flux rotation, i.e. a latitude-dependent zonal flux velocity. We call the solution with differential zonal flux rotation DFR; see Figure 5. The misfit is reduced to 411 nT, corresponding to a variance reduction (relative to SBFR-2 of 14%). We have, though, chosen a solution that has the same total number of degrees of freedom (567), spread across the magnetic field and flux velocity parts of the solution, as SBFR-2, even though it now incorporates seven additional degrees of freedom due to the zonal flux velocity: the spatial damping of the magnetic field has been increased slightly to reduce by six the number of degrees of freedom in the magnetic field part of the solution. Two points should be considered when interpreting this variance reduction. First, the fact that solid body flux velocity results in a larger variance reduction does not imply that the data contain little signal of differential flux rotation: the signal of solid-body flux velocity could be made arbitrarily large simply by choosing a different coordinate system but without diminishing the role of differential flux rotation. Second, the effects of a differential flux rotation may be localized, and so when incorporated into a measure of misfit across the entire dataset may be small, though larger locally.

Figure 5: DFR model: Radial component, $B_r$, of the magnetic field at $0.9R_J$ at 2016.5. The figure is otherwise the same as Figure 1. It is possible that some of the azimuthal structure present in this map may be spurious, despite the anisotropic regularization. However, the improvement in fit compared to the baseline model and the clearly improved meridional resolution, for example on the southern fringes of the NHFB, suggest that this is a worthwhile trade-off.

In Figure 6 we compare the flux rotational velocity for SBFR-2 and DFR. We identify three zones: first, and foremost, an equatorial jet in which the flux rotation is about 50% faster; second, at latitudes corresponding to the NHFB the difference in flux rotation is small; and, third, in the southern hemisphere, the flux rotation is much reduced, becoming slightly reversed. We can expect the difference in fit to be concentrated in the
first zone, around the GBS. There the difference in flux rotation is largest and the field azimuthal gradient is also large, so the difference in the solutions should be most apparent.

The equatorial flux velocity of the DFR model peaks at 0.86 cm s\(^{-1}\) which is one quarter of the peak velocity in the profile used by Moore et al. (2019) and the velocity inferred by Connerney et al. (2022). The peak velocity of Moore et al. (2019) occurs north of the equator: nearer to the equator, in other words where the GBS is most intense and hence from where the secular variation originates, it is more similar to that found here. However, the 4 cm s\(^{-1}\) equatorial velocity found by Connerney et al. (2022) differs substantially from that found here, though it seems possible that a result more similar could be obtained using their method given the limited azimuthal resolution of the field models compared with the small angular displacement of the GBS over this time interval.

![Flux rotational velocity for the solutions SBFR-2 (blue) and DFR (orange) as a function of latitude.](image)

**Figure 6:** Flux rotational velocity for the solutions SBFR-2 (blue) and DFR (orange) as a function of latitude. The x-axis is the flux velocity in cm/s and the y-axis is latitude in degrees. The gray line depicts the equator, highlighting the symmetry of the equatorial jet.

In Figure 7 we compare the residuals of this solution with those of the solid body rotation solution, SBFR-2 in the vicinity of the GBS. The residuals are, in general, smaller for the differential flux rotation model (DFR) than for the solid body rotation model (SBFR-2), though with the exception, in part, of PJ26, indicating that the enhanced flux velocity in the equatorial band better fits the data than a simple solid body flux velocity. The residuals corresponding to the NHFB are little changed, which is not surprising given that the zonal flux velocity itself is little changed there. Indeed, the evidence for a differential flux rotation is precisely the difference in flux velocity required for the equa-
torial GBS and the NHFB. One might then ask why the solid body flux velocity solution fits the NHFB in preference to the GBS? The answer is simply that more passes are sensitive to the NHFB than to the GBS.

![Diagram](image)

Figure 7: Residuals of the radial component of the magnetic field data along track in the vicinity of the GBS for SBFR-2 (top) and DFR (bottom). Note the change in scale of the residuals from Figures 2, 3 and 4.

In Figure 8 we show the change in the field over the time interval 2016.5 to 2021.5 (normalized per year) at 0.9 $R_J$ (a depth at which advection of the field by zonal winds is expected to become significant), at 1.0 $R_J$ (for comparison with Moore et al. (2019)) and at 1.15 $R_J$ (an altitude more representative of the Juno observations). Much, though not all, of the secular variation is concentrated near the Great Blue Spot (GBS), consistent with Moore et al. (2019) and Connerney et al. (2022).
Figure 8: Secular variation of the radial component of the field at 0.9 $R_J$ (top), at 1.0 $R_J$ (middle), and at 1.25 $R_J$ (bottom) from the DFR model. Note how the drift of the GBS, as expected, results in adjacent foci of oppositely-signed, intense secular variation, with a very large gradient in the secular variation between the foci. Note the changes in scale of the figures.

