

Self-heating of cryogenic HEMT amplifiers and the limits of microwave noise performance

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Abstract

The fundamental limits of the microwave noise performance of high electron mobility transistors (HEMTs) are of scientific and practical interest for applications in radio astronomy and quantum computing. Self-heating at cryogenic temperatures has been reported to be a limiting mechanism for the noise, but cryogenic cooling strategies to mitigate it, for instance using liquid cryogenes, have not been evaluated. Here, we report microwave noise measurements of a packaged two-stage HEMT amplifier immersed in normal and superfluid ⁴He baths and in vacuum from 1.6 – 80 K. We find that these liquid cryogenes are unable to mitigate the thermal noise associated with self-heating. Considering this finding, we examine the implications for the lower bounds of cryogenic noise performance in HEMTs. Our analysis supports the general design principle for cryogenic HEMTs of maximizing gain at the lowest possible power.

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I. INTRODUCTION

Microwave low-noise amplifiers (LNAs) based on III-V semiconductor high electron mobility transistor (HEMT) technology [1, 2] are a key component of high precision measurements across diverse fields in science and engineering such as radio astronomy [3, 4], deep-space communication [5] and quantum computing [6–10]. In these applications, LNAs serve as the first or second stage of amplification in the receiver chain, thereby making a decisive contribution to the noise floor of the entire measurement apparatus. Although marked improvements in noise performance have been achieved in recent decades [5, 11–17], the noise performance of HEMT LNAs remains a factor of 3 – 5 larger than the quantum limit [4, 18, 19].

The noise behavior of HEMT amplifiers is typically interpreted using the Pospieszalski model [20]. In this model, noise generators are assigned to the gate resistance at the input and the drain conductance at the output, parameterized by noise temperatures T_g and T_d , respectively. The drain temperature T_d lacks an accepted physical origin, with several theories having been proposed [21–24], and it is typically taken as a fitting parameter. The gate temperature T_g is assumed to be equal to the physical device temperature [1, 25]. In this interpretation, cryogenic cooling leads to improvements in the noise figure of the HEMT in part by decreasing the gate temperature and hence its thermal noise.

The monotonic decrease in noise figure with decreasing physical temperature has been observed to plateau below physical temperatures of 20 – 40 K (see Fig. 10 from Ref. [26], Fig. 1 from Ref. [27] and Fig. 2 from Ref. [28], for example). Recent numerical and experimental studies have attributed this plateau to heating of the gate caused by power dissipated in the active channel, referred to as self-heating [27, 29]. In more detail, optimal low-noise performance at cryogenic temperatures requires power dissipation on the order of milliwatts. At these temperatures, the observed thermal resistance from Schottky thermometry is consistent with that expected of phonon radiation for which the thermal resistance varies as T^{-3} [30]. Consequently, at physical temperatures $\lesssim 20$ K the rapid increase in thermal resistance with decreasing temperature leads to a plateau in the gate temperature, which produces a corresponding plateau in noise figure.

Mitigating the effect of self-heating by improving heat dissipation is desirable. However, cryogenic thermal management of the gate in modern devices with sub-micron gate lengths

and a buried gate structure is challenging. A possible approach is to submerge the heated surface in superfluid ^4He , a quantum fluid with the highest known thermal conductivity [31]. Such an approach is routinely used for cryogenic thermal management of superconducting magnets [32] and is actively employed in high-energy physics experiments [33–35]. However, the effectiveness of liquid cryogenics to mitigate self-heating in HEMTs has not been experimentally evaluated.

Here, we present noise measurements of a packaged HEMT amplifier immersed in normal (He I) and superfluid (He II) ^4He baths as well as in vapor and vacuum environments up to 80 K at various biases. We find that the liquid cryogenics are unable to mitigate self-heating owing to the thermal boundary resistance between the HEMT surface and the ^4He bath. We extract the gate temperature using a small-signal model of the device and show that the trends with physical temperature are generally consistent with those predicted by a phonon radiation model, regardless of the presence of liquid cryogenics. We use these observations to examine the lower bounds of noise performance in cryogenic HEMTs, accounting for the temperature and power dissipation dependence of the thermal noise at the input.

II. EXPERIMENT

A. Overview of measurement apparatus

We measured the microwave noise temperature T_e and gain G of the device under test (DUT), a common-source two-stage packaged amplifier comprised of OMMIC D007IH metamorphic HEMTs [17], each with a 70 nm gate length and a 4 finger 200 μm width double-mushroom gate structure consisting of an InGaAs-InAlAs-InGaAs-InAlAs epitaxial stack on a semi-insulating GaAs substrate with each stage biased nominally identically, using the cold attenuator Y-factor method [36]. An input matching network (IMN) was employed to match the optimal transistor impedance to the 50 Ω impedance of our measurement system over a 4–5.5 GHz bandwidth (see SI section S.1 and Ch. 5.1 of Ref. [37] for more details).

Figure 1(a) shows a schematic of the measurement setup designed for microwave noise characterization in a liquid ^4He dewar. The DUT, packaged 20 dB chip attenuator, temperature diodes and heater (mounted behind the stage and not shown) were screw-mounted to a copper mounting stage using indium foil. The stage was screw-mounted to a dipstick,

and a liquid level sensor was taped to the interior wall of the dipstick. The dipstick was designed to mount on the dewar neck and submerge the stage in the liquid bath. A custom vacuum fitting with hermetic SubMiniature-A (SMA) and DC feedthroughs was sealed to the dewar neck, allowing for evaporative cooling via pumping. SMA coaxial cables and phosphor-bronze cryogenic wires were used to transmit microwave and DC signals, respectively, between the stage and feedthroughs.

Noise power was generated by a packaged 2 – 18 GHz solid state SMA noise diode with 15 dB excess noise ratio (ENR), which was biased with an Agilent 33220A signal generator amplified by a MOSFET amplifier circuit. This scheme allowed for Y-factor sampling up to 100 kHz, limited by the RC time constant of the MOSFET and noise diode circuit. Following the cold attenuator method [38], this noise power was directed through coaxial cabling to the 20 dB attenuator thermally strapped to the stage. The resulting hot and cold noise temperatures presented to the input plane of the DUT under cryogenic conditions were 52 K and 7 K respectively, yielding a Y-factor of 7.4. The noise power amplified by the DUT was then directed to a room temperature measurement apparatus. The ^4He dewar was grounded, and inner-outer DC blocks were used to connect the hermetic feedthroughs to the noise source and detector to minimize low-frequency bias noise on the DUT.

Figure 1(a) also shows the room temperature backend measurement chain, which consisted of the following components. The microwave power emerging from the SMA feedthrough output first passed through a 3–6 GHz microwave isolator to improve impedance matching and minimize reflections. Two Minicircuits ZX60-83LN-S+ broadband amplifiers were placed sequentially following the isolator. A microwave filter was then used to limit the bandwidth of power amplified by the final gain stage, a Miteq AMF-3B-04000800-25-25P power amplifier. Next, a microwave switch was used to periodically switch between the signal path and a $50\ \Omega$ load, enabling re-calibration of the detector to correct for DC offset drifts. A variable attenuator and temperature-controlled YIG filter were then used to set the magnitude and frequency f of microwave noise power reaching the Herotek DT4080 tunnel diode detector, which linearly transduced this power into a DC voltage. Inner-outer DC blocks were placed at the diode detector’s input to eliminate unwanted biasing. The final DC signal was amplified and low-pass filtered by an SRS560 pre-amplifier and then digitized by an analog-digital converter for further data processing.

Figure 1(b) shows representative Y-factor voltage data spanning one duty cycle of data

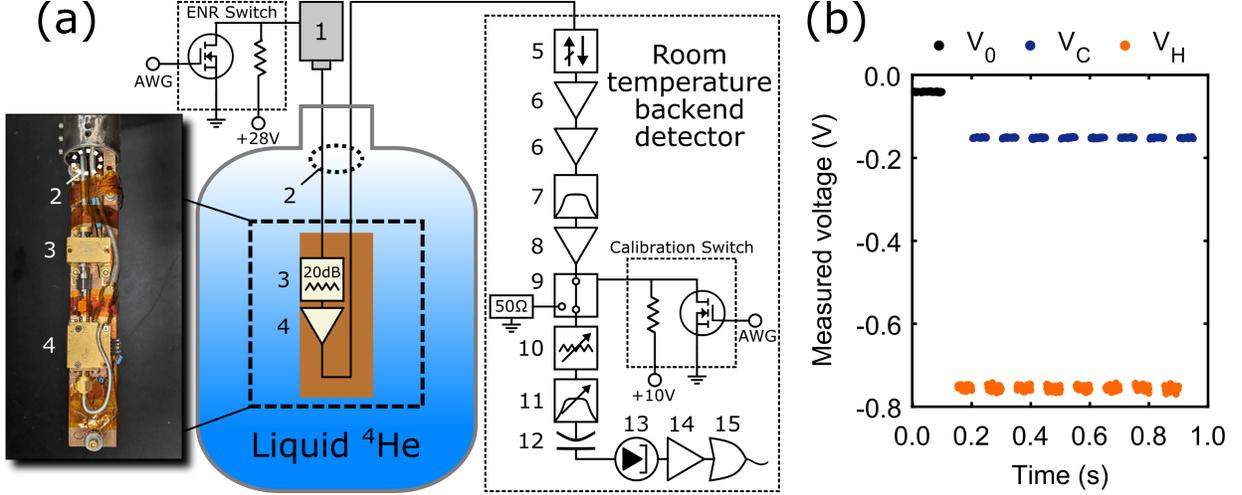


Fig. 1: (a) Schematic of the microwave noise characterization apparatus designed for insertion into a ^4He dewar. The image shows the copper mounting stage with the DUT, attenuator and temperature diodes mounted. A liquid ^4He level sensor was mounted behind and above the copper stage inside the dipstick. The components consisted of (1) 15 dB ENR 2-18 GHz solid state SMA packaged noise diode biased at 28 V through a MOSFET amplifier circuit, (2) input and output silver-plated stainless steel SMA coaxial cables each of length 1.3 m, (3) 20 dB packaged cryogenic chip attenuator with factory calibrated DT-670-SD diodes mounted directly on the attenuator substrate, (4) DUT, (5) Pasternak PE8327 isolator, (6) Minicircuits ZX60-83LN-S+ low-noise amplifiers, (7) Minicircuits filters with 3–6 GHz bandwidth, (8) Miteq AMF-3B-04000800-25-25P medium power amplifier, (9) RF switch for calibration, (10) 0 – 20 dB variable attenuator, (11) Micro Lambda MLFM-42008 20 MHz bandwidth tunable YIG filter, (12) Pasternak PE8224 inner-outer DC block, (13) Herotek DT4080 tunnel diode detector, (14) SRS560 low-noise preamp, (15) National Instruments NI6259-USB DAQ. The losses of the SMA cabling and attenuator pads are not shown. (b) Representative raw noise power data versus time. The diode detector DC offset voltage V_0 (black lines), hot voltage V_H (orange lines) and cold voltage V_C (blue lines) are all shown.

