# Quasiparticle Poisoning and Josephson Current Fluctuations Induced by Kondo Impurities

Lara Faoro,[1,2] Alexei Kitaev,[3] and Lev B. Ioffe[1]

[1]*Department of Physics and Astronomy, Rutgers University, 136 Frelinghuysen Road, Piscataway, New Jersey 08854, USA*

[2]*Laboratoire de Physique Théorique et Hautes Énergies, CNRS UMR 7589, Universités Paris 6 et 7, 4 place Jussieu, 75252 Paris, Cedex 05, France*

[3]*Institute for Quantum Information, California Institute of Technology, Pasadena, California 91125, USA*

(Received 25 January 2008; published 10 December 2008)

We introduce a toy model that allows us to study the physical properties of a spin impurity coupled to the electrons in the superconducting island. We show that, when the coupling of the spin is of the order of the superconducting gap $\Delta$, two almost degenerate subgap states are formed. By computing the Berry phase that is associated with the superconducting phase rotations in this model, we prove that these subgap states are characterized by a different charge and demonstrate that the switching between these states has the same effect as quasiparticle poisoning (unpoisoning) of the island. We also show that an impurity coupled to both the island and the lead generates Josepshon current fluctuations.

DOI: 10.1103/PhysRevLett.101.247002 PACS numbers: 85.25.Cp, 03.65.Yz, 73.23.−b

*Introduction.*—Superconducting circuits based on small Josephson junctions are promising candidates for the implementation of qubits [1–5] and for the development of a prototype quantum current standard [6]. Unfortunately, the performances of these devices are significantly limited by different types of noise whose sources remain mostly unknown. Particularly dangerous for single-Cooper-pair transistors and Cooper pair boxes is the noise produced by the incoherent tunneling of single quasiparticles into the superconducting island. Each tunneling event changes the island charge, thereby shifting the operation points of the device. An important requirement for the regular operation of these devices is that this tunneling is very rare. Despite significant experimental efforts to reduce quasiparticle poisoning [7–12], a complete understanding of its microscopic mechanisms is still missing. The goal of this Letter is to show that the mechanism of the charge noise discussed in Ref. [13] might also be responsible for the creation of the low energy quasiparticle traps and provides an explanation of the puzzling features observed in quasiparticle poisoning experiments [14].

The work [13] shows that Kondo-like traps located at the superconductor insulator (SI) interface might produce the charge noise in small Josephson charge qubits; a similar mechanism might be responsible for the critical current fluctuations in large superconducting contacts [15]. These Kondo traps are impurities with a singly occupied electron level that carry a spin degree of freedom. Each trap is characterized by an effective Kondo temperature $T_K$ that depends exponentially on its hybridization with the conducting electrons in the bulk superconductor. In this mechanism, both charge and critical current noise originate from the electrons tunneling between those Kondo traps with $T_K \sim \Delta$, where $\Delta$ is the superconducting gap. In this Letter, we show that Kondo-like traps might also be responsible for quasiparticle poisoning of the superconducting island. Further, we show that such traps located *close to* the Josephson junction generate additional sources of critical current fluctuations due to their coupling to the superconductors on both sides of the barrier. This mechanism for critical current noise provides the alternative to the conventional picture of fluctuators blocking conducting channels in the insulating barrier. In order to derive these results, we introduce a toy model that captures the essential physics of a spin impurity coupled to the superconducting electrons in the superconducting island. By computing the Berry phase that is associated with the superconducting phase rotations in this model, we show that two different low energy states of the impurity are characterized by a different charge. As a consequence, switching between these two low energy states has the same effect as quasiparticle unpoisoning (poisoning) of the island. Finally, we use this model to study the effect of the motion of electrons between the Kondo-like traps in a Josephson junction, and we prove that, if one of those traps is coupled to both the lead and the island, these processes result in critical current fluctuations. We begin with the review of the features of the Kondo physics that are relevant for the following and which provide justification of the toy model.

