peaks are of the same height except for the weakest lines of the patterns. The satellites appear on the microphotograms as subsidiary peaks where they are well-resolved and as points of inflection where the resolution is not very good. Of course, all the satellites do not appear. Some of them are too close for the resolving power available.

The Zeeman effect of the Hg lines is being studied in detail at the French Academy of Sciences[5] using their large magnet which provides fields up to about 60,000 gauss. Certain irregularities seem to show up, but there is a doubt in the author's mind of the reality of these, principally because of the effect of the unresolved h.f.s. components. If, for example, we label the principal components of λ4358 with the numbers 1 to 6, the components 2 and 5 separate proportionally with the field strength, while 1 and 6 seem to separate more rapidly. This would seem to indicate that something was affecting $^3S_1$ but not $^3P_1$. The same sort of thing happens with λ5461, indicating perhaps a similar cause for the behavior of $^3P_2$. The meagerness of the report makes it very difficult to comment upon, but in view of all of the work done on h.f.s., it would seem that any discussion of the peculiar behavior of the Zeeman components of a line cannot entirely neglect the effects of the Paschen-Back effect of the h.f.s. components, which tend to spoil any effect that concerns itself only with the linearity of position and field-strength.

In conclusion, the author wishes to express his thanks to Professor R. A. Loring, of the University of Louisville, for his assistance in part of the experimental work connected with this paper.

[5] Dupouy and Jacquinot, Comptes rendus **201**, 543 (1935). Jacquinot and Belling, ibid. **201**, 778 (1935).

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The Diffusion and Absorption of Neutrons in Paraffin Spheres*

DON M. YOST AND ROSCOE G. DICKINSON, *Gates Chemical Laboratory and Kellogg Radiation Laboratory, California Institute of Technology*

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Measurements are described of the activities imparted to silver and to indium detectors placed in paraffin spheres of various sizes with a radon-beryllium source of neutrons located at the center. The results are discussed with the aid of diffusion theory. Reasonably good quantitative agreement is obtained at distances from the source greater than about 8 cm. Various independent estimates are made of the mean free path of the neutrons in paraffin, their specific absorption rate constant, and the ratio of the latter to the diffusion constant. The mean free path is found as 0.53 cm; the ratio of the specific absorption rate constant to the diffusion constant as 0.026 cm^{-2}.

FOLLOWING the discovery of Fermi[1] and his co-workers that hydrogen containing substances are effective in slowing down the neutrons from a radon-beryllium source, several investigators have made experiments to determine the velocities of the slow neutrons produced. The results of various experimenters[1, 2, 3] have shown that the velocities are comparable with thermal velocities. Dunning[4] and others, by means of a velocity filter, found that the neutrons slowed down by paraffin had a velocity distribution at room temperature substantially that predicted by Maxwell's law. These results suggested that, under definite and simple geometrical conditions, the behavior of slow neutrons in paraffin might be more closely studied from the point of view of diffusion. The method of experimentation adopted consisted in deter-

* Contribution No. 545.

[1] Fermi, d'Agostino, Rasetti and Segrè, Proc. Roy. Soc. **A146**, 483 (1934); Amaldi, d'Agostino, Fermi, Pontecorvo, Rasetti and Segrè, Proc. Roy. Soc. **A149**, 522 (1935).

[2] Westcott and Bjerge, Proc. Camb. Phil. Soc. **31**, 145 (1935); Westcott and Niewodniczański, Proc. Camb. Phil. Soc. **31**, 617 (1935); Bjerge and Westcott, Proc. Roy. Soc. **A150**, 709 (1935).

[3] Moon and Tillman, Nature **135**, 904 (1935).

[4] Dunning, Pegram, Fink, D. P. Mitchell and Segrè, Phys. Rev. **48**, 704 (1935); Fink, Dunning, Pegram, and Mitchell, Phys. Rev. **49**, 103 (1936).

mining the activity produced in indium and silver detectors placed inside paraffin spheres of different diameters at measured distances from a source located at the center.

