Geophysical Research Letter

Supporting Information for

Two volcanic tsunami events caused by trapdoor faulting at a submerged caldera
near Curtis and Cheeseman Islands in the Kermadec Arc

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Contents of this file

Figures S1 to S18
Tables S1 to S3
Texts S1 to S7

Additional Supporting Information (Files uploaded separately)

Caption for Data Set S1

Introduction

This Supporting Information contains supplementary texts, figures, and tables. We
describe methodologies for estimation of the vertical sea-surface displacement (Text S1)
and for earthquake source modeling (Text S2), moment tensor representation of our
source model (Text S3), moment tensor analysis (Text S4), examinations of the model
uniqueness (Text S5), modeling results with either only a ring fault or only a horizontal
crack (Text S6), and an earthquake scaling relationship compared with geodetic moment
magnitudes (Text S7). Figures S1–S6 and Tables S1–S3 are mentioned in Main Text, while
Figures S7–S18 are mentioned only in this Supporting Information. Data Set S1 contains
the data of the best-fit source model of the 2017 earthquake.
Supplementary figures (mentioned in Main Text)

**Figure S1.** An assumed fault-crack composite source system, composed of a partial ring fault and a horizontal crack, viewed from the east (left) and above (right). This structure is discretized into triangular meshes.
Figure S2. Source models with different ring-fault dip angles: (a) 70°, (b) 78°, and (c) 85°. All the models have a horizontal crack at a depth of 3 km in the crust. (Left) Dislocations of the fault-crack source system determined by the tsunami waveform inversion. See the caption of Figure 3b in Main Text. (Right) Synthetic tsunami waveforms from this model (red), compared with observed waveforms (black). See the caption of Figure 4b in Main Text. Note that variations of the ring-fault dip angle do not change the tsunami waveforms.
Figure S3. Moment tensors and seismic waveforms from source models with a dip angle of (a) 70°, (b) 78°, and (c) 85°; all these models have a horizontal crack at a depth of 3 km in the crust, which are shown in Figure S2. Partial moment tensors of the horizontal crack and the ring fault are shown with the moment tensor of the model. Red and black lines represent synthetic and observed waveforms, respectively. Note that variations of the ring-fault dip angle largely change the seismic wave amplitudes.
Figure S4. Contributions to tsunami waves by the horizontal crack and the ring fault of the best-fit source model (Figure 3b in Main Text). (a–b) Vertical sea-surface displacements caused by (a) the horizontal crack and (b) the ring fault. Red and blue colors represent uplift and subsidence, respectively, with white contour lines plotted every 0.5 m. (c) Comparison of the synthetic tsunami waveforms from the horizontal crack (blue) and the ring fault (red), compared with the observed (black) waveforms. The gray line represents the time interval used for the inversion.
Figure S5. Contributions to long-period seismic waves by the best-fit source model (Figure 3b in Main Text; see Text S3 for detailed descriptions). Red lines in the right panels represent synthetic waveforms from (a) $M = M_{HC} + M_{RF}$, the partial moment tensors of (b) the horizontal crack $M_{HC}$ and (c) the ring fault $M_{RF}$, and (d) the moment tensor of the ring fault, excluding $M_{r\theta}$ and $M_{r\phi}$. Note that the smaller-amplitude waveforms from $M_{HC}$ have the reversed polarities relative to those from $M_{RF}$, reducing the seismic amplitudes of $M$, and that the main contributor to the long-period seismic waves is the limited moment-tensor components shown in d.
Figure S6. Resolvable moment tensors $M_{\text{res}}$ of the (a) 2017 and (b) 2009 Curtis earthquakes determined by the moment tensor analysis (see Text S4 for detailed descriptions). The orientation of the best double-couple solution is shown by thin curves, whose Null-axis direction coincides with that of $M_{\text{res}}$. The focal mechanisms are shown by projection of the lower focal hemisphere.
Supplementary tables (mentioned in Main Text)

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Depth</th>
<th>$M_w$</th>
<th>$M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 Feb. 2009</td>
<td>03:30:58.6</td>
<td>178.54°W</td>
<td>30.56°S</td>
<td>12.1 km</td>
<td>5.8</td>
<td>6.0</td>
</tr>
<tr>
<td>8 Dec. 2017</td>
<td>02:10:03.0</td>
<td>178.56°W</td>
<td>30.49°S</td>
<td>13.4 km</td>
<td>5.8</td>
<td>6.2</td>
</tr>
</tbody>
</table>

**Table S1.** Earthquake information reported in the GCMT catalogue. Note that the depth may be determined at a greater depth than the accurate depth to maintain the stability of solutions (Ekström et al., 2012).

