Comparison of Statistical Estimation Techniques for Mars Entry, Descent, and Landing Reconstruction

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Flight data from an entry, descent, and landing sequence can be used to reconstruct the vehicle’s trajectory, aerodynamic coefficients, and the atmospheric profile experienced by the vehicle. Past Mars missions have not contained instrumentation that would allow for the separation of uncertainties in the atmosphere and the aerodynamic database. The 2012 Mars Science Laboratory took measurements of the pressure distribution on the aeroshell forebody during entry and allows freestream atmospheric conditions to be partially observable. Methods to estimate the flight performance statistically using onboard measurements are demonstrated here through the use of simulated Mars data. A range of statistical estimators, specifically the extended Kalman filter and unscented Kalman filter, are used to demonstrate which estimator best quantifies the states and the uncertainties in the flight parameters. The techniques demonstrated herein are planned for application to the Mars Science Laboratory flight dataset.

Nomenclature

\[ \begin{align*}
A, B &= \text{process equation Jacobian matrices} \\
a &= \text{speed of sound, m/s} \\
\alpha_{x,y,b} &= \text{sensed acceleration in the body frame, m/s}^2 \\
\alpha_{x,y,b}^T, \alpha_{x,y,b} &= \text{axial and normal force coefficients} \\
F_N, F_F &= \text{aerodynamic forces in the normal and tangential directions, N} \\
g &= \text{local gravity, m/s}^2 \\
H &= \text{measurement equation Jacobian matrix} \\
h &= \text{nonlinear transformation} \\
l &= \text{identity matrix} \\
K &= \text{Kalman gain} \\
k &= \text{specific heat ratio} \\
M &= \text{Mach number} \\
m &= \text{mass, kg} \\
p &= \text{state covariance matrix} \\
p &= \text{pressure, Pa} \\
Q &= \text{process noise covariance matrix} \\
q &= \text{dynamic pressure, Pa} \\
q_i &= \text{quaternions (i = 0, 1, 2, 3)} \\
R &= \text{measurement covariance matrix} \\
R_{b, b} &= \text{rotation matrix from local horizontal to body frame} \\
r &= \text{planet-centric radius, m} \\
S &= \text{reference area of vehicle, m}^2 \\
t &= \text{time, s} \\
\mathbf{u}, v, w &= \text{velocity in body axis, m/s} \\
V &= \text{planet-relative velocity, m/s} \\
W &= \text{sigma points weighting factors} \\
\mathbf{x} &= \text{state vector} \\
\mathbf{y} &= \text{measurement} \\
\alpha &= \text{angle of attack, rad} \\
\alpha_{u,b}, \beta_{u,b} &= \text{unscented Kalman filter tuning parameters} \\
\beta &= \text{sideslip angle, rad} \\
\gamma &= \text{planet-relative flight-path angle, rad} \\
\theta &= \text{longitude, rad} \\
\nu &= \text{bank angle, rad} \\
\rho &= \text{density, kg/m}^3 \\
\sigma &= \text{standard deviation} \\
\phi &= \text{planet-centric latitude, rad} \\
\psi &= \text{planet-relative heading angle, rad} \\
\omega &= \text{planet rotation rate, rad/s} \\
\omega_{u,v}, \omega_{v,u}, \omega_{v,v} &= \text{angular rotation rates in body frame, rad/s} \\
\end{align*} \]

Subscripts

\[ \begin{align*}
b &= \text{backward} \\
f &= \text{forward} \\
k &= \text{time increment} \\
t &= \text{total value} \\
xy &= \text{cross covariance between the estimated state and measurement} \\
y &= \text{predicted measurement covariance} \\
\infty &= \text{freestream value} \\
0 &= \text{initial value} \\
\end{align*} \]

Superscripts

\[ \begin{align*}
a &= \text{posttransformation sigma point value} \\
\end{align*} \]
\( b \) = pretransformation sigma point value
\( i \) = state increment
\( T \) = transpose
\( \sim \) = estimated quantity
\( \hat{\cdot} \) = nominal estimate
\( + \) = best estimate
\( - \) = deviation value

I. Introduction

Since Viking 1 and 2 landed on Mars in 1976, the United States has successfully sent five other spacecraft to the Martian surface. However, failures have also accompanied the exploration of Mars, such as the 1999 Mars Polar Lander mission, which lost contact during entry, descent, and landing (EDL). Mitigating the large uncertainties that exist in engineering models used during Mars EDL design may reduce design conservatism and EDL system mass while avoiding some of the problems of these previous missions.