The secular variation near the GBS results from the spot’s eastward drift. In addition to strong secular variation around the GBS, we see patches of secular variation associated with the Northern Hemisphere Flux Band: these patches result from drift of the lobes of flux that together comprise the band, though the drift is largely consistent with solid body rotation. With the exception of the GBS, there is almost no secular variation in the southern hemisphere. We should, therefore, be cautious of interpreting the details of zonal drift velocity profile in Figure 6 south of the GBS. The spatial distribution of the secular variation is consistent with what we inferred directly from the residuals to the baseline model. At 1.15 $R_J$ the secular variation is, as expected, less spatially concentrated. Of note, though, is its amplitude on the order of several 1000 nT/yr. This shows that any inversion for Jupiter’s magnetic field using Juno data (for which perijoves are currently separated by 53 days, and adjacent perijoves may be separated by a year or more) should take account of secular variation. For example, in the time between PJ01 and PJ24 secular variation amounts to over 10,000 nT as those tracks pass over the GBS. As the Juno Extended Mission proceeds the need to account for secular variation will only become more important. To put this in perspective, by the end of the Extended Mission Jupiter’s magnetic field will have changed since the start of the mission by an amount comparable to the strength of the Earth’s main magnetic field.
In Figure 9 we show the change in the field over the period 2016.5 to 2021.5 (normalized per year) due to the non-solid body rotation part of the DFR model. This isolates the true secular variation from that due to a rotation of the reference frame. As expected, the secular variation is concentrated around the GBS, which highlights the role of the equatorial jet acting on the GBS to create the true secular variation.

Figure 9: Secular variation of the radial component of the field at $1.0 R_J$ for the DFR model minus the solid-body component. Note again the adjacent foci of oppositely-signed, intense secular variation, with a very large gradient in the secular variation between the foci.

As mentioned, the secular variation determined here is similar to that found for the epoch 1973–2017 by Moore et al. (2019), at least in the region around the GBS. Figure 10 is an adaptation of Figure 3 from Moore et al. (2019) in which we include the SBFR-2 and DFR models from this study. The SBFR-2, DFR and ZWA models match the Pioneer 11, Voyager 1 and Ulysses data well for the radial component of the field — the fit is slightly better for the DFR model than for the SBFR-2 model. See Moore et al. (2019) for a discussion of the fits to other components, and to Pioneer 10. The fits obtained here lend support to our approach of adding zonal drift to the magnetic field inversion since the zonal flux drift matches the secular variation derived from comparison with much earlier missions. Furthermore, the small improvement in fit seen with the DFR model suggests that features of that model (in particular the equatorial jet) have been persistent over a time span of several decades. However, this does not necessarily imply that zonal flows at this depth in Jupiter are steady since short period oscillations, such as torsional oscillations (Hori et al., 2019), could be superimposed on the steady zonal flow.

5 Summary and Discussion

We have shown that in order to more completely explain Juno magnetometer data the effects of secular variation due to a zonal flux velocity field should be included. A large part of the flux velocity field is simply a solid body rotation and so vanishes with an appropriate change of coordinate system. Given that the rotation rate of System III was determined from the magnetic field (Dessler, 1983) it is appropriate to suggest a correction to that rotation rate. We note that Higgins et al. (1996) suggest a revised rota-
Figure 10: *Fit to the secular variation inferred from earlier missions*. Based on Figure 3 of Moore et al. (2019) with the SBFR-2 and DFR models from this study included. For each mission, the black markers show the difference between the observations and the field inferred at those points in space from the Juno reference model JRM09 (Connerney et al., 2018). Overlaid on the figure is the difference that is predicated by the ZWA model of (Moore et al., 2019) and the SBFR-2 and DFR models from this study.

The rotation rate for the magnetic field corresponding to a prograde adjustment of 0.22°/yr. Our values are smaller (for SBFR-2 0.15°/yr and for DFR 0.11°/yr) but similar to those found by Ridley and Holme (2016) (0.13°/yr), Moore et al. (2019) (0.09°/yr) and Connerney et al. (2022) (0.11°/yr from Juno observations and 0.09°/yr from comparison with earlier missions). Our value for DFR is less than for SBFR-2 simply because the more complex solution places some of the power into higher harmonics. Furthermore, our solutions help explain observations from several decades earlier, and so the adjustment discussed here is persistent over that timescale. Any subsequent analysis of Juno magnetometer data should take account of the rotation of the magnetic field in System III coordinates, with the value from SBFR-2 being an appropriate choice if no other aspects of the zonal drift are included. The effect is large (up to 10000 nT yr⁻¹) along the Juno spacecraft track and cannot be ignored.
However, for other purposes any rotation rate for Jupiter is to a certain extent arbitrary, with the possible exception of the unknown frame in which the planet has no net angular momentum. The best fit rotation rate of the magnetic field does not necessarily represent the physical rotation rate at any depth in the planet’s interior. At best, if the frozen flux hypothesis holds, it represents the rotation rate at that hypothetical and likely non-existent depth at which frozen-flux becomes applicable; deeper in Jupiter’s interior the rotation rate will almost certainly be different. Accordingly, we consider that any proposed rotation rate for Jupiter’s deep interior that differs from System III by an amount within the likely range of zonal flows expected in the dynamo region, that is up to 10 cm s$^{-1}$ (Christensen & Aubert, 2006; Showman et al., 2011) corresponding to a $\Delta \Omega/\Omega \sim 10^{-5}$ to be equally plausible, and for that matter equally useful.