The behavior of a spin-$1/2$ impurity coupled antiferromagnetically with an exchange constant $J$ to an electron gas characterized by a constant density of states $\rho_0$ within a bandwidth $D$ is completely different at high and low temperature regimes. In the former, the electrons scatter off the impurity inelastically in a spin-flip process, while at low temperatures the impurity is screened by the electrons forming a bound singlet state leaving only elastic scattering. The crossover takes place at the energy scale of Kondo temperature $T_K \sim De^{-1/J\rho_0}$. All relevant physics is described by the Anderson Hamiltonian: $H = H_{\rm lead} + H_d + H_{\rm sd}$, where $H_{\rm lead} = \epsilon_\kappa c^\dagger_{\kappa,\sigma} c_{\kappa,\sigma}$ and

$$\begin{aligned} H_d &= \sum_\sigma \epsilon_d c^\dagger_{d\sigma} c_{d\sigma} + U n_{d\uparrow} n_{d\downarrow}, \\ H_{\rm sd} &= \sum_{\kappa\sigma} (V c^\dagger_{\kappa\sigma} c_{d\sigma} + \text{H.c.}). \end{aligned} \tag{1}$$

Here $c_{\kappa\sigma}$ ($c^\dagger_{\kappa\sigma}$) is the annihilation (creation) operator for an electron in the band, and $c_{d\sigma}$ ($c^\dagger_{d\sigma}$) is the annihilation (creation) operator for an electron on the impurity site. $U$ denotes the Coulomb on site repulsion of the trap and $n_{d\sigma} = c^\dagger_{d\sigma} c_{d\sigma}$. In the limit of strong on site repulsion, i.e., $U \gg \Gamma$, where $\Gamma = \pi \rho_0 V^2$ defines the hybridization to the conducting electrons, using the Schrieffer-Wolff transformation [16], we can map the Hamiltonian given in Eq. (1) to the Kondo model [17]:

$$H = H_{\rm lead} + \sum_{\kappa\kappa'} J_{\kappa\kappa'} \vec{S} \cdot c^\dagger_{\kappa\sigma_1} \vec{\sigma}_{\sigma_1\sigma_2} c_{\kappa'\sigma_2}. \tag{2}$$

Here $J_{\kappa\kappa'} = J = 8V^2/U$. This mapping neglects the particle hole asymmetry of the original problem, the effects that are small in $T_K/D$ but might have important physical consequences. Notice that in the limit $J/D \to 1$, the Kondo temperature becomes $T_K \approx J$.

A more complicated problem is presented by the impurity interacting with the conduction electrons in the superconductor. The competition between the Kondo temperature of the trap and the superconducting gap $\Delta$ of the lead results in three different regimes for the system (impurity + superconductor): (i) $T_K \ll \Delta$, where the ground state of the system is a doublet and is characterized by an odd number of electrons; (ii) $T_K \gg \Delta$, where the ground state of the system is a singlet, the electron of the impurity forms a bound state with the superconducting electrons, and the total number of electrons is even [18]. In both of these cases, the system is locked into one state. (iii) $T_K \sim \Delta$, where the singlet and doublet states become almost degenerate; it is in this regime that a new physics appears.

*Toy model and quasiparticle poisoning.*—In order to formulate a simplified model that captures the effects of an impurity interacting with the superconducting electrons, we notice that the Kondo physics can be viewed as a result of the "poor-man scaling" in which the high energy degrees of freedom are gradually integrated out, resulting in the logarithmic growth of the effective interaction as a function of the energy scale $\epsilon$. In the presence of the superconducting gap, the process of integration has to stop at $\Delta$. If at this moment the renormalized interaction $J_R(\epsilon) \sim \Delta$ is comparable with $\Delta$, a bound subgap state can be formed in agreement with the results described above. This shows that the essential physics can be captured by a simple model in which the spin interacts with a single electron mode with the coupling constant $J_R \sim \Delta$ and energy $\varepsilon < \Delta$ that is described by the BCS Hamiltonian:

$$H_{\rm BCS} = \varepsilon \sum_{\alpha = k\uparrow, -k\downarrow} c^\dagger_\alpha c_\alpha + \Delta(e^{i\theta} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + e^{-i\theta} c_{k\uparrow} c_{-k\downarrow}).$$

