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# Optical Depth Gauge for Laboratory Studies of Water Waves*

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The absorption of infrared light by water is used to measure depth. A collimated beam of light from an incandescent filament source is projected through the water from below, filtered, and detected by an infrared sensitive phototube. The quantitative performance of the gauge is assessed, and the effects of refraction and reflection at the the free surface are discussed. A circuit that linearizes the exponential dependence of the phototube output on depth is described. The sensitivity of the gauge and linearizing circuit is determined by calibration to be about 0.05 V/cm, and the smallest measurable wave amplitude, corresponding to unity signal-to-noise ratio, is about 0.07 mm. The accuracy for absolute depth measurement is about $\frac{1}{2}$ mm.

IN studies of water waves, the difficulties encountered in measuring relatively small and rapid changes of depth are well-known. This is particularly true with some unsteady flows (e.g., bores) if accurate local measurements with fast time response are required, and if large fluid velocities occur under the waves. Capacitance[1] and wire resistance[2] wave gauges are among the most sensitive instruments available, but they suffer from the difficulties common to all immersion devices attributable to the erratic dynamic behavior of the meniscus, the existence of a viscous film of fluid on the gauge as the free surface recedes, and large disturbances around the gauge when fluid velocities are large, e.g., the upward directed jet at the stagnation point and cavitation in the wake. This article reports a relatively simple and inexpensive optical depth gauge that is free of some of these disadvantages. It utilizes the absorption of infrared light by water.

## DESCRIPTION

As shown in Fig. 1, a collimated beam of light from an incandescent filament light source LS is projected from below through the (necessarily) glass bottom of the water channel. After passing through the water the attenuated beam is focused by lens L3 through an infrared filter F onto the photocathode of an infrared sensitive phototube PT.

The light source is a 30 V, 250 W quartz iodide lamp (type DXM). L1, a 25 mm diam by 20 mm focal length lens, images the filament onto the source slit S1 (0.5×0.25 mm). Two condenser lenses L2, each of 6.4 cm diam×6.4 cm focal length, are set with their combined focal point at S1. S2 (1.5×13 mm) defines the beam that passes through the water. L3 is another 6.4 cm diam×6.4 cm focal length lens positioned so the image of the light beam at the free surface is somewhat reduced in size and falls entirely on the photocathode of a type 6570 photodiode.

## ABSORPTION

The intensity $I$ of monochromatic light transmitted through a wave of elevation $\eta$ above the undisturbed water level (Fig. 1) is given by Lambert's law,

$$I=I_0\exp[-k(\lambda)\eta],\tag{1}$$

where $k(\lambda)$ is the absorption coefficient for light of wavelength $\lambda$. While $k$ is only about $7\times10^{-3}$ cm$^{-1}$ for visible light ($\mathrm{Na_D}$), it is about 0.07 cm$^{-1}$ for $\lambda=0.850\ \mu$ and 0.5 cm$^{-1}$ for $\lambda=1.0\ \mu$.[3] Thus, for laboratory studies of water

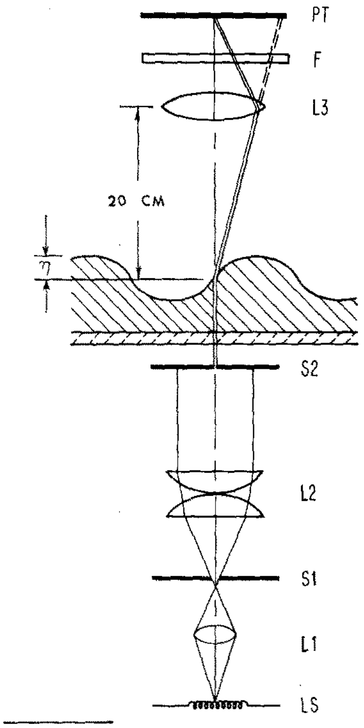

FIG. 1. Schematic diagram of the depth gauge.

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[1] M. J. Tucker and H. Charnock in *Proc. Fifth Conf. Coastal Engineering*, J. W. Johnson, Ed. (Council on Wave Research, University of California, Berkeley, California, 1955).

[2] J. R. Morison, Bull, Beach Erosion Bd. 3, No. 3, 16 (1949); and R. L. Wiegel, University of California Wave Project Report No. HE 116-269 (University of California, Berkeley, California, 1947).

[3] *International Critical Tables* (McGraw-Hill Book Company, Inc., New York, 1929), Vol. 5, p. 269.

waves, use of light in the near infrared is sufficient to give a rather sensitive measurement without requiring special techniques or components. A further advantage of infrared light is that the device can be used in an undarkened room.