acquisition and detector re-calibration. Here, the hot (noise source on), cold (noise source off) and zero-power offset ($50\ \Omega$ load) output voltages at time index k , denoted $V_{H,k}$, $V_{C,k}$ and $V_{0,k}$, respectively, were analog filtered at 1 kHz and over-sampled at $f_s = 1.1$ kHz to avoid aliasing artifacts. The noise source was pulsed at $f_{\text{ENR}} = 10$ Hz, and the zero-power offset was re-calibrated at $f_0 = 1$ Hz with a 10% duty cycle, yielding a voltage offset integration time of $t_0 = 0.1f_0^{-1} = 0.1$ s. Each half-pulse of hot (cold) voltage data was integrated for its duration $t_{\text{ENR}} = 0.5f_{\text{ENR}}^{-1} = 0.05$ s. The first and last half-pulses of each cycle were discarded due to the 20 ms switching time of the microwave switch, yielding 8 total pulses. The integrated zero-power offset was subtracted from each integrated hot (cold) half-pulse to give hot (cold) offset-corrected data $V'_{H,p}$ ($V'_{C,p}$) for each pulse p . This procedure yielded Y-factor data effectively sampled at f_{ENR} with 0.2 s of data skipped every 1 s. The expression

for the measured Y-factor for pulse p considering the above specifications is:

$$Y_p = \frac{N_{\text{HC}}^{-1} \left(\sum_{k=k_{\text{H},p}}^{k_{\text{HC},p}} V_{\text{H},k} \right) - N_0^{-1} \left(\sum_{k=0}^{k_0} V_{0,k} \right)}{N_{\text{HC}}^{-1} \left(\sum_{k=k_{\text{HC},p}}^{k_{\text{C},p}} V_{\text{C},k} \right) - N_0^{-1} \left(\sum_{k=0}^{k_0} V_{0,k} \right)} = \frac{V'_{\text{H},p}}{V'_{\text{C},p}} \quad (1)$$

where p ranges from 1 to 8 for each cycle, $N_{\text{HC}} = t_{\text{ENR}} f_s$ is the number of sampled points in each half-pulse, $N_0 = t_0 f_s = 2N_{\text{HC}}$ is the number of sampled points in each calibration pulse, $k_{\text{H},p} = (k_0 + 1) + 2(p - 1)N_{\text{HC}}$ is the time index at the start of the p th hot pulse, $k_{\text{HC}} = k_{\text{H},p} + N_{\text{HC}}$ is the time index at the center of the p th pulse when the noise source switches from hot to cold, and $k_{\text{C},p} = k_{\text{H},p} + 2N_{\text{HC}}$ is the time index at the end of the p th cold pulse. For all steady-state data shown in this paper, the Y-factor was further averaged over a total measurement time $t_{\text{fin}} = 4$ s.

A Rhode&Schwarz RSZVA50 VNA calibrated with a Maury 8050CK20 SOLT calibration kit was used for S_{21} measurements. To measure gain and noise temperature simultaneously through Y-factor measurements, the gain $G_{\text{full}} = GL_{\text{coax}}^{-1}L_2^{-1}$ of the entire system from the noise source output plane to the backend input plane was first measured using the VNA, where $L_{\text{coax}} \equiv L_1L_3$ is the total loss of the cables with input cable loss L_1 , output cable loss L_3 , and attenuator loss L_2 . The DUT gain G could then be extracted using:

$$G = G_{\text{full}}L_{\text{coax}}L_2 = \frac{V'_{\text{H}} - V'_{\text{C}}}{V_{\text{H}}^{\text{cal}} - V_{\text{C}}^{\text{cal}}} G_{\text{full}}^{\text{cal}} L_{\text{coax}} L_2 \quad (2)$$

where $V_{\text{H}}^{\text{cal}}$, $V_{\text{C}}^{\text{cal}}$ and $G_{\text{full}}^{\text{cal}}$ are hot and cold output calibration voltages and total calibration gain (including cabling and attenuator), respectively, all measured at a particular device bias. The noise temperature T_e was then determined by:

$$T_e = \frac{1}{L_1L_2} \left[\frac{T_0E}{Y - 1} - T_{\text{C}} - T_{\text{coax}}(L_1 - 1) - T_{L_2}(L_2 - 1)L_1 - \frac{T_{\text{coax}}(L_3 - 1)}{G_{\text{full}}L_3} - \frac{T_{\text{BE}}}{G_{\text{full}}} \right] \quad (3)$$

where $T_0 = 290$ K, E is the noise source excess noise ratio, T_{C} is the noise source physical temperature, T_{coax} is the lumped coaxial cable physical temperature, T_2 is the attenuator physical temperature and T_{BE} is the backend noise temperature (see SI section S.2 for a derivation of Eqs. (2) and (3)).

B. Measurement apparatus calibration

We now describe the calibration procedure for each term of Eqs. (2) and (3). First, the backend noise temperature T_{BE} was independently obtained using a liquid nitrogen cooled fixed load method. The noise source ENR was then calibrated by using the backend detector as a reference amplifier (see SI section S.3 for more details). Two separate liquid ^4He dewar baths were then used, one to calibrate the coaxial cable loss and temperature and then another to calibrate the attenuator loss.

The general procedure in each calibration dewar was as follows. The dipstick was used to submerge the mounting stage 2 cm from the bottom of a fully filled 60 L liquid ^4He dewar at 4.2 K and ambient pressure. After waiting 30 minutes for thermal equilibration, calibration measurements were taken. The dewar was then sealed, and a vacuum pump was used to evaporatively cool the liquid into the He II phase. A steady-state temperature of 1.6 K was reached after roughly 6 hours of pumping, and further calibration measurements were taken. A heater was then switched on for less than 2 hours to accelerate the boil-off rate of the remaining liquid, and switched off when a spike in stage temperature was observed which indicated that the liquid surface had dropped below the stage. Further calibration was performed after turning off the heater as the stage was allowed to warm from 1.6 K under the ambient heating power of the measurement apparatus (see SI section S.3 for more details). The dewar was then back-filled with ^4He exchange gas to facilitate thermalization to room temperature, at which point the dipstick was removed.

We now discuss the details of the measurements in each calibration dewar. In the first calibration dewar, the attenuator and DUT shown in the configuration of Fig. 1(a) were replaced with a short through which was thermally anchored to the copper stage. Figure 2(a) shows the total loss L_{coax} of the coaxial cables measured at room temperature in air and in the first calibration dewar at 4.2 K and 1.6 K. To isolate the individual cable losses L_1 and L_3 , these losses were also measured independently at room temperature, and their ratio was assumed to remain constant for all temperatures.

The lumped coaxial cable physical temperature T_{coax} , which is the effective temperature at which the cables radiate their noise power, was measured directly using the Y-factor method. Figure 2(b) shows the measured lumped temperature versus frequency with the cables dipped in a 4.2 K He I bath. We assumed that both cables were at the same physical

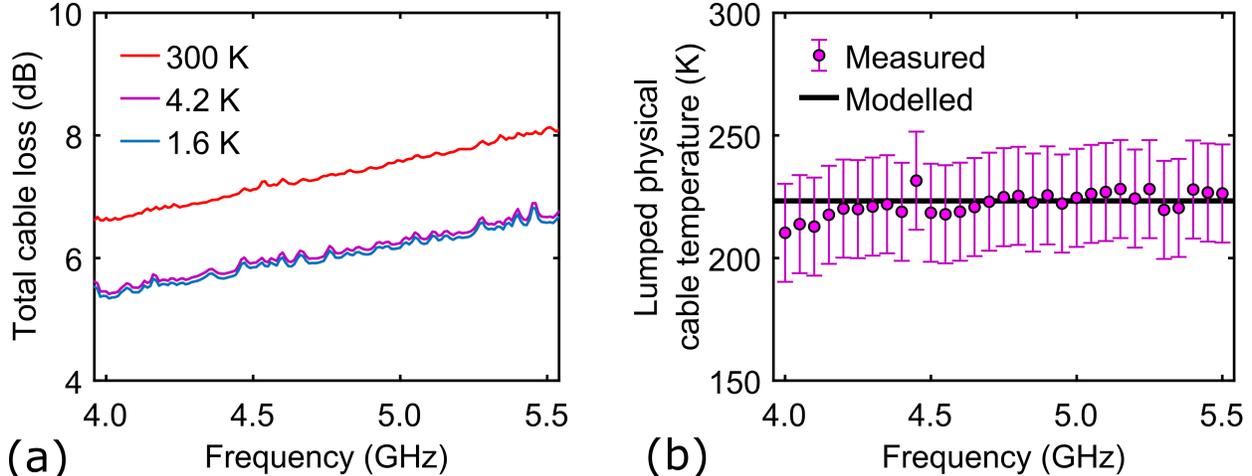


Fig. 2: (a) Total loss of input and output coaxial cables versus frequency measured at 300 K (red line), 4.2 K (magenta line) and 1.6 K (blue line) with a commercial VNA. (b) Lumped physical coaxial cable temperature versus frequency obtained from Y-factor measurements (magenta circles) and from a heat conduction model (black line). Error bars represent an estimate of the total uncertainty including systematic errors.

temperature. The 1.6 K He II calibration measurement is omitted for clarity as it is within 10 K of the He I measurement. To support the accuracy of the measured effective coaxial cable temperature, we estimated the value using an extension of the cable temperature model from Ref. [39]. This estimate is shown in Fig. 2(b) and is in agreement with the measured values (see SI section S.4 for more details).

