scattering[7] as for proton-proton scattering[8] at lower energies. In view of the similarity of proton-nucleus and proton-proton scattering at high energies demonstrated in the present paper, it is reasonable to assume that the ratio $\mathrm{Re}T(0)/\mathrm{Im}T(0)$ is negative at high energies also for proton-nucleus scattering, which gives a repulsive potential $V_0 = +12$ MeV.

The way in which the potential scattering has been introduced is only one of many possible ways, the essential point being that without potential scattering, $V_0 = 0$, we should have pure shadow scattering, $h_R = 0$ and $h_I = \frac{1}{2}[1-(1-\rho)^{1/2}]$. In order to investigate this point we also made an attempt to fit the experimental results using a complex potential, i.e., we put

$$h = (i/2)(1 - e^{-\eta - i\chi}), \tag{11}$$

where $\chi = 2V_0\beta^{-1}(R^2 - b^2)^{1/2}$ describes potential scattering while η is given by shadow scattering only, $e^{-2\eta} = 1 - \rho(b)$. The values of μ_0 and A are as before, but we had to choose $R_0 = 1.00$ fm, $R = 1.32A^{1/3}$ fm, and $|V_0| = 22$ MeV. The results are shown as dashed curves in Figs. 1 and 2 for Be, Cu, and Pb. The agreement with experimental results is approximately as good as with our first model.

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SIGN OF THE PROTON-NEUTRON MASS DIFFERENCE AND A GHOST SCALAR MESON*

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A new interpretation of the proton-neutron mass difference has been given independently by Pagels[1] and Fried and Truong.[2] In this note, it is shown that, if this model works, it implies a ghost scalar meson with $T = 1$ and $G = -1$. Use is made of the technique of spontaneous symmetry-breaking theory.[3] By identifiying the ghost with that of the Regge-pole model, one can crudely estimate the magnitude of the feedback by means of a linear extrapolation.

The physical mass of the proton, in the absence of electromagnetic interactions, may be written

$$m_p = m_0 + \Sigma_p(\not{p}, m_{0p}, m_{0n})\big|_{\not{p}=m_p}, \tag{1}$$

where Σ_p is the unrenormalized proper self-energy part. Its dependence on the bare masses of the proton and neutron are indicated explicity. It is also a function of the bare masses of other strongly interacting particles and of the bare strong-interaction coupling constants.

Now let us introduce electromagnetism. The proton mass will shift to $m_p + \delta m_p$, the function Σ_p will become $\Sigma_p + \delta\Sigma_p$, and we may write

$$\delta m_p = \frac{\partial \Sigma_p(\not{p}, m_{0p}, m_{0n})}{\partial \not{p}}\bigg|_{\not{p}=m_p} \delta m_p + \delta\Sigma_p(\not{p})\big|_{\not{p}=m_p}. \tag{2}$$

This may be rewritten in the form

$$\delta m_p = Z_p \delta\Sigma_p(\not{p})\big|_{\not{p}=m_p}, \tag{3}$$

using the fact that the proton wave-function renormalization constant is

$$Z_p = \left[1 - \frac{\partial \Sigma_p(\not{p}, m_{0p}, m_{0n})}{\partial p}\bigg|_{\not{p}=m_p}\right]^{-1}. \tag{4}$$

Similar equations can, of course, be written for the neutron, and we note that to lowest order in the fine-structure constant we may take $Z_p = Z_n = Z$.

The physical effect exploited by Pagels and by Fried and Truong is the feedback on the electromagnetic self-mass of electromagnetic mass shifts internal to various Feynman diagrams. To begin with, and to study this feedback, let us ignore all explicit effects of the electromagnetic interaction, and simply assume that the proton and neutron masses are caused to shift, for some unexplained reason. Then we may write the perturbation to the Lagrangian for the strong interactions as

$$\delta\mathcal{L} \approx \delta m_{0p}\bar{\psi}_p\psi_p + \delta m_{0n}\bar{\psi}_n\psi_n \equiv \bar{\psi}(\delta m_{0s} + \delta m_{0r}\tau_Z)\psi. \tag{5}$$

That is, the bare masses of the proton and neutron are shifted to $m_0 + \delta m_{0p}$ and $m_0 + \delta m_{0n}$, respectively, and nothing else occurs. This new term in the Lagrangian is also identical to the interaction of nucleons and two "scalar mesons," one with isospin one and one with isospin zero, with "coupling constants" δm_{0s} and δm_{0v}, where the scalar mesons are treated in lowest order and carry no four-momentum. All the usual strong interactions, it should be emphasized, are treated in all orders. The strong interactions have the effect of renormalizing the scalar "coupling constants" to the values $(Z/Z_{1s}^{(0)})\delta m_{0s}$ and $(Z/Z_{1s}^{(1)})\delta m_{0v}$, where $Z_{1s}^{(0)}$ and $Z_{1s}^{(1)}$ are the vertex renormalization constants for the isospin-zero and -one "scalar-meson" vertices. (The wave-function renormalization constants for the "scalars" are unity, because the "scalar mesons" interact only once.)