In trials with monochromatic light from an interference filter ($\lambda = 0.94\ \mu$, half-width$= 0.015\ \mu$) the gauge performed satisfactorily and according to Eq. (1). However, because of the narrow passband of the interference filter, the signals were not large. Indeed, with this arrangement it would be preferable to increase the sensitivity by using, e.g., a photomultiplier instead of the photodiode.

On the other hand, another way to increase the signal is to use a large passband filter. In this case, the total transmitted intensity is the integral of Eq. (1) over the passband. Since water selectively absorbs infrared light, if $k\eta \geq O(1)$ the water affects the long wavelength cutoff of the optical system. As the depth increases, the filtering action of the water tends to *decrease* the long wavelength cutoff, thereby decreasing the effective absorption coefficient. The result is that the gauge output is even more nonlinear than with purely monochromatic light.

Of course, in any case, if the filter is selected so that $k\eta \ll 1$, the output is linear, but, as the inequality implies, the signal is small. Thus, as with all nonlinear devices, the choice must be made between linear, but small, or nonlinear output. Since it is quite easy to compensate electronically for exponential nonlinearities, linearization of a large signal is preferable in this case to the more expensive alternative of amplification of the small signal, e.g., with a photomultiplier.

Therefore, in the remainder of this article the use of an infrared transmitting Wratten 87C filter (short wavelength cutoff at $\lambda \doteq 0.85\ \mu$) is described. In this case, the long wavelength cutoff of the optical system is determined entirely by the water. With the light source described above and the 87C filter, the intensity of the light falling on the photocathode is very nearly the maximum permissible for the 6570 tube, and the dark current is less than 1% of the phototube output.

A calibration of this gauge installed in the Harvard $\frac{1}{2}$ m wide$\times$1 m deep water channel is shown in Fig. 2, where the phototube current is plotted against depth. At large depths the passband is apparently narrow enough that the system behaves as though the light were monochromatic, with a constant absorption coefficient of 0.06 cm$^{-1}$. The additive correction due to the wider passband at small depths also seems to depend exponentially on depth, with an exponent 6 times larger than the deep water absorption coefficient. This latter, apparently fortuitous, behavior simplifies the problem of linearizing the phototube output (*cf.* below).

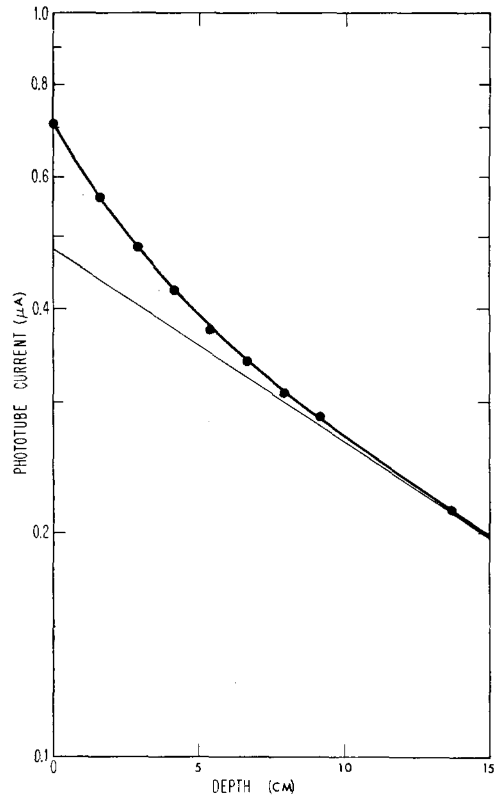

FIG. 2. Calibration of the depth gauge.

## REFRACTION AND REFLECTION

In addition to being absorbed by the water, the light beam is also refracted and reflected at the free surface. In principle, the motion, in the absence of lens L3, of the refracted beam along the photocathode should cause no difficulty so long as the size of the photocathode is sufficient to accommodate this motion (Fig. 1), but since in practice the luminous sensitivity of the type 6570 phototube varies by 15% over the length of the photocathode, a significant distortion of the wave profile is thereby introduced. Therefore, L3 is used to image the free surface onto the photocathode, virtually eliminating the motion of the light spot.

It should be noted that since the light spot always returns to the same position at the crests and troughs of waves, the gauge still gives accurate measurements of wave height even if the refraction is not compensated, so long as the distortion of the wave profile is not so large that the position of crests and troughs on the record can not be determined! On the other hand, very accurate traces of wave profile can be obtained using L3, if the spot is directed onto one of the several areas about $\frac{1}{2}$ cm long that can be found on the photocathode of each 6570 tube where the sensitivity does not vary by more than 2%.