In the second calibration dewar, the short through was replaced by the 20 dB attenuator. The total loss $L = L_{\text{coax}}L_2$ was measured using the same procedure as in the first calibration dewar, and the attenuator loss L_2 was extracted by dividing L by the previously measured L_{coax} . A measurement of the temperature of the attenuator using the Y-factor method was not possible in this case due to the larger loss of the attenuator. Instead, the temperature T_{L_2} of the attenuator was measured by a calibrated LakeShore DT-670-SD temperature diode indium-bonded directly to the attenuator chip. The diode calibration curve was provided by LakeShore and the saturated liquid temperature of He I at 4.23 K was used to correct for DC offsets. We assumed that negligible temperature differences existed between the attenuator, mounting stage and DUT, and we therefore took the DUT physical temperature to be $T_{\text{phys}} = T_{L_2}$. All temperature diodes were measured using a LakeShore 336 temperature controller, which converted the temperature to a voltage measured by the DAQ synchronously with all Y-factor measurements.

C. LNA noise and gain measurements

With the calibration data obtained, the noise temperature and gain of the DUT was measured using two additional liquid ^4He dewar baths at various frequencies, temperatures, and biases. The gate-source and drain-source bias voltages V_{GS} and V_{DS} , which were nominally applied equally to each individual transistor, were varied to yield transistor drain-source current densities I_{DS} from 35 mA mm^{-1} to 120 mA mm^{-1} , corresponding to dissipated power densities P_{DC} per transistor of 15 mW mm^{-1} to 150 mW mm^{-1} . The relevant measured values from the calibration procedure described above were used to extract G and T_e from Eqs. (2) and (3).

In the first measurement dewar, the device was mounted to the stage in the configuration shown in Fig. 1(a), and the cooldown procedure followed that of the calibration dewars described above. At each physical temperature, the DUT bias was varied and Y-factor measurements were taken versus frequency by adjusting the YIG filter frequency. In addition, the calibration measurement of $V_{\text{H}}^{\text{cal}}$, $V_{\text{C}}^{\text{cal}}$ and $G_{\text{full}}^{\text{cal}}$ was performed, with the DUT bias chosen arbitrarily to be its low-noise bias. Using these measurements, the device gain G could be extracted from Y-factor voltage measurements at any bias using Eq. (2) without requiring a separate VNA measurement. This calibration was found to be stable over several days, and was repeated each day before data acquisition.

In addition to measurements under specific liquid cryogen environments, continuous Y-factor measurements were also taken as the He II bath was pumped away, yielding noise data both before and after the DUT gate was submerged. The measurements were performed at a fixed bias of $P_{\text{DC}} = 80 \text{ mW mm}^{-1}$ and frequency of $f = 4.55 \text{ GHz}$. The bias was chosen to be sufficiently high that any self-heating mitigation would be readily observed without risking device damage due to prolonged biasing, and the frequency was chosen due to it being the optimum noise match frequency as determined by the IMN [37].

The He II film creep effect [40] was expected to cause the entire mounting stage, including the heated DUT region, to be coated in a superfluid film even after the liquid bath surface dropped below the DUT height. We used the sharp rise observed in the attenuator and stage temperature measurements to indicate the complete evaporation of He II from the stage. The DUT noise was also measured on warming from 1.6 K to 80 K in the vacuum space left after all liquid was pumped away.

In the second measurement dewar, the noise temperature was again measured in 4.2 K liquid using the same procedure as in the first measurement dewar but without the subsequent evaporative cooling step. Instead, the liquid bath was allowed to evaporate under the heating power of the dipstick, enabling measurements to be taken in a vapor environment at several temperatures after the liquid level dropped below the mounting stage. The vapor warmed sufficiently slowly (< 1 K/hour) such that all measurements were effectively taken in a steady state vapor environment. The calibrations used for these measurements were the same as for the 4.2 K liquid since the coaxial cable loss and temperature were observed to change negligibly up to 45 K stage temperature.

III. RESULTS

A. Microwave noise temperature versus frequency

We begin by showing the noise temperature and gain versus frequency at various temperatures, bath conditions and biases. Figure 3(a) shows T_e and G versus f with the device biased at its low-noise bias of $P_{\text{DC}} = 24.5 \text{ mW mm}^{-1}$ and immersed in three different bath conditions ranging from 1.6 K He II to 35.9 K vacuum, with measurements in 4.2 K He I and 8.2 K ^4He vapor omitted for clarity since they are within 0.2 K of the He II data. The noise varies by less than 2 K across all measured frequencies and temperatures at this bias. The noise increases monotonically with increasing physical temperature regardless of bath condition. The gain varies by approximately 4 dB over the measured frequency range, peaking at $f = 4$ GHz. The gain variation with physical temperature is less than 0.5 dB at fixed f and P_{DC} , and so only the He II gain data is shown for clarity.

Figure 3(b) shows T_e and G versus f with the device immersed in 1.6 K He II at three different device biases of $P_{\text{DC}} = 24.5 \text{ mW mm}^{-1}$, $P_{\text{DC}} = 79.5 \text{ mW mm}^{-1}$ and $P_{\text{DC}} = 120 \text{ mW mm}^{-1}$. At biases below $P_{\text{DC}} = 79.5 \text{ mW mm}^{-1}$ the noise temperature varies by less than 1.5 K for all frequencies, while above $P_{\text{DC}} = 79.5 \text{ mW mm}^{-1}$ the noise increases more rapidly with bias. The gain increases monotonically with increasing bias while retaining the same shape versus frequency, but it appears to asymptotically plateau at approximately the highest gain shown here, at a bias of $P_{\text{DC}} = 120 \text{ mW mm}^{-1}$.

To interpret these measurements, a microwave model of the full device including IMN,

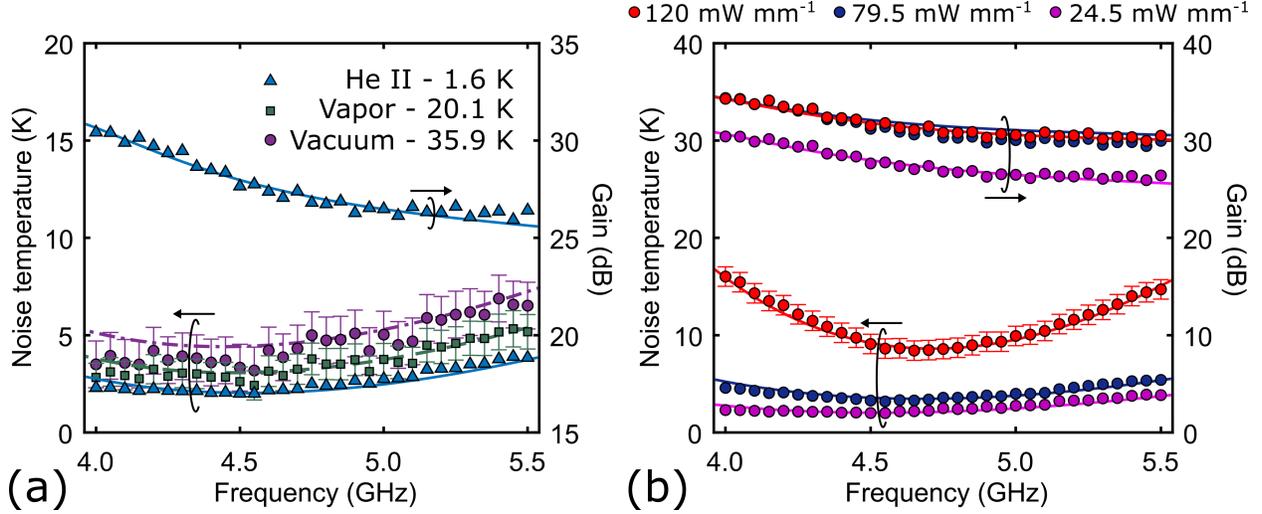


Fig. 3: (a) Noise temperature (left axis) and gain (right axis) versus frequency, measured at the device’s low-noise bias of $P_{\text{DC}} = 24.5 \text{ mW mm}^{-1}$ ($V_{\text{DS}} = 0.56 \text{ V}$, $I_{\text{DS}} = 43.9 \text{ mA mm}^{-1}$, $V_{\text{GS}} = -2.7 \text{ V}$) in the following cryogenic environments: 1.6 K He II (blue triangles), 20.1 K vapor (green squares) and 35.9 K vacuum (purple circles). Only the gain under He II conditions is shown for clarity since the gain varies by less than 0.5 dB across all temperatures. The small-signal model fits for He II (solid blue line), ^4He vapor (dashed green line) and vacuum (dash-dotted purple line) are also shown. (b) Noise temperature (left axis) and gain (right axis) versus frequency measured at biases of $P_{\text{DC}} = 24.5 \text{ mW mm}^{-1}$ (magenta circles; $V_{\text{DS}} = 0.56 \text{ V}$, $I_{\text{DS}} = 43.9 \text{ mA mm}^{-1}$, $V_{\text{GS}} = -2.7 \text{ V}$), $P_{\text{DC}} = 79.5 \text{ mW mm}^{-1}$ (dark blue circles; $V_{\text{DS}} = 1.0 \text{ V}$, $I_{\text{DS}} = 79.5 \text{ mA mm}^{-1}$, $V_{\text{GS}} = -2.7 \text{ V}$) and $P_{\text{DC}} = 120 \text{ mW mm}^{-1}$ (red circles; $V_{\text{DS}} = 1.2 \text{ V}$, $I_{\text{DS}} = 100.0 \text{ mA mm}^{-1}$, $V_{\text{GS}} = -2.7 \text{ V}$) with the DUT submerged in He II at 1.6 K. To vary the bias, the gate-source voltage was held constant at $V_{\text{GS}} = -2.7 \text{ V}$ while the drain-source voltage V_{DS} was varied. The small-signal model fits (solid lines) are also shown. Where omitted in both (a) and (b), the vertical error bars are equal to the height of the symbols.

