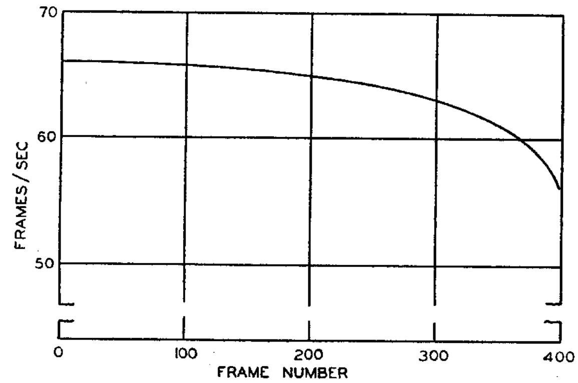

FIG. 3. Calibration curve for spring-driven camera with synchronizing switch.

The Strobolume type-B is shown schematically in Fig. 1. This instrument is manufactured by General Radio under license of Edgerton, Germeshausen, and Grier. The power supply, consisting of a conventional transformer and rectifier, charges the capacitors to about 2500 v. The capacitors remain charged until a pulse from the trigger circuit, operating through a spark coil, ionizes the gas in the flash lamp sufficiently to initiate the discharge of the capacitors through it.

For spring-driven cameras, a distinct advantage of the entire unit is the ease with which the framing rate can be calibrated against the number of frames exposed. The spring is fully wound and a good millisecond clock is photographed while the camera is being operated normally. The framing rate can be read from the angular displacement of the clock hand for a given number of frames. Figure 3 shows such a calibration curve obtained for the Paillard-Bolex camera referred to above.

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Experimental Procedure for the Determination of the Number of Paramagnetic Centers

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THE determination of the number of paramagnetic centers in a given crystal is usually performed by comparing the resonance signal of the unknown centers with that of a calibrated standard. The two most often used standards are $CuSO_4\cdot 5H_2O$ and DPPH. In the procedure described below the number of "spins" is obtained from a measurement of the reflection coefficient of a reflection cavity containing the spins; or more specifically from the change in the reflection coefficient between the "on resonance" and "off resonance" conditions.

The measurements can be performed with the aid of the conventional equipment for the measurement of reflection coefficients. Great simplification is realized when a variable coupling cavity[1] is used.

The reflection coefficient of a reflection cavity containing paramagnetic centers is given by[2]

$$\Gamma=\Gamma_0+\frac{8\pi Q_c\eta\beta}{(1+\beta)^2}\chi''-j\frac{8\pi Q_c\eta\beta}{(1+\beta)^2}\chi'. \tag{1}$$

In Eq. (1), the radio frequency is assumed equal to the resonance frequency of the cavity and the magnetic losses are a small fraction of the cavity losses, i.e., $|4\pi\eta Q_c\chi|\ll 1$,

$\Gamma_0=(\beta-1)/(\beta+1)$ is the reflection coefficient far away from resonance,

$\beta=$ cavity Q/external $Q=Q_c/Q_{ex}$,

$\eta=\int_{V_s}H_1^2dV\Big/\int_{V_c}H_1^2dV$ is the filling factor of the sample,

and χ' and $-\chi''$ are the real and imaginary parts of the rf susceptibility.

For operation near the critical coupling point $\beta\approx 1$, for a spectrometer tuned to χ'' only, Eq. (1) simplifies to

$$\Gamma=\Gamma_0+2\pi Q_c\eta\chi''$$

which at resonance becomes

$$\Gamma_{res}=\Gamma_0+2\pi Q_c\eta\chi''_{max}. \tag{2}$$

Consider an ion with effective spin S in a crystalline field of trigonal or higher symmetry. Let z be the coordinate along the symmetry axis. The interaction of such an ion with an external magnetic field $\mathbf{H}=(H_x,H_y,H_z)$ is given by the Hamiltonian

$$H_{\text{interaction}}=g_{\parallel}\beta H_zS_z+g_{\perp}\beta(H_xS_x+H_yS_y).$$

S_x, S_y and S_z are three components of the spin vector operator, β is the Bohr magneton, while $g_{\parallel}$ and $g_{\perp}$ are, respectively, the parallel and transverse components of the g tensor.[3] In the presence of a steady magnetic field applied in the z direction and a linearly polarized rf field of frequency ν_0 applied in the x-y plane, the χ'' due to the $M\leftrightarrow M-1$ transition is given by

$$\chi''_{max}=\frac{N_0\beta g_{\perp}^2h\nu_0(S+M)(S-M+1)}{2kT\Delta Hg_{\parallel}(2S+1)}, \tag{3}$$

where N_0 is the number of spins/cm^3 and cgs units have been used.

