Higher-order Bragg coupling in periodic media with gain or loss*

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We consider wave propagation in longitudinally periodic media with gain or loss. An extended coupled-waves (ECW) approach and Floquet approach are used to study index and gain (or loss) coupling in periodic media at the first few Bragg resonances by the use of Brillouin diagrams. Phase speeding or slowing that is dependent upon the type of coupling occurs in periodic media. Even and odd resonances display different dispersion characteristics. Several examples of coupling due to multiharmonic periodicities are also covered. Coupling or feedback strength can be continuously varied by changing the relative phase of multiharmonic periodicities.

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I. INTRODUCTION

Although the study of wave propagation in periodic structures has been a popular subject since late last century, most of the studies have dealt with passive and lossless periodic media. The recent development of distributed feedback (DFB) lasers[1–3] has extended the applicability of periodic structures to the case of complex media (i.e., media with a complex dielectric constant or perturbation).

Kogelnik and Shank,[1] in their analysis of DFB lasers using the coupled-waves approach, briefly discussed the change in the dispersion diagram near the first Bragg resonance as the gain of the medium starts increasing from zero or when the gain coefficient itself is modulated. A somewhat more detailed analysis of the dispersion curves for index or gain modulation in an active medium was presented by Wang.[2] Higher-order[4,5] DFB lasers have been demonstrated experimentally at the second and third Bragg resonance.

In this paper we investigate in detail the properties of the Brillouin diagram for a complex periodic medium using the exact Floquet approach and the extended coupled-waves (ECW) approach. Higher-order Bragg interactions refer to the case $N \geqslant 2$ in Bragg's law, $\lambda \approx 2\Lambda/N$, where λ is the wavelength of the propagating wave and Λ is the spatial period of the structure. We will consider both the index and gain (or loss) perturbations of active and lossy media and analyze the first- and higher-order Bragg regions. The ECW equations, which will be used here, have been discussed in detail by the authors[6] in a previous paper and are applicable to the first- and higher-order Bragg resonances.

In Sec. II we will use the Floquet results to plot the Brillouin diagrams for several cases. In Sec. III we will use the approximate but simple ECW solution to study in some detail the changes in the Brillouin diagram. In some of the illustrative numerical examples, we will use large values of gain or perturbation that most probably cannot be achieved in reality. However, the objective is to dramatize the effects of these parameters on the Brillouin diagram. In addition, the ECW examples are easily scaled for other values of gain or perturbation. In Sec. IV we discuss the case of multiharmonic periodicites. The conclusions are given in Sec. V.

Throughout this paper, we will assume an $\exp(-i\omega t)$ time dependence and will treat only the monochromatic case.

II. PROBLEM STATEMENT AND FLOQUET RESULTS

Let us consider a TEM wave propagating along the z axis in an unbounded medium with a periodically modulated relative dielectric constant,

$$\epsilon(z) = \epsilon_r + i\epsilon_i + \epsilon_r(\eta_r + i\eta_i)\cos Kz, \qquad (1)$$

where

$$\epsilon_r = \mathrm{Re}\{\epsilon\},$$
$$\eta_r = \mathrm{Re}\{\eta\},$$
$$\epsilon_i = \mathrm{Im}\{\epsilon\},$$
$$\eta_i = \mathrm{Im}\{\eta\},$$

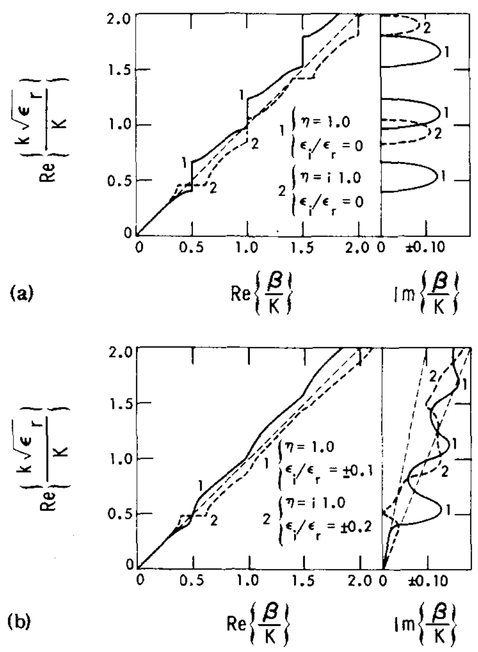

FIG. 1. Brillouin diagram for first three Bragg orders using the Floquet theory and real frequencies (a) index coupling (curve 1) and gain/loss coupling (curve 2) when $\epsilon_i/\epsilon_r = 0$, (b) the similar result for finite ϵ_i/ϵ_r. The light dashed lines are the unperturbed values ($\eta = 0$).

