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Chew-Low Model for Regge-Pole Couplings*†

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The couplings of a meson trajectory $\alpha(t)$ to the baryon octet B and the decimet Δ are studied in the Chew-Low model. The model predicts ratios, though not absolute magnitudes, for $SU(3)$-symmetric couplings of the 0^- octet Π, 1^- octet V, and 2^+ octet T trajectories at the small t of either sign for which static kinematics is applicable. For non-spin-flip, the V and T trajectories are predicted to couple to $\bar{B}B$ like $F+\frac{1}{4}D$, independent of t. For magnetic dipole terms, the Π, V, and T trajectories are all predicted to couple to $\bar{B}B$ like $D+\frac{2}{3}F$, and to $\bar{\Delta}B$ with the same relative strength as the 0^- octet, independent of t. The electric quadrupole couplings of the Π, V, and T trajectories are predicted to be small, independent of t. These results generally agree with existing data, improve Sawyer's explanation of the Johnson-Treiman relations, provide a partial justification of the recent suggestion that V and T couplings are similar, predict that T exchange produces large spin flips, and predict certain ratios such as $d\sigma/dt(\pi^-p\rightarrow\pi^0n)/(d\sigma/dt)(\pi^+p\rightarrow\pi^0N^{*++})$.

I. INTRODUCTION

IN the usual $SU(3)$ formulation of the Chew-Low model, one studies the reaction $\Pi+B\rightarrow\Pi+B$ with B and Δ exchange, where Π is the 0^- octet, B the $\frac{1}{2}^+$ octet, and Δ the $\frac{3}{2}^+$ decimet. The model gives the D to F ratio of $BB\Pi$ couplings and the ratio of $BB\Pi$ to $B\Delta\Pi$ couplings, these results being independent of the Π mass provided it is small enough for the static model to make sense.

Recently, the authors[1] generalized the model to the reaction $\Pi+B\rightarrow\theta+B$. where θ is an arbitrary meson state or current with a mass small compared with that of a nucleon. The requirement of self-consistency allows sizeable θ coupling in only a few states [all $SU(3)$ octets or singlets], and the D/F ratio and $BB\theta$-to-$B\Delta\theta$ coupling ratio were predicted for each state. Interpreting θ as a current, we showed that the known electromagnetic and weak couplings of B and Δ fit nicely into the list of self-consistent possibilities.

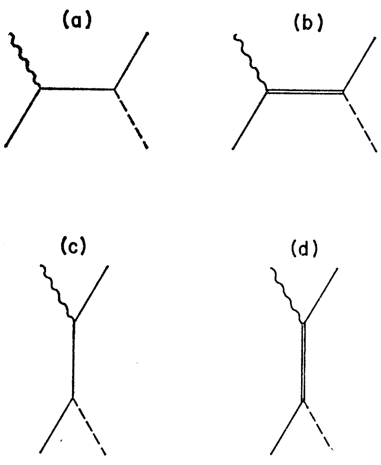

FIG. 1. Diagrams appearing in the $SU(3)$ Chew-Low model for the reaction $\Pi+B\rightarrow\theta+B$. The single solid line represents B, the double line is Δ, the dashed line is Π, and the wavy line is θ.

In the present paper we use the same results for $\Pi+B\rightarrow\theta+B$, this time interpreting θ as any of the Regge trajectories $\alpha(t)$ associated with the hadrons Π, V_8 (1^- octet), V_1 (1^- singlet), T_8 (2^+ octet), and T_1. The key technical points here are: (i) At small t [roughly, $|t|\lesssim 0.5$ (BeV)^2] along the trajectory, we can use the static model. (ii) The static crossing matrix for $\Pi+B\rightarrow\theta+B$ depends only on the *sum* $\mathbf{K}$ of θ's spin $\mathbf{S}$ and its orbital angular momentum $\mathbf{L}$,so that for a given K such as non-spin-flip ($K^P=0^+$), every point on a given Regge trajectory uses the same crossing matrix. Again one obtains predictions for the D to F ratio and the ratio of $BB\theta$ to $B\Delta\theta$ couplings, the results being independent of t provided it is small enough for the static model to make sense.