Equation (3) is a modification of an expression given by Bloembergen *et al.*,[4] applied to the case of an ion in a crystal. The following conditions are implicit in Eq. (3):

1. The radio frequency is resonant, i.e., $h\nu_0=|E_M-E_{M-1}|$;
2. The temperature T is high enough so that $h\nu \ll kT$;
3. The dc magnetic field is oriented along the crystal symmetry axis.
4. The χ'' vs H curve is a Lorentzian whose width at half maximum is ΔH. This width is then related to the peak of the normalized Lorentzian $f(\nu_0)$ by

$$f(\nu_0)=2h/\pi g_{\parallel}\beta H.$$

The filling factor η is related to the sample volume V_s by a proportionality constant whose value depends on the mode of resonance and the cavity dimensions

$$\eta=aV_s. \qquad (4)$$

Substitution of Eqs. (3) and (4) in Eq. (2) leads to

$$N_0=\left(\frac{k}{\pi\beta h}\right)\left(\frac{g_{\parallel}}{g_{\perp}^2}\right)\left(\frac{T\Delta H}{Q_c a\nu_0 N_0}\right)\frac{(2S+1)}{(S+M)(S-M+1)}\times[\Gamma_{\text{res}}-\Gamma_0]. \qquad (5)$$

Replacing the first factor on the right-hand side of Eq. (5) by its numerical value and multiplying by V_s results in

$$N=7.08\times10^{29}\left(\frac{T\Delta H g_{\parallel}}{Q_c a\nu_0 g_{\perp}^2}\right)\frac{(2S+1)}{(S+M)(S-M+1)}[\Gamma_{\text{res}}-\Gamma_0],$$

where $N=N_0V_s$ is the total number of "spins."

The usefulness of Eq. (6) is twofold. It can be used to estimate the optimum size of sample to be used in a paramagnetic resonance experiment, if the spin density is known. This will also involve a consideration of the system's sensitivity and the saturation power level. The second application, one with which this paper is chiefly concerned, is the determination of N.

After assuming a knowledge of $Q_c,\nu_0,a,T,\Delta H$, and the appropriate quantum numbers S and M for the transition considered, we have to measure $\Gamma_{\max}$ and Γ_0, the voltage reflection coefficients "on" and "off" resonance. This can be achieved by measuring the voltage standing wave ratios for the two conditions and using the relation

$$\text{VSWR}=\frac{1+|\Gamma|}{1-|\Gamma|}$$

to derive the reflection coefficients. An alternative method, and one actually used in our experiment, is to measure the reflection coefficients by measuring the ratio, power reflected/power incident, with a precision attenuator. An indication of the incident power is obtained by causing a total reflection to occur by extreme mismatching of the cavity (which was made possible by the variable coupling feature). Series attenuation is then introduced until the receiver indication is equal to that obtained "on" and "off" resonance. The necessary attenuation is equal to the reflection coefficient, expressed in db.

To test the procedure described above, we used it to determine the number of Gd^{3+} ions in a host $CaWO_4$ crystal. The pertinent data were; $Q_c=25\,600$, $a=3.04$ cm^{-3}, (a TE_{011} circular cavity having a volume of 3.12 cm^3 and a centrally placed sample) $\Delta H=1.8$ gauss, $T=20.4$°K, $g_{\perp}\cong g_{\parallel}\approx 2$, $\nu_0=23$ kMc, $S=7/2$, $M=-5/2$. The quantity $(\Gamma_{\text{res}}-\Gamma_0)$ was found to be 0.143. Substitution in Eq. (6) gives

$$N=1.15\times10^{15}.$$

This result was in good agreement with the result $N=1.00\times10^{15}$ obtained by an independent method based on measurement of the characteristic Gd^{3+} fluorescence and which is believed to be accurate to within $\pm10\%$.

The authors are indebted to J. M. Dziedzic for experimental assistance and to Dr. K. Nassau for growing the crystal used in the experiment.

[1] J. P. Gordon (to be published).
[2] Equation (1) is valid when the reflection coefficient is measured at the position of the "detuned short."
[3] K. D. Bowers and J. Owen, Repts. on Progr. in Phys. 18, 304 (1955).
[4] N. Bloembergen *et al.*, Phys. Rev. 73, 679 (1948).

Letters to the Editor

Prompt publication of brief reports of NEW ideas in measurement and instrumentation or comments on papers appearing in this Journal may be secured by addressing them to this department. No proof will be sent to the authors. Communications should not exceed 500 words in length. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Comments on "Erroneous Readings of Large Magnitude in a Bayard-Alpert Ionization Gauge and their Probable Cause"

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The intent of the paper recently published by Barnes[1] is to show, by indirect measurements of the ion current in a field ion vacuum gauge, that Bayard-Alpert gauges are unreliable. This writer believes that the purpose has not been accomplished. The objections are as follows: