

## Space-time duality between quantum chaos and non-Hermitian boundary effect

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Quantum chaos in Hermitian systems concerns the sensitivity of long-time dynamical evolution to initial conditions. The skin effect discovered recently in non-Hermitian systems reveals the sensitivity to the spatial boundary condition even deep in the bulk. In this Letter, we show that these two seemingly different phenomena can be unified through the space-time duality. The intuition is that the space-time duality maps unitary dynamics to nonunitary dynamics and exchanges the temporal direction and spatial direction. Therefore, the space-time duality can establish the connection between the sensitivity to the initial condition in the temporal direction and the sensitivity to the boundary condition in the spatial direction. Here, we demonstrate this connection by studying the space-time duality of the out-of-time-ordered correlator in a concrete chaotic Hermitian model. We show that the out-of-time-ordered correlator is mapped to a special two-point correlator of a non-Hermitian system in the dual picture. For comparison, we show that the sensitivity disappears when the non-Hermiticity is removed in the dual picture.

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*Introduction.* Chaos describes the phenomenon that the future is highly sensitive to any small perturbation at present, and this sensitivity can be more significant for a longer evolution time. During the past years, chaos in quantum systems has been extensively studied in terms of the out-of-time-ordered commutator (OTOC) and it shows that chaotic behavior is a general property in most quantum many-body systems [1–27]. As a separate development, the non-Hermitian skin effect has been discovered recently as a generic feature in non-Hermitian systems both theoretically [28–38] and experimentally [39–41]. The non-Hermitian skin effect states that the eigenstates of a non-Hermitian Hamiltonian can be highly sensitive to the spatial boundary condition. Unlike Hermitian systems, this sensitively holds even far from the boundary.

Although both effects concern the sensitivity to perturbations, they appear somewhat different at first glance. First, quantum chaos is mostly discussed in Hermitian systems, and the non-Hermitian skin effect is a unique feature of non-Hermitian systems. Second, quantum chaos concerns the sensitivity on the temporal domain, whereas the non-Hermitian skin effect concerns the sensitivity on the spatial domain. Therefore, previous discussions failed to bring out

the connection between these two effects, in addition to the possible equivalence between them.

In this Letter, we show that these two effects can be unified under the space-time duality of the quantum circuit. As we will review below, the space-time duality of the quantum circuit can map unitary dynamics to nonunitary dynamics and simultaneously exchanges the role of spatial direction and temporal direction. Therefore, it is very intuitive to understand that the space-time duality can bridge the gap between these two phenomena. Here, we demonstrate such intuition with a concrete example.

*Review of the space-time duality.* Before proceeding, let us first briefly review the space-time duality of a quantum circuit [42–55]. In the simplest case, let us consider a two-qubit gate  $\hat{u}$  operating on a two-qubit state  $|i_1\rangle \otimes |i_2\rangle$ , and denote  $\hat{u}|i_1\rangle \otimes |i_2\rangle = u_{i_1, i_2}^{j_1, j_2} |j_1\rangle \otimes |j_2\rangle$ . Note that here we have fixed a set of bases. In this case two qubits represent the spatial direction and the incoming and outgoing represents the temporal direction. By exchanging the role of spatial and temporal directions, we define another operator  $\hat{v}$ , which acts as  $\hat{v}|i_1\rangle \otimes |j_1\rangle = u_{i_1, i_2}^{j_1, j_2} |i_2\rangle \otimes |j_2\rangle$ . That is to say,  $\hat{v}$  is called the space-time duality circuit of  $\hat{u}$  if  $v_{i_1, j_1}^{i_2, j_2} = u_{i_1, i_2}^{j_1, j_2}$ . One example is shown in Fig. 1. If we choose

$$\hat{u} = e^{iJ_z \hat{\sigma}_1^z \hat{\sigma}_2^z} e^{iJ_x (\hat{\sigma}_1^x + \hat{\sigma}_2^x)} e^{iJ_z \hat{\sigma}_1^z \hat{\sigma}_2^z}, \quad (1)$$

