Dissipation-Driven Quantum Phase Transition in Superconductor-Graphene Systems

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We show that a system of Josephson junctions coupled via low-resistance tunneling contacts to graphene substrate(s) may effectively operate as a current switching device. The effect is based on the dissipation-driven superconductor-to-insulator quantum phase transition, which happens due to the interplay of the Josephson effect and Coulomb blockade. Coupling to a graphene substrate with gapless excitations further enhances charge fluctuations favoring superconductivity. The effect is shown to scale exponentially with the Fermi energy in graphene, which can be controlled by the gate voltage. We develop a theory that quantitatively describes the quantum phase transition in a two-dimensional Josephson junction array, but it is expected to provide a reliable qualitative description for one-dimensional systems as well.

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Artificially fabricated Josephson-junction arrays (JJA) offer a unique playground for studying quantum phase transitions (QPT) [1]. The transitions in JJA occur due to the competition between the Josephson coupling E_J, which favors a globally ordered state, and the charging energy E_c, which leads to Coulomb blockade of Cooper-pair tunneling and enhances quantum fluctuations of the superconducting (SC) phase. At zero temperature, the QPT from a globally superconducting to an insulating phase [2–5] occurs, roughly, when the Josephson energy E_J becomes smaller than the charging energy. Another key factor in determining the ground state is dissipation, which is present whenever the SC system is connected to a reservoir of gapless single-particle excitations [6]. The main effect of the dissipation in JJA is a suppression of quantum phase fluctuations. Taking into account the phase-charge uncertainty relation, the dissipation enhances fluctuations of the charge and, hence, stabilizes the SC phase [7]. This type of dissipative QPT has been considered previously by Feigel'man and Larkin [8] in the model of a regular 2D proximity-coupled JJA and by Galitski and Larkin [9] in a disorder-induced random Josephson network. In both cases, it was found that the effect of dissipation on the transition point is *exponential*; i.e., the critical Josephson coupling scales exponentially with the Andreev conductance. The Andreev conductance and hence the degree to which the stabilizing effect of dissipation is important obviously depend on the density of states of gapless excitations. Thus, by controlling the latter, one can tune transitions between a global superconductor and an insulator. This observation provides strong motivation for studying superconductors coupled to a graphene substrate, where the density of carriers can be tuned by gate voltage from essentially zero at the Dirac point (no "Ohmic" dissipation) to large values in the electron-doped graphene (strong dissipation).

In this Letter, we propose to study the superconductor-insulator phase transition (SIT) in a JJA in a tunneling contact with a graphene layer (or layers) (see Fig. 1), which acts as a source of gapless quasiparticles. The graphene substrate provides a unique possibility to control the dissipation strength via the gate voltage and thereby tune the dissipation-driven QPT. Hence, the system may be used as a current switching device. While the physics of the underlying effect is intuitively clear, the formal description of

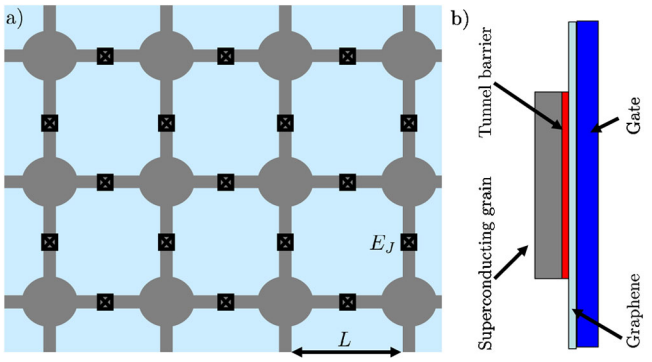

FIG. 1 (color online). (a) JJA on top of a graphene sheet is shown. The grains are coupled by the Josephson junctions with the coupling strength E_J. The distance between the grains L is assumed to be large compared to the SC coherence length ξ. Thus, the coherent transport through the graphene is neglected here. Alternatively, SC grains in the array can be coupled to different graphene sheets. Then, there is no coherent transport through the graphene due to physical separation between the sheets. (b) The superconductor-graphene interface. The graphene and superconductor are connected through the tunnel barrier. The chemical potential in graphene can be tuned with the gate voltage V_g.

