Voting over Economic Plans

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We review and provide motivation for a one-sector model of economic growth in which decisions about capital accumulation are made by a political process. If it is possible to commit for at least three periods into the future, then for any feasible consumption plan, there is a perturbation that is majority-preferred to it. Furthermore, plans that minimize the maximum vote that can be obtained against them yield a political business cycle. If it is impossible to commit, voters select the optimal consumption plan for the median voter. (*JEL* D72, E61, E62, H43)

This paper summarizes and provides intuition for a model of political choice over economic plans. The formal development of the model is reported elsewhere, in work together with John Ledyard (Boylan et al., 1996). That work studies the consumption plans generated by a political process operating in the temporal environment of a one-sector growth model. The model identifies conditions leading to "neoclassical" growth plans versus political business cycles, finding that length of commitment is an important variable.

There is a long history of work in the economics literature on the problem of economic growth. The problem was originally formulated by Frank P. Ramsey (1928), was taken up again by Robert M. Solow (1956), and considered from the point of view of optimal economic policy by David Cass (1965), Tjalling C. Koopmans (1965), Lionel W. McKenzie (1976), and others. The initial work focused on the one-sector growth model, and the main results showed that, under sufficient convexity conditions on production and social preferences, there is a unique solution, which is characterized by the "turnpike" theorems.[1]

While the problem of economic growth has received a lot of attention, very little has been done to incorporate political institutions into such models. One exception is the work of Nathaniel Beck (1978), who studied political behavior in a continuous-time, one-sector model of economic growth, where voters differ only in their time preferences. Beck shows that if the set of feasible plans is limited to consumption plans that are optimal for at least one voter, then the plan that is optimal for the voter with the median discount factor is a majority core. He conjectures that the median-voter plan is no longer a majority core if all plans are feasible. He also argues that the plan for the median voter is a "local equilibrium" in which no majority can agree on an instantaneous deviation, assuming that the optimal plan for the median voter will be followed after that instant.

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[1]The turnpike theorems imply that the consumption plan does not cycle. Periodic and chaotic dynamics have also been found to occur in the optimal solution for the standard growth model when there are multiple sectors (see e.g., Michele Boldrin and Luigi Montrucchio, 1986; Raymond Deneckere and Steve Pelikan, 1986; Makul Majumdar and Tapan Mitra, 1994; Kazuo Nishimura et al., 1994).

When one leaves the setting of growth theory, there is a fair amount of work that has attempted to characterize the type of fiscal and monetary policy that would be generated by political processes. A recurrent theme in this literature is that, if politicians are allowed to make economic decisions, they will generate "political business cycles"—business cycles coinciding with the term of office of the politicians. William D. Nordhaus (1975) first derived such results in a model in which the incumbent officeholder must choose among different points along a Phillips curve. He also presented some empirical evidence supporting the existence of political business cycles in some countries.[2] Nordhaus's theoretical argument depends crucially on voter myopia. Subsequent papers by Kenneth Rogoff (1990) (see also Kenneth Rogoff and Anne Sibert [1988]) and Alberto Alesina (1987) have derived political business cycles without having to assume voter myopia. Rogoff (1990) and Rogoff and Sibert (1988) show that the introduction of asymmetric information over the competency of political candidates can generate a political business cycle. In this model a business cycle emerges as a signaling equilibrium in which the size of the cycle is used by the candidate to signal competency to the voters. Alesina (1987) assumes that different political parties have different relative preferences over the trade-off between inflation and unemployment levels. He then gets political business cycles emerging even when voters have rational expectations, due to the fact that the election provides a random shock. Both of the above models are partial-equilibrium models. Rogoff's economy does not have the capability of real growth, while Alesina's political parties have exogenously given policy positions.

The Boylan et al. (1996) paper examines a discrete-time version of Beck's model: a one-sector growth model in which voters differ only in their time preferences, and in which they vote over the optimal plan. Unlike Beck's model, all plans, rather than just plans that are optimal for one voter, are available.

We begin, in Section I, by introducing the details of the standard one-sector growth model and providing some background on optimal consumption plans for those not familiar with this framework.

Section II considers the political stability of consumption plans if voters can choose between different plans to be followed. If all voters have different discount factors and the time horizon is at least three periods, Beck's conjecture—that the median-voter plan is not a majority-rule core when plans that are not optimal for some voter are available—is true. In fact, a stronger result is true; namely, there is no majority-rule core if nonoptimal plans are feasible, and as long as the horizon is at least three periods, then any neoclassical optimal plan (a plan that could be optimal for a voter with time-consistent preferences) can be defeated almost unanimously.