Trajectory, atmosphere, and aerodynamic coefficient reconstructions from flight data allow quantification of the uncertainties in the vehicle performance and the Martian environment. Most of the past reconstructions of Mars vehicle data have been conducted using a deterministic methodology [1–5], in which the flight data, such as on-board sensed accelerations and angular rates, were used to reconstruct the trajectory without explicitly considering uncertainties in the measurement values or in the flight dynamics. In such a situation, off-nominal data could lead to the divergence of the estimated trajectory. Deviation in the trajectory estimation can spread to the reconstruction of other performance parameters.

Atmosphere reconstruction efforts for the past missions have lacked measurement of the freestream atmosphere. Thus, the atmospheric reconstruction generally has been based on the onboard inertial measurement unit (IMU) data, deterministic reconstruction of velocity, and an assumption of perfect knowledge of the aerodynamic coefficients [6–8]. This assumption has led to confounding of atmospheric and aerodynamic coefficient uncertainties. However, the 2012 Mars Science Laboratory (MSL) mission includes instruments that provide a direct measurement of the surface pressure distribution over the entry body’s aeroshell during EDL. The pressure data can be combined with information from the IMU and radar altimeter to produce a simultaneous reconstruction of trajectory, atmosphere, and aerodynamic parameters.

Whereas past EDL reconstruction efforts have been largely deterministic and have rarely blended disparate EDL datasets together for parameter estimation, this study demonstrates a comprehensive statistical estimation methodology that incorporates all onboard EDL data types of the MSL mission in the parameter reconstruction and uncertainty quantification process. Datasets with random noise are created from simulated Mars EDL trajectories, and then the estimator is used to reconstruct the original EDL parameters.

II. Mars EDL Reconstruction Background

A major objective for Mars EDL reconstruction is to verify the performance of the vehicle and to quantify any off-nominal behavior. It is also useful if the reconstruction methods can quantify uncertainty in the estimated parameters that can be used in the design process. Stiepe et al. [9], who performed a conceptual design analysis of the MSL mission, show that among a long list of design parameters two major sources of uncertainties lie in the knowledge of the aerodynamic coefficients of the vehicle (summarized in Table 1) and the atmospheric profile it encountered. Some of the previous EDL reconstructions have attempted to quantify these uncertainties with limited success.

A. Past Trajectory Reconstructions

1. Deterministic Trajectory Reconstructions

The reconstruction techniques have been mostly limited to deterministic estimation methods. These estimation techniques are similar to strap-down reconstruction methods, in which the inertial measurements are integrated using the nonlinear equations of motion without considering the measurement uncertainty in the estimation process [10]. Table 2 summarizes the various measurements taken during the EDL phase by past and upcoming U.S. missions [1–5,7,11–14]. Results from the deterministic trajectory reconstructions can be found in the literature for Viking 1 and 2 [1], Mars Pathfinder [2], Mars Exploration Rovers (MERs) [3,7], and Phoenix [4,5]. These reconstructions do not consider measurement and process uncertainties, and they are unable to quantify the uncertainty of the estimated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>3σ (99.7%) uncertainty/range</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial-force coefficient (Knudsen ( \geq 0.1 ))</td>
<td>5%</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Axial-force coefficient (Mach &gt; 10)</td>
<td>3%</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Axial-force coefficient (Mach &lt; 5)</td>
<td>10%</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Normal-force coefficient (Knudsen ( \geq 0.1 ))</td>
<td>10%</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Normal-force coefficient (Mach &gt; 10)</td>
<td>5%</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Normal-force coefficient (Mach &lt; 5)</td>
<td>8%</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Supersonic parachute drag coefficient</td>
<td>±10%</td>
<td>Uniform</td>
</tr>
<tr>
<td>Subsonic parachute drag coefficient</td>
<td>±5%</td>
<td>Uniform</td>
</tr>
<tr>
<td>Axial-force coefficient (Mach &lt; 0.8)</td>
<td>±20%</td>
<td>Uniform</td>
</tr>
<tr>
<td>Normal-force coefficient (Mach &lt; 0.8)</td>
<td>±0.03</td>
<td>Uniform</td>
</tr>
<tr>
<td>Pitching-moment coefficient (Mach &lt; 0.8)</td>
<td>±0.03</td>
<td>Uniform</td>
</tr>
</tbody>
</table>

*Note: 3σ uncertainties are for Gaussian distributions, and ranges are for Uniform distributions.*

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometer</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<tr>
<td>Three-axis gyroscope</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Radar altimeter</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Thermal protection system recession</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressure (during EDL)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>In-depth temperature (during EDL)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

*Pathfinder only took pressure measurements during subsonic parachute descent. X is to signify that a mission did have a specific instrument, while blank space means that the vehicle did not have that instrument.*
B. Past Atmospheric Reconstructions

Some past EDL trajectory reconstruction efforts have used statistical estimation techniques. Kalman filtering was first used for Mars EDL data to estimate the trajectories for Viking 1 and 2 by Euler et al. [11], who integrated the equations of motion using the inertial data and then used radar altimeter and terminal landing Doppler data to correct the estimate of the trajectory. A Kalman filter was also used for the Mars Pathfinder reconstruction by Spencer et al. [2], who used radar altimeter data to correct a nominal trajectory based on the integration of IMU data. Spencer et al. also used a smoothing algorithm to combine trajectory reconstruction from forward and backward runs of the data, but used only translational equations of motion. Hence, angular parameter calculations assumed a priori knowledge of the vehicle’s aerodynamics.