and fix the basis as the eigenbasis of  $\hat{\sigma}^z$ , it can be shown that the corresponding  $\hat{v}$  has the same form as  $\hat{u}$ ,

$$\hat{v} = e^{i\tilde{J}_z \hat{\sigma}_1^z \hat{\sigma}_2^z} e^{i\tilde{J}_x (\hat{\sigma}_1^x + \hat{\sigma}_2^x)} e^{i\tilde{J}_z \hat{\sigma}_1^z \hat{\sigma}_2^z}, \quad (2)$$

and the parameters  $\tilde{J}_x$  and  $\tilde{J}_z$  are given by [56]

$$\tilde{J}_x = \arctan(-ie^{-2iJ_z}), \quad \tilde{J}_z = -\frac{\pi}{4} + \frac{i}{2} \ln(\tan J_x). \quad (3)$$

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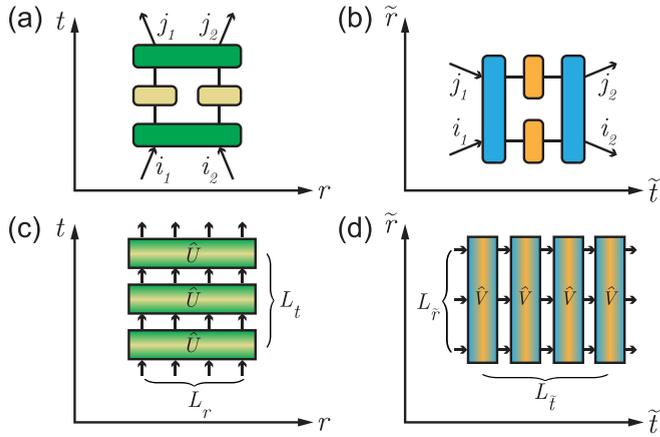


FIG. 1. Schematic of the space-time duality of a quantum circuit. (a), (b) The space-time duality between two-qubit quantum circuits  $\hat{u}$  (left) and  $\hat{v}$  (right) with  $v_{i_1, j_1}^{i_2, j_2} = u_{i_1, i_2}^{j_1, j_2}$ . The green and blue boxes respectively represent two-qubit gates  $e^{iJ_z \hat{\sigma}_1^z \hat{\sigma}_2^z}$  and  $e^{i\tilde{J}_z \hat{\sigma}_1^z \hat{\sigma}_2^z}$ . The yellow and orange boxes respectively represent single-qubit gates  $e^{iJ_x \hat{\sigma}_x}$  and  $e^{i\tilde{J}_x \hat{\sigma}_x}$ , with the relation between  $J_z$  and  $\tilde{J}_z$ , and relation between  $J_x$  and  $\tilde{J}_x$  given by Eq. (3). (c), (d) The space-time duality between two general operators  $\hat{U}$  and  $\hat{V}$ , with  $L_{\tilde{r}} = L_t$  and  $L_{\tilde{t}} = L_r$ .

When  $J_x$  and  $J_z$  are both real, that is to say  $\hat{u}$  is unitary,  $\tilde{J}_x$  and  $\tilde{J}_z$  are in general complex numbers, and that means  $\hat{v}$  is a nonunitary evolution.

More generally, as shown in Figs. 1(c) and 1(d), let us consider a unitary operator  $\hat{U}$  repeatedly acting  $L_t$  steps on this system with  $L_r$  qubits, so we can introduce a circuit  $\hat{V}$  as the space-time dual of  $\hat{U}$ , which repeatedly acts  $L_{\tilde{r}}$  steps on a system with  $L_{\tilde{t}}$  qubits. Here,  $L_{\tilde{r}} = L_t$  and  $L_{\tilde{t}} = L_r$ . For example, if we choose  $\hat{U}$  as