In Sec. II we list the θ which have self-consistent solutions, and describe the extension of the static model to meson trajectories. The predictions made are collected together in Table I. In Sec. III the predictions for couplings of trajectories are compared with experiment.

II. $SU(3)$ STATIC MODEL FOR MESON COUPLINGS

Consider the scattering amplitude for $\Pi+B\rightarrow\theta+B$, where θ is anything which can be treated as a "particle", i.e., a state of definite mass and angular momentum. In the $SU(3)$ version of the Chew-Low model, one has the diagrams in Fig. 1: B and Δ exchange and B and Δ poles in the direct channel. Note that each diagram in the model contains just one θ coupling, so that the model can predict ratios of θ couplings, but not the absolute magnitude of the coupling. Because of this linear feature, the model applies both when θ is weakly coupled ($\theta=$photon or lepton pair) and when it is strongly coupled ($\theta=$the Regge trajectories associated with Π, V, or T).

The couplings appearing in the static model can be classified as follows.[1] Let $\mathbf{S}$ denote the spin of θ in its rest frame, $\mathbf{L}$ its orbital angular momentum with respect

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[1] R. Dashen and S. Frautschi, Phys. Rev. 143, 1171 (1966).

TABLE I. Self-consistent $BB\theta$ and $B\Delta\theta$ couplings in the $SU(3)$ version of the static model.

K	Properties of θ: $SU(3)$ Representation	Details of couplings	Physical examples in electromagnetism and weak interactions	Physical examples in strong interactions
1	8	(i) $BB\theta$ couplings $\approx D+\frac{2}{3}F$ (ii) $g^{BB\theta}/g^{B\Delta\theta}=g^{BB\Pi}/g^{B\Delta\Pi}$	(i) Magnetic moments ($S=1$, $L=1$, $K=1$) (ii) Axial currents ($K=1$ with various S and L) (iii) Induced pseudoscalar term ($S=0$, $L=1$, $K=1$)	Magnetic dipole couplings of: (i) Π trajectory ($K=1$ with $S=0$ and $L=1$, $S=2$ and $L=1$ or 3, etc.) (ii) V trajectory ($K=1$ with $S=L=1, 3, 5, \ldots$) (iii) T trajectory ($K=1$ with $S=K=2, 4, 6, \ldots$)
0	8	$BB\theta$ couplings $\approx F+\frac{1}{4}D$	(i) Electric form factors ($S=1$, $L=1$, $K=0$) (ii) Weak vector form factors ($S=1$, $L=1$, $K=0$)	Non-spin-flip couplings of $SU(3)$ octets: (i) V trajectory ($K=0$ with $S=L=1, 3, 5, \ldots$) (ii) T trajectory ($K=0$ with $S=L=2, 4, 6, \ldots$)
0	1			Non-spin-flip couplings of $SU(3)$ singlet V and T trajectories

to B, and $\mathbf{K}=\mathbf{L}+\mathbf{S}$. Then since B has $J^P=\frac{1}{2}^+$ and Δ has $J^P=\frac{3}{2}^+$, the only possible couplings are $BB\theta$ with $K^P=0^+$ or 1^+, and $B\Delta\theta$ with $K^P=1^+$ or 2^+.

Let us now make the definition of these $BB\theta$ and $B\Delta\theta$ couplings more precise. Since the mass of θ is assumed to be small compared with a baryon mass, we can pick a frame where the heavy particles at the $BB\theta$ and $B\Delta\theta$ vertices are nonrelativistic. In this frame we describe $J=\frac{1}{2}^+$ baryons by two component spinors B_i ($i=1, 2$) and the $J=\frac{3}{2}^+$ resonances by the vector-spinor objects Δ_i^α ($i=1, 2$; $\alpha=1, 2, 3$) which satisfy[2] $\sum_{\alpha i}\sigma_{ji}^\alpha\Delta_i^\alpha=0$. For a given value L of the orbital angular momentum of θ, the general $BB\theta$ coupling has the form

$$s(L)B^\dagger B+v^\alpha(L)B^\dagger\sigma^\alpha B\,, \tag{1}$$

where s is a scalar and $\mathbf{v}$ is an axial vector constructed from the spin and angular momentum of θ. The general $B\Delta\theta$ vertex is

$$v'^\alpha(L)B^\dagger\Delta^\alpha+q^{\alpha\beta}(L)B^\dagger\sigma^\alpha\Delta^\beta\,, \tag{2}$$

where v'^α is again an axial vector and $q^{\alpha\beta}$ is the spin 2 part of a tensor. In the above $\mathbf{S}$, $\mathbf{L}$, $\mathbf{K}$ classification, it is easy to see that s corresponds to $K=0$, v and v' to $K=1$, and q to $K=2$. The most familiar names for these couplings come from the case where θ is a photon: s is then electric coupling, $\mathbf{v}$ and $\mathbf{v}'$ are magnetic dipoles, and q is the electric quadrupole. When discussing Regge poles, we will refer to $\mathbf{v}$ and s as spin-flip and non-spin-flip, respectively.

The crossing matrices for our model were treated in Ref. 1.[3] We refer the reader to this reference, especially Appendix B and Table I, for details, but the key points are: (i) Couplings with a given K cross only to couplings with the same K, so the K's can be discussed separately. (ii) The crossing matrix depends only on K, not on how K is built up from $\mathbf{S}$ and $\mathbf{L}$. Note that a given K can apply to many different S and L; for example, $K^P=1^+$ applies to the magnetic couplings of $\Pi(S^P=0^-, L^P=1^-)$, $V(S^P=1^-$, with $L^P=1^-$ to get the parity right), $T(S^P=2^+, L^P=2^+)$, and their possible Regge recurrences (e.g., $S^P=4^+$, $L^P=4^+$). It will turn out that this simplification is the reason we can treat Regge trajectories easily. (iii) Dynamical self-consistency requires that the "potential" associated with the θ coupling to exchange diagrams [Figs. 1(a) and 1(b)] must generate the θ coupling to the direct channel poles [Figs. 1(c) and 1(d)]. For the particular conditions of our model, this implies the crossing matrix must have a unit eigenvalue. The coupling ratios are then given by the eigenvector of the unit eigenvalue. Eigenvalues approximately equal to one were found in Ref. 1 for $K=0$ [$SU(3)$ singlet and octet] and $K=1$ [$SU(3)$ octet]. These cases and some applications of them are listed in Table I.

One type of prediction implied by these results is negative—since θ's belonging to the **10**, $\overline{\mathbf{10}}$, and **27** representation fail to achieve self-consistency, they should couple weakly to baryons. Similarly, since electric quadrupole couplings fail to achieve self-consistency, they should be small for all $SU(3)$ representations.

A second type of prediction, following from the eigenvectors of the unit eigenvalues, gives the D/F and $BB\theta/B\Delta\theta$ coupling ratios for the self-consistent θ couplings.

The case where θ is an electromagnetic or weak current was discussed in Ref. 1 and the predicted coupling ratios and smallness of quadrupole terms were found to agree roughly with experiment. These results are summarized in Table I.

We now turn to the main topic of the present paper: the case where θ is a strongly interacting Regge pole. Consider, for example, the trajectory associated with the V octet. It is actually a pole in an amplitude such as $\Pi+B\to 2\Pi+B$. If we define the 2Π state to have en-

[2] This is the nonrelativistic form of the Rarita-Schwinger formalism for a $J=\frac{3}{2}$ particle. We take the normalization of $\mathbf{\Delta}$ to be such that $\sum_\alpha\Delta^{\alpha\dagger}\Delta^\alpha=1$.