C. Mars Science Laboratory

The 2012 MSL mission carried instruments that took in situ measurements of the pressure and temperature distribution on the aeroshell. The instrumentation, known as MEDLI, consisted of the Mars entry atmospheric data system (MEADS) to take pressure measurements and MEDLI integrated sensor plug (MISP) to take aerothermodynamic data [14]. MEADS provides a dataset that allows the estimation of atmospheric properties without confounding the uncertainties in the knowledge of the aerodynamic coefficients [14] by collecting pressure data from seven pressure transducers located around the forebody of the aeroshell (see Fig. 1).

The original science objective of MEADS was to reconstruct trajectory and atmospheric parameters within certain bounds as shown in Table 3. The accuracy requirement for Mach number has since been dropped, as the calculation of Mach number to such strict bounds requires an accurate characterization of the speed of sound, which is not observable to such accuracy using the MEADS dataset alone. The issue regarding Mach number reconstruction is discussed in more detail in the results section of this paper. A methodology to use MEDLI data for reconstruction is described here, whereas Edquist et al. [19] and Mahzari et al. [20] describe how MISP data could be used for aerothermodynamic reconstruction.

III. MEDLI Simulation

MEDLI-like datasets are simulated in this study to demonstrate the effectiveness of a statistical reconstruction methodology that incorporates disparate data types and estimates trajectory, atmospheric parameters, and aerodynamic coefficients. The program to optimize simulated trajectories 2 (POST2) [21] is used to generate two Mars EDL trajectories, which are shown in Fig. 2. There is a nominal EDL trajectory and a dispersed case of the nominal trajectory with perturbations in the vehicle’s aerodynamic database, planetary atmosphere, and winds. Both of these trajectories represent the truth data for each of the test cases.

The POST2 outputs are used to generate IMU, radar altimeter (when the altitude is less than 10 km), and pressure transducer data (when dynamic pressure $q_d$ is greater than 850 Pa). Random Gaussian noise is applied to the simulated data to model measurement noise and create a dataset for analysis (see Fig. 3). The uncertainty of the noise is based on MEDLI or past Mars EDL instrumentation specifications as shown in Table 4.

The data sample rate used for reconstruction was chosen after a sensitivity study of the rms of the error in the estimate of the MEDLI parameters of interest, which is shown in Fig. 4. Although both the IMU and MEDLI data are available at higher sample rates, it was found that the error does not decrease significantly if the sample rate is increased from 25 Hz for the IMU data and 4 Hz for the MEDLI data.
IV. EDL Reconstruction Methodology

The methodology used for reconstructing Mars EDL vehicle flight parameters (as seen in Fig. 5) involves taking EDL sensor measurements and using an estimation method to reconstruct the vehicle trajectory and atmospheric profile. Using the outputs of the estimation method, one can also calculate the aerodynamic coefficients of the vehicle and its uncertainties. Certain formulations of the estimation methods can also allow reconstruction of the sensor calibration data. However, sensor calibration reconstruction is not described in this study.

The estimation method in Fig. 5 will consist of two types of statistical estimators: 1) an extended Kalman filter (EKF) or an 2) unscented Kalman filter (UKF). The reconstruction starts from an initial condition that is propagated to the time the next measurement is available at which the states are updated (Fig. 6). Christian et al. [22], Dutta et al. [23,24], Karlgaard et al. [25], and Wells et al. [26,27]
have conducted EDL trajectory and atmosphere reconstruction using EKF. However, these studies did not test the methodology on a MEDLI-like dataset, and most of these studies did not use the UKF, which has been shown to improve the parameter and uncertainty estimates compared with linearized estimators, such as the EKF [28].