monolithic microwave integrated circuit (MMIC) components and transistor small-signal model was made using Microwave Office. The model used micrograph measured values of the IMN, foundry schematic values for the MMIC, and independently measured small-signal model values from nominally identical discrete transistors from Ref. [41]. The small-signal model and IMN parameters were manually tuned by less than 20 % from these starting values to fit both the gain and noise temperature curves (see SI section S.1 for more details). Representative frequency-dependent results of the model are plotted in Figs. 3(a) and 3(b). The modelled and measured gain are in agreement over the full frequency range, and the model captures the overall trend in noise temperature as a function of temperature and bias.

We used this model to extract the gate temperature T_g under various conditions. Figure 4(a) shows the extracted T_g versus T_{phys} at the device’s low-noise bias of $P_{\text{DC}} = 24.5 \text{ mW mm}^{-1}$. T_g is elevated above T_{phys} below 20 K even in the presence of superfluid, and follows T_{phys} above 20 K, behavior which is in agreement with prior reports [26–28]. Figure 4(b)

shows T_g versus P_{DC} at $T_{\text{phys}} = 1.6$ K. Here, T_g changes by less than 2 K for bias powers below $50 - 75$ mW mm⁻¹, after which T_g increases more rapidly.

We compare these results with an equivalent circuit radiation model of the HEMT structure developed in Ref. [29]. The explicit functional form for the gate temperature derived from this model is:

$$T_g(T_s, P_{\text{DC}}) = \left(T_s^4 + \frac{P_{\text{DC}} \mathcal{R}_{cs} \mathcal{R}_{gs}}{\sigma_p (\mathcal{R}_{cs} + \mathcal{R}_{gc} + \mathcal{R}_{gs})} \right)^{1/4} \quad (4)$$

where T_s is the substrate temperature, $\sigma_p = 850$ W m⁻² K⁻⁴ is the equivalent Stefan–Boltzmann constant for phonons in GaAs, and $\mathcal{R}_{ij} = A_i F_{ij}$ is the space resistance between nodes i and j with emitting line length A_i and view factor F_{ij} which quantifies the fraction of power emitted from surface i that intercepts surface j . The subscripts g , c and s represent the gate, channel, and substrate, respectively. Following Ref. [30], we take $A_g = A_c = 70$ nm and compute $F_{gc} = 0.3$.

The predictions of Eq. (4) are also shown in Figs. 4(a) and 4(b). In Fig. 4(a) the data and model are in quantitative agreement over the full range of physical temperatures. The model captures the elevated gate temperature behavior at fixed bias. In Fig. 4(b) the data and model agree at biases below 50 mW mm⁻¹, but the data deviates from the model above some critical bias between 50 – 75 mW mm⁻¹. The origin of this discrepancy is presently unclear. A possible explanation is that other noise sources are being attributed to gate thermal noise leading to artificially high extracted gate temperatures. This additional noise may signal the onset of impact ionization [42, 43], however the usual indications of this process, such as increased gate-leakage current and decreased gain, were not observed. The gain plateaued at 33 dB at the highest measured bias, as shown in Fig. 3(b), and the gate current remained less than 100 μ A at all biases. This discrepancy remains a topic of investigation.

B. Noise temperature dependence on cryogenic environment

We obtain further insight into how liquid cryogenics impact HEMT noise performance by examining the DUT noise temperature measured continuously in a changing cryogenic environment. Figure 5(a) shows the time series of both T_e and T_{phys} measured continuously as the He II was pumped out of the measurement dewar, with $t = 0$ minutes chosen as a

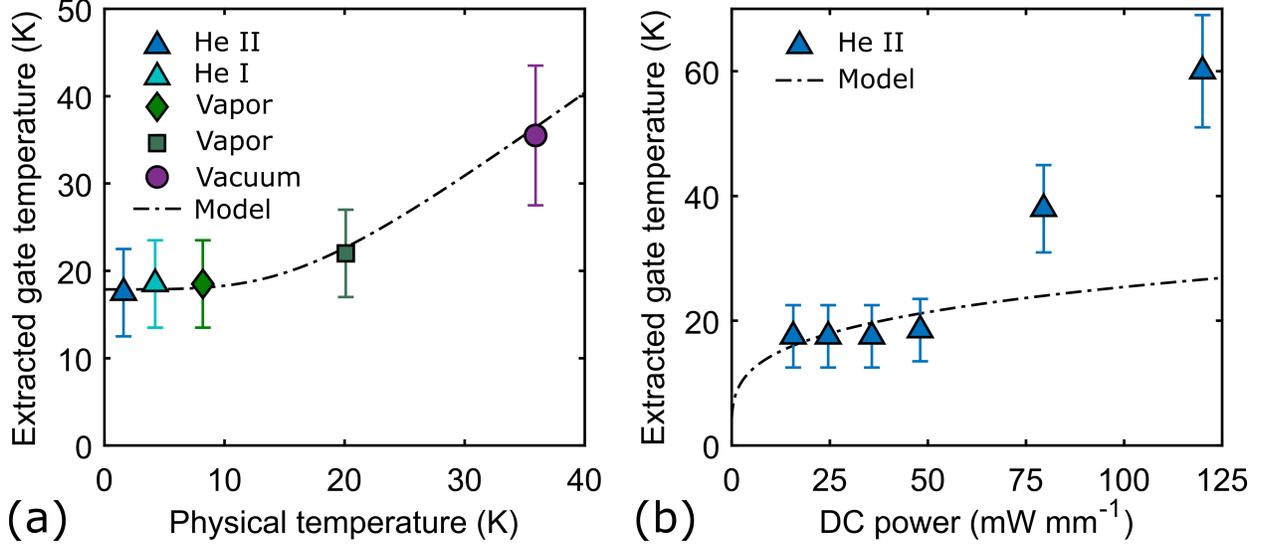


Fig. 4: (a) Extracted gate temperature versus physical temperature at the device’s low-noise bias of $P_{\text{DC}} = 24.5 \text{ mW mm}^{-1}$. Symbols indicate extracted values and represent the same conditions as in Fig. 3(a), along with extracted values in 4.2 K He I (cyan triangles) and 8.1 K vapor (green diamonds). The radiation model is also shown (dash-dotted black line). (b) Extracted gate temperature versus bias at 1.6 K physical temperature (blue triangles). The radiation model is also shown (dash-dotted black line). In both (a) and (b) the error bars were generated by determining the range of gate temperatures that accounted for the uncertainty in the frequency-dependent noise temperature data.

reference time at which a rise in T_{phys} was observed, interpreted as the departure of superfluid from the attenuator and device. A corresponding feature in the DUT noise temperature is absent. After $t = 0$ minutes the device thermalized with the surrounding ^4He vapor, and T_e was observed to increase smoothly with increasing physical temperature. After 20 minutes, the remaining He II liquid below the stage fully evaporated, and the warming rate increased as the mounting stage and DUT passively warmed to room temperature through the mounting apparatus.

In Fig. 5(b) the warming curve of T_e plotted against T_{phys} is shown from 1.6 K to 80 K, taken from the time series in Fig. 5(a). Again, the noise temperature measured in vacuum exhibits no sharp features, instead smoothly varying with physical temperature. Also plotted are noise temperatures measured separately under various bath conditions, at the same bias and frequency. We note that the liquid, vapor and vacuum data all lie within the reported error bars of the warming curve. These observations suggest that the liquid and vapor cryogen environments provide no self-heating mitigation beyond maintaining a fixed ambient temperature.