<table>
<thead>
<tr>
<th>Moment tensor</th>
<th>$M_w$</th>
<th>$M_\theta$</th>
<th>$M_\phi$</th>
<th>$M_r\theta$</th>
<th>$M_r\phi$</th>
<th>$M_\theta\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composite, $M$</td>
<td>6.24</td>
<td>2.87</td>
<td>3.59</td>
<td>0.95</td>
<td>0.85</td>
<td>-0.18</td>
</tr>
<tr>
<td>Horizontal crack, $M_{HC}$</td>
<td>6.19</td>
<td>2.44</td>
<td>3.02</td>
<td>1.18</td>
<td>1.18</td>
<td>0.00</td>
</tr>
<tr>
<td>Ring-fault, $M_{RF}$</td>
<td>5.96</td>
<td>1.11</td>
<td>0.57</td>
<td>-0.23</td>
<td>-0.34</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

**Table S2.** Moment tensors of our best-fit source model (Figure 3b in Main Text) of the 2017 earthquake: the moment tensor of the model $M (= M_{HC} + M_{RF})$, and the partial moment tensors of the horizontal crack $M_{HC}$ and the ring fault $M_{RF}$.

<table>
<thead>
<tr>
<th>Event</th>
<th>$M_w$</th>
<th>$M_\theta$</th>
<th>$M_\phi$</th>
<th>$M_r\theta$</th>
<th>$M_r\phi$</th>
<th>$M_\theta\phi$</th>
<th>$t_c = t_h$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>6.45</td>
<td>0.615</td>
<td>-0.276</td>
<td>-0.339</td>
<td>-2.201</td>
<td>-5.468</td>
<td>0.103</td>
</tr>
<tr>
<td>2017</td>
<td>6.45</td>
<td>0.505</td>
<td>-0.186</td>
<td>-0.320</td>
<td>-2.441</td>
<td>-5.437</td>
<td>0.134</td>
</tr>
</tbody>
</table>

**Table S3.** Deviatoric moment tensor inversion results for the 2009 and 2017 earthquakes using long-period seismic data. We assume that the centroid time shift $t_c$ and the half duration $t_h$ are the same. Note that two components representing the vertical dip-slip ($M_\theta$ and $M_{\theta\phi}$) are poorly determined because of their weak excitation of long-period seismic waves (Kanamori & Given, 1981; Sandanbata, Kanamori, et al., 2021).
Supplementary texts

Text S1. Methodology for vertical sea-surface displacement modeling

S1.1. Tsunami waveform inversion for vertical sea-surface displacement

We estimate the vertical sea-surface displacement of the 2017 Curtis earthquake using a tsunami inversion method. We set a tsunami source area of 25 km × 25 km square around Curtis and Cheeseman Islands and distribute 181 unit sources of vertical sea-surface displacement with 1.25 km intervals (Figure S7), each of which is formulated as:

\[ \eta^k(x, y) = 0.25 \times \left[ 1.0 + \cos \frac{\pi (x - x^k)}{L} \right] \times \left[ 1.0 + \cos \frac{\pi (y - y^k)}{L} \right], \]

\[ \left( |x - x^k|, |y - y^k| \leq L \right) \quad (S1) \]

where \( \eta^k \) is the vertical sea-surface displacement (in meter) of the \( k \)-th unit source (\( k = 1, ..., K \); here \( K = 181 \)) with the central location at \((x^k, y^k)\) (in km) with a source size of \( L \) (1.25 km, here).

We compute the Green’s function \( g = g_j^k \), relating the \( k \)-th unit source to tsunami waveform at the \( j \)-th station (\( j = 1, ..., J \); here \( J = 4 \)). We use the simulation code JAGURS (Baba et al., 2015) to solve the standard Boussinesq-type equations (Peregrine, 1972). The rise time for each unit source is assumed as 10 s, which is similar to a source duration of 6 s determined by the moment tensor analysis (see Text S4; Table S2). Bathymetry data in broad regions is modeled with GEBCO_2014 (Weatherall et al., 2015) with 30 arcsec grid spacing, and the New Zealand Regional Bathymetry with 250 m grid spacing downloaded from the National Institute of Water and Atmospheric Research (NIWA) in New Zealand.

Figure S7. Unit sources of vertical sea-surface displacement around Curtis & Cheeseman Islands. Black dots represent central locations of the unit sources. Each unit source has a cosine-tapered shape with a horizontal source size of 1.25 km x 1.25 km. Contour lines of the water depth are plotted every 50 m.
To include nearshore effects around tide gauges, we use finer bathymetry data (~28 m) obtained by combining digital topographic data on land and bathymetric data digitized and interpolated from analogue charts of the Land Information of New Zealand (LINZ) department. Because the tsunami speed is reduced by the elasticity of the Earth, the compressibility and the density stratification of seawater, and the gravitational potential change due to tsunami motions (Ho et al., 2017; Sandanbata, Watada, et al., 2021; Watada et al., 2014), we approximately incorporate the effects by delaying the synthetic waveforms by 25 s at LOTT and GBIT with epicentral distances of ~830 km. The delay time is based on the estimation by Sandanbata, Watada, et al. (2021), who calculated that short-period tsunamis with a period of 500 s in water of 1-km depth are delayed by about 3 s every 100-km distance.

We then solve the observation equation by the damped least-squares method (pp. 695–699 in Aki & Richards [1980]):

$$[d]_0 = [g_1]m,$$  
(S2)

where $d = [d_1(t) \ldots d_J(t)]^T$ is the column vector of the observed tsunami data at the $j$-th station, $g = \begin{bmatrix} g_1^1(t) & \cdots & g_J^1(t) \\ \vdots & \ddots & \vdots \\ g_1^K(t) & \cdots & g_J^K(t) \end{bmatrix}$ is the Green’s function, $m = [m_1^1 \ldots m_J^K]^T$ is an unknown column vector of the amplitude factor of the $k$-th unit source, $I$ is the identity matrix, and $\alpha$ is the damping parameter to obtain a smooth source model, which we assume as 0.02. We set the data length to include several wave crests and troughs of the tsunami signal. By the superposition of the unit sources $\eta^k$ weighted by $m_i^k$, we obtain the vertical sea-surface displacement model. Additionally, we obtain an uplift source model without subsidence, by solving Equation S2 with the non-negative condition (i.e., $m \geq 0$).

### S1.2. Resolution tests

Our modeling is mainly based on tsunami data from the tide gauge records with a low sampling rate (one sample per 60 s) and limited azimuthal coverage of stations. To investigate the resolution of our tsunami waveform inversion, we conduct two resolution tests, in which we prepare two target models: (1) a checker-board distribution (Figure S8a), and (2) an uplift distribution near Curtis caldera (Figure S9a). Synthetic tsunami waveforms from the two target models are computed by the tsunami simulation method explained in Text S1.1, and resampled with a time interval of 60 s; additionally, the waveforms of LOTT and GBIT are delayed by 25 s. We then apply the tsunami waveform inversion to these synthetic waveform data.

For the checker-board distribution (Figure S8a), the tsunami waveform inversion yields a solution (Figure S8b), which is somewhat different from the target, although the target waveforms are reproduced well (Figure S8c). This shows that complex distribution patterns near the caldera can be poorly constrained by our inversion based on the tide-gauge tsunami data. On the other hand, the inversion for the uplift distribution near the caldera (Figure S9a) yields uplift distribution over the caldera similar to the target model (Figure
S9b), reproducing the target waveforms (Figure S9c). We note that the peak location is well estimated, whereas the horizontal size and amplitude are estimated with slight difference.

These results suggest that, although our inversion cannot resolve complex pattern of sea-surface displacement, a simple-shaped displacement focused near the caldera can be resolved well with good resolutions on its location and overall shape. We emphasize that the observed tsunami waveforms have long-period characters (Figure 1b in Main Text) and are similar to the waveforms computed from the uplift distribution (Figure S9c) rather than those from the checker-board distribution (Figure S8c). This indicates that the actual sea-surface displacement caused by the earthquake and the uplift distribution were alike (Figure S9a); this was also proposed by Gusman et al. (2020). Therefore, it is plausible to consider that our vertical displacement models with a localized uplift in the western part of the caldera, which are estimated in Section 4.1 (Figures 2b and 2d) in Main Text, reasonably reflect the actual sea-surface displacement due to the earthquake.

**Figure S8.** Resolution test of the tsunami waveform inversion for vertical sea-surface displacement: a case of a checker-board distribution. (a) Target model, and (b) inverted model. (c) Synthetic tsunami waveforms from the target (red) and inverted models (black). The gray line represents the time interval used for the inversion.
Text S2. Methodology for dislocation modeling of the fault-crack composite source