A. Process Equations

The estimators need dynamic equations of motion, as seen in Eqs. (1–9), to propagate the estimate of the states in time. The states consist of the vehicle’s position, velocity, attitude, freestream pressure $p_\infty$, and density $\rho_\infty$. The position is in terms of planet-centric radius $r$, latitude $\phi$, and longitude $\theta$, whereas velocity $V$, flight-path angle $\gamma$, and heading angle $\psi$ are defined relative to the planet surface and are based on the vehicle-carried local horizontal frame [29]. The heading angle is defined in the horizontal plane, where due east is 0 deg and due north is 90 deg. The attitude states are given in terms of the quaternion $(q_0, q_1, q_2, q_3)$ that defines the orientation between the vehicle-carried local horizontal frame and the body frame [30,31]. The planetary rotation rate is $\omega$:

$$\dot{r} = V \sin \gamma$$  \hspace{1cm} (1)

Table 4 Measurement noise uncertainties for the simulated dataset

<table>
<thead>
<tr>
<th>Measurement</th>
<th>$\sigma$ uncertainty (normal)</th>
<th>Sample rate used, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-axis sensed acceleration [22]</td>
<td>$100 \mu g$ rms</td>
<td>25</td>
</tr>
<tr>
<td>Three-axis angular rate [5]</td>
<td>$0.03$ deg /h rms</td>
<td>—</td>
</tr>
<tr>
<td>Radar altimeter altitude [22]</td>
<td>$0.3$ m</td>
<td>1</td>
</tr>
<tr>
<td>Pressure transducers [14]</td>
<td>$1%$ reading/transducer</td>
<td>4</td>
</tr>
</tbody>
</table>
\( \dot{\gamma} = 1 \left( \frac{F_N \cos \nu}{m} \right) - g \cos \gamma + \frac{V^2}{r} \cos \gamma + 2aV \cos \phi \cos \psi + \frac{a^2r \cos \phi \cos \gamma \sin \phi \sin \psi}{r} \)  

\( \dot{\psi} = \frac{1}{V} \left( \frac{F_N \sin \nu}{m \cos \gamma} - \frac{V^2}{r} \cos \gamma \cos \psi \tan \phi \right) \)  

\( \dot{\phi} = \frac{V \cos \gamma \sin \psi}{r} \)  

\( \dot{\theta} = \frac{V \cos \gamma \cos \psi}{r \cos \phi} \)

The matrix \( R_{ib} \), which is solely a function of the quaternion, defines the rotation from the local horizontal frame to the body frame and is defined in the literature [31]. The angular rates in the body frame (\( \omega_x \), \( \omega_y \), and \( \omega_z \)) come from the onboard gyroscopes, whereas \( g \) is the altitude-dependent local gravitational acceleration based on a spherical mass distribution. \( F_N \) and \( F_T \) represent the normal (lift) and tangential (drag) aerodynamic forces in the body axis, and lift modulation is modeled in the equations using a bank angle (\( \phi \)). The dynamical equations for the freestream pressure and density are derived from the hydrostatic equation and the perfect gas law, and the derivation is described in [23] and [25]. Equations (8) and (9) use an isothermal assumption that is valid over small changes in the altitude. Because the freestream pressure and density rate equations are used as process equations and are propagated over small time steps, this assumption is reasonable. Note that the process noise chosen for the reconstruction process is tuned to compensate for potential issues with these equations.

The process equations used here are not the same equations used internally in POST2. Thus, there is a process uncertainty between how the simulated data is generated and how the estimator predicts the values of the states. In a simulation, one can modify the estimator’s process equations to match POST2’s dynamics, but in the

Fig. 4 Effect of sample rate of IMU and MEDLI data on the estimated parameter residual from the truth.

Fig. 5 Flow diagram of the overall reconstruction methodology for Mars EDL flight parameters.
case of real data the simulation models are never perfect. Thus, the difference in the dynamics between POST2 and this estimation methodology provides a test of the unmodeled uncertainties expected in the actual data.

B. Measurement Equations

Measurement equations are used by the statistical estimator to predict the measurement value based on the current estimate of the state. The EKF and UKF assume that the measurement has a Gaussian error distribution, and the expectation of the error is zero. The EKF approximates the measurement equation by linearizing about a point (the nominal estimate of the state). The measurement sensitivity (Jacobian) matrix is involved in the linearization, and evaluation of this matrix is computationally intensive. This Jacobian matrix is not necessary for the UKF.

When using the EKF, accurate computations of the measurement Jacobian equations generally require custom derivations for every measurement type included in the estimators. Christian et al. [22], Karlgaard et al. [25], and Jazwinski [32] provide detailed expressions for the measurement sensitivity equations pertaining to accelerometer and radar altimeter measurements. For MEDLI data, a measurement equation has to give a predicted pressure coefficient for the measurement sensitivity equations pertaining to accelerometers and radar altimeters. For MEDLI data, a measurement equation has to give a predicted pressure coefficient for the measurement sensitivity equations pertaining to accelerometers and radar altimeters.

C. Statistical Estimators

1. Extended Kalman Filter

An EKF is a modification of the linear Kalman filter. The algorithm for this well-known filter is summarized next [33,34]:

1) Initialize the state vector and the state covariance matrix at time $t_{k-1} = t_0$ and let $k = 1$, where $k$ is an index of the epoch when a measurement is first available.