the transition developed in this Letter is technically nontrivial: First, we use the tunneling Hamiltonian formalism and elements of random matrix theory to derive the effective phase fluctuation action of a small SC grain coupled to a graphene substrate. The dissipation kernel $K(\tau)$ shows a crossover from the Ohmic dissipation behavior $K(\tau) \propto \tau^{-2}$ in the electron-doped graphene to extremely weak dissipation $K(\tau) \propto \tau^{-4}$ at the Dirac point. Second, we develop a mean-field theory of the SIT and show that the quantum critical point is determined by the single-grain phase correlator. To calculate the phase correlator, we use the two-loop renormalization group (RG) results from a related spin model and determine a critical voltage V_c at which the transition occurs: For $V > V_c$, the system is a superconductor; for $V < V_c$, the system is an insulator.

Our theoretical model is an array of SC grains connected with each other with the Josephson junctions and connected via tunnel contacts to a graphene substrate; see Fig. 1. For $T \to 0$, one can neglect massive fluctuations of the amplitude of the order parameter Δ in the grain, and describe the dynamics of the system in terms of the phase-only imaginary-time effective action ($\hbar = 1$)

$$S_C + S_J = \sum_i \int d\tau \frac{\dot{\varphi}_i^2(\tau)}{E_c} - \sum_{\langle ij \rangle} \int d\tau E_J \cos[\varphi_i(\tau) - \varphi_j(\tau)] \tag{1}$$

with φ_i being the phase of the order parameter on the ith grain. Here, for simplicity, we assume that the Josephson and charging energy are the same for all grains. However, this assumption is not essential for our results.

We now consider the effect of graphene gapless excitations on the phase coherence of a single SC grain. We study here the situation when the SC grain lies on top of the graphene sheet; see Fig. 1. In this planar geometry the tunnel junction does not break internal symmetries of graphene, and thus does not modify the spectrum of the excitations. In the limit of low transparency tunnel barrier, the transport between superconductor and graphene can be described by the tunneling Hamiltonian $H_T = t\sqrt{d_z}\sum_\sigma \int_A d^2\boldsymbol{r}[\Psi_\sigma^\dagger(\boldsymbol{r})\Psi_\sigma^{(g)}(\boldsymbol{r}) + \text{H.c.}]$ with $\Psi_\sigma^{(g)}(\boldsymbol{r})$ and $\Psi_\sigma(\boldsymbol{r})$ being the electron operators in graphene and superconductor, respectively. Here d_z is the thickness of the grain, t and A are the tunneling matrix element and the area of the junction, respectively.

Assuming that the SC gap energy is sufficiently large, the main contribution to the subgap transport originates from Andreev processes, which involve correlated tunneling of two electrons from or to the graphene. In the fourth order perturbation theory in tunneling t (see Fig. 2), the contribution of the Andreev processes to the dynamics of the phase is given by the following effective action

$$S_A = 24t^4 d_z^2 \int_A \prod_{i=1,\ldots,4} d\boldsymbol{x}_i \,\mathrm{Re}[F^*(\boldsymbol{x}_1, \boldsymbol{x}_2)F(\boldsymbol{x}_3, \boldsymbol{x}_4)G^{(g)}(\boldsymbol{x}_1, \boldsymbol{x}_3)G^{(g)}(\boldsymbol{x}_2, \boldsymbol{x}_4)], \tag{2}$$

with $\boldsymbol{x} = \{\boldsymbol{r}, \tau\}$. Here $G^{(g)}(\boldsymbol{x}, \boldsymbol{x}')$ and $F(\boldsymbol{x}; \boldsymbol{x}')$ are imaginary-time Green's functions for graphene and superconductor, respectively. The latter is defined as

$$F^*(\boldsymbol{x}_1, \boldsymbol{x}_2) = -e^{i[\phi(\tau_1)+\phi(\tau_2)]/2}\sum_n \chi_n(\boldsymbol{r}_1)\chi_n(\boldsymbol{r}_2)u_n v_n[\Theta(\tau_1 - \tau_2)e^{-E_n(\tau_1-\tau_2)}\Theta(E_n) - \Theta(\tau_2 - \tau_1)e^{-E_n(\tau_1-\tau_2)}\Theta(-E_n)].$$

Here $E_n = \sqrt{\varepsilon_n^2 + \Delta^2}$, u_n and v_n are Bogoliubov coherence factors $u_n^2/v_n^2 = (1 \pm \varepsilon_n/E_n)/2$, and ε_n and $\chi_n(\boldsymbol{r})$ are the eigenvalues and eigenfunctions of the single-particle Hamiltonian of the grain, which includes random potential due to impurities and boundaries of the island.