$$\hat{U} = e^{i \sum_{r=1}^{L_r} J_x \hat{\sigma}_r^x} e^{i \sum_{r=1}^{L_r} (J_z \hat{\sigma}_r^z \hat{\sigma}_{r+1}^z + h \hat{\sigma}_r^z)}, \quad (4)$$

up to a constant, the corresponding  $\hat{V}$  is given by [56]

$$\hat{V} = e^{i \sum_{r=1}^{L_{\tilde{r}}} \tilde{J}_x \hat{\sigma}_r^x} e^{i \sum_{r=1}^{L_{\tilde{r}}} (\tilde{J}_z \hat{\sigma}_r^z \hat{\sigma}_{r+1}^z + h \hat{\sigma}_r^z)}, \quad (5)$$

with the same relation between  $\tilde{J}_x$ ,  $\tilde{J}_z$  and  $J_x$ ,  $J_z$  as given by Eq. (3). In general,  $\hat{V}$  is a nonunitary circuit when  $\hat{U}$  is unitary.

*The space-time duality of OTOC.* Here, we first consider a function  $\mathcal{F}(\phi)$  defined as

$$\mathcal{F}(\phi) = \frac{1}{2^{L_r}} \text{Tr}_{L_r} [\hat{O}(L_t) e^{i\phi \hat{W}} \hat{O}^\dagger(L_t) e^{-i\phi \hat{W}}]. \quad (6)$$

Here, we choose  $\hat{W} = \sum_r \hat{w}_r$  as an operator that uniformly acts on all spatial sites with  $\hat{w}_r$  denoting an operator  $\hat{w}$  acting on site  $r$ , and  $\hat{O}$  to be a spatially local operator. The reasons we consider this correlation function are multifold. First, this quantity is directly related to the OTOC. It can be shown that

$$\left. \frac{\partial^2 \mathcal{F}(\phi)}{\partial \phi^2} \right|_{\phi=0} = -\frac{1}{2^{L_r}} \text{Tr}_{L_r} [([\hat{O}(L_t), \hat{W}]^2)], \quad (7)$$

and the right-hand side of Eq. (7) is the OTOC. Thus, for quantum chaos, the OTOC is larger for larger  $L_t$ , which means that  $\mathcal{F}(\phi)$  should sensitively depend on  $\phi$  even for larger  $L_t$ . Second, this quantity is closely related to the multiple