As indicated in Fig. 3, the increase of Fresnel reflection

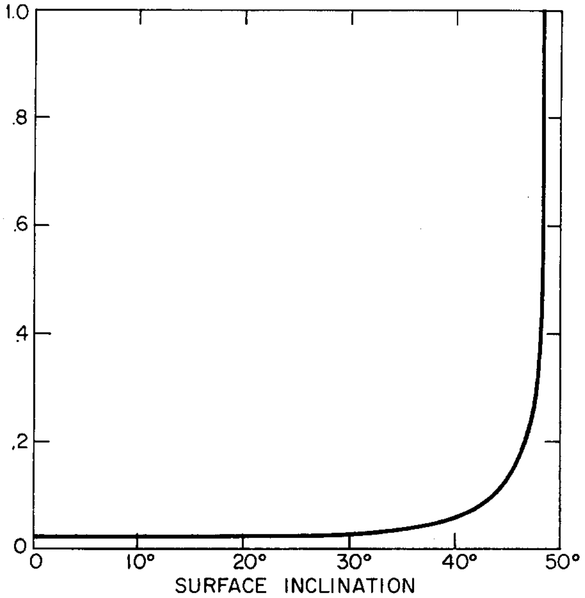

FIG. 3. Fraction of incident light reflected at water-air interface.

of unpolarized light[4] for free surface inclinations normally encountered in wave studies is negligible (only about 1% for surface slopes of 35°) but grows rapidly for larger slopes and becomes catastrophic at the angle of total reflection 48.6°. At this point the (vanishing) transmitted beam has been deflected 41.4°. Note that with the gauge depicted in Fig. 1, if the beam is deflected more than 9°, corresponding to a surface inclination of about 25°, it is deflected off lens L3. Therefore, the effects of reflection are never significant with this depth gauge.

## LINEARIZATION OF GAUGE OUTPUT

The curve of Fig. 2 can be represented analytically by

$$i = i_0(e^{-kd} + Ae^{-Bkd}), \qquad (2)$$

where $i$ is the phototube current; $d$ the depth; $k$, $A$, and $B$ constants; and the functional form of the second term on the right has been determined empirically.[5] As a consequence of this behavior, it is possible to convert the phototube current to an output voltage that is directly proportional to $d$ by making use of the exponential current-voltage characteristics of thermionic and solid state diodes, e.g.,

$$i/i_s = 1 - \exp(qV/kT), \qquad (3)$$

where $q$ is the electronic charge, $k$ Boltzmann's constant, and $T$ the temperature. Such behavior is typical of devices in which the current is space charge limited. The magnitude of the currents in the present application are such that $qV \gg kT$.

Equation (2) can be solved electronically for $d$ using the function inverting capability of operational amplifiers (*cf.* Fig. 4): If the diode is placed in the feedback loop of the amplifier, then, since the input current to operational amplifiers is very small, the entire phototube current flows through the diode (the amplifier insures this by generating an output voltage sufficient to draw that current through the feedback loop). Since the summing point at the input to the amplifier is a virtual ground, the output voltage is the voltage across the diode, and is therefore related to the input current by Eq. (3).

The occurrence of two exponential terms in Eq. (2) requires two parallel sets of diodes in the feedback loop, the leg representing the first term in the equation (the first leg) having $B$ times more diodes than the second leg. The constant $A$ in Eq. (2) fixes the bias voltage that must be applied to one of the legs to give the linear output.

The 25 kΩ resistor at the input of the circuit is a limiting resistor in case of short circuit and plays no role in the linearization. The diodes can be replaced by the 2 MΩ feedback resistor with switch S2 to give an uncompensated output voltage directly proportional to the phototube current.

For a given phototube current, the output voltage can be increased by adding more diodes to each leg. The circuit of Fig. 4 is such that the correspondence between the amplitude of a wave and the amplitude of its (unamplified) trace on a recorder of 0.05 V/cm sensitivity is about 1:1.

A calibration of the gauge and linearizing circuit is shown in Fig. 5. The uncompensated response of the gauge

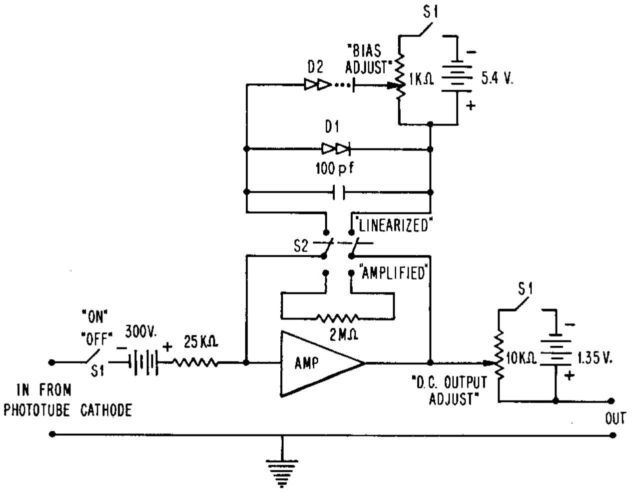

FIG. 4. Schematic diagram of phototube supply and linearizing circuit. AMP—Nexus type SA1 operational amplifier; D1—two Texas Instrument G130 silicon diodes in series; and D2—twelve G130 diodes in series.