Experimental Procedure

Four paraffin spheres were used; their diameters were 20, 25, 35, and 51 cm, respectively. The smaller ones were spherical to within two or three millimeters and the largest to within five millimeters. A cylindrical hole, 7 cm in diameter, was bored radially to the center of each sphere, and fitting easily into this hole was a stack of paraffin cakes. The two larger surfaces of each cake was so curved that a thin flexible detector placed between any two cakes would have all points on its surface equidistant from the source located at center of the sphere.

The detectors were thin, circular disks of silver and indium with several small sectors cut out in order to make them conformable to the curved surfaces of the paraffin cakes. The thickness of the indium detector was 0.005 cm and that of the silver 0.02 cm. The spheres and detectors are shown in Fig. 1.

A 3-mm×5-mm cylindrical brass capsule containing powdered beryllium and radon (300 millicuries or less) served as the source of neutrons.

In an experiment the detector used was left in some one position for a time sufficient for the activity to become practically saturated (7 to 12 half-lives). The activity was measured with a quartz fiber electroscope. Readings were begun at a fixed time (Ag, 30 sec.; In, 60 sec.) after removal of the detector from the sphere and were continued at suitable intervals for a definite period of time (Ag, 6 min.; In, 15 min). The total reading for the period, after correcting for nonlinearity of scale, for background, and for radon decay, was taken as the activity, β, for that experiment.

The half-lives of the two activities of silver were determined with the electroscope used and were found to be 23.5 sec. and 2.15 min., respectively.

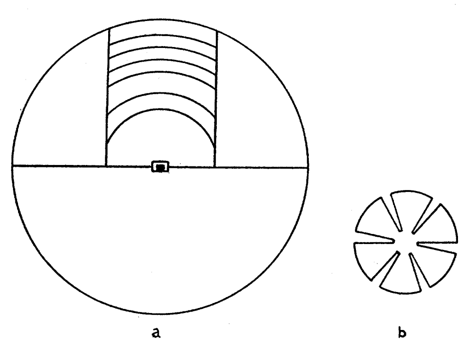

FIG. 1. (a) Cross section of the 10-cm paraffin sphere showing arrangement of the cakes between which the detector was placed. (b) The detector drawn to the same scale.

Results of the Experiments

The results of the experiments are shown graphically in Figs. 2 and 3 where βr is plotted against r the distance of the detector from the center of the sphere in cm. Because of the strong activity and relatively long half-life of indium the best accuracy was obtained with it as the detector.

Discussion of the Results

If it is assumed that the neutrons diffuse through the paraffin according to the ordinary diffusion laws, and that the absorption by the thin detector does not cause appreciable disturbances, and further that the neutrons are absorbed in the paraffin at a rate, ac, proportional to the concentration, then

$$D\nabla^2 c - ac = \partial c/\partial t$$

where D is the constant diffusion coefficient and a is the specific absorption rate constant. Under the steady state conditions of the experiments $\partial c/\partial t = 0$. The equation becomes then

$$D\nabla^2 c - ac = 0. \tag{1}$$

The general solution, for the present case of spherical symmetry, may be written in either of the forms

$$\begin{aligned} cr &= (A/\alpha)\sinh \alpha r + Be^{-\alpha r} \\ &= B \sinh \alpha(r+k_1)/\sinh \alpha k_1; \quad \alpha^2 = a/D. \end{aligned} \tag{2}$$

A, B and k_1, are constants of integration and are related to each other by the equation

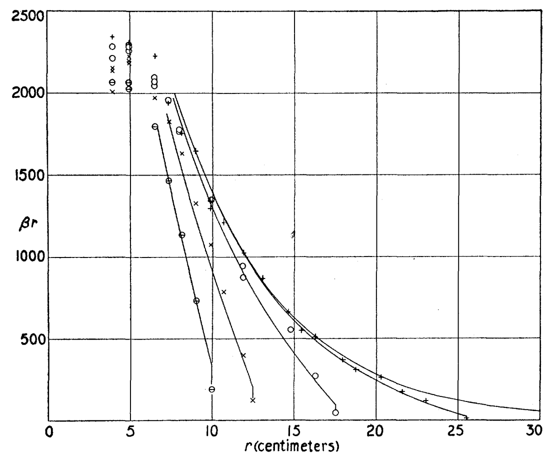

FIG. 2. Plots of βr *vs.* r for the indium detector in spheres of radii 10, 12.5, 17.55, and 25.55 cm, respectively, from left to right. The curves for the spheres of various radii were drawn using Eq. (5). The top curve is for a sphere of infinite radius.