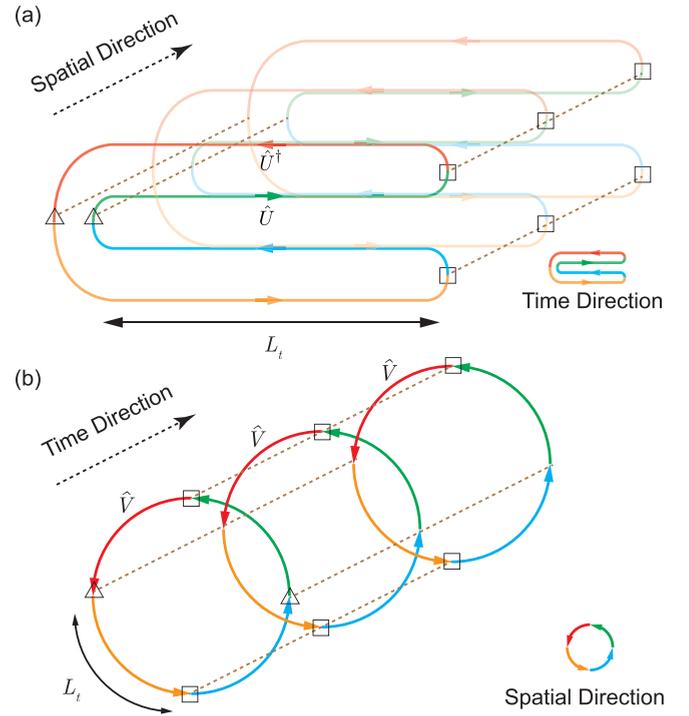


FIG. 2. (a) Correlation function  $\mathcal{F}(\phi)$  defined in Eq. (8). The solid line represents the temporal direction. Different branches of forward and backward evolutions in the double Keldysh contour are distinguished by different colors, and each part has a length  $L_t$ . (b) The space-time dual of  $\mathcal{F}(\phi)$ . The solid line now represents the spatial direction with  $L_{\tilde{r}} = 4L_t$ . Squares and triangles are respectively labeled as  $e^{\pm i\phi \hat{W}}$  and  $\hat{O}$  acting on the double Keldysh contour in (a) or their dual circuit acting on different spatial points on the spatial contour in (b). The distance between the triangle and square is  $L_t$  in both cases.

quantum coherence that can be directly measured in NMR and trapped ion systems [13,57]. Third, this quantity possesses a clear physical interpretation after performing the space-time duality on the basis diagonal in  $\hat{w}_r$ , as we will discuss from the following three aspects.

(i) Length of the dual spatial contour: In Eq. (6),  $\hat{O}(L_t)$  is given by  $(\hat{U}^\dagger)^{L_t} \hat{O} (\hat{U})^{L_t}$ , and explicitly,  $\mathcal{F}(\phi)$  can be written as

$$\frac{1}{2^{L_r}} \text{Tr}_{L_r} [(\hat{U}^\dagger)^{L_t} \hat{O} (\hat{U})^{L_t} e^{i\phi \hat{W}} (\hat{U}^\dagger)^{L_t} \hat{O} (\hat{U})^{L_t} e^{-i\phi \hat{W}}]. \quad (8)$$

Unlike the unidirectional evolution discussed above,  $\mathcal{F}(\phi)$  contains two forward evolutions  $(\hat{U}^\dagger)^{L_t}$  and two backward evolutions  $(\hat{U})^{L_t}$ . In other words, it contains two Keldysh contours. They are marked by different colors in Fig. 2(a). The length of each evolution is  $L_t$ . Therefore, after performing the space-time duality, the length  $L_{\tilde{r}}$  of the spatial contour should be  $4L_t$ . In Fig. 2(b), we stretch the spatial contour into a circle, which is correspondingly marked by the same set of colors.

(ii) Boundary operators: In Eq. (8),  $e^{\pm i\phi \hat{W}}$  is an operator that uniformly acts on all spatial sites. Then, after performing space-time duality, the dual operator again takes the form  $e^{\pm i\phi \hat{w}}$  that acts as a time-independent operator. In the double Keldysh contour,  $e^{i\phi \hat{W}}$  and  $e^{-i\phi \hat{W}}$  are separated by  $2L_t$ . Therefore, after space-time duality,  $e^{\pm i\phi \hat{w}}$  acts on two endpoints of