and η_r is such that $\mathrm{Re}\{\epsilon(z)\} > 0$. The average relative dielectric constant is ϵ_r, η is the perturbation or modulation coefficient, and $K = 2\pi/\Lambda$. The case $\eta_i = 0$ is referred to as index coupling and $\eta_r = 0$ refers to gain/loss coupling. The corresponding wave equation for the transverse electric field E is

$$\frac{d^2E}{dz^2} + k^2\epsilon(z)E = 0, \tag{2}$$

and $k = \omega/c = 2\pi/\lambda$ where c is the speed of light in vacuum. The solution is similar to the case of passive-lossless media,[6-8] except that the medium parameters are now complex.

An example of the Brillouin diagram for passive-lossless periodic media (i.e., $\epsilon = \epsilon_r$, $\eta = \eta_r$) is given in curve 1 of Fig. 1(a) for the extreme case $\eta = 1$. Note the noninverting band gaps (i.e., stop bands in $k\epsilon_r^{1/2}/K$) and the upward band-gap shift which causes the curve to be shifted above the unperturbed value ($\eta = 0$) shown by the light dashed line.

The Brillouin diagram changes significantly for gain/loss coupling when the perturbation is imaginary. The case $\epsilon = \epsilon_r$ and $\eta = \eta_i$ is shown in curve 2 of Fig. 1(a) for real frequencies. This case corresponds to a medium with successive amplifying and lossy layers while the average gain/loss is zero. In contrast to the previous case, note the pattern of noninverting and inverting band gaps (i.e., stop bands in β/K) which occur at even and odd Bragg orders, respectively. The downward band-gap shift can be understood from the expression for the average phase velocity,[6]

$$\langle v \rangle = \frac{c}{\epsilon^{1/2}}\left(1 + \frac{3\pi}{16}\eta^2 + O(\eta^4)\right) \tag{3}$$

for $\eta < 1$. It is apparent that phase speeding (upward band-gap shift) occurs for $\eta = \eta_r$ and phase slowing (downward band-gap shift) occurs for $\eta = i\eta_i$.

Let us now consider the case of a complex medium where only the real part of the dielectric constant is sinusoidally perturbed but average gain or loss is present (i.e., $\epsilon = \epsilon_r + i\epsilon_i$, $\eta = \eta_r$). The corresponding Brillouin diagram for real frequencies is shown in curve 1 of Fig. 1(b). For $\epsilon_i \neq 0$, we observe that $\mathrm{Re}\{\beta/K\}$ is no longer constant across a Bragg region and the effective spatial gain[2] or loss[9] increases appreciably near the Bragg frequencies from the unperturbed value.

When the gain or loss is sinusoidally perturbed, the spatial gain or loss diminishes near the odd-order Bragg resonances and is enhanced near the even-order resonances. This case is illustrated in curve 2 of Fig. 1(b) where there is an average imaginary dielectric constant and imaginary perturbation.

Thus, the spatial gain can be either enhanced or diminished near Bragg resonances. This behavior depends upon the Bragg order and the type of coupling or perturbation.

The signs of $\mathrm{Im}\{\beta/K\}$ and $\mathrm{Im}\{k\epsilon_r^{1/2}/K\}$ (not shown) have not been specified because the correct root of the dispersion relation is dependent upon the stability of the waves. For passive media $\mathrm{Im}\{\epsilon(z)\} > 0$, and the correct signs indicate spatially and temporally decaying waves since no sources are present. Hence, the medium is stable and the correct root of the dispersion relation is determined. However, absolute instabilities similar to those of backward wave osillators may arise when $\mathrm{Im}\{\epsilon(z)\} < 0$. In this case, an analysis of wave packets[10] in the periodic medium must be made. The wave-packet analysis has been given for active periodic media elsewhere.[11] However, in this paper we will restrict ourselves to the monochromatic case.