Section III discusses the political stability of consumption plans when there are periodic decision points, such as elections, when policy can be committed for multiple periods. If the plan is restricted to be stationary over every decision period, a minmax plan (a plan that can be defeated by the smallest majority) exhibits a "political business cycle." The cycles arise from coalitions between patient and impatient voters. We compute an example which suggests that the shape of the cycles is different than predicted by previous theoretical work. In the example, the cycles yield postelection (rather than preelection) bursts in consumption to attract impatient voters, followed by long-term investment to attract patient voters.

In Section IV, we consider what consumption plans are chosen if it is impossible

[2]Subsequent empirical evidence for cycles is mixed. Bennett McCallum (1978) and David G. Golden and James M. Poterba (1980) do not find evidence of business cycles in the United States. But other studies find evidence of cycles in other countries. Empirical evidence for cycles can be found in Martin Paldam (1979), Stuart D. Allen et al. (1986), Byung Hee Soh (1986), Manfred W. Keil (1988), Lawrence S. Davidson et al. (1990), David W. Findlay (1990), and Thomas F. Cargill and Michael M. Hutchinson (1991).

to commit credibly to multiperiod consumption plans, but possible instead just to commit one period at a time. In this case, voters select the consumption plan that is optimal for the voter with the median discount rate. In essence, Beck's intuition is correct in the discrete-time setting.

It follows that political business cycles can arise in models that require no myopia or incomplete information and that the existence and severity of political business cycles may be related to the length of the time horizon that the political system can commit.[3] These results also raise questions as to whether neoclassical optimal plans are desirable from a welfare point of view, since they can always be defeated by virtually unanimous majorities. In particular, with finite planning horizons, cyclical plans are majority-preferred to them and can be defeated by smaller majorities.

I. The Political-Economic Growth Model

The simplest possible dynamic framework is used to model the economy: a one-sector growth model. The single good is a public good which can be consumed or invested.

We first review the basic setup and results of the one-sector growth model: let $F: \mathbb{R}_+ \to \mathbb{R}_+$ be a twice continuously differentiable concave production function with $F(0)=0, F'(0)=+\infty, F'(\infty)=0$.[4] Let k_t be the per capita capital stock at the beginning of date t, let c_t be the consumption per capita on date t, and let $T \in \mathbb{N} \cup \{\infty\}$ be the length of the time horizon. Given an initial capital $\bar{k} > 0$, the technology can be summarized in the fundamental equation of growth theory: for $t = 0, 1, 2, \ldots, T$,

$$(1) \qquad c_t + k_{t+1} = F(k_t),$$

where

$$(2) \qquad k_0 = \bar{k} \qquad k_t \geq 0$$
$$c_t \geq 0.$$

Thus in each period, the output of production is divided between consumption and capital for use in next-period production. Any plan $\mathbf{c} = \{c_t\}_{0 \leq t \leq T}$ which is a feasible solution to (1) and (2) is called a *feasible consumption plan*. Let C denote the set of feasible consumption plans.

The one-sector growth model has been studied extensively in the case where a particular social-welfare function is defined. The primary interest has been in solving for a feasible consumption plan that maximizes the welfare function. In most cases the welfare function has been assumed to be temporally separable with impatience represented by a discount factor, δ. The problem to be solved is then

$$(3) \qquad \max_{\mathbf{c} \in C} \sum_{t=0}^{T} \delta^t u(c_t)$$

where $\delta \in (0,1)$ and $u: \mathbb{R}_+ \to \mathbb{R}$ satisfies $u'(\mathbf{c}) > 0$, $u'(0) = \infty$, and $u''(\mathbf{c}) < 0$. A *neoclassical optimal plan* is a solution to problem (3) (for some δ).

The solution to problem (3), $\{c_t^*\}$, is Markov; this means that there is a family of functions $\{g_t(k;T)\}$, such that $g_t(k;T)$ is the consumption at time t, given the capital in the previous period. By equation (1), corresponding to any solution there is an optimal capital plan $\{k_t^*\}$ and functions $\{h_t(k;T)\}$ that express the optimal capital at time t as a function of capital at time $t-1$. For the infinite-horizon model, the solution can be expressed in terms of a single pair of functions $h(k) = h_t(k;\infty)$ and $g(k) = g_t(k;\infty)$.

[3] In connection with this result, the findings of Paldam (1979) are quite intriguing. He studies 49 stable governments in 17 OECD countries, finding significant business cycles in four-year governments, but no significant cycles in three-year governments. Thus, longer terms in office lead to less stable consumption plans.

[4] The production function $F(k)$ is frequently assumed to be of the form $f(k) = f(k) + (1-\lambda)k$, where λ is the depreciation rate of capital stock, and $f(k)$ is the net output. Hence, $(1-\lambda)k$ is the undepreciated capital.