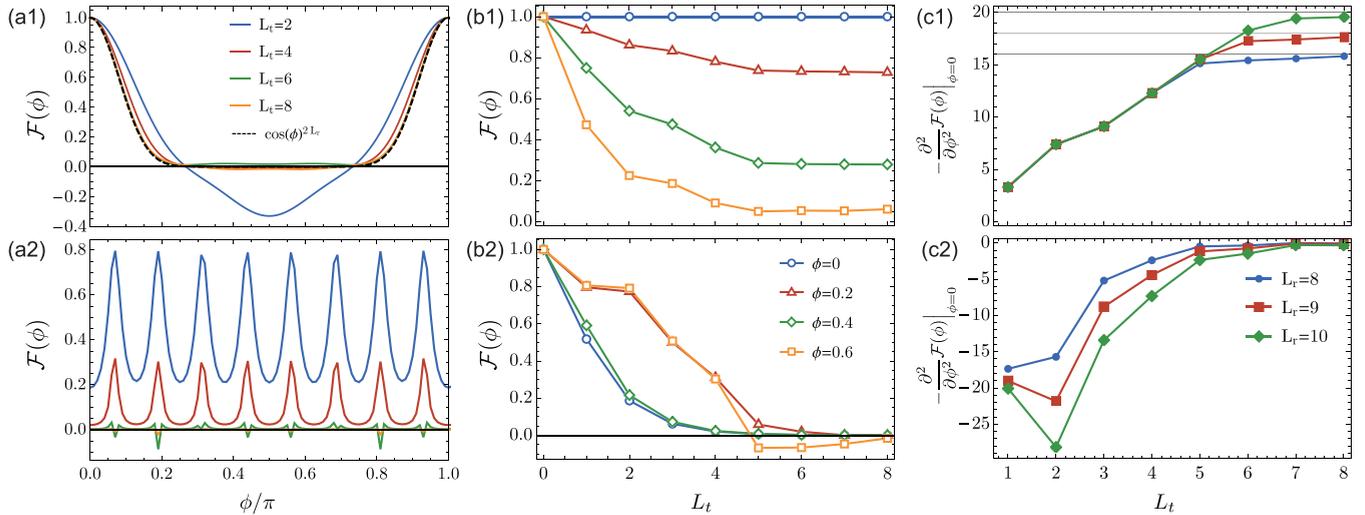


FIG. 3. (a1), (a2)  $\mathcal{F}(\phi)$  as a function of  $\phi$  for different  $L_t$ ; (b1), (b2)  $\mathcal{F}(\phi)$  as a function of  $L_t$  for different  $\phi$ ; (c1), (c2)  $\partial^2 \mathcal{F}(\phi)/\partial\phi^2|_{\phi=0}$  as a function of  $L_t$  for different  $L_r$ .  $L_r = 8$  for (a1), (a2) and (b1), (b2). The top row plots the function given by Eq. (8), or equivalently Eq. (9), with  $\hat{O} = \hat{\sigma}_1^z$  and  $\hat{W} = \sum_{r=1}^{L_r} \hat{\sigma}_r^z$ . We choose parameters  $\{J_x, J_z, h\} = \{1, 1, 0.5\}$ . The bottom row plots a modified Eq. (9) which eliminates the non-Hermiticity (see text for details).

a diameter in the spatial contour, and it is denoted by a square in Fig. 2. Therefore, these two operators are considered as the boundary operators in the dual picture. When  $\phi = 0$ ,  $e^{i\phi\hat{W}}$  and  $e^{-i\phi\hat{W}}$  are both identity operators. When we use Eq. (8) to diagnose quantum chaos, we concern the sensitivity of  $\mathcal{F}$  when  $\phi$  deviates from zero. In the dual picture,  $e^{\pm i\phi\hat{W}}$  becomes the boundary operators, and this measures the sensitivity to boundary conditions, which is attributed to the non-Hermitian boundary effect.

(iii) Equal-time correlator: In Eq. (8),  $\hat{O}$  is a spatial local operator. Therefore, the space-time dual of Eq. (8) can be viewed as an equal-time correlator of two bulk operators  $\hat{O}$  that are the space-time dual of  $\hat{O}$ . Two  $\hat{O}$  operators are separated by  $2L_t$  in the double Keldysh contour, and after applying the space-time duality, they also sit at two endpoints of a diameter, as denoted by triangles in Fig. 2. The spatial separation between the bulk operator  $\hat{O}$  and the boundary operator  $e^{\pm i\phi\hat{W}}$  is  $L_t$ . In quantum chaos, we are concerned with the sensitivity to  $\phi$  for long evolution steps  $L_t$ . Therefore, in the dual picture, we are concerned with the sensitivity for large spatial separation  $L_t$  between the bulk operator and the boundary operator.