We want to choose the masses δm_{0p} and δm_{0n} such that the physical masses appearing after strong interaction renormalization effects are included on the bare propagators; thus we have

$$\tfrac{1}{2}(\delta m_p + \delta m_n) = (Z/Z_{1s}^{(0)})\delta m_{0s},$$
$$\tfrac{1}{2}(\delta m_p - \delta m_n) = (Z/Z_{1s}^{(1)})\delta m_{0v}. \tag{6}$$

Let us now return to Eq. (3). Clearly, when only mass shifts are included in the perturbed Lagrangian, the only effect on the proper self-energy part is to change $\Sigma_p(p, m_{0p}, m_{0n})$ into $\Sigma_p(p, m_{0p} + \delta m_{0p}, m_{0n} + \delta m_{0n})$. Therefore,

$$\delta\Sigma_p(p) = \frac{\partial\Sigma_p(\not{p}, m_{0p}, m_{0n})}{\partial m_{0p}}\delta m_{0p} + \frac{\partial\Sigma_p(\not{p}, m_{0p}, m_{0n})}{\partial m_{0n}}\delta m_{0n} \equiv J_{pp}\delta m_{0p} + J_{pn}\delta m_{0n}. \tag{7}$$

Equation (3) becomes

$$\delta m_p = Z(J_{pp}\delta m_{0p} + J_{pn}\delta m_{0n}). \tag{8}$$

The analogous equation for the neutron is

$$\delta m_n = Z(J_{nn}\delta m_{0n} + J_{np}\delta m_{0p}), \tag{9}$$

and we note that $J_{pp} = J_{nn}$ and $J_{pn} = J_{np}$ to lowest order in the fine-structure constant. We therefore have

$$\delta m_p - \delta m_n = Z_{1s}^{(1)}(J_{pp} - J_{pn})(\delta m_p - \delta m_n), \tag{10}$$

where we have used Eq. (6).

To complete the derivation, we need only to observe that

$$\left(\frac{\partial\Sigma(p, m_{0p}, m_{0n})}{\partial m_{0p}} - \frac{\partial\Sigma_p(p, m_{0p}, m_{0n})}{\partial m_{0n}}\right)\Bigg|_{\not{p} = m_p} = \Gamma_s^{(1)}(0, m_{0p}, m_{0n}) - 1, \tag{11}$$

where $\Gamma_s^{(1)}(0, m_{0p}, m_{0n})$ is the unrenormalized proper vertex function for the "scalar meson" with isospin one coupled to nucleons and evaluated at zero momentum transfer. This is evidently an identity to all orders in the strong interactions. Now, by definition,

$$\Gamma_s^{(1)}(0, m_{0p}, m_{0n}) = 1/Z_{1s}^{(1)}, \tag{12}$$

and furthermore, the vertex renormalization constant is equal to the D function, normalized to unity at infinite energy, for ${}^3P_0$ channel with $T = 1$ of nucleon-antinucleon scattering, evaluated at zero total energy.[4] If we put all this together, we find that

$$\delta m_p - \delta m_n = [1 - D(0)](\delta m_p - \delta m_n). \tag{13}$$

Spontaneous breakdown thus requires $D(0) = 0$; that is, there must exist a massless bound state of spin $J^{PG} = 0^{+-}$ and isospin one in the nucleon-antinucleon system.[5]

Now let us return to the beginning of our discussion, and include other effects of electromagnetism, in addition to the shift of internal nucleon masses. There is then a new electromagnetic term in the Lagrangian besides the $\bar{\psi}(\delta m_{0s} + \delta m_{0v}\tau_Z)\psi$ which we have already discussed. Consequently, Eq. (7) will be modified by the presence of an additional term, which we call $\delta\Sigma_p'$, on the right-hand side, and Eq. (13) will become

$$\delta m_p - \delta m_n$$
$$= [1-D(0)](\delta m_p - \delta m_n) + Z(\delta\Sigma_p' - \delta\Sigma_n'). \quad (14)$$

We therefore conclude that

$$\delta m_p - \delta m_n = [Z(\delta\Sigma_p' - \delta\Sigma_n')/D(0)]. \quad (15)$$

The effect of the mass feedback is to introduce the denominator $D(0)$ in Eq. (15), and thus to modify whatever other electromagnetic effects we may wish to include in $\delta\Sigma_p' - \delta\Sigma_n'$.[6] It is clear that the feedback effect will change the sign of the mass difference from what it would be in the absence of feedback, provided $D(0) < 1$. However, since $D(-\infty) = 1$ in virtue of the normalization of D, there must then be a zero of D at some negative squared energy. (We assume D is continuous for negative squared energy.) There is, then, a ghost.

It is important to remark that we use the phrase "a ghost exists" to mean simply that the D function vanishes; the ghost will not appear physically if any of the ghost-killing mechanisms suggested in connection with Regge poles applies.[7] No contradiction is therefore implied.

The ghost we find here might, for example, be identified with a ghost on the trajectory passing through the A_2 meson ($J^{PG} = 2^{+-}$, $T = 1$), which would occur if this trajectory went through zero. Assuming the usual straight line for the trajectory (with the same slope that goes with the Pomeranchuk trajectory if it passes through the f_0), the ghost occurs at -1.4 (GeV)^2. We could try to fit D in this region with a single pole located, say, near threshold at about $+4$ (GeV)^2. If we do this, we get $D(0) \sim -0.3$; however, such numerical estimates are very crude and should not be taken seriously.

In conclusion, let us restate our results. We find that if the effect of the mass-feedback mechanism is to change the sign of the proton-neutron mass difference from what it would be ignoring this mechanism, then there exists a ghost meson with $J^{PG} = 0^{+-}$ and $T = 1$. The derivation of this result involves no assumptions other than elastic unitarity for the nucleon-antinucleon system in the $^3P_0$ channel. In particular, therefore, the Pagels and Fried-Truong explanation of the sign of the mass difference requires such a ghost. This does not, however, contradict nature since the ghost may not appear physically, and the ghost could even be identified with the point at which the A_2 trajectory passes through zero.

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