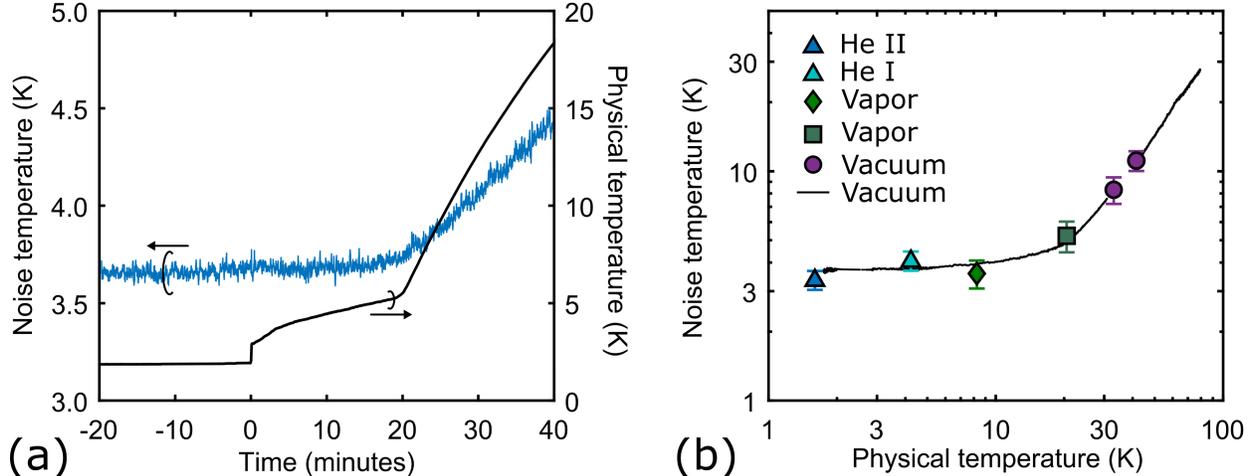


Fig. 5: (a) Noise temperature (left axis, blue line) and physical temperature (right axis, black line) versus time in an evaporating He II bath sampled at $f_{\text{ENR}} = 10$ Hz and digitally filtered at 1 Hz, taken at a fixed bias $P_{\text{DC}} = 80$ mW mm $^{-1}$ ($V_{\text{DS}} = 1.0$ V, $I_{\text{DS}} = 80$ mA mm $^{-1}$, $V_{\text{GS}} = -2.8$ V) and frequency $f = 4.55$ GHz. The sharp kink in the physical temperature at time $t = 0$ minutes, interpreted as the time at which superfluid is no longer present on the attenuator and device, is not reflected in the noise temperature. (b) Noise temperature (black line) versus physical temperature obtained from the transient data shown in (a). Symbols show independently measured noise temperatures representing the same bath conditions as in Fig. 3(a), and the same bias conditions as in (a). The presence of liquid cryogenics does not affect the noise temperature within the measurement uncertainty.

IV. DISCUSSION

A. Limits on thermal conductance at the ^4He -gate interface

We consider our finding that He II is unable to mitigate self-heating in the context of prior studies of He II heat transport properties. We estimate the heat flow \dot{Q} between the He II bath and the HEMT gate surface using $\dot{Q} = hA\Delta T$ where h is the thermal boundary conductance, A is the surface area of the gate, and ΔT is the steady-state temperature difference. We note that considerable uncertainty exists in all of these parameters owing to the complex heat transfer regime involving film boiling of superfluid ^4He [44] and its dependence on the surface conditions, as well as the effective area for heat transfer of the HEMT. We therefore expect our estimate to give an order-of-magnitude indication of the heat flux.

To begin, we take the surface area to be that of the gate head, $1 \mu\text{m} \times 200 \mu\text{m}$, which we take as an estimate of the heat transfer area of the dielectric passivation layer which covers the device. A representative value of the temperature difference if the liquid cryogenics measurably decreased the surface temperature is $\Delta T \sim 10$ K. To estimate the heat transfer

coefficient, we take $h = 6.2 \text{ kW m}^{-2} \text{ K}^{-1}$, which is the highest measured He II thermal boundary conductance in the film boiling regime, taken from the compiled data in Table 7.5 from Ref. [31]. It is possible that h could exceed this value owing to the micron-scale dimensions of the gate per Eq. 7.113 of [31], but we neglect this effect here. Assuming a temperature difference $\Delta T = 10 \text{ K}$, we estimate $\dot{Q} \sim 12 \text{ } \mu\text{W}$. This value is two orders of magnitude less than the milliwatts of power required to optimally bias the device in our experiment. The absence of any observable effect of cooling by liquid cryogenics in our experiment therefore indicates that the particular surface conditions of the HEMT do not increase the boiling heat flux sufficiently to make a measurable impact on the microwave noise temperature.

B. Implications for noise performance of cryogenic HEMTs

We now examine the implications of our finding that self-heating in HEMTs cannot be mitigated for their noise performance. The only method to reduce gate noise below the observed low-temperature plateau at approximately 20 K is to reduce the dissipated DC power. However, lower power also reduces the gain, which in turn leads to an increase in the contribution of both drain noise and any noise source originating after the gain stage of the HEMT, when referred to the input.

We first explore how T_{\min} from the Pospiezalski model [20] varies with bias while including the explicit bias dependencies of both T_{d} and T_{g} . In the limit $f \ll f_T$, T_{\min} is given by [1]:

$$T_{\min} = g_0 \frac{\sqrt{T_{\text{g}} T_{\text{d}}}}{g_{\text{m}}} \quad (5)$$

where we have explicitly introduced the prefactor $g_0 = 4\pi f (C_{\text{gs}} + C_{\text{gd}}) \sqrt{(r_i + R_{\text{G}} + R_{\text{S}}) g_{\text{ds}}}$ which is bias independent. We assume $T_{\text{phys}} = 4.2 \text{ K}$, a gate-source capacitance $C_{\text{gs}} = 150 \text{ fF}$, a drain-source capacitance $C_{\text{ds}} = 28 \text{ fF}$, frequency $f = 5 \text{ GHz}$, a parasitic gate resistance $R_{\text{G}} = 1 \text{ } \Omega$, a parasitic source resistance $R_{\text{S}} = 1 \text{ } \Omega$, an intrinsic input resistance $r_i = 1 \text{ } \Omega$, an intrinsic drain-source conductance $g_{\text{ds}} = 15.4 \text{ mS}$, and a transconductance g_{m} obtained by taking a finite-difference approximation of the derivative of $I_{\text{DS}} - V_{\text{DS}}$ data for different V_{GS} separated by 20 mV. All values are taken from Ref. [41]. We also assume a drain noise temperature that varies linearly from $T_{\text{d}} = 20 \text{ K}$ at $P_{\text{DC}} = 0 \text{ mW mm}^{-1}$ to $T_{\text{d}} = 1000 \text{ K}$

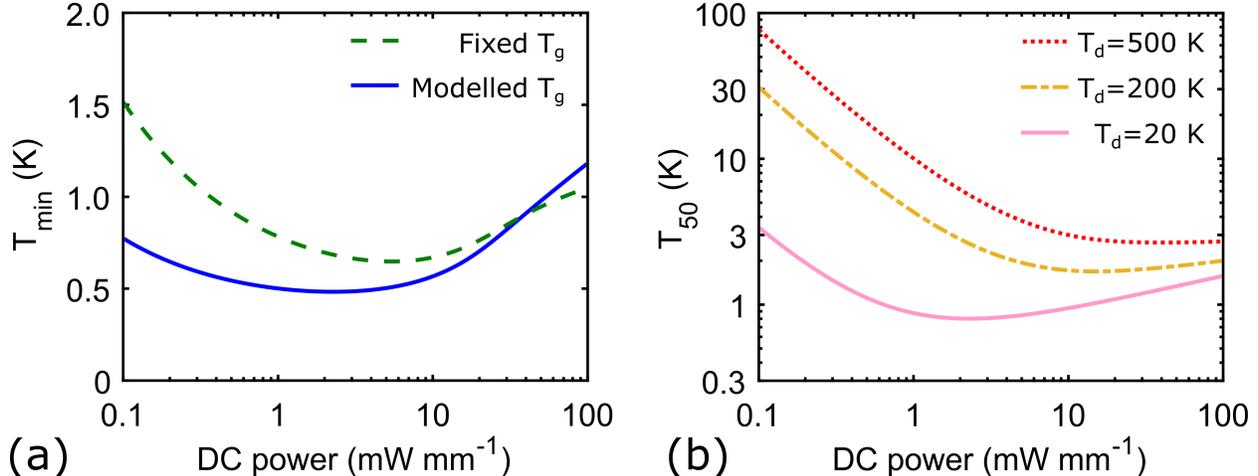


Fig. 6: (a) Modelled T_{\min} versus bias, shown for a fixed gate temperature $T_g = 20$ K (dashed green line) and for a gate temperature with bias dependence determined by a radiation model (solid blue line). The radiation model predicts a gate temperature below 20 K for powers below 40 mW mm^{-1} , which is reflected in T_{\min} (b) Modelled T_{50} noise temperature versus bias, shown for $T_d = 500$ K (dashed red line), $T_d = 200$ K (dash-dotted gold line) and $T_d = 20$ K (solid pink line). Both the minimum T_{50} and the power required to achieve this minimum decrease with decreasing T_d .

at $P_{\text{DC}} = 68.5$ mW mm^{-1} , an approximation of the bias dependence measured in Ref. [41], while taking $T_d = 20$ K as the zero-bias limit.

Figure 6(a) shows T_{\min} versus dissipated power both with and without the $T_g \propto P_{\text{DC}}^{1/4}$ dependence predicted by the radiation model. For the case of fixed gate temperature we assume $T_g = 20$ K. The radiation model predicts a lower T_{\min} than the fixed T_g model up to a power of $P_{\text{DC}} = 40$ mW mm^{-1} , above which self-heating raises the gate temperature above 20 K. The minimum T_{\min} predicted by the radiation model is lower than that predicted by the fixed model by 0.16 K, and the bias at which the minimum T_{\min} occurs is also lower by 2.7 mW mm^{-1} . Below this optimal bias, both models predict an increase in T_{\min} with decreasing power, indicating where the gain is insufficient to overcome drain noise.

A reduction in drain noise at low biases is evidently beneficial. We demonstrate the effect such a reduction has on the overall noise temperature by using a phenomenological model that both accounts for self-heating and separates the input and output noise contributions additively, a feature of noisy amplifiers which is not captured in the expression of T_{\min} . At low enough frequencies such that $f \ll f_T$, the noise temperature of a HEMT with a 50 Ω source impedance is:

$$T_{50} = \frac{T_g(r_i + R_G + R_S) + T_d g_{\text{ds}} g_m^{-2}}{50 \Omega} \quad (6)$$

as derived in Ref. [37] in the limit of open C_{gs} and C_{gd} . For illustrative purposes, we assume a constant T_d , $T_{\text{phys}} = 4.2$ K, and all other parameter values identical to those in the T_{min} model.

Figure 6(b) shows the modelled T_{50} noise temperature versus dissipated power at different drain temperatures. Drain temperatures of 500 K and 200 K were chosen to approximate state-of-the-art low-power performance in GaAs devices [41] and InP devices [9], respectively. A knee is observed in each curve, the location of which indicates where the gate noise and input-referred drain noise become comparable in magnitude. As T_d is decreased, both the T_{50} value at the knee and the bias at which the knee is observed decrease. This feature is explained as follows. As T_d decreases, less gain is required to achieve the same contribution of T_d to the overall noise, which implies that less power is required to bias the device, ultimately leading to less self-heating and therefore a lower T_g . In this way we see that reducing T_d leads to a simultaneous improvement in noise temperature and reduction in optimal low-noise bias.

V. SUMMARY

We have presented noise measurements of a packaged low-noise GaAs HEMT amplifier immersed in various cryogenic baths. The measured noise temperature and extracted gate temperature trends are generally consistent with those expected from heat dissipation by phonon radiation, independent of the presence of liquid cryogenes. This finding indicates that self-heating of cryogenic HEMTs cannot be mitigated. We explored the consequences of this result on overall noise performance for various values of the drain temperature. A decreased drain temperature was found to enable simultaneous improvements in noise performance by reducing the necessary gain and hence dissipated power, thereby reducing the self-heating. This result supports the general design principle of cryogenic HEMTs of maximizing gain at the lowest possible power.

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Supplementary Information

Self-heating of cryogenic HEMT amplifiers and the limits of microwave noise performance

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S.1. DEVICE MODELLING

Cadence AWR Microwave Office was used to create a model of the packaged device. Figure S.1 shows a micrograph image of the 2-stage MMIC, including the external IMN, which are all housed inside a gold-plated copper chassis. All components of both the IMN and MMIC were included in the model.

Model parameter fitting to the gain and noise temperature data was performed manually. The transistor small-signal model parameters were taken from nominally identical OMMIC devices measured in a separate study [1]. All parameters were constrained to change by less than 20% from these starting values. The gain data from several datasets were used first to tune the IMN microstrip geometry which determined the shape of the gain versus frequency. The gain data from each dataset was then used to tune the small-signal model parameters C_{gs} , C_{gd} , g_m and g_{ds} . Finally the noise data was used to tune and extract T_g and T_d .

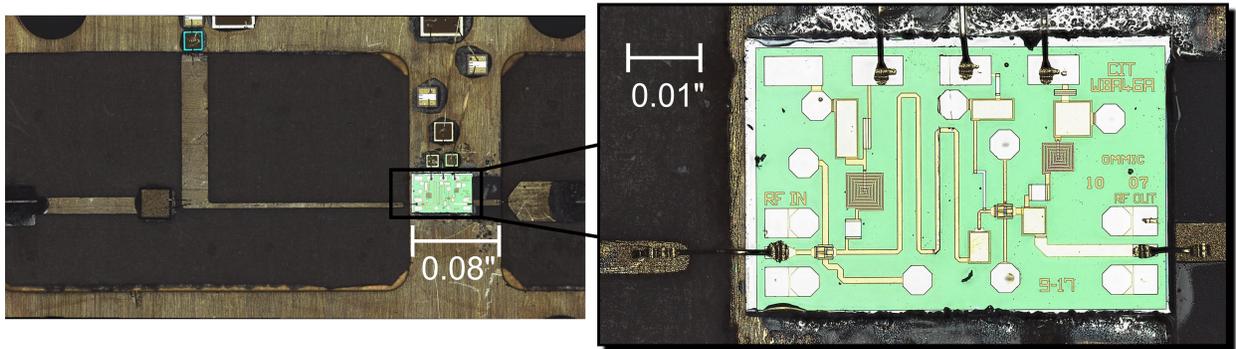


Fig. S.1: High-resolution micrograph image of the packaged device including the input matching network and MMIC. The inset shows a zoom of the MMIC fabricated by OMMIC.

S.2. NOISE TEMPERATURE AND GAIN EQUATION DERIVATIONS

We present a derivation of Eqs. (2) and (3) from the main text, starting from the definition of the measured Y-factor:

$$Y \equiv \frac{P_H}{P_C} \quad (\text{S.1})$$

where P_H and P_C are the noise powers presented to the input of the backend detector with the noise source turned on and off respectively. Using the definition of noise temperature $T_N = P_N(Bk_B)^{-1}$ where P_N is the noise power, B is the measurement bandwidth and k_B is the Boltzmann constant, we can write:

$$\frac{P_H}{Bk_B} = \left(T_H \frac{G}{L_1 L_2 L_3} + T_{L_1} \frac{G(L_1 - 1)}{L_1 L_2 L_3} + T_{L_2} \frac{G(L_2 - 1)}{L_2 L_3} + T_e \frac{G}{L_3} + T_{L_3} \frac{(L_3 - 1)}{L_3} + T_{\text{BE}} \right) \quad (\text{S.2})$$

$$\frac{P_C}{Bk_B} = \left(T_C \frac{G}{L_1 L_2 L_3} + T_{L_1} \frac{G(L_1 - 1)}{L_1 L_2 L_3} + T_{L_2} \frac{G(L_2 - 1)}{L_2 L_3} + T_e \frac{G}{L_3} + T_{L_3} \frac{(L_3 - 1)}{L_3} + T_{\text{BE}} \right) \quad (\text{S.3})$$

where each term in Eqs. (S.2) and (S.3) represents the noise power added by successive elements in the measurement apparatus as defined in Section II A of the main text, and $T_H = T_0 E + T_C$ is the noise temperature of the noise source when switched on. We have also used the fact that the input-referred noise temperature T_{in} of a matched attenuator with loss L and physical temperature T_L as derived in Ref. [2] is:

$$T_{\text{in}} = (L - 1)T_L \quad (\text{S.4})$$

Plugging Eqs. (S.2) and (S.3) into Eq. (S.1) and solving for T_e yields Eq. (3) from the main text.

To derive Eq. (2) from the main text, we first recall that P_H and P_C are linearly transduced by the diode detectors with some power-to-voltage gain K so that we can write the measured voltages as $V_H = KP_H$ and $V_C = KP_C$ [3]. Plugging these into Eqs. (S.2) and (S.3),

subtracting Eq. (S.2) from Eq. (S.3) and rearranging yields:

$$\frac{V_H - V_C}{G_{\text{full}}} = KBk_B T_0 E \quad (\text{S.5})$$

Since the right side of this equation is constant, we see that the ratio $\frac{V'_H - V'_C}{G_{\text{full}}}$ must also be constant. This allowed us to calibrate the output voltage difference $V'_H - V'_C$ directly to the VNA measurement of the total gain G_{full} by fixing the DUT bias to some arbitrarily chosen calibration value and measuring V_H^{cal} , V_C^{cal} and the total gain $G_{\text{full}}^{\text{cal}}$ at this bias. The total gain at any other bias could then be extracted by measuring only $V'_H - V'_C$ and using Eq. (S.5). Equation (2) from the main text follows from setting the left side of Eq. (S.5) equal to itself at two different DUT biases and rearranging for G .

S.3. CALIBRATION DETAILS

Figure S.2(a) shows the calibration data for the backend noise temperature T_{BE} versus frequency. This measurement was performed using a nitrogen cooled fixed load method, where the noise power of the backend terminated with a $50\ \Omega$ load was first measured at room temperature, and then measured again with the $50\ \Omega$ load submerged in a small liquid nitrogen bath at 77 K. Thermal insulation was placed over the bath to minimize vapor cooling of the 3.5 inch long stainless steel coaxial cable connecting the backend to the $50\ \Omega$ load. The time between hot and cold measurements was approximately 1 minute, the time it took to dip the load into the nitrogen bath and allow its temperature to stabilize as indicated by a plateauing of the measured noise power. All backend amplifiers, tunnel diode detectors and DC pre-amplifiers were wrapped in thermal insulation to mitigate drifts in gain and noise temperature.

Figure S.2(b) shows the calibration data for the noise source ENR versus frequency, measured by taking Y-factor measurements with the noise source connected directly to the calibrated backend detector. The ENR was extracted using the following equation:

$$E = \frac{Y - 1}{T_0}(T_C + T_{\text{BE}}) \quad (\text{S.6})$$

The noise source chassis, which was wrapped in thermal insulation to promote thermal equilibration between the chassis and the internal noise diode, was monitored at all times using a type T thermocouple, and its temperature was found to vary negligibly under all experimental conditions. We took the chassis temperature to be equal to the internal diode temperature T_C .

Figure S.2(c) shows the attenuator loss versus frequency measured at room temperature, 4.2 K and 1.6 K. The measurement procedure is described in Section II B of the main text. The losses varied by less than 0.1 dB between each temperature, which is consistent with other measurements of similar chip attenuators [4].

Figure S.2(d) shows the lumped physical coaxial cable temperature versus stage temperature during the warming phase of the calibration measurements. Both of these quantities were measured independently as a time series, and the final calibration curve shown here was generated by fitting a smoothing spline to the raw data.

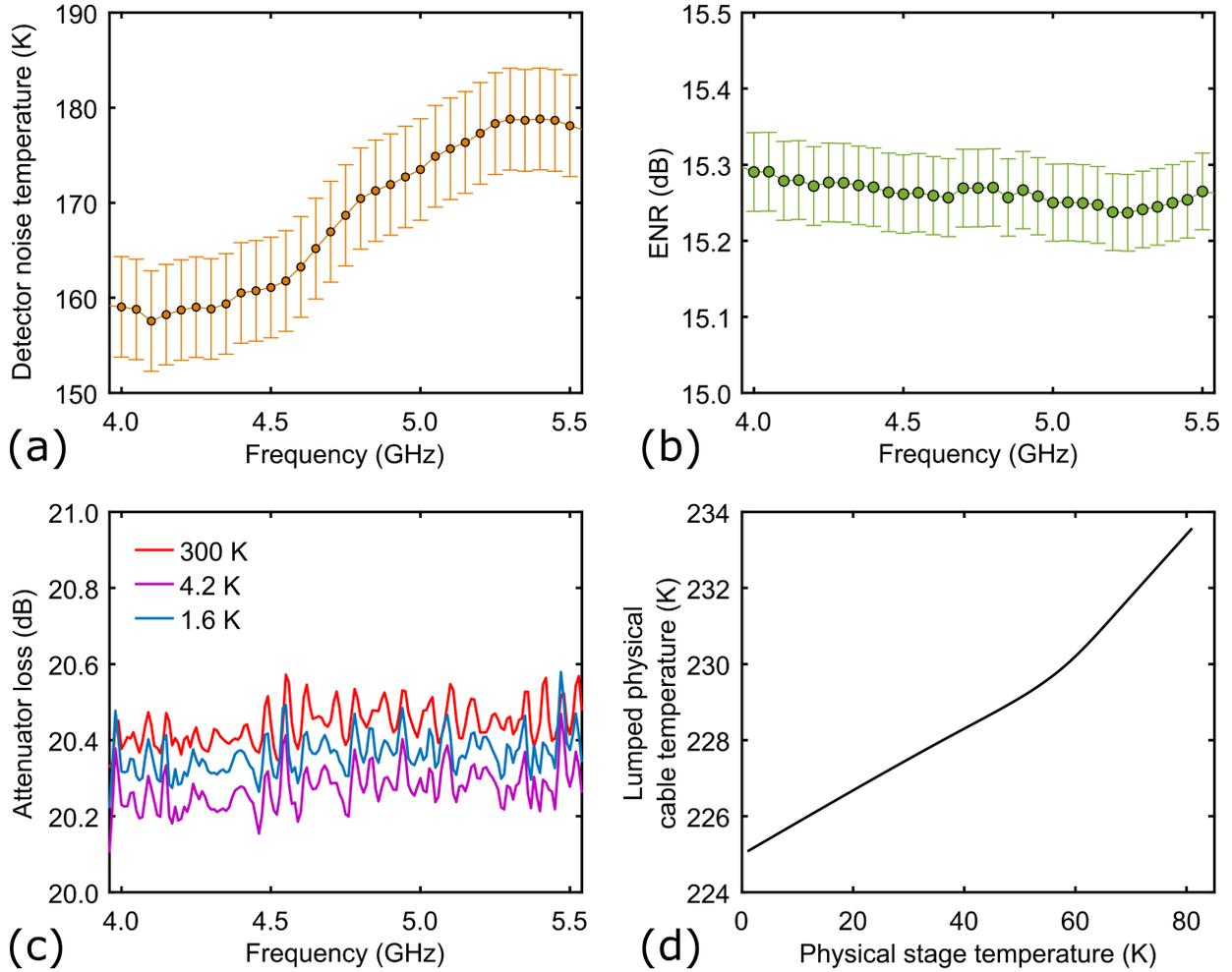


Fig. S.2: (a) Backend detector noise temperature versus frequency. Error bars reflect the uncertainty in the temperature of the cable connecting the $50\ \Omega$ load to the backend. (b) Noise source ENR versus frequency. Error bars reflect the error propagated from uncertainty in the backend noise temperature. (c) Attenuator loss versus frequency at room temperature (red line), 4.2 K (magenta line) and 1.6 K (blue line). (d) Lumped physical coaxial cable temperature versus physical stage temperature, taken from time series data of the cable temperature and stage temperature during the warming phase of the calibration measurements, and used as the calibration curve for the warming data shown in Fig. 5(b) in the main text.

S.4. CABLE TEMPERATURE CALIBRATION VIA THE Y-FACTOR METHOD

We measured the lumped input and output coaxial cable temperature T_{coax} directly using the Y-factor method. Recalling Eq. (S.4), we can write the measured hot and cold powers from the Y-factor measurements of a single attenuator with loss L and physical temperature T_L as:

$$P_H = \frac{T_0 E + T_C}{L} + \frac{T_L(L-1)}{L} + T_{\text{BE}} \quad (\text{S.7})$$

$$P_C = \frac{T_C}{L} + \frac{T_L(L-1)}{L} + T_{\text{BE}} \quad (\text{S.8})$$

taking the ratio $Y = \frac{P_H}{P_C}$ and solving for T_L gives:

$$T_L = \frac{1}{L-1} \left(\frac{T_0 E}{Y-1} - T_C - T_{\text{BE}} L \right) \quad (\text{S.9})$$

To support the accuracy of this method, we developed a thermal model of the cables used in our experiment. This model extends the work from Ref. [5] to include the effect of heat transfer between the cables and the surrounding fluid. We consider infinitesimal cross-sectional slices of the cable in contact with the surrounding bath. Assuming steady-state where the net heat flux is zero we can write:

$$\dot{Q}(x+dx) = \dot{Q}(x) + d\dot{Q}(x) \quad (\text{S.10})$$

where $\dot{Q}(x)$ is the heat flux at height x along the cable, and $d\dot{Q}(x)$ is the differential convective heat flux along the slice of length dx . Applying Fourier's law along the length of the cable and using the definition of the convective heat transfer coefficient [6] we can write:

$$\dot{Q}(x) = C_s(x) \left. \frac{dT}{dx} \right|_x \quad (\text{S.11})$$

$$d\dot{Q}(x) = PH_{g,1}(x) \left(T(x) - T_{g,1}(x) \right) dx \quad (\text{S.12})$$

$$C_s(x) = \frac{\kappa_{\text{steel}}(x)A_{\text{steel}} + \kappa_{\text{ptfe}}(x)A_{\text{ptfe}}}{\kappa_{\text{steel}}(x)\kappa_{\text{ptfe}}(x)A_{\text{steel}}A_{\text{ptfe}}} \quad (\text{S.13})$$

where $C_s(x)$ is the thermal conductance arising from the parallel stainless steel and teflon heat conduction channels in the cable, P is the perimeter of the cable, $H_{g,l}(x)$ and $T_{g,l}(x)$ are the convection coefficient and temperature of the surrounding gas and liquid baths, respectively, and $\kappa_{\text{steel}}(x)$, $\kappa_{\text{ptfe}}(x)$, A_{steel} and A_{ptfe} are the thermal conductivities and cross-sectional areas of the inner stainless steel conductor and teflon dielectric in the cable, respectively. The temperature dependence of the thermal conductivities κ_{steel} and κ_{ptfe} were taken from compiled data from NIST [7, 8]. Massaging the above equations and rearranging, we arrive at:

$$C_s(T(x))\frac{d^2T}{dx^2} + \frac{dC_s}{dT}(T(x))\left(\frac{dT}{dx}\right)^2 - PH_{g,l}(x)\left(T(x) - T_{g,l}(x)\right) = 0 \quad (\text{S.14})$$

A schematic of the model is shown in Fig. S.3(a), where we have defined $R_{\text{steel}} = (\kappa_{\text{steel}}A_{\text{steel}})^{-1}$, $R_{\text{ptfe}} = (\kappa_{\text{ptfe}}A_{\text{ptfe}})^{-1}$, and $R_{g,l} = (PH_{g,l}dx)^{-1}$. Equation (S.14) was solved numerically using the `bvp4c` routine in Matlab with fixed boundary values of 301 K and 4.2 K and assuming a liquid surface height of 10 cm above the bottom of the cable. Table I lists the remaining assumed parameter values, and Fig. S.3(b) shows the modelled coaxial cable temperature profile. Averaging over this curve yields an effective lumped physical temperature of the stainless steel coaxial cable of $T_{\text{steel}} = 208$ K.

We now determine the effective lumped physical temperature of the full cable by including the additional SMA cabling which connected the stainless steel cables to the noise source and backend detector. We make the simplifying assumptions that the entire stainless steel cable radiates its noise at T_{steel} with a loss $L_{\text{steel}} = 2.34$ dB and that the additional cabling radiates entirely at $T_{\text{cab}} = 301$ K with a loss of $L_{\text{cab}} = 0.65$ dB. The additional cabling loss was measured directly, and the stainless steel cable loss was found by subtracting L_{cab} from the total loss measured at 4.2 K shown in Fig. 2(a), at 5 GHz. Cascading the noise from these two cables and applying Eq. (S.4), we find:

$$T_{\text{coax}} = \frac{L_{\text{cab}}L_{\text{steel}}}{L_{\text{cab}}L_{\text{steel}} - 1} \left(\frac{T_{\text{cab}}(L_{\text{cab}} - 1)}{L_{\text{cab}}L_{\text{steel}}} + \frac{T_{\text{steel}}(L_{\text{steel}} - 1)}{L_{\text{steel}}} \right) \quad (\text{S.15})$$

$$T_{\text{coax}} = 223.3 \text{ K}$$

which was used for the value of the horizontal line shown in Fig. 2(b).

| Parameter | Value | Source |
|-------------------------|--------------------------------------|----------|
| κ_{steel} | T dependent | [7] |
| κ_{ptfe} | T dependent | [8] |
| A_{steel} | 1.30 mm ² | Measured |
| A_{ptfe} | 18.83 mm ² | Measured |
| P | 1.12 mm | Measured |
| H_g | 30 Wm ⁻² K ⁻¹ | [9] |
| H_l | 15 kWm ⁻² K ⁻¹ | [10] |

Table I: Table of parameters used in solving the coaxial cable thermal model.