In order to derive low-energy effective action S_A due to the Andreev processes, it is convenient to separate the fast and slow times κ and τ: $\tau_1 = \tau + \kappa/2$ and $\tau_2 = \tau - \kappa/2$. Since the SC Green's function decays exponentially on the time scale of order Δ^{-1}, one can neglect κ in the dynamics of the phase, which evolves on much longer time scales. Performing the integral over the fast time scales, we obtain for S_A

$$S_A \approx 24t^4 d_z^2 \int_A \prod_{i=1,\ldots,4} d\boldsymbol{r}_i d\tau d\tau' \cos[\phi(\tau) - \phi(\tau')] \times F_{12}F_{34}G_{13}^{(g)}(\tau - \tau')G_{24}^{(g)}(\tau' - \tau). \tag{3}$$

Here F_{ij} is the anomalous Green's function at zero frequency, i.e., $F_{ij} = -\sum_n \chi_n(\boldsymbol{r}_i)\chi_n(\boldsymbol{r}_j)\Delta/E_n^2$.

It is well known that Andreev transport is sensitive to disorder [10]. Therefore, in order to calculate the effective action, one has to take into account spatial correlations [10,11], and average S_A over the random realization of the wave functions in the SC grain resulting from the scattering of electrons from the grain boundaries and impurities. We perform this averaging using exact eigenstate method assuming that the grain is sufficiently small. Our approach accounts for the enhancement of the tunneling rate due to

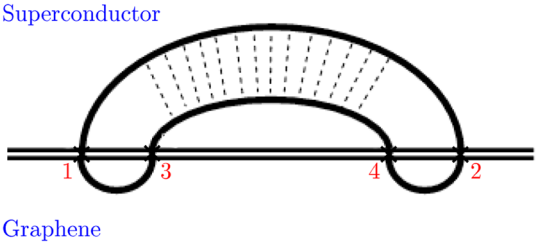

FIG. 2 (color online). The diagrams describing the correlated two-electron tunneling process (Andreev process) are shown.

the coherent backscattering of electrons to the tunnel junction. The correlation function $\langle F_{12}F_{34}\rangle$ in the grain consists of reducible and irreducible parts: $\langle F_{12}F_{34}\rangle \approx \langle F_{12}\rangle \times \langle F_{34}\rangle + \langle F_{12}F_{34}\rangle_{ir}$. The reducible part can be easily calculated

$$\langle F_{12}\rangle = \frac{\pi}{2}\nu_F f_{12}, \quad \text{and} \quad f_{12} = \frac{\sin(k_F|\boldsymbol{r}_1 - \boldsymbol{r}_2|)}{k_F|\boldsymbol{r}_1 - \boldsymbol{r}_2|} e^{-(|\boldsymbol{r}_1 - \boldsymbol{r}_2|/2l)}, \tag{4}$$

where k_F, l, and ν_F are the Fermi wave vector, mean free path, and density of states at the Fermi level, respectively.

The irreducible correlation function is obtained assuming that the Thouless energy E_T is the largest relevant energy scale in the problem $E_T \sim D/R^2 \gg \Delta, E_c, E_J$, where D and R are the diffusion constant and radius of the grain. In this limit, the electron diffusion time in the grain $\tau_D = 1/E_T$ is much smaller than the time the system dwells in the virtual state with one unpaired electron $\sim 1/\Delta$. Since an electron in the virtual state covers the entire available phase space, one can calculate the irreducible correlation function within random matrix theory [12] finding that it acquires the universal form $\langle F_{12}F_{34}\rangle_{ir} = \frac{\pi}{4}\frac{\nu_F^2\delta}{\Delta}(f_{14}f_{23} + f_{13}f_{24})$. Combining this equation with Eq. (4), we obtain $\langle F_{12}F_{34}\rangle = \frac{\pi^2}{4}\nu_F^2 f_{12}f_{34} + \frac{\pi}{4} \times \frac{\delta}{\Delta}\nu_F^2(f_{14}f_{23} + f_{13}f_{24})$, where δ is level spacing in the grain. Then, by substituting the above expression into Eq. (3), and carrying out the spatial integrals, we obtain ensemble-averaged S_A:

$$S_A \approx -G\int d\tau d\tau' K(\tau - \tau')\cos[\phi(\tau) - \phi(\tau')]. \tag{5}$$