The interaction between the impurity and the superconducting electrons is described by the spin exchange coupling given in Eq. (2):

$$H_{\rm toy} = H_{\rm BCS} + J_R \vec{S} \cdot \hat{c}^\dagger \vec{\sigma} \hat{c}, \tag{3}$$

where $\hat{c}^\dagger = (c^\dagger_{k\uparrow}, c^\dagger_{-k\downarrow})$ and we assume that the coupling is isotropic: i.e., $J_R \approx T_K \sim \Delta$. The Hamiltonian (3) can be readily diagonalized. We choose the state basis: $\{|i\rangle_{k\uparrow}, |j\rangle_{-k\downarrow}, |\sigma\rangle_{\rm imp}\}$, where $i, j = 0, 1$ denotes, respectively, the absence or presence of the quasiparticle in the single electron mode while $\sigma = \Uparrow, \Downarrow$ represents the spin configuration up or down of the electron in the trap, and find the lowest eigenvalues:

$$E_0 = \varepsilon - \sqrt{\Delta^2 + \varepsilon^2}, \qquad E_1 = \varepsilon - \tfrac{3}{2} T_K, \tag{4}$$

corresponding, respectively, to the (non-normalized) doublet and singlet states:

$$\begin{aligned} |D_\sigma\rangle &= \left[ -\frac{(\varepsilon + \sqrt{\Delta^2 + \varepsilon^2})}{|\Delta|} e^{-i\theta} |00\rangle + |11\rangle \right] |\sigma\rangle, \\ |S\rangle &= -|01\Downarrow\rangle + |10\Uparrow\rangle. \end{aligned} \tag{5}$$

As expected, the spin impurity interacting with the conduction electrons in the superconductor leads to the formation of weak Kondo subgap states. Notice that the subgap states have different properties: The doublet is characterized by an odd number of electrons, its degeneracy is due to the spin degree of freedom of the trap, while the singlet state is a maximally entangled state with an even number of electrons. Depending on the ratio $T_K/\Delta$, the ground state of the system can be either a doublet or a singlet. At a special value $T_K^* = \frac{2}{3}\sqrt{\varepsilon^2 + \Delta^2}$ singlet and doublet states are degenerate, while for traps with $T_K \approx T_K^* \sim \Delta$ singlet and doublet states are almost degenerate.

We show now that singlet and doublet states differ in the induced offset charge on the superconducting island. Because the number of electrons fluctuates in the superconducting state, the induced charge is defined modulus $2e$. To compute the residue, we recall that the operator $\hat{n}$, describing the excess number of Cooper pairs on the island, and the operator $\hat{\theta}$, representing the superconducting phase, are conjugate variables, i.e., $[\hat{\theta}, \hat{n}] = i$, and that the Hamiltonian of the island/box

$$H_{\rm isl/box} = E_c(\hat{n} - n_g)^2 + H_{\rm toy}$$

is invariant with respect to the local gauge transformation: $U^{-1} H_{\rm isl/box} U$, where $U = e^{i n_g \theta}$ and $n_g = C_g V_g / 2e$ is the offset charge induced in the island, which plays a role similar to the vector potential appearing in the

Hamiltonian of an electron in a magnetic field. $E_c = e^2/2C$, $e$ is the electron charge, $C$ is the total island capacitance, and $C_g$ is the gate capacitance.