The discussions above highlight the main feature of the space-time duality of  $\mathcal{F}(\phi)$ . It can be shown more rigorously that the space-time duality of  $\mathcal{F}(\phi)$  can be written as [56]

$$\mathcal{F}(\phi) = \frac{\text{Tr}_{L_r} [[\hat{V}\hat{B}(\phi)]^{L_t} \hat{O}_{L_t+1} \hat{O}_{3L_t+1}]}{\text{Tr}_{L_r} [[\hat{V}\hat{B}(\phi)]^{L_t}]} \quad (9)$$

Here,  $\hat{B}(\phi) = e^{i\phi\hat{w}_{2L_t+1}} e^{-i\phi\hat{w}_1}$  denotes the boundary operator in the dual picture.  $\hat{V}$  in Eq. (9) is related to  $\hat{U}$  in Eq. (8) via the space-time duality. Hence, we have now mapped Eq. (8) into an equal-time correlator of a non-Hermitian system with a boundary term. Nevertheless, we note that this correlator is not a standard two-point correlator in real time [58]. Quantum chaotic behavior in Eq. (8) is mapped to the sensitivity on the

boundary parameter for the large separation between bulk and boundary operators.

*Numerical results.* Here, we set  $\hat{U}$  as given by Eq. (4) and  $\hat{V}$  behaves as Eq. (5) [56]. Moreover, we choose  $\hat{O}$  as  $\hat{\sigma}_1^z$  and  $\hat{W}$  as  $\sum_{i=1}^{L_r} \hat{\sigma}_i^z$ . The numerical results of  $\mathcal{F}(\phi)$ , as well as  $\partial^2 \mathcal{F}(\phi)/\partial\phi^2|_{\phi=0}$ , are shown in Figs. 3(a1), 3(b1), and 3(c1) [59]. We can see the  $\mathcal{F}(\phi)$ 's sensitivity to  $\phi$  even for large  $L_t$ .

Here, we would like to provide further evidence that this sensitivity of  $\mathcal{F}(\phi)$  to  $\phi$  can be interpreted as the non-Hermitian boundary effect. To this end, we can artificially change the parameters  $\hat{J}_x, \hat{J}_z$ , and  $h$  in  $\hat{V}$  to be purely imaginary, such that  $\hat{V}$  behaves as  $e^{-\hat{H}}$  in which  $\hat{H}$  is a Hermitian operator. Thus, the modified Eq. (9) can be viewed as the equal-time correlator of a statistical Hermitian system, and this modification eliminates the non-Hermiticity in Eq. (9). We plot this modified  $\mathcal{F}(\phi)$ , as well as  $\partial^2 \mathcal{F}(\phi)/\partial\phi^2|_{\phi=0}$ , in Figs. 3(a2), 3(b2), and 3(c2), and we should contrast Figs. 3(a2), 3(b2), and 3(c2) with Figs. 3(a1), 3(b1), and 3(c1).

(i) In Fig. 3(a1), we plot  $\mathcal{F}(\phi)$  as a function of  $\phi$  for different  $L_t$ . One can see that  $\mathcal{F}(\phi)$  approaches  $\cos(\phi)^{2L_t}$  for large  $L_t$ . This is also consistent with the final saturation value of OTOC, as illustrated in Fig. 3(c1). Theoretically, we can obtain these two results in the fully scrambled limit. In this limit,  $\hat{\sigma}_1^z(L_t)$  uniformly populates the entire operator space, and then  $\text{Tr}_{L_r} [|\hat{O}(L_t), \hat{W}|^2]/2L_r$  in Eq. (7) approaches  $2L_r$ . In contrast, we show in Fig. 3(a2) that when  $L_t$  is large enough, the modified  $\mathcal{F}(\phi)$  approaches a constant independent of  $\phi$ .

(ii) In Fig. 3(b), we plot  $\mathcal{F}(\phi)$  as a function of  $L_t$  for different  $\phi$ . It is quite clear in this plot that, even for large  $L_t$ ,  $\mathcal{F}(\phi)$  also strongly depends on  $\phi$ . In contrast, Fig. 3(b2) shows that, for the modified  $\mathcal{F}(\phi)$ , the differences between  $\mathcal{F}(\phi)$  of different  $\phi$  become smaller as  $L_t$  increases.