An important question is whether the differential flux rotation, in particular that within the equatorial belt ($\pm 20^\circ$), results from the surface winds persisting to this depth or whether it results from deeper flow within the dynamo region. Although the projected surface winds (Kaspi et al., 2018; Moore et al., 2019) are predominantly prograde in the equatorial belt, they are not symmetric about the equator, instead peaking at around 10$^\circ$N. Here, we find that within the equatorial belt the flow is symmetric about the equator, more similar to that found in numerical dynamo models (Jones, 2014; Gastine et al., 2014; Yadav et al., 2020). Furthermore, the flux velocities inferred here are within the range of flow speeds for the dynamo derived from scaling laws (Christensen & Aubert, 2006; Showman et al., 2011) based on the surface heat flux. Given that the secular variation of the field necessarily originates in a region in which the effects of electrical conductivity are significant, it seems more likely that secular variation results from flows associated with magnetoconvection rather than the penetration of surface zonal winds into this region, especially given the factor of 10$^4$ difference in velocity inferred from the flux velocity and the surface winds.

An enduring question regarding Jupiter’s magnetic field is why the magnetic field is concentrated into the NHFB and the GBS. It is possible that the shearing associated with zonal flows in Jupiter’s semiconducting region acts to concentrate field into the NHFB. If we remove the solid body component from DFR, we see shear both to the north and the south of the GBS (as can be readily inferred from Figure 6). This shear may concentrate flux into the NHFB through flux expulsion by the mechanism first described by Weiss (1966).

The GBS sits largely within the equatorial band and so is drifting mostly intact towards the east, albeit with some indication of shearing along its southern edge, as described by Moore et al. (2019) and Connerney et al. (2022). The westward, plume-like tail from the southern edge of the GBS that likely results from this shearing extends over about 30 degrees of longitude. We assume that the length of the plume is determined by a balance between shearing by zonal winds and Ohmic decay; in other words, it is a tail left behind the GBS that, in time, decays Ohmically. The timescale on which the GBS drifts eastward through 30$^\circ$ degrees is approximately $3 \times 10^9$ s, so we set that as the diffusion time for the plume, assuming its current length represents a steady state. Thus $L^2/\eta \approx 3 \times 10^9$ s, where L is the characteristic lengthscale of the plume and $\eta$ the magnetic diffusivity. Choosing $L \approx 10^7$ m (representing the width of the plume), we obtain $\eta \approx 3 \times 10^4$ m$^2$s$^{-1}$ which corresponds to an electrical conductivity $\sigma \approx 30$Sm$^{-1}$;
this corresponds to the region of rapidly increasing conductivity from around 0.95 $R_J$ to 0.90 $R_J$ (French et al., 2012). If we were to double the lengthscale over which it is smeared (there is some suggestion in Figure 5 that it extends twice the distance assumed above) then the diffusive timescale would be doubled, resulting in a doubling of the electrical conductivity, moving the depth a little deeper within the zone from 0.95 $R_J$ to 0.90 $R_J$. This suggests that the secular variation of Jupiter’s magnetic field arises from both advective and diffusive effects. We should caution though that this analysis is a gross simplification given the multiple lengthscales likely to be present owing to complications such as the rapid change in electrical conductivity or strong radial shearing of the field.

Two aspects of Juno’s Extended Mission, in particular, bode well for addressing the issues that arise from this study. First, the latitude of Juno’s closest approach continues to precess northwards at approximately 1° per orbit, significantly enhancing resolution of the NHFB and its northern extent. We hope that this may also help determine why PJ06 and PJ25 are not as well fit as nearby orbits to the west of the NHFB flux lobe. It is plausible that our solution is currently too simple around the northern edge of the NHFB. Second, and most importantly, the Extended Mission includes two sets of (nearly) sequential orbits targeting the GBS, which will greatly aid in separating secular variation from other sources of between-track field variability, and, in particular, in determining the speed of the equatorial jet.

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Data Availability Statement
All the Juno magnetometer data used in this study are available from the NASA Planetary Data System (Connerney, 2017). The models produced in this study, together with a Python script for reading them, are archived in the Harvard Dataverse (Bloxham, 2022).

References


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