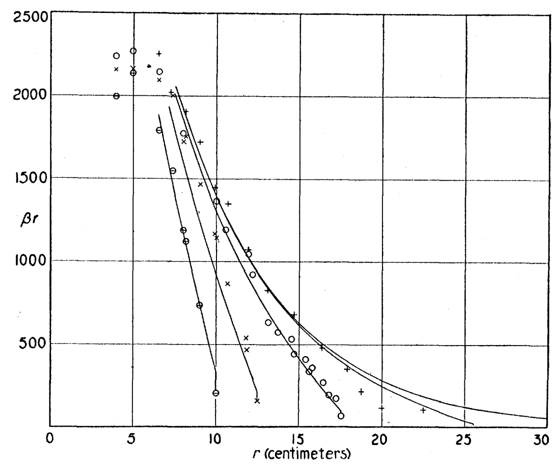

FIG. 3. Plots of βr *vs.* r for the silver detector. The solid curves are identical with those of Fig. 2.

$A = \alpha B/\sinh \alpha k_1$. B is directly proportional to the strength of the source when the source is placed at the center of the sphere. In case $a = 0$, $cr = Ar + B$.

There are two particular solutions contained in (2) which will be required later, namely,

$$cr = Be^{-\alpha r} \tag{3}$$

when the source is central and the sphere is of infinite radius, and

$$cr = (A'/\alpha) \sinh \alpha r \tag{4}$$

when the source is uniformly distributed over the surface of a finite sphere.

In order to relate the concentrations in one sphere with a central source to those in a sphere of different radius a boundary condition may be introduced as follows. The net outward flux of neutrons across a spherical surface just inside the paraffin is equal to the rate of escape of neutrons from the surface. The first quantity is equal to $-4\pi r^2 D\partial c/\partial r$; if the rate of "evaporation" of the neutrons per unit area be assumed proportional to the concentration prevailing a short distance under the surface, then

$$-D(\partial c/\partial r)_{r=r_0} = k_2 c_{r=r_0},$$

where r_0 is the radius of the sphere. The use of this boundary condition leads to the particular solution

$$cr = B \sinh \alpha(r + l - r_0)/\sinh \alpha(l - r_0), \tag{5}$$

where $l = (1/\alpha) \sinh^{-1}[\alpha/\{(1/r_0 - k_2/D)^2 - \alpha^2\}^{\frac{1}{2}}]$.

The equations derived involve the concentration of neutrons, while the measured quantity is β, the activity of a detector. The number of neutrons per unit time entering the detector will depend on both the concentration in its neighborhood and the velocity distribution; of the neutrons entering, the fraction captured by the detector may again depend on the velocity distribution.[5] Proportionality between β and c may safely be expected, then, only in regions of constant velocity distribution; this doubtless excludes regions close to the central source. Moreover, in regions where the velocity distribution is not constant it is not likely that D, a, and k_2 remain constant as assumed in Eqs. (1) to (5). However, in order to see to what extent the observations are in harmony with the hypothesis, we have assumed Eqs. (1) to (5) and have placed $\beta = k'c$, with k' constant for a given detector. Similarly, with the detector placed on the surface of a sphere the activity β_0 is assumed proportional to f_0, the flux of neutrons from the surface: $\beta_0 = k_0' f_0$. If one assumes that the capture cross section of indium or silver is a constant, and further that the velocity distribution of the neutrons is Maxwellian, then it may easily be

[5] For recent experiments and theory on absorption see Rasetti, Segrè, Fink, Dunning and Pegram, Phys. Rev. **49**, 104 (1936). Breit and Wigner, Phys. Rev. **49**, 519 (1936).