(iii) In Fig. 3(c1), we show the OTOC obtained from  $\partial^2 \mathcal{F}(\phi)/\partial\phi^2|_{\phi=0}$ . The OTOC increases as  $L_t$  increases until it saturates to a finite nonzero value for large enough  $L_t$ , and

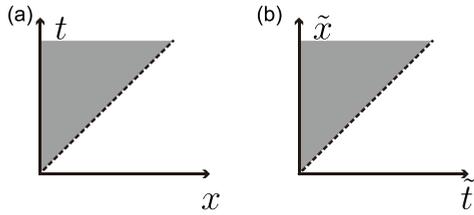


FIG. 4. The duality between the light cone in (a) unitary dynamics and in (b) nonunitary dynamics. The timelike regime [shaded area in (a)] is translated into a spacelike regime [shaded area in (b)] after space-time duality. The correlation is nonzero in the shaded regimes.

this is due to the finite-size effect. The saturation value  $2L_r$  is consistent with the fully scrambled limit of the finite Hilbert space. In contrast, Fig. 3(c2) shows that, for the modified  $\mathcal{F}(\phi)$ , the derivative  $\partial^2 \mathcal{F}(\phi)/\partial \phi^2|_{\phi=0}$  approaches zero as  $L_t$  increases.

All these results show that, when the non-Hermiticity effect is mostly eliminated, the correlator of two bulk operators is no longer sensitive to the boundary parameter  $\phi$  as long as the separation  $L_t$  between the bulk operators and the boundary is large enough. This is consistent with our intuition of a Hermitian system where the boundary effect should not significantly affect properties deep in the bulk. In other words, it supports the claim that the interpretation of the results shown in Figs. 3(a1), 3(b1), and 3(c1) are due to the non-Hermiticity in the dual picture.

*Implications.* Although we illustrate the connection between quantum chaos and the boundary effect using a concrete example, this duality generally holds. This correspondence can provide more insights for discovering different phenomena, and here we highlight a few implications of this duality.

First, the non-Hermitian skin effect has been mostly studied in noninteracting systems so far. Here, we note that the dual nonunitary dynamics cannot be viewed as free dynamics, revealing the boundary effect as a generic feature of non-Hermitian systems beyond single-particle physics. In other words, this duality can also be viewed as an alternative route to generalize the skin effect to interacting non-Hermitian systems.

Second, for chaotic unitary dynamics, there is an information scrambling timescale at which the out-of-time-order

TABLE I. Correspondence between quantum chaos and the non-Hermitian boundary effect via the space-time duality.

Quantum chaos	Boundary effect
Unitary dynamics	Nonunitary dynamics
Initial condition sensitivity	Boundary operator sensitivity
Scrambling time	Penetration depth
Information spreading	Information erasing

correlator drops significantly. This timescale can be translated into a length scale in nonunitary dynamics, such as a penetration depth. The Lyapunov exponent will lead to a characteristic exponent associated with this length scale.

Third, it is known that the OTOC behaves differently between chaotic and nonchaotic unitary systems, such as systems with many-body localization [60–63]. This difference can also manifest itself in the dual nonunitary dynamics.

Finally, when both  $\hat{W}$  and  $\hat{O}$  are taken as local operators, the front of OTOC exhibits a light-cone structure, as a manifestation of causality in the information scrambling. This light cone divides the space-time coordinates into spacelike and timelike regions, and the correlator is nonzero only in the timelike region in the unitary model, as schematically shown as the shaded area in Fig. 4(a). The space-time duality exchanges the timelike region and the spacelike region. Thus, as shown in Fig. 4(b), an initially encoded spatial correlation remains nonzero in the spacelike regime. The nonunitary dynamics, such as measurements, gradually erase the initial information, yielding almost zero correlation in the timelike region. That is to say, the information spreading process in unitary dynamics is dual to the information erasing process in nonunitary dynamics. Interestingly, this duality reveals that the information erasing also depends on the separation of spatial coordinates: The larger the spatial separation, the longer time is required to erase the information.

In summary, we summarize the correspondence in Table I, and we hope that the insights provided by this duality can inspire more understanding of the nonunitary dynamics.

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