2) Read in the measurement at time $t_k$.

3) Calculate nominal state $\hat{x}_k^+$ at time $t_k$ by integrating the nonlinear equations of motions [Eqs. (1–9)] with $\hat{x}_{k-1}$ as the initial condition. Although the nominal state at time $t_k$ is a discrete state, it is equivalent to the value of integrating the continuous-time, nonlinear, ordinary differential equations at time $t_k$.

4) Calculate nominal state covariance matrix $\hat{P}_k^+$ by integrating the Riccati equations [Eq. (14)]. Similar to step 3, $\hat{P}_k^+$, the covariance matrix for a discrete state, is equivalent to the final value of integrating the continuous-time, nonlinear, Riccati matrix ordinary differential equations.

5) Calculate measurement residual vector $y_k$, measurement sensitivity matrix $H_k$ and Kalman gain $K_k$ using the nominal state and state covariance [Eq. (15)].

6) Calculate the best estimate of state $\hat{x}_k$ and state covariance $\hat{P}_k$ using Eqs. (16) and (17).

7) Increment counter $k$ and go back to step 2 until measurements at all times have been processed:

$$\dot{\hat{P}} = AP + PA^T + BQB^T$$

(14)

$$K_k = \hat{P}_k^+ H_k^T (H_k \hat{P}_k^+ H_k^T + R_k)^{-1}$$

(15)

$$\hat{x}_k^+ = \hat{x}_k^- + K_k (y_k - h(\hat{x}_k^-))$$

(16)

$$\hat{P}_k^+ = (I - K_k H_k) \hat{P}_k^+ (I - K_k H_k)^T + K_k R_k K_k^T$$

(17)

Measurement covariance matrix $R$ is defined at time $k$, and information from Table 4 is used for this matrix. The covariance of state noise vector $Q$ consists of noise variables in the process equations, such as the sensor uncertainty of the angular rate gyroscopes or tuning parameters for the velocity vector equations. The state noise vector values are summarized in a later section in Table 5. $A$ is the Jacobian of the process equations with respect to the state vector, whereas $B$ is the Jacobian of the process equations with respect to the state noise vector. The state noise vector for EDL reconstruction comes from uncertainties in the process equations.
such as aerodynamic and atmospheric uncertainties. Matrix $I$ in the covariance update equation is the identity matrix.

2. Unscented Kalman Filter

Instead of using a linearized approximation to update the state and covariance matrix, the UKF is based on the idea that a transformation of a probability distribution can be approximated with multiple direct evaluations of an arbitrary nonlinear function [28]. Just like the EKF, the UKF assumes that the state variables are Gaussian distributions in which the state estimates are the means and the state uncertainties are the standard deviations of the distributions. The UKF propagates a set of specially chosen state vectors called sigma points to characterize the standard deviations of the distribution. The forward transformation of the state probability distribution.

\[ \sum_{i=0}^{2n} W^{(i)}(\tilde{x}^{(i)}_k - \hat{x}_k)(\tilde{y}^{(i)}_k - \hat{y}_k)^T + R_k \] (28)

\[ K_k = P_{xy} \gamma^{-1} \] (29)

\[ \hat{x}^+_k = \hat{x}_k + K_k(y_k - \hat{y}_k) \] (30)

\[ \hat{P}^+_k = \hat{P}_k - K_k P_k K_k^T \] (31)

Unlike the EKF, UKF does not require the calculation of Jacobians and other derivative terms that are often computationally difficult and are sources of numerical ill conditioning. Additionally, it should be noted that other derivative-free filters, such as the divided-difference filters, are essentially variants of the UKF with minor differences in the tuning parameters for selecting the sigma points [37].

3. Statistical Smoothing

The reconstruction can start from the atmospheric entry (forward pass) or a projected landing location (backward pass). The forward pass starts its estimate from an initial state and covariance that is found independent of the trajectory reconstruction process, and the reconstruction is conducted in a chronological manner. The backward pass has the advantage of starting at a smaller uncertainty value, as it begins from the end of the forward estimate. The forward and backward pass estimates (denoted by subscripts $f$ and $b$, respectively) can be combined using the Fraser–Potter smoothing solution [38], which is shown in Eqs. (32) and (33). It is advantageous to combine both the forward and backward estimates in finding an optimal estimate of the trajectory [22, 25]. The forward pass estimate at time $k$ uses the measurement data from entry to $k$, whereas the backward pass uses the measurement data from landing time to $k$. The combined smoothed estimate can use measurement data at all times to create the estimate at $k$ and is similar to a batch least-squares solution [39]:

\[ \hat{P}_k = [\hat{P}^T_{f,k} + \hat{P}^T_{b,k}]^{-1} \] (32)