[3] The X matrix of Ref. 1 is, for the particular conditions of our model, just a crossing matrix. In Ref. 1 it was calculated by the methods of S-matrix perturbation theory, but since the amplitude is linear in θ coupling, the crossing matrix remains the same even when θ is a hadron.

ergy $\sqrt{t}$ in its rest frame, the pole appears at a variable spin $\alpha(t)$ in the 2Π production amplitude. Let us project this term in the amplitude onto physical 2Π spins S; the trajectory contributes to all odd S in the 2Π amplitude. To study the crossing of the V trajectory, then, we need the crossing matrices for all odd S. But for a given K (e.g., $K^P=1^+$ as reached by $S^P=1^-$, $L^P=1^-$, and $S^P=3^-$, $L^P=3^-$, and ....) we have seen that all S have the *same* crossing matrix in the static model. Therefore, the crossing matrix is independent of $\alpha(t)$ and the results of Table I apply to the meson trajectories.[4]

III. APPLICATIONS OF THE REGGEIZED MESON COUPLINGS

In the previous section, we showed that couplings of meson trajectories will have the properties listed in Table I. These properties imply that (i) All electric quadrupole couplings are small. (ii) All couplings to **10**, $\overline{\mathbf{10}}$, and **27** meson trajectories are small. (iii) Non-spin-flip coupling is large for **1** and **8** meson trajectories, and the octet coupling is $\approx F+\frac{1}{4}D$. (iv) Magnetic dipole coupling is large for **8** meson trajectories. The $BB\theta$ coupling is $\approx D+\frac{2}{3}F$. The ratio to $B\Delta\theta$ coupling is the same as in the static model for Π mesons; for example, $v'_{pN^{*++}\pi^+}=(3/\sqrt{2})v_{pp\pi^0}$.

Because of the static kinematics used in the model, we cannot apply these predictions directly to physical 1^- or 2^+ mesons with their large masses. But the static kinematics *is* very appropriate to the trajectories at small t, and we now proceed to compare the predictions with the evidence at small negative t.

A. Johnson-Treiman Relation

The Johnson-Treiman relation[5] involves differences $\sigma^{\text{total}}(\Pi B)-\sigma^{\text{total}}(\bar{\Pi}B)$. These differences are related through the optical theorem to the forward *non-spin-flip* amplitude $T(\Pi B\to\Pi B)-T(\bar{\Pi}B\to\bar{\Pi}B)$.

Only $C=-$ exchanges (for example, V but not T) can contribute to the Johnson-Treiman relations, since $C=+$ exchange has the same sign for particles and antiparticles. Sawyer has shown[6] that the Johnson-Treiman relations follow from the assumption that $C=-$ exchanges are dominantly octet with pure F coupling to the baryons, thus providing an alternative to the original $SU(6)$ derivation.

Our results justify Sawyer's assumption. The only $\Pi\Pi$ states with $C=-1$ are $\mathbf{8}_A$ and $\mathbf{10}-\overline{\mathbf{10}}$. Since we have found that **10** and $\overline{\mathbf{10}}$ will not have large couplings to the baryons, we are left only with **8** for which, in non-spin-flip, we predict a coupling pattern of $F+\frac{1}{4}D$. This is close to Sawyer's pure F; actually, the $\frac{1}{4}D$ admixture provides a correction to the Johnson-Treiman relation which, according to Barger and Olsson,[7–9] significantly improves the agreement with experiment.