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[4] *cf.*, e.g., J. M. Stone, ***Radiation and Optics*** (McGraw-Hill Book Company, Inc., New York, 1963), pp. 396–401.

[5] For the type DXM lamp (color temperature 3400°K) $B \doteq 6$. In general $B$ is found to increase with decrease of lamp temperature.

is also plotted for comparison. The rms deviation of the data from the best fit straight line is 3 mV, which implies that the gauge measures depth with an accuracy of 0.6 mm. If it is assumed that the data are randomly scattered about the best fit straight line, then it follows that the value of the sensitivity measured in this calibration is accurate to about 1%, and therefore that wave amplitudes can be measured with 1% accuracy.[6] The smallest wave amplitude that can be measured with this gauge, corresponding to unity signal-to-noise ratio in the (unfiltered) gauge output, is about 0.07 mm.

One disadvantage of the linearizing circuit shown above is that the diode characteristic [Eq. (3)] is very sensitive to changes of ambient temperature. Indeed, the accuracies quoted above can not be achieved without electronic thermal compensation if room temperature changes are larger than 1°C. The measured time response of the circuit is, at worst, $\frac{1}{2}$ msec. After a change from slightly cloudy, rusty water to fresh tap water in the channel the gauge sensitivity changed by less than 1%.

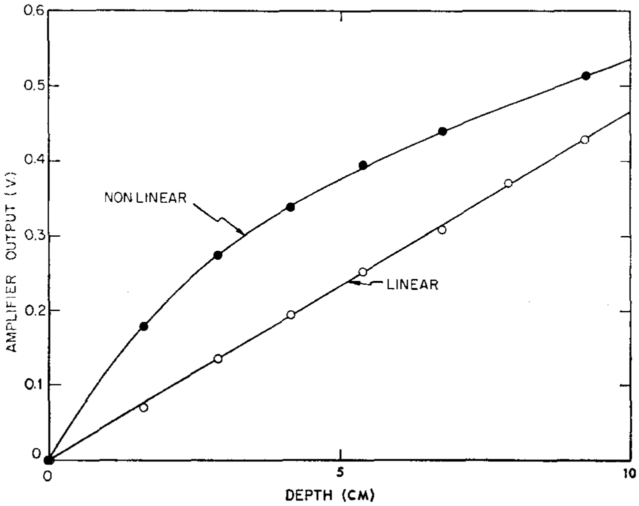

FIG. 5. Calibration of the depth gauge and linearizing circuit. Origin of ordinate arbitrary.

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[6] This result is true only if the spurious signals arising from variations of photocathode sensitivity discussed above are sufficiently small.

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# Stepper Magnet with 20 Channels for Preventing Superimposition of Tracks in Bubble Chamber*

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This report describes a 20 channel "stepper" magnet assembly designed to cause the incoming Bevatron particle beam to scan across the 1.83 m hydrogen bubble chamber in discrete steps. The purpose of this device is to avoid superimposing tracks. Stepping was triggered by an "events" counter placed in the beam. The system was built with low voltage, silicon controlled rectifiers. Magnet pulsing was accomplished in two parts, rapid rise of field was produced by a fast circuit and pulse length was maintained by a slow circuit. Cost was approximately $15 000, exclusive of the surplus components.

## INTRODUCTION

A NEED existed for the incoming particle beam to be swept across the 1.83 m hydrogen bubble chamber so as to prevent superimposed tracks. Uniform spacing of tracks would make the photographs clearer and would simplify automation of the track scanning system. It was proposed to do this scanning by a pulsed magnet, using either a simple sinusoidal sweep or a stepped sweep triggered by an "events" counter. The counter triggered system with a maximum of 20 steps was selected because it was more compatible with a random time distribution of incident particles.

A second choice had to be made between a high voltage system using ignitrons or a low voltage system employing silicon controlled rectifiers. The CERN system used ignitrons.[1] We chose the SCR system because it cost less.[2]

## OBJECTIVE

The objective was to build a pulsed magnet that would move the beam laterally after the arrival of each particle in a beam spill containing as many as 20 particles (Fig. 1). The Bevatron was to spill the beam for 200 $\mu$sec once every 5 sec. The magnet was to be made in 20 segments, each

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* Work done under the auspices of the U. S. Atomic Energy Commission.

[1] E. Chesi and G. Petrucci, CERN/TC/200 65-9 (5 June 1965).

[2] K. Aaland, Lawrence Radiation Laboratory Report LER 442-2 (March 1962).