### Table 5 Process noise uncertainties used for the reconstruction process

<table>
<thead>
<tr>
<th>State</th>
<th>3σ uncertainty (normal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (planet-centric)</td>
<td>—</td>
</tr>
<tr>
<td>Latitude (planet-centric)</td>
<td>—</td>
</tr>
<tr>
<td>Longitude</td>
<td>—</td>
</tr>
<tr>
<td>Velocity (relative)</td>
<td>0.003 m/s</td>
</tr>
<tr>
<td>Flight-path angle (relative)</td>
<td>0.0825 deg</td>
</tr>
<tr>
<td>Heading angle (relative)</td>
<td>0.0825 deg</td>
</tr>
<tr>
<td>Quaternation</td>
<td>Based on angular rate measurement noise</td>
</tr>
<tr>
<td>Freestream pressure</td>
<td>0.3 $p_\infty$</td>
</tr>
<tr>
<td>Freestream density</td>
<td>0.3 $p_{\infty}$</td>
</tr>
</tbody>
</table>

### Table 6 Initial state uncertainties used for the reconstruction process

<table>
<thead>
<tr>
<th>State</th>
<th>3σ uncertainty (normal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (planet-centric)</td>
<td>5100 m</td>
</tr>
<tr>
<td>Latitude (planet-centric)</td>
<td>0.12 deg</td>
</tr>
<tr>
<td>Longitude</td>
<td>0.03 deg</td>
</tr>
<tr>
<td>Velocity (relative)</td>
<td>2.9 m/s</td>
</tr>
<tr>
<td>Flight-path angle (relative)</td>
<td>0.06 deg</td>
</tr>
<tr>
<td>Heading angle (relative)</td>
<td>0.06 deg</td>
</tr>
<tr>
<td>Euler angles (related to the quaternion)</td>
<td>0.03 deg /angle</td>
</tr>
<tr>
<td>Freestream pressure</td>
<td>10$P_{\infty}$</td>
</tr>
<tr>
<td>Freestream density</td>
<td>10$P_{\infty}$</td>
</tr>
</tbody>
</table>
\[ \dot{x}_k = \hat{P}_k^{-1} \left[ f_{x,k} + \hat{P}_k^{-1} A \right] \] (33)

D. Aerodynamic Calculation

The aerodynamic coefficients of the vehicle can be constructed after trajectory and atmospheric states have been estimated. As seen in Eq. (34), aerodynamic force coefficients (in this case, axial force coefficient \(C_A\)) can be reconstructed from axial acceleration measurements \(a_{x,b}\) and the estimated freestream density and velocity values. With the estimation process complete, uncertainty values are known for the accelerations and the freestream properties. In this manner, the aerodynamic and atmospheric uncertainties can be separated. The uncertainty in the estimate of aerodynamic force coefficients \(\sigma_{C_A}\) can be calculated by applying the chain rule to Eq. (34) and using the already calculated uncertainties of the estimator’s state vector as seen in Eq. (35):

\[ C_A = \frac{ma_{x,b}}{0.5 \rho \omega V^2 \omega} \] (34)

\[ \sigma_{C_A} = \frac{ma_{x,b}}{0.5 \rho \omega V^2 \omega} \left( \frac{\sigma_{a_{x,b}}}{a_{x,b}} - \frac{\sigma_{\rho \omega}}{\rho \omega} - \frac{\sigma_{V \omega}}{V \omega} \right) \] (35)

Fig. 7  Reconstructed MEDLI-related parameters’ deviation from the truth using the nominal dataset.
V. Nominal Dataset Reconstruction

The reconstruction process for both datasets (nominal and dispersed) begins with the same initial conditions and initial covariance values. The initial uncertainties in the state variables are listed in Table 6 and are based on the initial conditions at entry interface from recent Mars missions. The initial state covariance matrix is calculated from these values. The process noise covariance is calculated using the uncertainty information given in Table 5. Process noise improves the estimator’s ability to reconstruct parameters from noisy data and to model uncertainties in the process equations [32–35]. The high process noises for freestream pressure and density demonstrate the relatively high uncertainty in the process equations, so that the estimator is biased toward the more certain measurements from the accelerometer and MEDLI-like pressure transducers.

The process noise uncertainties are tuning parameters for a filter, and in this case the values were chosen to tune the EKF reconstruction using the nominal dataset. The process noise could have been varied for the UKF reconstructions and when the dispersed dataset was used; however, the noise was kept constant for the other reconstructions to demonstrate the robustness of the methodology. Fine tuning of filter parameters might not be feasible when analyzing real-life data that display off-nominal behavior.