Further, h satisfies: $h' > 0$ and $h(k) < k^*$ for $k < k^*$, and $h(k) > k^*$ for $k > k^*$. The steady-state level of capital, k^*, is defined by $F'(k^*) = 1/\delta$.[5]

The above results imply that the optimal plan of capital begins at k_0 and converges monotonically to k^*. Also, the optimal plan of consumption converges monotonically to $c^* = F(k^*) - k^*$. Similar results hold for the finite-horizon case. Here, one gets the so-called "turnpike" theorem: for any $\varepsilon > 0$ there is a $T_\varepsilon > 0$ such that, if $T > T_\varepsilon$, then $|k_t^* - k^*| > \varepsilon$ for at most T_ε periods (see e.g., D. Gale, 1970).

II. Political Stability of Economic Plans

This section discusses the political stability of various consumption plans. There is a set $\mathcal{N}$ of n voters who all have the same one-period utility for consumption but differ in their time preferences. Voter i's utility function over consumption plans is $U_i(\mathbf{c}) = \sum_{t=0}^{T} \delta_i^t u(c_t)$, where $u: \mathbb{R}_+ \to \mathbb{R}$ satisfies $u'(\mathbf{c}) > 0$, $u'(0) = \infty$ and $u''(\mathbf{c}) < 0$ for all $\mathbf{c} \in \mathbb{R}_+$. We consider both the case of finite and infinite T. We will assume throughout that any two distinct voters have distinct discount factors.

One might worry about the distribution of c_t across voters, but we will treat this as a public good. That is, voters pick c_t, the amount that each voter consumes, yielding voter i a utility level of $u(c_t)$ for that period. Most of the results we show in this paper extend to the case where the good is private with one-period utility functions being logarithmic. If the good is private, individuals will lend and borrow at the market-clearing interest rate, but in a model where government can influence such a rate, voters will have different preferences over government's actions (see Boylan, 1995).

The first proposition, which is proved in Boylan et al. (1996), states that neoclassical optimal plans can always be defeated by large majorities.

PROPOSITION 1: *If $T \geq 2$, then for any neoclassical optimal plan, $\mathbf{c}^* \in C$, there is an alternative plan $\mathbf{c} \in C$, which defeats $\mathbf{c}^*$ by at least $n-1$ votes. If $\mathbf{c}^*$ is not optimal for any voter, then it can be defeated by n votes. The same result holds if we restrict the set of alternative plans to those that differ from $\mathbf{c}$ at no more than three consecutive periods.*

The above result may seem surprising at first glance. One might think that, when utility functions differ only by one parameter, the median voter theorem would apply, implying that the optimal plan for the voter with the median discount factor would be a majority core point. In fact, the optimal plan for the median-discount-factor voter is defeated by a plan supported by a coalition including patient and impatient voters. This plan has more consumption in earlier periods (to satisfy the impatient voters), more consumption in later periods (to satisfy patient voters), and less consumption in intermediate periods (to make the plan feasible). To illustrate this, we give an example showing how the optimal plan for the median voter can be defeated.

Example 1: Assume the one-period utility is $u(c) = \ln(c)$ and the production function is $F(k) = k^{1/2}$. There are three voters with discount factors $\delta_1 = 0.5$, $\delta_2 = 0.05$, and $\delta_3 = 0.95$. Let c^* and k^* be the steady-state values of consumption and capital on the optimal plan for the voter with the median discount factor, namely, voter 1. As

[5] To see this, let v^* be a function satisfying

$$v^*(k) = \max_c [u(c) + \delta v^*(F(k) - c)]$$

$$= \max_h [u(F(k) - h) + \delta v^*(h)].$$

Then $v^*(k) = u(F(k) - h(k)) + \delta v^*(h(k))$, where for all k, $h(k)$ satisfies $\partial v^*/\partial h = 0$, and consequently $u'(F(k) - h(k)) = \delta v'(h(k))$. By the envelope theorem,

$$v^{*\prime}(k) = \partial v^*/\partial k = u'(F(k) - h(k))F'(k).$$

Hence, $u'(F(k) - h(k)) = \delta v^{*\prime}(h(k)) = \delta u'(F(h(k)) - h(h(k)))F'(h(k))$. This implies that $u'(g(k)) = \delta u'(g(h(k)))F'(h(k))$. Note that if $h(k) = k$, then $\delta u'(g(k)) = u'(g(k))/F'(k)$ and $F'(k) = 1/\delta$. The value function, v^*, is continuous, differentiable, strictly increasing, and strictly concave. For more details the reader can consult Milton Harris (1987).