TABLE I. *Total fluxes measured by indium.*

SPHERE RADIUS, r_0	ACTIVITY, β_0 (Mean)	FLUX, $r_0^2\beta_0$
10.0	18.7	1870
12.5	9.83	1540
17.55	2.61	800
25.55	0.53	340

shown that $k'=bn\sigma(2kT/\pi m)^{\frac{1}{2}}$ and $k_0'=bn\sigma$ where n is the number of nuclei per cm^2 in the detector and b is an instrument constant. If, on the other hand, the capture cross section $\sigma=\sigma_0/v$, then $k'=bn\sigma_0/2$ and $k_0'=(bn/2)(\pi m/2kT)^{\frac{1}{2}}$. In either case $k'/k_0'=(2kT/\pi m)^{\frac{1}{2}}$.

There appears to be some justification for the assumption of constancy of k' in the fact that the curves for silver and indium are superposable throughout. Silver and indium are known to be activated by neutrons of different qualities.[6] Also the ratio of the 23.5-sec. activity of silver to the 2.15-min. activity remains substantially constant. It does not follow, but it does appear possible that the neutrons which cause activation are all at the same "temperature," and, from Dunning's velocity filter experiments, the temperature corresponds to that of the paraffin. It is recognized, of course, that in the region near the source the velocity distribution is presumably not Maxwellian.

On the assumption that k', D and a are constants, independent of r, it may be immediately concluded that, since the values for βr do not fall on a straight line, some absorption of the neutrons in the paraffin occurs. Moreover, since the total outward flux, $r_0^2\beta_0$, as measured by detectors placed on the surfaces of the spheres, decreases with increasing radius of the spheres it follows that absorption takes place in the paraffin if k_0' remains constant after the neutrons have traversed 10 cm or more of paraffin. The total fluxes are presented in Table I.

In order to compare the predictions of the theory with the observations in the region $r \geqq 8$ cm, the following procedure was used. Eq. (2) was fitted to a smoothed curve drawn through the points for the 51-cm sphere. A satisfactory fit was obtained with 0.16 cm^{-1} as the value for α. Eq. (5) was then used to construct the predicted curves for the other spheres. The agreement with the observations may be seen in Figs. 2 and 3. It is satisfactory for all but the 25-cm sphere where the deviations are somewhat greater than the experimental error. Evidently then, had independent values of α been conversely calculated from the relative activities near the surfaces of the various spheres, substantially the same value of α as before would have been obtained.

A value for α may be obtained in still another manner as follows. At a given distance r from the center, the activity produced in a detector increases with the radius r_0 of the sphere. The increase must be due to neutrons which have been at larger distances and then diffused back to r. The difference in the activity (times r) at r for two different spheres of radii r_0 and r_0' will evidently be

$$\Delta=\beta' r-\beta r=\frac{A''(r_0')-A''(r_0)}{\alpha}\sinh \alpha r$$

and

$$(1/\Delta)(d\Delta/dr)=\alpha \coth \alpha r. \tag{6}$$

Application of relation (6) to the observations yielded an average value of 0.23 for α. This is appreciably higher than the value given above, but the errors involved are also considerably greater. For the present it can only be said that the two results are quite closely of the same order of magnitude.

An estimate of the value of a was made in the following manner. The total rate of absorption of neutrons in a sphere is equal to $\int_0^{r_0}4\pi r^2 acdr$; and the total flux of neutrons from the sphere equal to $4\pi r_0^2 f_0$. The sum of these is equal to the rate of issue of neutrons from the source, so that for two experiments with equal sources but different spheres of radii r_0 and r_0' the sums are equal. Assuming a constant

$$a=\frac{r_0'^2 f_0'-r_0^2 f_0}{\int_0^{r_0}r^2cdr-\int_0^{r_0'}r^2c'dr}$$

$$=\frac{r_0'^2\beta_0'-r_0^2\beta_0}{\int_0^{r_0}r^2\beta dr-\int_0^{r_0'}r^2\beta' dr}\frac{k'}{k_0'}.$$

The last member also assumes constancy of k_0' and k'. At first sight these assumptions seem especially objectionable since the integrations extend from the origin; however, at the smaller

[6] C. H. Collie, Nature **137**, 614 (1936).