III. EXTENDED COUPLED WAVES THEORY–COMPLEX MEDIA

Following the derivation for the previous ECW results,[6] we find that the ECW equations for complex periodic media at the Nth-order Bragg resonance are

$$\begin{aligned} &F_1'(z) - i\delta_N F_1(z) = i\chi_N B_1(z), \\ &-B_1'(z) - i\delta_N B_1(z) = i\chi_N F_1(z), \end{aligned} \tag{4}$$

where

$$\delta_N = \frac{k^2\epsilon\{1 - \zeta_N(\frac{1}{2}\eta)^2[N^2/2(N^2-1)]\} - \beta_0^2}{2\beta_0}, \tag{5}$$

$$\chi_N = (-1)^{N+1}\frac{k^2\epsilon_r\eta^N}{2^{N+1}\beta_0}\,\frac{1}{\Pi^*\{[4n(n-N)/N^2]^2\}}, \tag{6}$$

$$\begin{aligned} \zeta_N &= 0 \quad \text{for } N=1 \\ &= 1 \quad \text{otherwise,} \end{aligned}$$

$$\begin{aligned} \Pi^* g(n) &= \prod_{n=1}^{(N-1)/2} g(n) && \text{for } N \text{ odd} \\ &= \prod_{n=1}^{N/2} g(n) && \text{for } N \text{ even} \\ &= 1 && \text{for } N = 1, \end{aligned}$$

$$\beta_0 = \tfrac{1}{2}NK,$$

$$\epsilon = \epsilon_r + i\epsilon_i,$$

$$\eta = \eta_r + i\eta_i,$$

and the primes denote differentiation with respect to the coordinate z. The waves F_1 and B_1 have positive and negative phase velocities, respectively. The solution to Eq. (4) is of the form

$$\exp(\pm i\Delta\beta_N z),$$

where

$$(\Delta\beta_N/K)^2 = (\delta_N/K)^2 - (\chi_N/K)^2 \tag{7}$$

is the dispersion relation at the Nth-order Bragg region.

The ECW theory requires that

$$|\eta_r|, \ |\eta_i|, \ |\epsilon_i/\epsilon_r| \ll 1$$

although the results may still hold with reasonable accuracy for

$$|\eta_r|, \ |\eta_i|, \ |\epsilon_i/\epsilon_r| \to 1.$$

Most features of the Floquet results can be predicted by the following ECW examples.

It can be shown that at the first Bragg order. $\mathrm{Im}\{\epsilon\}$ only affects the phase mismatch δ_1 and $\mathrm{Im}\{\eta\}$ only affects the coupling χ_1. For index coupling, the maximum