discussed in Section I, $F'(k^*) = \delta_2$, so $k^* = 0.0625$ and $c^* = F(k^*) - k^* = 0.1875$. For simplicity, we assume that initial capital is at the steady state, k^*, so that the optimal plan is $\mathbf{c}^* = (c^*, c^*, c^*, \ldots)$. Now consider the plan $\mathbf{k}$, which changes the capital stock only at periods 1 and 2 to $k_1 = 0.0600$, $k_2 = 0.0675$, and then returns to k^*. This yields the consumption plan $\mathbf{c} = (c_0, c_1, c_2, c^*, c^*, c^*, \ldots)$, with $c_0 = 0.1900$, $c_1 = 0.1774$, and $c_2' = 0.1973$. It is easily checked that this plan is feasible. Further, both voters 2 and 3 prefer $\mathbf{c}$ to $\mathbf{c}^*$: $U_2(\mathbf{c}) = -1.751 + \delta_2^3 U_2(\mathbf{c}^*) > -1.762 + \delta_2^3 U_2(\mathbf{c}^*) = U_2(\mathbf{c}^*)$, and $U_3(\mathbf{c}) = -4.768 + \delta_3^3 U_3(\mathbf{c}^*) > -4.775 + \delta_3^3 U_3(\mathbf{c}^*) = U_3(\mathbf{c}^*)$.

Figure 1 gives an intuition for Proposition 1. Let $\mathbf{c}$ be a consumption plan that agrees with the optimal path for the median voter except at times 0, 1, and 2. The interior of the curve F and the axis describes the set of such consumption plans that are feasible. The consumption at times 0 (horizontal axis) and 2 (vertical axis) together with the fundamental equation of growth theory, equation (1), determine consumption in period 1, c_1. This is summarized by a function $c_1 = g(c_0, c_2)$. Thus, the curve F represents the set of points (c_0, c_2) such that $g(c_0, c_2) = 0$.

The curve P represents the preferred choices of (c_0, c_2) for voters with different values of δ; each point on P corresponds to a different value of δ. Higher values of δ map to points that are further to the left on P. Let c^* be the optimal choice for the median-discount-factor voter, m. The slope of the indifference curve of individual $i \neq$ m through the point c^* is

$$(4)\quad \left.\frac{dc_2}{dc_0}\right|_{U_i(c_0, g(c_0,c_2), c_2, \ldots) = U_i^*} = -\frac{1}{\delta}\frac{u'(c_0^*) + \delta u'(c_1^*) g_0(c_0^*, c_2^*)}{u'(c_1^*) g_2(c_0^*, c_2^*) + \delta u'(c_2^*)}$$

where $U_i^* = U_i(c_0^*, c_1^*, c_2^*, \ldots)$ and $g_i = dg/dc_i$. Since $\mathbf{c}^*$ is optimal for the median voter, the first-order conditions must hold, namely,

$$(5)\quad u'(c_0^*) + \delta_m u'(c_1^*) g_0(c_0^*, c_2^*) = 0$$
$$u'(c_1^*) g_2(c_0^*, c_2^*) + \delta_m u'(c_2^*) = 0.$$

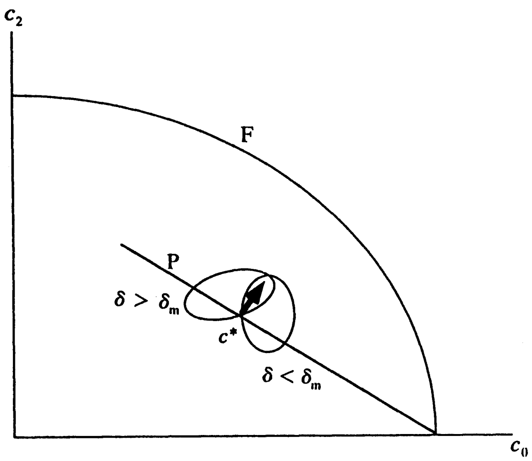

FIGURE 1. ILLUSTRATION OF PROPOSITION 1

Notes: The axes measure the consumption at times 0 and 2. Consumption after time 2 is fixed, while consumption at time 1 is computed as the residual. F is the frontier of the feasible set. A point on P represents the choice of (c_0, c_2) most preferred by a voter with some discount factor δ. Points on P that are further northwest correspond to higher values of δ. The optimal consumption for the median discount factor is c^*. The circles represent indifference curves of a patient individual ($\delta > \delta_m$) and an impatient individual ($\delta < \delta_m$). A small change in consumption in the direction of the arrow makes all individuals better off (except for the median voter).

By combining (4) and (5), we get

$$\left.\frac{dc_2}{dc_0}\right|_{U_i(c_0, g(c_0,c_2), c_2, \ldots) = U_i^*} = -\frac{1}{\delta}\left[\frac{u'(c_1^*) g_0(c_0^*, c_2^*)}{u'(c_2^*)}\right].$$

Note that dc_2/dc_0 is larger for smaller δ. Thus indifference curves cross as shown in Figure 1. Hence, a small deviation in the direction of the arrow shown on this figure makes everyone better off, except for the median-discount-factor voter.