Finally, we note that our derivation of these relations is independent of whether a single Regge pole or several Regge poles and possibly cuts are contributing to the cross sections; each $C=-1$ Regge object with large couplings to the baryons will produce the same pattern. This fact allows us to understand why the Johnson-Treiman relations continue to hold at lower energies where a single Regge pole would not be expected to dominate.[10]

B. Similarity of V and T Couplings

Our model predicts that V and T have the same coupling ratios. An analysis of total cross sections by Barger and Olsson[8,9] yields $F/D\approx 2$ for the forward non-spin-flip couplings to both V and T, in rough agreement with our prediction.[11]

We further predict the new features that both V and T have small electric quadrupole couplings, and that their magnetic couplings have the same D/F and $BB\theta/B\Delta\theta$ coupling ratios.[11] The prediction that T as well as V has a large spin-flip (i.e., magnetic dipole) coupling to baryons seems to be in accord with the recent discovery of a secondary peak in $p\bar{p}\to p\bar{p}$ scattering:[12] If this secondary peak represents a large spin flip, like the πN secondary peak[13,14] which it so closely resembles,[15] then the absence of any corresponding peak in $pp\to pp$ implies that large V $(C=-)$ and T $(C=+)$ spin flips are somehow working together, tending to add in $p\bar{p}$ and

[4] In other words, although S_{Regge} and L become complex, K remains a fixed integer, allowing us to use the same crossing matrix all along a trajectory.

[5] K. Johnson and S. Treiman, Phys. Rev. Letters **14**, 189 (1965).

[6] R. Sawyer, Phys. Rev. Letters **14**, 471 (1965).

[7] V. Barger and M. Olsson, Phys. Rev. Letters **15**, 930 (1965).

[8] V. Barger and M. Olsson, Phys. Rev. **146**, 1080 (1966).

[9] The negative value of F/D reported by Barger and Olsson is due to a different sign convention and does not represent a disagreement with our prediction.

[10] At energies so low that direct channel resonances are important, the sum over exchanged Regge poles converges *very* slowly. Here, "coherence" among the different exchanges may become more important than the pattern associated with any individual exchange, in which case our predictions no longer necessarily apply.

[11] Recently, A. Ahmadzadeh, Phys. Rev. Letters **16**, 952 (1966) [following the exchange-degeneracy hypothesis of R. Arnold, Phys. Rev. Letters **14**, 657 (1965)], has made the stronger assumption that the *magnitude and signs* of the non-spin-flip couplings are the same (except for signature factors) and the trajectories are degenerate. He reports reasonable agreement with the $\pi^\pm p$, $K^\pm p$, $K^\pm n$, pp, pn, $\bar{p}p$, and $\bar{p}n$ total cross sections. Also R. Arnold, Report, 1966 (unpublished), Argonne reports reasonable agreement between this model (extended to include helicity-flip) and experiment for cases where charge or hypercharge is exchanged. Note that the present paper is weaker; we cannot make any statement on over-all magnitudes and signs in our model, and we do not assume or derive $\alpha_V(t)=\alpha_T(t)$.

[12] B. Barish, D. Fong, R. Gomez, D. Hartill, J. Pine, A. Tollestrup, A. Maschke, and T. Zipf, Phys. Rev. Letters **17**, 720 (1966).

[13] D. Damouth, L. Jones, and M. Perl, Phys. Rev. Letters **11**, 287 (1963).

[14] C. Coffin, N. Dikman, L. Ettlinger, D. Meyer, A. Saulys, K. Terwillinger, and D. Williams, Phys. Rev. Letters **15**, 838 (1965).

[15] S. Frautschi, Phys. Rev. Letters **17**, 722 (1966).

cancel in pp.[16]

C. N-N^* Coupling Ratio

The predictions of Regge theory become especially simple when only one trajectory can be exchanged. Such is the case for $\pi N \to \pi N$ charge exchange and $\pi N \to \pi N^*$ charge exchange (pure ρ exchange) and $\pi N \to \eta N$ and $\pi N \to \eta N^*$ (pure A_2 exchange). Our predictions for A_2 and ρ exchange are similar, but we shall discuss mainly the ρ-exchange reactions here because more data are available for them.