One consequence of these observations is that we can deduce the value of the offset charge induced by the Kondo impurity in the singlet and doublet states from the value of the Berry phases associated to these states. We find that

$$\oint \bar{n} \cdot d\theta = i \int_0^{2\pi} \langle S| \frac{\partial}{\partial\theta} |S\rangle d\theta = 0,$$
$$\oint \bar{n} \cdot d\theta = i \int_0^{2\pi} \langle D_\sigma| \frac{\partial}{\partial\theta} |D_\sigma\rangle d\theta = \pi\left(1 + \frac{\varepsilon}{\sqrt{\varepsilon^2 + \Delta^2}}\right),$$

where $\bar{n}$ denotes the average. Notice that a Kondo impurity induces a nonzero offset charge in the superconducting island *only* when the system (impurity + superconductor) is in the doublet state, while in the singlet state the electron charge is absent. Moreover, when $\varepsilon \to 0$, we find that $\bar{n} \to 1/2$; i.e., exactly one electron is induced on the superconducting island. Let us now consider two Kondo traps with $T_K \sim \Delta$ located at the SI interfaces: one on the superconducting island and another on the lead within distance $\xi$ from each other. The electron tunneling process across the junction couples these traps. When the Kondo trap located at the SI interface in the island switches between singlet and doublet, the parity of the island changes from even to odd. For this process to be physically relevant, the energy difference between these states should be smaller than $T$. Thus, the pairs of Kondo subgap states with close energy levels might be responsible for the quasiparticle poisoning in superconducting devices. Notice that, if the two traps are located on the same side of the barrier, the switching between singlet and doublet caused by the tunneling of quasiparticles through the superconductor results in charge fluctuations; i.e., the entire process can be viewed as a new type of a charge fluctuator.

We now discuss the implications for the recent experiments where quasiparticle tunneling rates were measured with microsecond resolution [10,14,19] in a micrometer-sized island with a capacitive gate electrode that was probed by two Josephson junctions. The island charging energy is modulated by the gate as $E_c^n(n_g) = E_c(n - n_g)^2$. At $n_g = 1$ the electrostatic energy of the system is minimized when unpaired electrons reside in the superconducting island. Thus, at $n_g = 1$ the island is a trap for a quasiparticle with depth $\delta E = E_c - E_J/2 + \Delta_l - \Delta_i$. Here $\Delta_i$ and $\Delta_l$ are the superconducting gap of the island and lead, and $E_J$ is the Josephson energy. A model suggested by Aumentado *et al.* [7] explains many features of quasiparticle poisoning of the island. In this model, some unknown nonequilibrium source of quasiparticles produces them in the leads. Quasiparticles are able to tunnel onto the island which acts as a trap. Subsequently, the trapped quasiparticle is thermally excited (unpoisoning) out of the trap, and the island returns to its even state. Relevant implications of this model are that the quasiparticle poisoning can be reduced by putting normal metal leads (quasiparticle traps) close to the junctions in order to filter the quasiparticles and by making $\Delta_i > \Delta_l$, because it works as a barrier, which prevents nonequilibrium quasiparticles in the leads from entering the island. Experiments showed that these ideas help to reduce quasiparticle poisoning but do not eliminate it. In particular, the effect of quasiparticle traps have been recently studied in Ref. [14]. In these experiments, two similar Cooper pair boxes were fabricated with (QT) or without (NT) quasiparticle traps attached to the leads. The island was biased at $n_g = 1$, and the dynamics of the quasiparticles captured by the island was characterized by their incoming ($t_{\rm even}$) and outgoing ($t_{\rm odd}$) rates. One expects that the incoming process involves quasiparticle tunneling into the island and its relaxation to the bottom of the well, while the reverse process involves thermal excitation. As a result, the outgoing rate should be smaller by a factor $\propto e^{-\delta E/KT}$ than the incoming rate. This is in contrast with the data that show that the trapping and escape rates are roughly equal and temperature independent below $T \lesssim 200$ mK. However, their values are dramatically different in the devices with or without traps: $t_{\rm even} \sim t_{\rm odd} \approx (10^2\text{–}10^3)\ \mu$s (QT) and $t_{\rm even} \sim t_{\rm odd} \approx (0.1\text{–}1)\ \mu$s (NT).