S2.1. Source structures of the fault-crack composite source system

We assume a fault-crack composite source system, composed of an inward-dipping ring fault connected to a horizontal crack at a depth of 3 km in the crust, which is discretized with triangular meshes (Figure S1). Given the focused uplift estimated in Section 4.1 in Main Text, we assume a partial ring fault on the western side of the caldera with a central angle of 150° that extends from the seafloor to the crack edge. Although the detailed geometry is unknown, the ring fault is assumed to be along an elliptical line; this ellipse is with the center at (178.56°W, 30.542°S), the major axis oriented S60°E, and the horizontal size of 3.4 km × 2.8 km on the seafloor. For the ring fault, we assume a uniform inward dip angle, varied from 65° to 85°. Only inward dip angles are considered, because the vertical-T CLVD moment tensor can be generated when the caldera floor uplifts along with an inward-dipping ring fault (see Figure 1 in Sandanbata, Kanamori, et al. 2021; Figure 9 in Shuler, Ekström, et al., 2013). Thus, we prepare tens of source structures with different ring-fault dip angles.

Figure S9. Same as Figure S8, but for a case of an uplift distribution near Curtis caldera.
We discretize the source system into triangular source elements. The ring fault is divided into elements with an arc angle of 30° along the circumference and 1.5 km along the depth, and a trapezoid composed of two neighboring triangular elements with the same dip and strike angles is considered as a sub-fault. The horizontal crack is discretized using the DistMesh code (Persson & Strang, 2004), each of which is considered as a sub-crack. By a tsunami waveform inversion explained later, we will determine amounts of the reverse slip of each sub-fault and the opening/closure of each sub-crack, denoted by \( \mathbf{s} = [s_1 \ \cdots \ s_{N_3}]^T \) and \( \mathbf{\delta} = [\delta_1 \ \cdots \ \delta_{N_5}]^T \), respectively. Because the dislocations of the ring fault and the horizontal crack should be similar to each other at their contacts, we link the vertical component of the sub-fault slip at bottom to the sub-crack opening/closing at edge adjacent to the sub-fault by imposing a kinematic condition:

\[
s_p \sin \Delta_p = \delta_q
\]

where \( \Delta_p \) is the dip angle of the \( p \)-th sub-fault to which the \( q \)-th sub-crack is adjacent.

**S2.2. Tsunami waveform inversion of dislocations of the fault-crack composite source**

For each source structure assumed above, we perform a tsunami waveform inversion to obtain a fault-crack composite source model. We use the same tsunami data, as described in Section 3.1 of Main Text.

To efficiently compute the Green’s function relating each sub-fault slip or sub-crack opening to the tsunami waveforms, we use the method proposed in a previous study (Sandanbata et al., 2022), which is summarized as follows. First, we compute the vertical sea-surface displacement excited by unit dislocation of the \( i \)-th source element (that is, 1-m reverse slip of sub-fault or 1-m vertical opening of sub-crack; \( i = 1, \ldots, I \); here \( I \) depends on the source structure). We calculate vertical seafloor displacement due to each source element by the triangular dislocation method (Nikkhoo & Walter, 2015) assuming flat seafloor and Poisson’s ratio of 0.25, and we convert it into vertical sea-surface displacement by applying the Kajiura’s filter (Kajiura, 1963). The water depth of 400 m is used for this filter. We thus compute the sea-surface displacement from the \( i \)-th source element \( h_i(x,y) \). Second, we approximate the vertical sea-surface displacement \( h_i(x,y) \) of the \( i \)-th source element by a linear combination of the unit sources \( \eta^k(x,y) \) used in Section 4.1 of Main Text and Text S1 (Equation S1; Figure S7):

\[
h_i(x,y) \approx \sum_{k=1}^{K} m_i^k \eta^k(x,y),
\]

where the amplitude factors \( m_i^k \) are obtained by a least-squares method. Third, we compute the Green’s functions relating the \( i \)-th source element to the tsunami data at the \( j \)-th station by superimposing the Green’s functions of the unit sources \( g_j^k \) multiplied by the amplitude factors \( m_i^k \):

\[
G_{ij}(t) = \sum_k m_i^k g_j^k(t).
\]
Finally, to obtain a source model, we determine the dislocations of the fault-crack composite source system by solving the observation equation with the damped least-squares method:

\[
\begin{bmatrix} d \\ 0 \end{bmatrix} = \begin{bmatrix} G \\ K \end{bmatrix} \begin{bmatrix} S \\ \delta \end{bmatrix},
\]

(S6)

where \(d\) is the observed tsunami data at the \(j\)-th station, and \(G = \begin{bmatrix} G_{11}(t) & \cdots & G_{13}(t) \\ \vdots & \ddots & \vdots \\ G_{1f}(t) & \cdots & G_{1j}(t) \end{bmatrix}\) is the matrix of the Green’s functions \(G_{ij}\). \(s\) is an unknown column vector of reverse slip amounts for sub-faults, for which we impose the non-zero condition \((s \geq 0)\), and \(\delta\) is an unknown column vector of opening amounts for sub-cracks, for which we allow either positive (opening) or negative (closing) values. The linear equation of \(KC\delta = 0\) represents the kinematic condition of Equation S3. \(\beta\) is the damping parameter, which we set at 0.015 by taking a balance between the solution smoothness and the waveform fit (Figure S10).