The reconstruction results of the MEDLI-related parameters using the nominal dataset are shown in Fig. 7. The deviations of the reconstructed parameters are from the true values shown in Fig. 2. For the most part, it appears that the trajectory and atmospheric parameters are reconstructed close to the MEDLI objectives, especially in the region in which MEDLI data are available. The Mach number estimate does stray from the original MEDLI objective bounds for both the EKF and UKF, but the UKF estimate’s residual is lower than EKF estimate’s residual during the time MEDLI data are available. The original MEDLI science goals were to reconstruct $M_{\infty}$ within ±0.1 of the truth, whereas the science goals for the other parameters are less stringent (e.g., the $\alpha$ reconstruction goal is to estimate within ±0.5 deg). The Mach number value is dependent on the calculated speed of sound, which in turn relies on the estimated freestream pressure and density. These parameters are estimated using the MEDLI data that peak around the time period (Fig. 3e) when the Mach number is outside the MEDLI objective bounds.

The MEDLI data have simulated noises that are a percentage of the nominal measurement, and the data are noisiest in this region. This nonlinearity manifests itself in the reconstructed freestream atmospheric parameters, speed of sound, and Mach number.

The estimated uncertainties presented are the 99.7% ($3\sigma$) confidence interval, because the states are assumed to be Gaussian distributions and the residual between the estimated states and the truth fall within these confidence bands. In practice, Monte Carlo analyses are performed for a given trajectory, and the possible deviations in trajectories will be compared with the estimated uncertainties. However, for this study there are no Monte Carlo data available to compare with the estimated uncertainties. Such analyses are planned for future work.

Because of a lack of the true uncertainty values, it is hard to determine if the UKF provides a better estimate of uncertainties than the EKF, as predicted in the literature. Quantifying uncertainties of the POST2 simulated dataset using techniques such as Monte Carlo analysis and linear covariance analysis could give an estimate of the true uncertainty values, which could then be compared with the EKF and UKF estimated uncertainties. Such analysis is planned for future work. However, even without comparison with the true uncertainty values, it can be seen that the estimate’s residual magnitude for the UKF is smaller for some variables (such as Mach number) relative to the EKF’s estimate residual magnitude. Thus, the UKF demonstrates a better ability in state estimation than the EKF as the literature predicts.

Trajectory parameters such as planet-centric radius and planet-relative velocity are also reconstructed in the estimation process. The percent deviation from the truth for the trajectory parameters can be seen in Fig. 8. Both the EKF and the UKF do a good job of estimating the radius (within 0.2%) and the velocity, although the UKF estimate’s residual magnitude is lower than the EKF estimate’s residual. Note that the radius and its uncertainty estimation improve significantly with the introduction of radar altimeter measurement around 220 s.

Figure 9 shows the reconstructed aerodynamic force coefficients for the time span that MEDLI data were simulated, because the freestream pressure and density are directly observable for only this time period. The reconstructed axial force coefficient appears to be very close to the truth, as the coefficient’s deviations for both the EKF...
and UKF estimates mostly lie within ±0.02 during the time period of interest. However, true normal force coefficient \( C_N \) has a very small value (Fig. 2g), which raises a numerical issue, as both estimators do not estimate \( C_N \) to the same percentage accuracy as they estimate \( C_A \). Although the residuals for \( C_A \) and \( C_N \) are of the same order of magnitude, the residual for \( C_N \) is only one order of magnitude lower than its nominal value. The estimates for \( C_A \) and \( C_N \) are both accurate (demonstrated by their low residuals), but the \( C_N \) estimate is better when considering the relative percentage accuracy.

The 3\( \sigma \) bounds show that the \( C_A \) and \( C_N \) uncertainty estimates are approximately 1 and \( 1 \times 10^{-2} \). For nominal \( C_A \) values of about one, an error band of one may be adequate, but for nominal \( C_N \) values, which are of the order \( 1 \times 10^{-3} \), an error band of \( 1 \times 10^{-2} \) is large. Such incongruity reinforces the need for further research with tuning parameters to improve upon the uncertainty estimation, especially for parameters with nominal values close to zero, such as \( C_N \). As mentioned earlier, future work is planned to understand and calibrate such uncertainty estimates by comparing EKF and UKF reconstructions with Monte Carlo analysis-generated uncertainties, where known truth trajectories are perturbed according to known process and measurement noises and the same process and measurement equations used in the reconstruction process.

Table 7 summarizes the rms deviation of the MEDLI-related parameters from the truth for the time period MEDLI data are available. The reconstruction values for the dispersed case are also shown.

### VI. Dispersed Dataset Reconstruction

The estimates of the MEDLI-related parameters using the dispersed simulated dataset are shown in Fig. 10. The performances of both EKF and UKF estimations are shown for comparison.

Compared with the reconstructed parameters from the nominal dataset, the deviations of the estimates from the truth appear to be larger and noisier, which could be reflecting the perturbations in aerodynamics and atmosphere modeled in this trajectory or the fact that the process noise is not tuned for this dataset. Once again, the estimated values of attack angle, sideslip angle, and dynamic pressure meet the MEDLI science objectives for the most part, whereas the estimated Mach number (Figs. 10e and 10f) strays from the MEDLI objective. The rationale for this deviation is similar to what was previously stated. Note that in this case even the true Mach number profile shows significant variation in this region (Fig. 2b) for the dispersed profile with less of the perturbations in aerodynamics and atmosphere modeled in this trajectory.

The percent deviation from the truth for the trajectory parameters can be seen in Fig. 11. Once again, both the EKF and the UKF estimate the radius and the velocity accurately, although the UKF estimate’s residual magnitude is lower than the EKF estimate’s residual for radius. The reconstructed aerodynamic force coefficients for the dispersed dataset are shown in Fig. 12. The small values for the normal coefficient again raise the same numerical issues previously discussed.

<table>
<thead>
<tr>
<th>State</th>
<th>EKF nominal</th>
<th>UKF nominal</th>
<th>EKF dispersed</th>
<th>UKF dispersed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of attack (deg)</td>
<td>0.192</td>
<td>0.145</td>
<td>0.570</td>
<td>0.334</td>
</tr>
<tr>
<td>Sideslip angle (deg)</td>
<td>0.211</td>
<td>0.140</td>
<td>0.506</td>
<td>0.242</td>
</tr>
<tr>
<td>Mach number</td>
<td>0.190</td>
<td>0.073</td>
<td>1.379</td>
<td>0.393</td>
</tr>
<tr>
<td>Dynamic pressure (Pa)</td>
<td>38.51</td>
<td>16.27</td>
<td>43.56</td>
<td>25.73</td>
</tr>
<tr>
<td>Axial force coefficient</td>
<td>0.017</td>
<td>0.006</td>
<td>0.022</td>
<td>0.018</td>
</tr>
<tr>
<td>Normal force coefficient</td>
<td>0.0015</td>
<td>0.0014</td>
<td>0.003</td>
<td>0.001</td>
</tr>
</tbody>
</table>
VII. Comparison Between the Estimators

It appears that the EKF and UKF have generally similar performances. However, there are some situations where the UKF outperforms the EKF. Looking at the reconstruction of the angle of attack and sideslip angle (Figs. 7a–7d and 10a–10d), one can see that the UKF reconstruction largely stays within the MEDLI science objective goals, whereas the EKF has a large deviation near the end of the time period that the MEDLI dataset is available. This type of behavior is expected from the EKF, as the errors due to linearization lead to divergence from the truth when the errors propagate over a long time period.

Looking at the overall rms deviation of the MEDLI-related parameters (as was shown earlier in Table 7), it is clear that the UKF has a smaller residual for all of the MEDLI-related parameters and would be the preferred type of statistical estimator based solely on this metric. However, the EKF residual is really not that far off from the UKF residuals (except for a few cases, such as Mach number deviation for the dispersed dataset). Of course, if the true uncertainties were known, then one could perform a similar comparison between the two estimators. However, in lieu of such information, it is hard to differentiate between the EKF and UKF.

Fig. 10 Reconstructed MEDLI-related parameters’ deviation from the truth using the dispersed dataset.
If the truth information for the states were not known, it seems that either method would be acceptable for reconstruction. EKF will be expected to have divergence issues if the reconstruction is conducted over a long time period. UKF is computationally more time consuming, because $2n + 1$ sigma points have to be propagated in time instead of just the propagation of the state vector in the EKF. However, if the truth is unknown, perhaps the best approach would be to use both estimators and then compare results. Both estimators had good performance in estimating trajectory, atmosphere, and aerodynamics from the simulated MEDLI dataset, and the same type of performance would be expected from the estimators for an unknown dataset.

VIII. Conclusions

Measurements from the Mars Science Laboratory (MSL) entry, descent, and landing (EDL) instrumentation (MEDLI) suite provide valuable data that can be used in EDL parameter reconstruction. With
the Mars entry atmospheric data system data and a statistically based reconstruction methodology shown here, one can simultaneously reconstruct a vehicle’s trajectory, atmospheric parameters, and aerodynamic coefficients. Because of the statistical nature of the estimator, this process also produces information about the uncertainty in the estimated parameters (providing, for example, valuable science regarding the Martian atmosphere). Sample reconstruction results from two statistical estimators, extended Kalman filter (EKF) and unscented Kalman filter (UKF), were shown for two simulated Mars entry datasets. The UKF residuals were smaller than those associated with EKF reconstruction, but not significantly. Generally, the two estimators reconstructed the data within the goals of the MEDLI science objectives. The largest errors were observed for estimation of the Mach number and normal force coefficient. The methodology demonstrated here can be applied to the MSL dataset.

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References


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