Given the result of Proposition 1, the natural question to ask is whether there are any plans that cannot be defeated by majority rule. Proposition 2, which is proved in

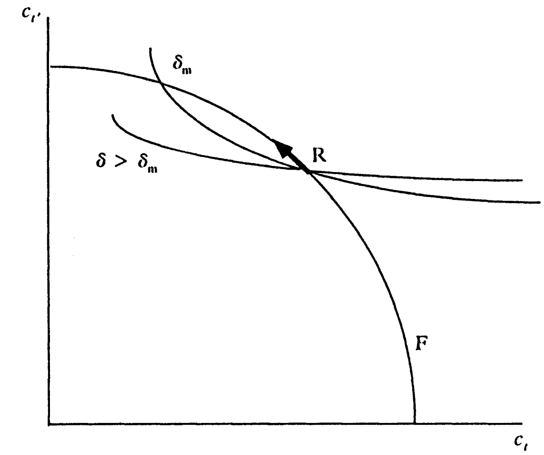

FIGURE 2. ILLUSTRATION FOR PROPOSITION 2

Notes: On the axes we have the consumption at time t and t'. F is the set of feasible choices given fixed consumption in all other periods. The point R denotes a choice that is not optimal for the median voter. Two curves describe the indifference curves of the median voter (δ_m) and of a patient voter ($\delta > \delta_m$). A small change in the direction of the arrow is preferred by the median voter and all voters with higher discount factors.

Boylan et al. (1996), states that the answer is negative.

PROPOSITION 2: *Suppose the time horizon, T, is at least 2 and the number of voters, n, is odd and greater than 2. Then, for any feasible consumption plan,* **c**, *there is an alternative feasible plan preferred to* **c** *by a majority of voters; in other words, there is no majority-rule core.*

Figure 2 gives an intuition for Proposition 2. Suppose there is a plan, **c**′, that cannot be defeated by majority rule. By Proposition 1, there are two periods, t and t', with $t > t'$, such that there is a feasible change in consumption during these two periods (leaving consumption in other periods fixed) that makes the median-discount-factor voter strictly better off. In Figure 2, F is the set of feasible choices $(c_t, c_{t'})$. The point R denotes the choice $(c'_t, c'_{t'})$. Given the choice of t and t', the indifference curve of the voter with the median discount factor must cross F at the point R. Suppose that this indifference curve is flatter than F, as drawn. Since higher values of δ correspond to even flatter indifference curves, those curves must also intersect F. Thus, there is a majority of voters who prefer a small change in consumption in the direction of the arrow over **c**′. If the indifference curve of the median-discount-factor voter is steeper than F, then the conclusion follows by an analogous argument.

III. Political Stability with Periodic Elections

The above results consider the political stability of long-term plans when it is possible to offer any possible alternative to a given plan. In this section we assume that the political institution allows commitment of policy at most for a fixed term, say, the length of the term of office of a given administration, which we take to be L. One administration cannot commit the economic policy of the next administration, except insofar as its policy can determine the initial economic conditions of the following administration. Thus, if we wish to characterize the kinds of policy that would emerge in political systems with periodic elections, we should be concerned with the stability of plans against L-period deviations. We want to find the plan for a T-period horizon that is maximally stable against attempts to amend it during periodic elections every L periods.

To deal with this case, there are two problems to confront. The first involves what voters conjecture will happen in the future, after the period to which they are committing, if they end up delivering a different initial capital stock for the future. The second is what policy to assume is selected for the L periods of commitment in the absence of an equilibrium.

Regarding the first problem, in order to deal formally with finite-length commitment one must specify expectations about what will happen after the period to which one can commit. The view we take here is that the problem at time jL is identical to the problem at time 0. The distribution of discount factors is the same at both points in time. Thus, it is reasonable to assume that the decision rule that is used at time jL is the same for all j. Hence, we look for L-period stationary policies, which are identical functions of the underlying preferences at each decision point.

Regarding the second problem, we already know from Propositions 1 and 2 that in cases when $T \geq 2$, neoclassical optimal plans can be beaten by large majorities, and there is no majority core among the set of feasible plans. Since these results only depend on perturbations of length 3, it is obvious that the same results will be true when perturbations can differ from the original plan for periods of length at least $L = 3$. So what do we assume candidates do in the absence of a majority core?

In the absence of a majority core, one reasonable assumption is that candidates will choose a plan that is as "safe" as possible. There are many ways of formulating such ideas in the social-choice literature. The one we adopt here is the idea of an α-majority set, the set of policies that can be defeated by at most a majority of size α, and the related idea of the minmax set, the set of policies that can be defeated by the smallest possible majority. Use of the minmax set would be justified if decisions must be made by supermajorities (e.g., because of the existence of political institutions such as bicameral legislatures and the executive veto). For a justification of the minmax set as the outcome of models of two-candidate competition, see Gerald H. Kramer (1980).