To evaluate the waveform fit between the observed tsunami waveforms and synthetic waveforms from an obtained source model, we quantify the normalized root-mean-square (NRMS) misfit of the tsunami waveforms, which we call tsunami NRMS misfit:

\[
\sqrt{\frac{\sum_j \|c^f_j - d^f_j\|^2}{\sum_j \|c^f_j\|^2}},
\]

(S7)

where \(c^f_j\) and \(d^f_j\) are the observed waveform and synthetic waveforms within the inversion time window at the \(j\)-th station, respectively. \(\| \|\) denotes the L2 norm of data vector.

### S.2.3. Forward computation of long-period seismic waveforms

For validation of the fault-crack composite source models obtained by the tsunami waveform inversion, we compute long-period seismic waveforms from the source models and compare them with the seismic data. The moment tensor of the models \(M\) is written by:

\[
M = M_{RF} + M_{HC} = \sum p M_{RF}^p + \sum q M_{HC}^q,
\]

(S8)

where \(M_{RF}\) and \(M_{HC}\) represent moment tensors of the ring fault and the horizontal crack, respectively, and \(M_{RF}^p\) and \(M_{HC}^q\) are moment tensors of the \(p\)-th sub-fault and the \(q\)-th sub-crack, respectively. We compute \(M_{RF}^p\) with the slip amount and strike, dip, and rake \((90^\circ)\) angles (Box 4.4 in Aki & Richards, 1980), with the seismic moment of \(\lambda + 2\mu\), where \(\lambda\) is rigidity, or Lamé’s constant. We calculate \(M_{HC}^q\) by:

\[
M_{HC}^q = \begin{bmatrix} M_{rr} & M_{r\theta} & M_{r\phi} \\ M_{r\theta} & M_{\theta\theta} & M_{\theta\phi} \\ M_{r\phi} & M_{\theta\phi} & M_{\phi\phi} \end{bmatrix} = \delta_q \times A_q \times \begin{bmatrix} \lambda + 2\mu & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix},
\]

(S9)
where $\delta_0$ and $A_0$ are the opening amount and area, respectively (Kawakatsu & Yamamoto, 2015). We assume Lamé’s constants of $\lambda$ and $\mu$ as 34.2 GPa, and 26.6 GPa, respectively, calculated with the P- and S-wave velocities and the density in the shallowest crust layer of 1-D Preliminary Reference Earth Model (PREM) (Dziewonski & Anderson, 1981). The total seismic moment is calculated by

**Figure S10.** Source models inverted with different damping parameters $\beta$ of (a) 0.005, (b) 0.015, and (c) 0.030. (Left) Dislocations of the fault-crack source system determined by the tsunami inversion. See captions of Figure 3b in Main Text. (Right) Synthetic tsunami waveforms from the models (red), compared with observed waveforms (black). See the caption of Figure 4b in Main Text. By taking the balance between the solution smoothness and the tsunami waveform fit, we use $\beta$ of 0.015.
\[ M_0 = \sqrt{\sum_{ij} M_{ij} M_{ij}} / 2, \]  
(Down and Tromp, 1998; Silver and Jordan, 1982) and the moment magnitude by
\[ M_w = \frac{2}{3} (\log_{10} M_0 - 9.10), \]
with \( M_0 \) in the N m scale (Hanks and Kanamori, 1979; Kanamori, 1977).

Using the W-phase package (Duputel et al., 2012; Hayes et al., 2009; Kanamori and Rivera, 2008), we compute long-period seismic waveforms from the moment tensor \( \mathbf{M} \). Green’s functions of seismic waves are calculated with the PREM model by the normal mode method (Takeuchi and Saito, 1972). The centroid is at the center of Curtis caldera (178.56°W, 30.54°S) and at the depth of 0.5 km below the seafloor. Both the half duration and the centroid time shift are assumed to be 5 s, which are comparable to the values (3 s and 3 s, respectively) obtained by the moment tensor inversion (Text S4; Table S3). We apply the same filter to the synthetic waveforms, as used for the seismic data.