In our model, the $F+\frac{1}{4}D$ combination makes $NN\rho$ non-flip somewhat small [it becomes much smaller when $SU(3)$-breaking corrections are added]. $NN\rho$ and $NN^*\rho$ magnetic dipole couplings are predicted to be large with the ratio given at the beginning of this section. $NN^*\rho$ electric quadrupole coupling is small. Thus, $\pi N \to \pi N$ charge exchange and $\pi N \to \pi N^*$ charge exchange are dominated by magnetic dipole couplings and we can predict their ratios.[16a] Working out the appropriate isospin factors and sums over final spins, we obtain predictions such as

$$\left.\frac{d\sigma/dt(\pi^- p \to \pi^\circ n)}{d\sigma/dt(\pi^+ p \to \pi^\circ N^{*++})}\right|_{\text{same } s \text{ and } t} = \tfrac{2}{3}. \qquad (3)$$

Similarly, since π exchange and A_2 exchange give the same ratios in our theory, we obtain

$$\left.\frac{d\sigma/dt(\pi^- p \to \rho^\circ n)}{d\sigma/dt(\pi^+ p \to \rho^\circ N^{*++})}\right|_{\text{same } s \text{ and } t} = \tfrac{2}{3}. \qquad (4)$$

Experiments covering a range of small t near 4 BeV/c are very roughly consistent with both Eq. (3)[17,18] and Eq. (4).[18,19]

Another test of whether a reaction such as $\pi N \to \pi N^*$ is dominated by magnetic dipole coupling is furnished by the density matrix of N^* decay. At small t, we predict that the amplitude in the lab frame for $\pi + N \to \pi + N^*$ will have the form $\mathbf{v}\cdot N^+\mathbf{N}^*$, where $\mathbf{v}$ is an axial vector constructed from the various momenta and spins in the problem. Since the π's are spinless, $\mathbf{v}$ must be $\mathbf{q}\times\mathbf{q}'$, where $\mathbf{q}$ and $\mathbf{q}'$ are the initial and final momenta of the pions. A theory with a fixed spin-one ρ exchange and pure magnetic coupling at the $NN^*\rho$ vertex would predict the same spin dependence (but not the same dependence on s and t) as Regge theory. Stodolsky and Sakurai,[20] starting with a fixed-spin ρ meson, have worked out the resulting density matrix and obtain results in rough agreement with experiment.[21] It appears that the t dependence predicted by a fixed spin exchange is, however, at odds with experiment. We predict, of course, the same density matrix, but a Regge t dependence.

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[16] A possible difficulty arises in connection with the secondary peak in πN elastic scattering. It seems to receive a big contribution from isoscalar T spin-flip (see Ref. 15) although T_1 does not have a large spin-flip coupling in our model and the $D+\frac{2}{3}F$ combination also gives the isoscalar member of the T octet a weak spin-flip coupling to nucleons.

[16a] Some of the predictions of this subsection were obtained from Chew-Low theory in a somewhat different way by Y. Hara, Phys. Rev. **140**, B178 and **140**, B1649 (1965).

[17] $d\sigma/dt(\pi^- p \to \pi^\circ n)$ is given by Sonderegger *et al.* [Phys. Letters **20**, 75 (1966)].

[18] $d\sigma/dt(\pi^+ p \to \pi^\circ N^{*++})$ and $d\sigma/dt(\pi^+ p \to \rho^\circ N^{*++})$ are given by the Aachen-Berlin-Birmingham-Born-Hamburg-London (I.C.)-München Collaboration, Phys. Rev. **138**, B897 (1965).

[19] $d\sigma/dt(\pi^- p \to \rho^\circ n)$ is given by the Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Nuovo Cimento **31**, 729 (1964).

[20] L. Stodolsky and J. Sakurai, Phys. Rev. Letters **11**. 90 (1963).

[21] Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-Müchen Collaboration, Phys. Letters **10**, 229 (1964).