Here $G = \frac{3t^4}{2\pi\delta\Delta k_F^2\gamma^2}$ is the dimensionless constant defining the transparency of the tunnel barrier and γ is the graphene Fermi velocity. The dissipation kernel $K(\tau - \tau')$ is defined in terms of the graphene Green's functions (see, e.g., Ref. [13]); i.e., $K(\tau - \tau') \propto G_{11}^{(g)}(\tau - \tau')G_{22}^{(g)}(\tau' - \tau)$. Assuming that the chemical potential $\mu \geq 0$, the kernel $K(\tau)$ is given by the function $K(\tau) = \frac{1}{k_F^2\gamma^2\tau^4}\{(\mu\tau)^2 + 2[1 - \mu|\tau|]e^{-\mu|\tau|} - 1\}$, which exhibits a crossover from τ^{-4} at the Dirac point (here we neglect smearing of the Dirac point due to charge disorder fluctuations [14]) to the usual Ohmic behavior τ^{-2} at $\mu|\tau| \sim 1$. For realistic experimental parameters μ^{-1} is much smaller than the characteristic time scale for the phase dynamics $\tau \lesssim E_c^{-1}$, i.e., $\mu \gg E_c$. Hence,

$$S_A \approx -\eta\int d\tau d\tau' \frac{\cos[\phi(\tau) - \phi(\tau')]}{(\tau - \tau')^2}, \qquad \eta = G\frac{\mu^2}{k_F^2\gamma^2}. \tag{6}$$

The important difference between dissipative action (6) and the one describing resistively shunted Josephson junction is that action (6) is "compact," and thus correctly describes the fact that the charge on the grain is quantized and can change by $2e$ only. From Eq. (6), we see that graphene as a source of gapless excitations provides the possibility to change the dissipation strength directly by changing μ, which is tied to the gate voltage.

Dissipation-driven QPT.—Combining Eqs. (1) and (6), we obtain the full action for the system: $S = S_C + S_J + S_A$. To derive effective action describing the transition in the JJA, we first write a partition function in path integral representation and then use Hubbard-Stratonovich transformation to decouple the Josephson term by introducing an auxiliary field $\psi_i(\tau)$ coupled linearly to $e^{i\varphi_i(\tau)}$. Then, the partition function becomes $Z = Z_0\int \mathcal{D}\psi \exp(-S[\psi])$, where $S[\psi]$ is given by

$$S[\psi] = \int d\tau \frac{1}{2}\sum_{i,j}\psi_i^*(\tau)w_{ij}^{-1}\psi_j(\tau) - \ln\left\langle \exp\left[\frac{1}{2}\int d\tau\sum_i e^{i\varphi_i(\tau)}\psi_i^*(\tau) + \text{H.c.}\right]\right\rangle_0. \tag{7}$$

Here we introduced the symmetric matrix w_{ij}, which describes Josephson tunneling: matrix elements of w_{ij} are equal to E_J for nearest neighbors and zero otherwise. The expectation value in Eq. (7) is taken with respect to the single-site action $S_0 = S_C + S_A$. To study QPT at the mean-field level, we perform cumulant expansion of the second term in the action $S[\psi]$ in powers of ψ and arrive at the effective complex ψ^4 field theory [15]. The phase boundary between macroscopically superconducting and insulating state of JJA can be obtained by setting the coefficient r in front of $|\psi|^2$ to zero: $r \propto \frac{1}{zE_J} - \frac{1}{2} \times \int d\tau\langle e^{i\varphi(\tau) - i\varphi(0)}\rangle_0 = 0$. (Here z is the coordination number of the lattice and averaging is taken with respect to S_0.) When calculating the correlation function we assume that the dissipation strength $\eta \gg 1$. Then, the second term in S_0 dominates at low frequencies and one can neglect the influence of the charging energy term S_C, which serves as the ultraviolet cutoff. Under these conditions, the correlation function can be mapped on the long-range ferromagnetic spin chain [16–18] first considered by Kosterlitz [16]. Later, the critical behavior was studied in Ref. [18], where the asymptotic behavior of the spin-spin correlation function was obtained using two-loop RG. Adopting the results of Refs. [8,16,18] to our problem, we get