To quantify the seismic waveform fit, we calculate the root-mean-square (RMS) misfit of the seismic waveforms, which we call seismic RMS misfit:
\[ \sqrt{\sum_j \| c^s_j - d^s_j \|^2}, \]
where \( c^s_j \) and \( d^s_j \) are the synthetic and observed seismic waveforms at the \( j \)-th station. The data length for this calculation includes P, S, and surface waves.

**Text S3. Contribution to long-period seismic waves**
The moment tensor of our best-fit source model \( \mathbf{M} \) is highly isotropic with \( M_w 6.24 \) (Figure 4c in Main Text), whereas the deviatoric moment tensor of the 2017 Curtis earthquake reported in the GCMT catalogue is a vertical-T CLVD type with \( M_w 5.8 \) (Figure 1a in Main Text). Here we discuss the reason for the moment tensor difference. Figure S5a shows synthetic seismograms at representative stations from the moment tensor \( \mathbf{M} (= \mathbf{M}_{HC} + \mathbf{M}_{RF}) \) of our model. For comparison, we show synthetic seismograms from the horizontal crack \( \mathbf{M}_{HC} \) and the ring fault \( \mathbf{M}_{RF} \) in Figures S5b and S5c, respectively. Although \( \mathbf{M}_{HC} \) has a larger moment magnitude \( (M_w 6.19) \) than \( \mathbf{M}_{RF} \) \( (M_w 5.96) \), the seismic amplitudes from \( \mathbf{M}_{HC} \) are much smaller than those from \( \mathbf{M}_{RF} \). This is because the vertical motion of a horizontal crack at a shallow depth has only a low efficiency of long-period seismic radiation (Fukao et al., 2018; Sandanbata, Kanamori, et al., 2021). Note that the polarities of seismograms from \( \mathbf{M}_{HC} \) are reversed to those from \( \mathbf{M}_{RF} \), which is known as the trade-off between the vertical-CLVD and isotropic components at shallow depth (Kawakatsu, 1996; Sandanbata, Kanamori, et al., 2021; compare Figures S5b and S5c). Hence, \( \mathbf{M}_{HC} \) slightly reduces the seismic amplitudes from \( \mathbf{M} \) but does not change the waveform shapes much.

To further examine radiations from the ring fault, we show in Figure S5d synthetic seismograms from \( \mathbf{M}_{RF} \) excluding two components representing vertical dip-slip, \( M_{\theta} \) and \( M_{\varphi} \) (i.e., from \( M_{rr}, M_{\theta\theta}, M_{\varphi\varphi} \), and \( M_{\theta\varphi} \) of \( \mathbf{M}_{RF} \)). Although the moment magnitude decreases
by 0.2 compared to that of $M_{RF}$, the synthetic seismograms are similar to those from $M_{RF}$. This demonstrates that the excluded components, $M_{r\theta}$ and $M_{r\phi}$, at a shallow depth are inefficient in radiating long-period seismic waves (Sandanbata, Kanamori, et al., 2021). Hence, long-period seismic waves of the earthquakes mainly arise from the four components, $M_{rr}$, $M_{\theta\theta}$, $M_{\phi\phi}$, and $M_{\theta\phi}$, of $M_{RF}$. The moment tensor composed of these components is of a vertical-T CLVD type, which is similar to the GCMT solution (Figure 1a in Main Text).

**Text S4. Moment tensor analysis**

We use the W-phase code (Duputel et al., 2012; Hayes et al., 2009; Kanamori & Rivera, 2008) to perform the deviatoric moment tensor (MT) inversion analysis for the 2009 and 2017 earthquakes using long-period seismic data. This analysis is independent of the source modeling in Main Text. For the two earthquakes, we download seismic records of LH and BH channels at stations within 5°–30° from seismic networks (network codes: II, IU, AU, NZ, and G). For computation of the Green’s function of seismic waveforms, we use the normal mode method (Takeuchi & Saito, 1972) with the 1-D Preliminary Reference Earth Model (PREM) (Dziewonski & Anderson, 1981). The time window includes P, S, and surface waves. We impose the zero-trace constraint, $M_{rr} + M_{\theta\theta} + M_{\phi\phi} = 0$. We assume the centroid location at (178.56°W, 30.54°S) and the depth at 2.5 km in the crust, and apply the same filter, as done in the source modeling (see Text S2.3). We start the inversion with a half duration $t_h$ and a centroid time shift $t_c$ reported in the GCMT catalog, and grid-search optimal values for $t_h = t_c$. During the inversion process, we select seismic data yielding a single-record normalized root-mean-square (NRMS) misfit $\leq 1.0$, which is calculated by 

$$\sqrt{||c_i^s - d_i^s|| / ||c_i^s||},$$

where $c_i^s$ and $d_i^s$ are synthetic and observed data in the inversion window at the $i$-th station, respectively. The selected datasets are composed of 29 and 33 records of the 2009 and 2017 earthquakes, respectively (Figures S1 and S12).

Table S3 shows the moment tensor solutions of the 2009 and 2017 earthquakes. The seismic moments and moment magnitudes are much larger than those in the GCMT catalogue (Table S1), because for such shallow earthquakes $M_{r\theta}$ and $M_{r\phi}$ cannot be estimated accurately (Kanamori & Given, 1981; Sandanbata, Kanamori, et al., 2021).

Following a previous study (Sandanbata, Kanamori, et al., 2021), we estimate the ring-fault geometries of the 2009 and 2017 earthquakes using resolvable moment tensor $M_{res}$. We obtain $M_{res}$ by removing $M_{r\theta}$ and $M_{r\phi}$ from the estimated deviatoric moment tensor, decompose $M_{res}$ into two components, i.e., vertical-CLVD component $M_{CLVD}$ and vertical strike-slip component $M_{SS}$, and calculate the moment ratio of $M_{CLVD}$ to $M_{SS}$, or the CLVD ratio $k_{CLVD}$. Figure S6 shows thus-obtained $M_{res}$ and the CLVD ratios of the two earthquakes. Using the relationships between the Null-axis direction of $M_{res}$ and the ring-fault orientation, and between $k_{CLVD}$ and the arc angle of the ring fault (see Figure 4 in Sandanbata, Kanamori, et al., 2021), we estimate that the 2009 and 2017 earthquakes occurred with ring faults with arc angles of $\sim 100°$ and $\sim 120°$, respectively, both of which are oriented in the NNW–SSW direction.
Figure S11. Model performance of the MT analysis for the 2009 earthquake. Red and black lines represent synthetic and observed waveforms, respectively. The time window for the inversion is indicated by red dots.
Figure S12. Same as Figure S11, but for the 2017 earthquake.
**Text S5. Examinations of the uniqueness of the source geometry**

To examine the uniqueness of our source model proposed in Section 4.2 in Main Text, we additionally perform the source modeling with some modifications in geometries, and assumption of the fault-slip direction.

**S5.1. Source geometry**

We examine how our analysis constrains the source geometries of trapdoor faulting, i.e., crack depth and ring-fault length. As proposed below, models with slightly different geometries can explain the tsunami and seismic data overall, suggesting that our analysis have only weak constraints on the two parameters.

1. **Depth of a horizontal crack**
   We first test a fault-crack composite source system with a deeper horizontal crack at a depth of 6 km in the crust (Figure S13). The tsunami waveform inversion yields a dislocation pattern of the fault-crack source system (Figure S13a) similar to that presented in our main results (Figure 3b in Main Text), and the tsunami waveform fit is overall good (tsunami NRMS of 0.74; Figure S13b). Long-period seismic waveforms computed with this model also show good agreements with the observed seismic data (seismic RMS of 0.96 μm; Figure S13e).

2. **Length of a ring fault**
   We also test a fault-crack source system with a longer ring fault (with a central angle of 240°), as shown in Figure S14a. The modeling results show that the estimated dislocations of the source system support the trapdoor faulting mechanism, and the tsunami and seismic waveform data are explained sufficiently (the tsunami NRMS of 0.68 and seismic RMS of 0.91 μm; Figures S14b and S14e).

**S5.2. Slip direction of a ring fault**

In Main Test, we assume only reverse slip on the ring fault. Here, we instead assume a fully elliptic ring fault and perform the tsunami waveform inversion by allowing both reverse and normal slips on the ring fault. In this case, we obtain a source model that contains reverse slips on the northern, western, and southern faults but normal slips on the eastern fault, as shown in Figure S15a. This model can explain the tsunami and seismic data overall, with a tsunami NRMS of 0.68 and seismic RMS of 0.97 μm; Figure S15b and S15e). This suggests that the trapdoor faulting perhaps involved normal faulting associated with the magma flow within the crack, but our analysis cannot determine whether a fault on the eastern side slipped or not.
Figure S13. Fault-crack composite source model with a horizontal crack at a depth of 6 km in the crust. (a) Dislocations of the fault-crack source system determined by the tsunami waveform inversion. See the caption of Figure 3b in Main Text. (b) Synthetic tsunami waveforms from this model (red), compared with observed waveforms (black). See the caption of Figure 4b in Main Text. (c–e) Results of long-period seismic waveform modeling. See the captions of Figures 4c–e in Main Text.
Figure S14. Fault-crack composite source model with a longer ring fault (with a central angle of 240°) and a horizontal crack at a depth of 3 km in the crust. See the caption of Figure S13.
Figure S15. Same as Figure S14, but for a fault-crack composite source model with a fully elliptic ring fault, on which reverse or normal slips are allowed.
Text S6. Models of only a ring fault or only a horizontal crack