As shown in Boylan et al. (1996), any policy in the minmax set maximizes a Samuelson-Bergson social-welfare function $\Sigma_{i \in \mathcal{N}} \lambda_i U_i(c)$, where $\lambda_i \geq 0$. Therefore, the consumption plan selected is optimal for a fictitious voter with preferences $\Sigma_t \delta_t u(c_t)$, where $\delta_t \equiv \Sigma_{i \in \mathcal{N}} \lambda_i \delta_i^t$. Notice that the discount factor ratio δ_t / δ_{t-1} is increasing in t; the more patient individuals have more weight in determining society trade-offs further out in time. Since preferences change over time, for any time periods $s < t < u$, the preferred time-u consumption, c_u, at time s is different from the preferred c_u at time t. The *time inconsistency* of preferences was first pointed out by Robert H. Strotz (1956). For this reason, Bezalel Peleg and Menahem E. Yaari (1973) define equilibrium consumption plans as consumption plans such that an individual cannot change consumption at time t (while keeping consumption at every other time period the same) and be better off.

In a growth model, the definition of equilibrium consumption plan has to be more complex, because a change in consumption in a particular period may make future consumption infeasible. In an *equilibrium consumption plan*, **c**, at the beginning of each policy period, t,

$$(c_t, \ldots, c_{t+L-1}) \in \operatorname{argmax} \sum_{k=0}^{L-1} \delta_k u(c_{k+t}) + \delta_L V(k_{t+L})$$

where $V(k_{t+L})$ is the continuation value. A *steady-state equilibrium consumption plan* is a plan where for all t and a, $k_t = k_{t+aL}$.

By the Euler equation,

$$u'(c_t) = (\delta_{t+1} / \delta_t) u'(c_{t+1}) F'(k_{t+1}).$$

Since δ_{t+1} / δ_t is increasing, it follows that c_t cannot be constant. Formally, $\{k_t\}_{t=0}^{L}$ exhibits a *political business cycle* if $k_0 = k_L \neq k_t$ for some $0 < t < L$.

PROPOSITION 3: *Any steady-state equilibrium consumption plan yields a political business cycle.*

More details regarding the proof, the definition of the continuation value, and the minmax set are provided in Boylan et al. (1996). In general, we do not know whether steady-state consumption plans exist. However, computation for simple examples suggests that they do. We close with an example illustrating equilibrium consumption plans that converge to steady-state equilibria.

Example 2: In this example, we compute (up to computer accuracy) an equilibrium consumption plan for the case in which $u(c) = \ln(c)$ and $F(k) = \sqrt{k}$, there are 100

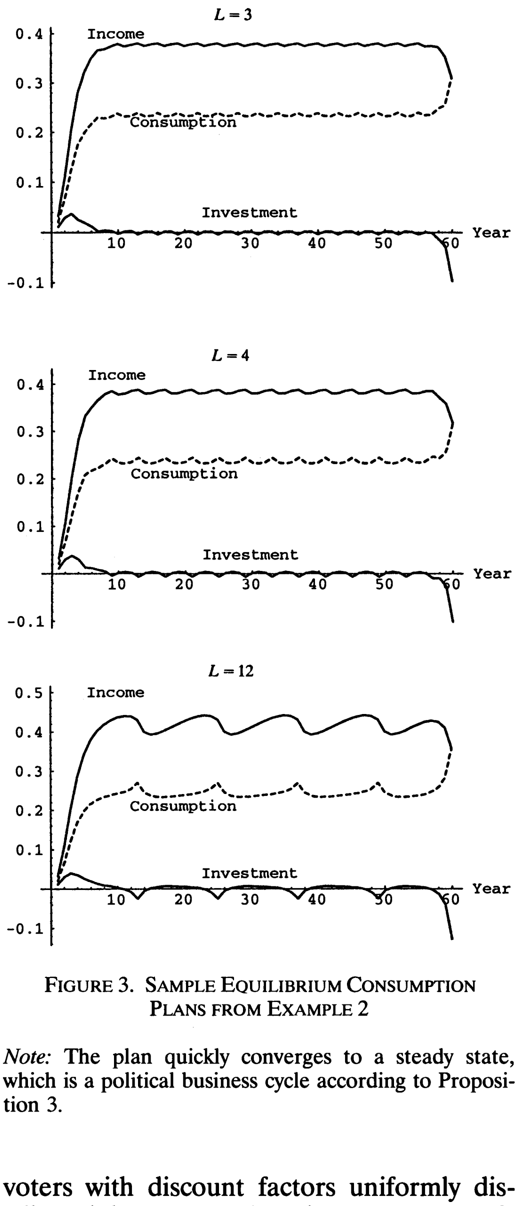

FIGURE 3. SAMPLE EQUILIBRIUM CONSUMPTION PLANS FROM EXAMPLE 2

Note: The plan quickly converges to a steady state, which is a political business cycle according to Proposition 3.

voters with discount factors uniformly distributed between 0.5 and 1.0, $\lambda_i = 1/100$, and $k_0 = 0.01$. Figure 3 illustrates an equilibrium consumption plan for the cases when $L = 3$, 6, and 12. (The Appendix describes the algorithm that computes such a plan.) In each graph the top line is output $[y_t = F(k_t)]$, the center line is consumption $[c_t = F(k_t) - k_{t+1}]$, and the bottom line is gross investment $[\iota_t = k_{t+1} - k_t]$. We note that the political business cycles have the feature that output and consumption peak at the beginning and end of the electoral term, while investment peaks in the middle of the term. Also we note that the amplitude of the cyclical behavior increases as the length of the term increases.