$$\langle e^{i\varphi(\tau) - i\varphi(0)}\rangle_0 \sim \begin{cases} (\frac{\tau_c}{\tau})^{1/2\pi^2\eta} & \Lambda^{-1} \ll \tau \ll \tau_c \\ (\frac{\tau_c}{\tau})^2 & \tau \gg \tau_c, \end{cases} \tag{8}$$

where $\Lambda \sim 2\pi E_c\eta$ is the ultraviolet cutoff and τ_c is the correlation time, which can be calculated using the RG for η. Since the compact dissipation term proportional to η is not Gaussian, it gets renormalized when integrating out high-frequency degrees of freedom resulting in the following flow equations:

$$\frac{d\eta}{d\ln(\Lambda\tau)} = -\frac{1}{2\pi^2} - \frac{1}{(2\pi^2)^2\eta}. \tag{9}$$

Here the right-hand side is the beginning of a Taylor

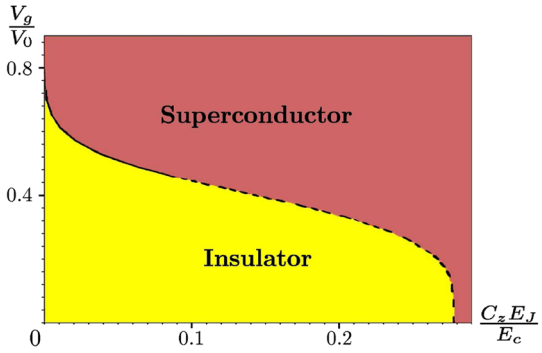

FIG. 3 (color online). Phase diagram for the SIT is shown. By tuning the applied gate voltage V_g, one can change the density of states of gapless excitations in graphene, and thus drive the phase transition between globally superconducting and insulating states of JJA. The solid (black) line represents the region of the validity of RG.

expansion in $1/\eta$. By integrating this equation between the initial value $\eta(0) \equiv \eta$ and final value $2\pi^2\eta(\ln[\Lambda\tau_c]) \sim 1$, we can estimate the correlation time as

$$\tau_c \sim (4\pi^3 E_c \eta^2)^{-1} \exp(2\pi^2\eta). \tag{10}$$

Using Eqs. (8) and (10) and carrying out the time integration, we find an expression defining the phase boundary $C_z E_J = 4\pi^3 E_c \eta_c^2 \exp(-2\pi^2\eta_c)$, where C_z is a numerical constant depending on z. The right-hand side of the above equation corresponds to the effective charging energy of the grain E_c^* renormalized by the dissipation [19]. By inverting the transcendent algebraic equation above, we can express the critical voltage V_c at which the QPT occurs in terms of the Lambert W function

$$\frac{V_c}{V_0} = \frac{1}{\pi}\sqrt{-W_{-1}\left(-\sqrt{\frac{\pi C_z E_J}{4E_c}}\right)}, \tag{11}$$

where $V_0 = \frac{\gamma k_F}{e\sqrt{G}}$. The phase boundary between globally superconducting and insulating phases of JJA is shown in Fig. 3. For the theory based on the RG procedure to be formally valid, we need $\ln(E_c/E_J) \gg 1$. This, however, is a mathematical rather than physical constraint, and for all practical purposes (i.e., experiment) what is important is the existence of the transition itself; the applicability of RG methods and the exact location of the nonuniversal "critical voltage" are not essential. A sharp transition should certainly exist if $E_c \gg E_J$ and perhaps even for $E_c \geq E_J$. In the opposite limit the system is already superconducting without any substrate and there is no dissipation-driven effect. Since the technology for making SC grains with required E_c/E_J is well developed, the observation of the QPT is certainly feasible.

We also emphasize here that since the dissipation-driven QFT is intrinsically local, it is expected to survive in one-dimensional chains (where the mean-field theory breaks down and the QPT is of Kosterlitz-Thouless type [20]). Again, the exact location of the transition point and the critical behavior would be different, but the effect itself should be present. Moreover, the same local argument suggests that a sharp voltage-induced crossover (cf. with the model of a shunt resistor coupled locally to the SC grain [21,22]) in the IV curves should be present even in finite chains proximity coupled to graphene, similar to those that are already being experimentally investigated [23,24]. We propose that experiments be carried out in the SC-graphene system to directly confirm our QFT and current switching predictions.

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