We test whether the fault-crack composite source model is preferable to models of only a ring fault or only a horizontal crack. We conduct the source modeling by assuming only a fully elliptical ring fault (with reverse slip), or only a horizontal crack (with vertical opening or closure), as shown in Figures S16 and S17, respectively. The model of only a ring fault explains the tsunami waveform data worse (Figure S16b), while the long-period seismic data are well explained (Figure S16c). On the other hand, the horizontal crack opening alone reproduces the tsunami data well (Figures S17b), but the model excites far smaller-amplitude seismic waves with flipped polarities compared to those of the observed waveforms (Figures S17c). These suggest that our model combining a ring fault and a horizontal crack is more plausible for the earthquake source model, compared to only a ring-faulting or only a crack opening.
Figure S16. Results of the source modeling for only a fully elliptic ring fault. (a) Dislocations of the ring fault determined by the tsunami waveform inversion. (b) Synthetic tsunami waveforms from this model (red), compared with observed waveforms (black). (c) Results of long-period seismic waveform modeling. (Left) moment tensor of the model, and (right) comparison between synthetic and observed seismic waveforms at representative stations.
Figure S17. Same as Figure S16, but for only a horizontal crack. In (a), dislocations of the horizontal crack determined by the tsunami waveform inversion are shown.
Text S7. Scaling relationships between the maximum fault slip with the geodetically estimated moment magnitude

In Section 5.3 and Figure 5 in Main Text, we discuss the scaling relationship between the maximum fault slips and the seismic magnitudes for trapdoor faulting events at three calderas. Yet, previous studies suggested that there are gaps between seismic and geodetic estimates of moment magnitudes of trapdoor faulting events (Amelung et al., 2000; Jónsson, 2009; Zheng et al., 2022). Hence, instead of seismically estimated magnitudes, we here compare geodetically estimated moment magnitudes with the maximum fault slips.

For the Sierra Negra caldera cases, the source models (Zheng et al., 2022; Yun, 2007) were geodetically estimated using data from Global Positioning System (GPS) sensors and Interferometric Synthetic Aperture Radar (InSAR). Zheng et al.’s (2022) model for the 16 April 2005 earthquake had a geodetic moment magnitude of $M_w$ 5.3 for the fault slip component, while Yun’s (2007) model for the 22 October 2005 event had $M_w$ 5.4. In both cases, a shear modulus $\mu$ of 10 GPa was assumed.

For the two submarine calderas, Curtis and Sumisu, the source models were mainly determined based on tsunami data by this study and Sandanbata et al. (2022). For these two models, we here calculate moment magnitudes for the fault slip component by $\sum_p(\mu s_p A_p)$; since tsunamis are generated by co-seismic seafloor deformation, the magnitudes based on tsunami data can be considered as “geodetic” magnitudes. The model of this study for the 2017 Curtis earthquake has a geodetic moment magnitude of $M_w$ 6.0 for the fault slip part, while Sandanbata et al.’s (2022) model for the 2015 Sumisu earthquake has $M_w$ 6.3. In these cases, a shear modulus $\mu$ is assumed as ~30 GPa. If a lower value is used, the geodetic moment magnitudes become smaller; for example, if we assume a three-times smaller value (~10 GPa), the magnitude becomes smaller by 0.32 ($= (2/3)\log_{10}(1/3)$), according to Equation S11.

In Figure S18, we show the scaling relationship between the maximum fault slips and the geodetic moment magnitudes for the trapdoor faulting events. This demonstrates that, even considering the discrepancies between seismic and geodetic estimates of moment magnitudes, the scaling relationship for trapdoor faulting events deviates from the empirical relationship proposed by Wells & Coppersmith (1994). This supports the idea that trapdoor faulting events are atypical compared to tectonic earthquakes.
Figure S18. Earthquake scaling relationship for trapdoor faulting events, with geodetic moment magnitudes. Circles represent the maximum slip amount and the geodetic magnitudes for four trapdoor faulting events at Curtis, Sumisu, and Sierra Negra. The black line represents the relationship for tectonic earthquakes proposed by Wells and Coppersmith (1994).
Caption for Data Set S1

Data Set S1. Fault-crack composite source model (separate file), including a source model presented in Figure 3b in Main Text. This dataset is also available from a repository, Zenodo (https://doi.org/10.5281/zenodo.7502680).

References