TABLE II. *Mean free path of neutrons in paraffin from indium measurements.*

r_0(cm)	β_0	$-(\partial\beta/\partial r)$	λ(cm)
10.0	18.7	48.9	0.58
12.5	9.83	26.9	0.55
17.55	2.61	8.3	0.47
25.55	0.53	1.5	0.53
		Mean	0.53 cm

r's, β and β' do not greatly differ, and the contribution to the denominator of distances under 6 or 7 cm is at most only a few percent. From graphical evaluation of the integrals, values of a of from 0.99 to $1.22\times 10^{-2}k'/k_0'$ with a mean of $1.13\times 10^{-2}k'/k_0'$ were obtained. With k'/k_0' equal to that for a Maxwellian distribution of 295°K, a becomes $1.41\times 10^3 \pm 0.08$ sec.$^{-1}$.

Finally, from a consideration of the behavior of the neutrons at and near the surface of a sphere, a value of their mean free path λ and the diffusion coefficient, $D=\frac{1}{3}(8kT/\pi m_n)^{\frac{1}{2}}$, may be calculated. By kinetic theory considerations it may be shown that the net flux, in the direction of increasing r, of neutrons through unit area of a detector placed normal to the concentration gradient $\partial c/\partial r$ is $-\frac{1}{3}\lambda v \partial c/\partial r$, where v is the mean velocity; similarly the gross flux is $\frac{1}{2}cv$. The net flux is measured by a detector placed on the surface of the sphere and the gross flux by a detector just inside the surface. From this it can readily be shown that

$$\lambda = -(3/2)\beta_0/(\partial\beta/\partial r),$$

where the derivative is taken a short distance inside the surface. The values of λ obtained for the several spheres are given in Table II. The fact that λ is about the same for all spheres does not necessarily imply a constancy of the temperature of the slow neutrons. On the assumption that $T=295°$, $D=4.4\times 10^4$.

From the independently determined values of a and D, a value of $\alpha=(a/D)^{\frac{1}{2}}=0.18$ cm^{-1} was calculated which is in very good agreement with the value 0.16 cm^{-1} found above.

The measurements also permit calculations of cross sections for scattering, and cross sections for capture.[7] That for scattering, σ_s, may be obtained from the mean free path through the relation $\sigma_s=\frac{3}{4}N\lambda$ where N is the number of scattering nuclei per unit volume. If hydrogen alone be assumed to scatter, then $\sigma_s=18\times 10^{-24}$ cm^2. If, however, a scattering cross section of 4×10^{-24} be allowed for carbon,[8] that for hydrogen becomes 16×10^{-24}. The cross section for absorption may be obtained from the specific absorption rate constant through the relation $\sigma_c=(a/N)(\pi m_n/8kT)^{\frac{1}{2}}$ where m_n is the mass of the neutron. The results of Fleischmann[9] and of Kikuchi, Husimi and Aoki[10] indicate that protons are mainly responsible for capture in paraffin. On the assumption of capture by hydrogen alone, our value of a leads to $\sigma_c=7.3\times 10^{-26}$ cm^2; Kikuchi obtained 3.0 to 8.3×10^{-26} cm^2. If we take $\sigma_c=\sigma_0/v$ then $\sigma_0=a/N$, independently of any assumed temperature. The value calculated becomes $\sigma_0=1.8\times 10^{-20}$ cm^2 sec.$^{-1}$, and this would be the cross section for neutrons of unit velocity. These values for σ_c and σ_0 are presumably lower limits.

The writers are indebted to Professor C. C. Lauritsen for helpful suggestions and the use of an electroscope, and to Mr. J. E. Walstrom for assistance in the preparation of the neutron sources.

[7] It has been emphasized by A. C. G. Mitchell that, in most cases, absorption and scattering cross sections have not been separated. Mitchell, Murphy and Langer, Phys. Rev. **49**, 400 (1936).

[8] Dunning, Pegram, Fink and D. P. Mitchell, Phys. Rev. **48**, 265 (1935). Dana P. Mitchell, Phys. Rev. **49**, 453 (1936). A. C. G. Mitchell, Murphy and Langer, Phys. Rev. **49**, 400 (1936).

[9] Fleischmann, Zeits. f. Physik **97**, 242 (1935).

[10] Kikuchi, Husimi and Aoki, Proc. Phys. Math. Soc. Japan **18**, 35 (1936); Nature **137**, 30 (1936).