IV. One-Period Commitment

The results in the previous sections show that, if voters can choose between multiperiod proposals, no majority-rule core exists. However, it can be argued that multiple-period commitments are not credible. Because of the temporal nature of the decision, period-t decisions must be implemented before period-$(t+1)$ decisions. But once period-t decisions are implemented, there is always the opportunity to revise the period-$(t+1)$ decisions. In making the period-t decision, all voters would realize this aspect of the problem, and they would make the period-t decision conditional on the belief that the period-$(t+1)$ decision will be made subject to preferences at that point in time. This means that multiperiod deviations from a proposed "status quo" consumption plan can only occur if players will want to continue with the deviation even in the later periods of the deviation. For example, if a coalition supporting a deviation contains members with both high and low discount factors, then for the deviation to benefit both groups, it may be necessary that the groups get their benefits at different times. Once the players with low discount factors have received their benefits from higher initial levels of consumption, they may no longer be willing to support the investment necessary to help their coalition partners with higher discount factors. Of course, if the coalition members had realized this problem when they were contemplating the proposed change, they would never have formed the coalition in the first place. This realization by the individuals that coalition members may want to back out of their agreements later in the process will make it harder to find proposals that can beat the status quo.

The ideas discussed in the previous paragraph are formalized by the concept of subgame perfection; that is, the voting equi-

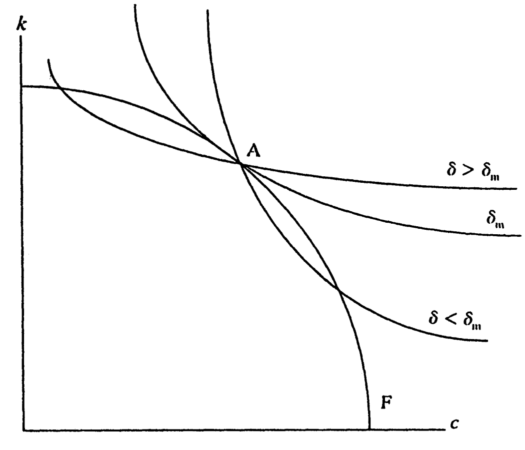

FIGURE 4. ILLUSTRATION FOR PROPOSITION 4

Notes: On the axes, we have the consumption at a given period and capital at the next period. F is the feasible set. A is the optimal choice of the median voter. Any movement away from A can be defeated by a majority.

librium must be an equilibrium at every time period. In other words, no precommitment is possible. However, it is not straightforward to formalize this intuition. In order to define the notion of subgame perfection, a specific noncooperative game needs to be defined. Such a game needs to specify exactly how voters make decisions and exactly what type of information the participants in the decision-making process have. In Boylan et al. (1996), a model of two-candidate competition in which both voters and candidates behave strategically is defined. In this model, at each period candidates simultaneously precommit to consumption for that time period. Voters then simultaneously vote for one of the candidates. The candidate with the most votes is elected and implements the announced consumption. Candidates do not care about what consumption they select, but only care about being elected. Voters do not care about which candidate is elected, but only care about their consumption stream. Given such a model, it is shown in our work with John Ledyard (Boylan et al., 1996) that a voting equilibrium can be defined which satisfies the following property.

PROPOSITION 4: *The optimal consumption plan for the median voter is a voting equilibrium. If the time horizon, T, is finite, then it is the unique equilibrium.*

In this discussion we do not present the definition of the game, but instead focus on the properties of such a "voting equilibrium." Let $\{c^*_{t,T}(k)\}$ be the optimal plan for the median voter, given an initial capital k and a length of the time horizon T. We want to provide intuition for why $\{c^*_{t,T}(k)\}$ is a voting equilibrium. Here we restrict ourselves to Markov plans, so at time t, voters have a utility that depends on only the current consumption, c, and capital in the next period, k; specifically, utility of an individual with discount factor δ is $u(c)+v_t^T(k,\delta)$, where $v_t^T(k,\delta)=\sum_{s=1}^{T-t}\delta^s u(c^*_{s,T-t}(k))$.[6] By monotonicity of $c^*_{s,T-t}(k)$ (which is proved in Boylan et al. [1996]), if $\delta > \delta'$, then $\partial v_t^T/\partial k(k,\delta) > \partial v_t^T/\partial k(k,\delta')$.