The presence of subgap states in the lead and the island provides a different scenario where two new processes are present: Quasiparticles with energies above $\Delta_i - E_c + E_J/2$ tunnel from a subgap state in the lead to the continuum in the island, while quasiparticles below these energies tunnel between the subgap states in the island and in the lead (see Fig. 1). The rate of the former process is $\Gamma_{\rm sc} = G\delta$, where $G \sim 1$ is the conductance of the barrier in the units of $e^2/\hbar$ and $\delta = 1/V_i\nu_{\rm Al}$ is the typical level spacing in the island. For a typical island of volume $V_i = 750 \times 125 \times 7$ nm$^3$ and a typical Al electron density of states $\nu_{\rm Al} \sim 35/{\rm eV\,nm}^3$, we estimate $\Gamma_{\rm sc} \approx 10^{-7}$ s$^{-1}$. The rate of the exchange process between subgap states is much

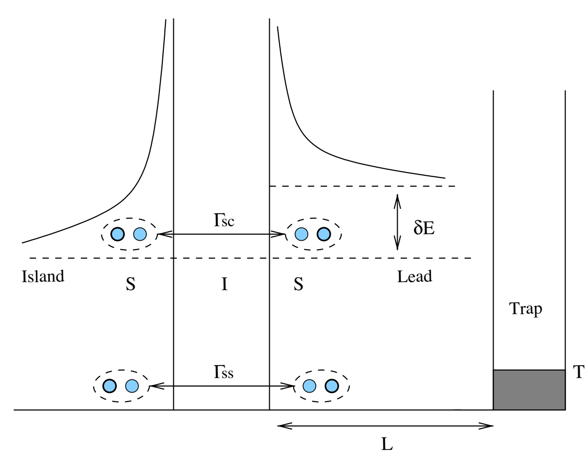


FIG. 1 (color online). Sketches of quasiparticle poisoning and unpoisoning due to quasiparticle tunneling between weak Kondo subgap states located at the lead-island SI interfaces.

slower $\Gamma_{ss} \ll \Gamma_{sc}$, because it occurs between two localized states and it depends on the level width of the state. In both cases the tunneling does not involve a significant energy transfer, so we expect these rates to be temperature independent. In the presence of quasiparticles with energy larger than $\Delta_i - E_c + E_J/2$, the first process dominates and the observable rate is $\Gamma_{sc}$. The presence of quasiparticle traps attached to the leads eliminates high energy quasiparticles, and the rate decreases to $\Gamma_{ss}$ in agreement with observations. This scenario can be checked by fabricating QT devices with normal metal traps attached to the leads at different distances $L \sim \xi$. The presence of these traps at sufficiently small distances broadens the levels of the subgap states in the leads, and consequently it should increase the tunneling rate into the island as $\propto e^{-L/\xi}$, where $\xi$ is the coherence length of the superconductor. The estimates of the rate assume that only a few subgap states are active at the same time. A large number of these states would make the effective rate higher; we do not know the density of these states and their occupation in a realistic system.

*Josephson current fluctuations.*—Kondo-like traps with $T_K \sim \Delta$ provide an additional source of the critical current fluctuations when they are located close to the Josephson junction barrier, because in this case the spin of the Kondo trap is coupled to the electrons both in the island and in the lead. This can be easily seen by including in our toy model the electrons of the lead and a tunneling between the lead and the island. The Hamiltonian becomes $H = H^{(1)}_{\rm toy} + H^{(2)}_{\rm BCS} + H^{\rm qp}_T$, and the quasiparticle tunneling through the junction barrier is given by

$$H^{\rm qp}_T = |T_{k_1,k_2}|[c^\dagger_{k_1\uparrow}c_{k_2\uparrow} + c^\dagger_{-k_1\downarrow}c_{-k_2\downarrow} + \text{H.c.}].$$

We assume that the superconducting leads are equal, and we calculate the correction to the lowest energy eigenvalues and eigenvectors at the second order in perturbation theory in the tunneling $|T_{k_1,k_2}| \approx \mathcal{T}$. We find that the correction to the singlet state depends on the phase difference $\varphi = \theta_1 - \theta_2$. The dependence on $\varphi$ implies an additional contribution to the Josephson current. A straightforward but lengthy calculation gives the contribution of the Kondo impurity in the barrier to the Josephson current:

$$\delta I_c \approx \frac{\mathcal{T}^2}{J}\frac{\Delta^2}{\varepsilon^2 + \Delta^2}\sin\varphi. \qquad (6)$$

In physical systems, $\mathcal{T}^2$ and $J$ are related. To find the ratio $\mathcal{T}^2/J$ that describes the physical situation in which a many-channel junction couples a small metallic island to a large superconducting lead, we compare the pairing field induced on the state $k_1$ in the islands by the superconducting order in the leads in the realistic situation ($\nu\mathcal{T}^2 \sim G\delta$) with the corresponding quantity in the simplified model ($\mathcal{T}^2/\Delta$) and get $\frac{\mathcal{T}^2}{J} = G\delta$. Notice that this reasoning holds for small superconducting islands whose size is less than the superconducting correlation length $\xi$. For larger islands, the impurities at a distance larger than $\xi$ from the junction are coupled exponentially weakly to the superconductor on the other side of the barrier.

*Conclusions.*—We have shown that subgap states generated by magnetic impurities due to the competition between superconducting pairing and the Kondo effect act as very efficient quasiparticle traps. We argued that the presence of such states in a typical single-Cooper-pair transistor and Cooper pair box might explain the results of recent experiments where unexpected poisoning/unpoisoning rates were observed. We have also shown that the same subgap states generate critical current noise.

This work was supported by the National Security Agency (NSA) under Army Research Office (ARO) Contract No. W911NF-06-1-0208 and NSF ECS 0608842.

[1] Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).
[2] D. Vion *et al.*, Science **296**, 886 (2002).
[3] I. Chiorescu *et al.*, Science **299**, 1869 (2003).
[4] A. Wallraff *et al.*, Nature (London) **431**, 162 (2004).
[5] J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, Phys. Rev. Lett. **89**, 117901 (2002).
[6] M. W. Keller, A. L. Eichenberger, J. M. Martinis, and N. M. Zimmerman, Science **285**, 1706 (1999).
[7] J. Aumentado, M. W. Keller, J. M. Martinis, and M. H. Devoret, Phys. Rev. Lett. **92**, 066802 (2004).
[8] T. Yamamoto, Y. Nakamura, Y. A. Pashkin, O. Astafiev, and J. S. Tsai, Appl. Phys. Lett. **88**, 212509 (2006).
[9] S. Corlevi, W. Guichard, F. W. J. Hekking, and D. B. Haviland, Phys. Rev. B **74**, 224505 (2006).
[10] O. Naaman and J. Aumentado, Phys. Rev. B **73**, 172504 (2006).
[11] B. S. Palmer *et al.*, Phys. Rev. B **76**, 054501 (2007).
[12] A. M. Savin *et al.*, Appl. Phys. Lett. **91**, 063512 (2007).
[13] L. Faoro and L. B. Ioffe, Phys. Rev. Lett. **96**, 047001 (2006).
[14] N. A. Court, A. J. Ferguson, R. Lutchyn, and R. G. Clark, Phys. Rev. B **77**, 100501 (2008).
[15] L. Faoro and L. B. Ioffe, Phys. Rev. B **75**, 132505 (2007).
[16] J. R. Schrieffer and P. A. Wolff, Phys. Rev. **149**, 491 (1966).
[17] J. Kondo, Prog. Theor. Phys. **32**, 37 (1964).
[18] L. Glazman and K. A. Matveev, JETP Lett. **49**, 659 (1989).
[19] A. J. Ferguson, N. A. Court, F. E. Hudson, and R. G. Clark, Phys. Rev. Lett. **97**, 106603 (2006).