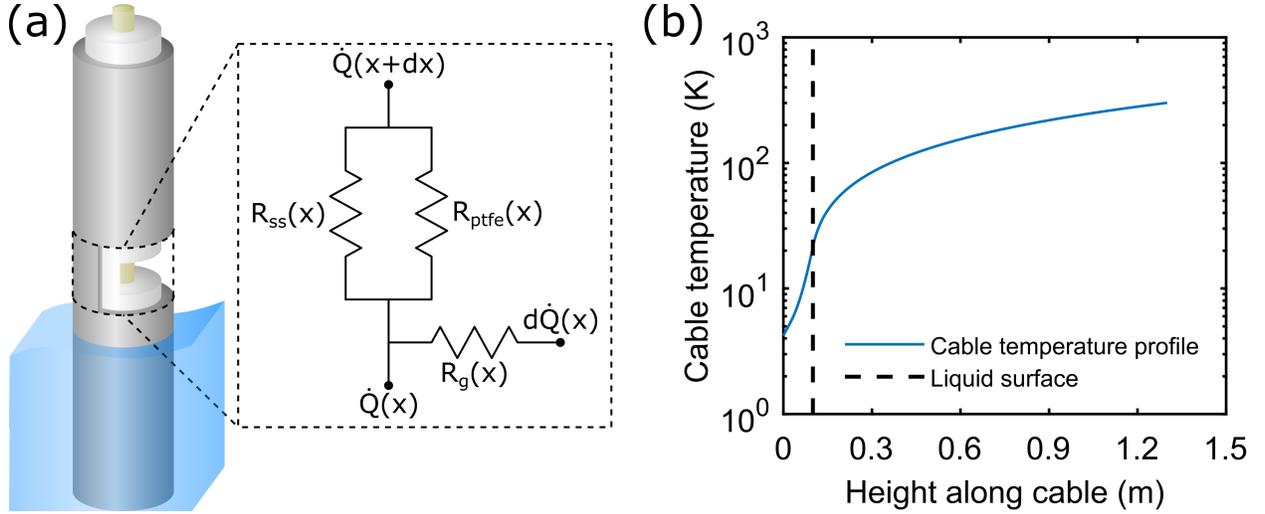


Fig. S.3: (a) Schematic of the coaxial cable temperature model showing a slice of length dx along the cable in contact with the gas environment. (b) Coaxial cable temperature profile (blue line) versus height along the cable. The height of the liquid surface is also shown (vertical dashed black line).

S.5. ERROR ANALYSIS

Here we estimate the contribution of each quantity in Eq. (3) from the main text to the overall noise temperature measurement uncertainty ΔT_e , as given by the standard error propagation formula:

$$\Delta T_e^X = \left| \frac{\partial T_e}{\partial X} \right| \Delta X \quad (\text{S.16})$$

where X denotes the measurement error source. All individual error sources are assumed to be independent unless otherwise stated, and are added in quadrature to estimate the overall uncertainty. Numerical estimates listed below assume a noise temperature of $T_e = 2$ K. We assume that each VNA loss measurement has an uncertainty of ± 0.01 dB, which is the magnitude of the variation in the measured loss versus frequency of the calibration cable immediately after calibration. We also assume that the variance in the measured losses across different calibration and measurement dewars is 0.03 dB.

A. Attenuator

The uncertainty in the attenuator loss ΔL_2 is found by adding the uncertainty from the VNA measurements in calibration dewars 1 and 2 in quadrature, so that $\Delta L_2 = 0.05$ dB.

$$\left| \frac{\partial T_e}{\partial L_2} \right| = \frac{T_e + T_{L_2}}{L_2}$$

$$\Delta T_e^{L_2} = 0.036 \text{ K}$$

The uncertainty in the attenuator temperature T_{L_2} is determined by the temperature diode calibration. The calibrated Lake Shore DT-670-SD bonded to the attenuator chip has a manufacturer-reported temperature uncertainty of $\Delta T_{L_2} = \pm 20$ mK.

$$\left| \frac{\partial T_e}{\partial T_{L_2}} \right| = \frac{L_2 - 1}{L_2}$$

$$\Delta T_e^{T_{L_2}} = 0.020 \text{ K}$$

B. Coaxial Cables

The uncertainties in the coaxial cable losses ΔL_1 and ΔL_3 are found by adding the uncertainties from three separate VNA loss measurements: the measurement of $L_{\text{coax}} = L_1 L_3$ in calibration dewar 1, and the measurements of L_1 and L_3 at room temperature to determine the loss asymmetry between the two cables, yielding $\Delta L_1 = 0.06$ dB and $\Delta L_3 = 0.07$ dB.

$$\left| \frac{\partial T_e}{\partial L_1} \right| = \frac{T_e + T_{L_1} L_2^{-1} + T_{L_2} (L_2 - 1) L_2^{-1}}{L_1}$$

$$\Delta T_e^{L_1} = 0.088 \text{ K}$$

$$\left| \frac{\partial T_e}{\partial L_3} \right| = \frac{T_{L_3}}{G_{\text{full}} L_1 L_2 L_3^2}$$

$$\Delta T_e^{L_3} = 0.006 \text{ K}$$

The uncertainty in the cable temperatures ΔT_{L_1} and ΔT_{L_3} are derived from error analysis of Eq. (S.9). We estimate $\Delta T_{\text{coax}} = \pm 20$ K.

$$\left| \frac{\partial T_e}{\partial T_{L_1}} \right| = \frac{L_1 - 1}{L_1 L_2}$$

$$\Delta T_e^{T_{L_1}} = 0.100 \text{ K}$$

$$\left| \frac{\partial T_e}{\partial T_{L_3}} \right| = \frac{L_3 - 1}{G_{\text{full}} L_1 L_2 L_3}$$

$$\Delta T_e^{T_{L_3}} = 0.032 \text{ K}$$

C. Gain

The uncertainty in the total gain ΔG_{full} comes directly from the uncertainty of a single VNA loss measurement so that $\Delta G_{\text{full}} = \pm 0.01$ dB.

$$\left| \frac{\partial T_e}{\partial G_{\text{full}}} \right| = \frac{T_{\text{coax}}(L_3 - 1)L_3^{-1} + T_{\text{BE}}}{L_1 L_2 G_{\text{full}}^2}$$

$$\Delta T_e^{G_{\text{full}}} = 0.002 \text{ K}$$

D. Back-end Detector

The uncertainty in the backend detector noise temperature ΔT_{BE} is determined by the temperature and loss uncertainties in the coaxial cable connecting the cooled load to the backend. We estimate $\Delta T_{\text{BE}} = \pm 5$ K.

$$\left| \frac{\partial T_e}{\partial T_{\text{BE}}} \right| = \frac{1}{G_{\text{full}} L_1 L_2}$$

$$\Delta T_e^{\text{BE}} = 0.016 \text{ K}$$

E. Noise Source

The uncertainty in the noise source ENR comes from error analysis of Eq. (S.6). We estimate $\Delta E = \pm 0.040$ dB.

$$\left| \frac{\partial T_e}{\partial E} \right| = \frac{T_0}{(Y - 1)L_1 L_2}$$

$$\Delta T_e^E = 0.073 \text{ K}$$

The uncertainty in the noise source diode temperature ΔT_{C} comes from the uncertainty in the Type T thermocouple temperature measurement of the noise source chassis. We estimate $\Delta T_{\text{C}} = 1$ K.

$$\left| \frac{\partial T_e}{\partial T_{\text{C}}} \right| = \frac{1}{L_1 L_2}$$

$$\Delta T_e^E = 0.005 \text{ K}$$

F. Y-Factor Power

The Y-factor measurement uncertainty ΔY accounts for all uncertainty sources originating after the transduction of microwave power to DC voltage. We report an effective normalized Y-factor error $\frac{\Delta Y}{Y}$ of better than 3×10^{-4} for a 4 s integration time, which used for all steady-state data presented in this paper.

$$\left| \frac{\partial T_e}{\partial Y} \right| = \frac{T_0 E}{L_1 L_2 (Y - 1)^2}$$
$$\Delta T_e^Y = 0.003 \text{ K}$$

G. Cable Mismatch

There is error introduced from the difference in noise source impedance between the on and off state, which causes a changing reflection coefficient between the noise source and the first component in the measurement chain (in our experiment this is the input coaxial cable). In cases where the impedance match at the output of the noise source is poor, this error must be considered, and can be corrected for if the full S-parameters of the noise source in the on and off state and of the cable are known. In our experiment this error was found to contribute negligibly to the overall uncertainty.

H. Overall Uncertainty

The uncertainty budget is shown in Table II. The uncertainty analysis shown in this section was used to generate the error bars in the primary noise temperature datasets.

| Error Source | Value | Estimated Error | Contribution to T_e |
|-------------------|----------|-----------------|-----------------------|
| L_2 | 20.00 dB | 0.05 dB | 0.036 K |
| T_2 | 1.600 K | 0.020 K | 0.020 K |
| L_1 | 3.25 dB | 0.06 dB | 0.088 K |
| L_3 | 3.44 dB | 0.07 dB | 0.006 K |
| T_1 | 223 K | 20 K | 0.100 K |
| T_3 | 223 K | 20 K | 0.032 K |
| G_{full} | 1.98 dB | 0.01 dB | 0.002 K |
| T_{BE} | 170 K | 5.0 K | 0.016 K |
| E | 15.0 dB | 0.040 dB | 0.073 K |
| T_{C} | 301.0 K | 1 K | 0.009 K |
| Y | 6.8 | 0.002 | 0.003 K |
| RMS Sum | | | 0.162 K |

Table II: Table of parameters used to extract T_e , and their associated uncertainties.

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