Figure 4 illustrates the decision in a particular time period and for a particular level of capital. F is the graph of capital level in next period, k, as a function of current consumption c (specifically, $k=F(\tilde{k}-c)$, where $\tilde{k}$ is the current level of capital). Point A denotes the optimal choice for the median-discount-factor voter. Hence, at point A, the indifference curve labeled δ_m is tangent to the feasible set. By the monotonicity of $\partial v_t^T/\partial k$, the indifference curves for voters with $\delta > \delta_m$ must be "flatter," and indifference curves for voters with $\delta < \delta_m$ must be "steeper." Thus, A defeats any feasible point to the left of A by a majority of voters (the median voter and those less patient than the median voter), and A also defeats any feasible point to the right of A by a majority of voters (the median voter and those more patient than the median voter).

Suppose that, after time t, voters select the continuation path that is optimal for the

[6]For the infinite-horizon model the notation simplifies: utility of an individual with discount factor δ is $u(c)+v(k,\delta)$, where $v(k,\delta)=\sum_{s=1}^{\infty}\delta^s u(c_s^*(k))$.

median voter. Then, the argument in the previous paragraph shows that at time t voters select the consumption level that is optimal for the median voter. Consequently, the optimal consumption plan for the median voter is a voting equilibrium. Suppose the time horizon is finite. In the last period there is a unique equilibrium, namely, consume everything. By the argument in the previous paragraph and by induction, at every time period, the continuation value is unique, and hence the equilibrium is unique.

V. Conclusion

The growth literature analyzes optimal policies and competitive outcomes from the point of view of a representative consumer. This paper discusses how the political process aggregates preferences of voters with different time preferences. The paper suggests that if there are large numbers of voters and there is either a finite planning horizon or periodic elections, the outcome of the political process will be different than the prediction given by the representative-agent model. These results are formally stated and proved in Boylan et al. (1996).

In the case of periodic elections, steady-state equilibrium consumption plans behave like political business cycles. Unlike the model of Nordhaus (1975), the political business cycles are not caused by voter myopia, but by how the majority relation aggregates preferences, and cyclic consumption plans would be selected even at the beginning of the term of office. Since the cycles arise because of coalitions between patient and impatient voters, they generate different patterns than other models of political business cycles. Computations on simple examples suggest that the cycles begin with a postelection (not preelection) burst of consumption to appeal to impatient voters, followed by a period of reinvestment to appeal to patient voters.

Our analysis also raises questions as to whether neoclassical optimal plans are desirable from a welfare point of view, since they can always be defeated by virtually unanimous majorities. In particular, cyclical plans are majority-preferred to them and, if the planning horizon is short, can only be defeated by smaller majorities than neoclassical optimal paths.

Appendix

This appendix describes how we computed the plans in Figure 3. Let $\delta_t = (1/100)\Sigma_{i=0}^{99}[0.5 + i(0.5)(1/99)]^t$. Let $h = (h_1, \dots, h_L)$: $\mathbb{R} \to \mathbb{R}^L$, represent the L-period policy function, where $h_t(k)$ represents the capital at the beginning of period t if k is the initial capital stock at time $t = 0$. For notational convenience, we write $h_0(k) = k$. For any integer j, define $h_{jL+t}(k) = h_t(h_L^j(k))$, and for any h, define

$$v_h(k) = \sum_{t=0}^{\infty} \delta_{t+L} u\big(F(h_t(k)) - h_{t+1}(k)\big).$$

For any h: $\mathbb{R} \to \mathbb{R}^L$, v: $\mathbb{R} \to \mathbb{R}$, and $k \in \mathbb{R}$, define

$$w(k; h, v) = \sum_{t=0}^{L-1} \delta_t u\big(F(h_t(k)) - h_{t+1}(k)\big) + v\big(h_L(k)\big).$$

To compute a solution, we choose a grid on k (capital), and for any given L we proceed as follows:

1. Start with an initial guess $v(k)$ of $v_h(k)$ on the grid.
2. Compute $\hat{h}(k) \in \operatorname{argmax}_{\hat{h}} w(k; \hat{h}, v)$ for each k on the grid.
3. Set $h(k) = \hat{h}(k)$ on the grid.
4. Compute $\hat{v}(k) = v_h(k) = \Sigma_{t=0}^{T} \delta_{t+L} \times u(F(h_t(k)) - h_{t+1}(k))$.
5. If $|\hat{v} - v| < \tau$ (in the ℓ_1 norm), stop.
6. Otherwise, set $v(k) = \hat{v}(k)$ and go to 2.

We use as our grid $H = \{k = 0.002 \times j : j$ an integer with $0 \le j \le 200\}$. We set $T = 150$ and $\tau = 1.0 \times 10^{-10}$. In all the examples we have tried, we find the above program converges (satisfies the condition in step 5 above).

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