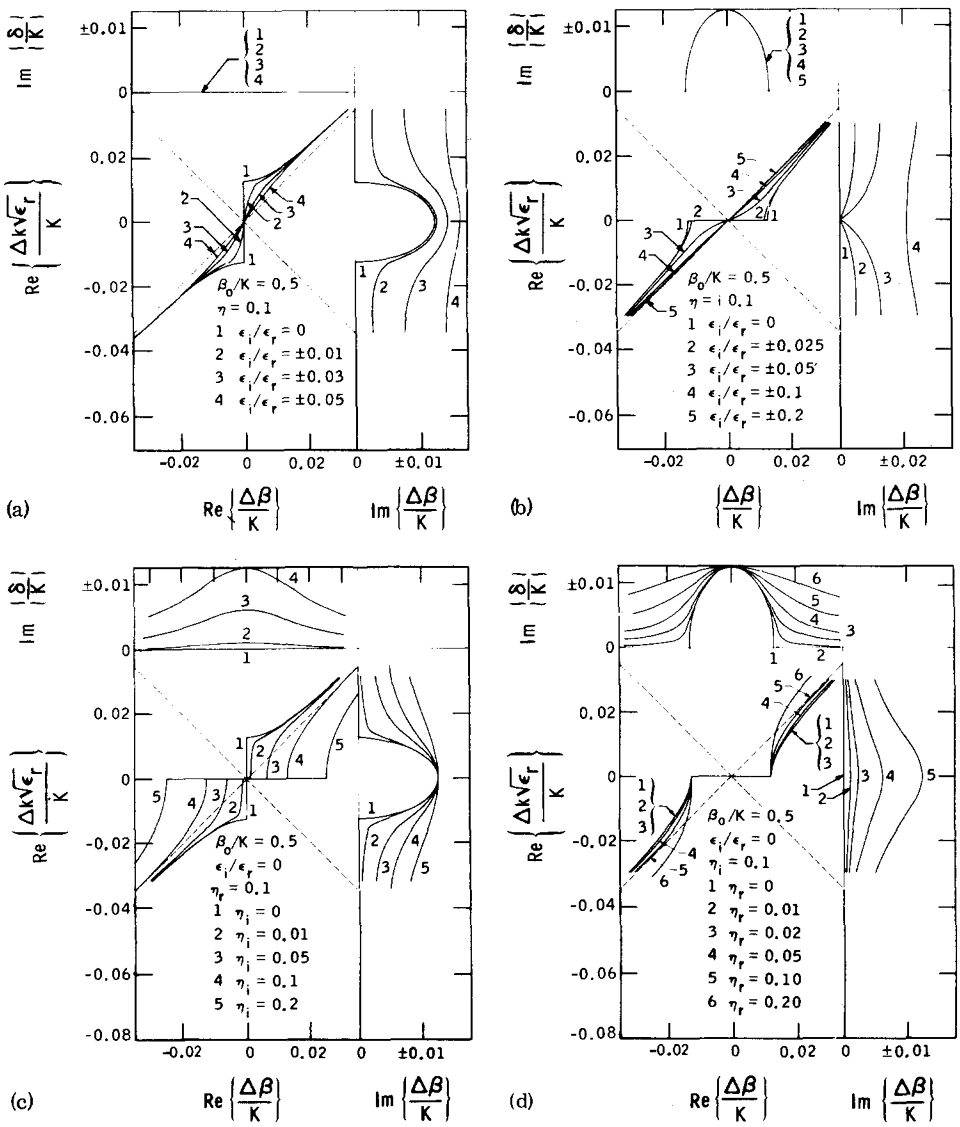

FIG. 2. Brillouin diagram at first Bragg order using: (a) index coupling, (b) gain/loss coupling, and (c) and (d) both couplings in the ECW theory.

value of $\mathrm{Im}\{\Delta\beta_1/K\}$ occurs exactly at the Bragg resonance $\Delta k_1\epsilon_r^{1/2}/K=0$. The value is

$$\mathrm{Im}\left\{\frac{\Delta\beta_1}{K}\right\}_{\max}=\pm\left(\frac{\epsilon_i^2}{16\epsilon_r^2}+\frac{\eta_r^2}{64}\right)^{1/2}.$$

Hence, for index coupling, the effect of spatial gain or loss and the perturbation add as the sum of their squares near the Bragg resonance. This enhancement of spatial gain or loss has been observed in the Floquet solution of Fig. 1(b) and is shown in Fig. 2(a) for the ECW solution for several values of ϵ_i/ϵ_r when $\eta=0.1$. Far from the Bragg resonance $\mathrm{Im}\{\Delta\beta_1/K\}=\pm\,\delta_1/K$, the unperturbed value. The temporal gain or loss, $\mathrm{Im}\{\Delta k_1\epsilon_r^{1/2}/K\}$, remains constant across the band gap with the value $\pm\epsilon_i/4\epsilon_r$.

For gain/loss coupling, χ_1^2 changes sign and, hence, $\Delta\beta_1/K$ is real if $\epsilon_i=0$. This produces an inverting band gap as shown in the Floquet results of Fig. 1(a). The ECW results are shown in Fig. 2(b) for several values of ϵ_i/ϵ_r when $\eta=i0.1$. Two classes of behavior appear that depend upon the sign of

$$A=(|\epsilon_i/\epsilon_r|-|\tfrac{1}{2}\eta_i|).$$

For $A<0$, a band gap of width $2[\eta_i^2/64-\epsilon_i^2/(4\epsilon_r)^2]^{1/2}$ appears in $\mathrm{Re}\{\Delta\beta_1/K\}$ and $\mathrm{Im}\{\Delta\beta_1/K\}=0$ at Bragg resonance. The two roots of $\mathrm{Im}\{\Delta k_1\epsilon^{1/2}/K\}$ are of opposite signs. For $A>0$, there is no band gap, and the gain or loss coupling only produces a decrease in the effective spatial gain or loss near Bragg resonance. This is due to the fact that the coefficients of η_i^2 and ϵ_i^2 have opposite signs in the dispersion relation. In this case, the roots of $\mathrm{Im}\{\Delta k_1\epsilon^{1/2}/K\}$ are of the same sign.

Thus, the effect of finite ϵ_i can either enhance ($\eta=\eta_r$) or reduce ($\eta=i\eta_i$) the effective spatial gain or loss near the Bragg resonance depending on the relative phase of ϵ_i and η_i. For both cases of coupling, increased ϵ_i/ϵ_r tends to mask the effect of the coupling.

Figures 2(c) and 2(d) show the effect of having both index and gain/loss coupling in a media with no average gain or loss. As the ratio of η_i/η_r is increased, the band-gap region changes from the shape typical of index coupling to the shape typical of gain or loss coupling for $\mathrm{Re}\{\Delta\beta_1/K\}$ and $\mathrm{Re}\{\Delta k_1\epsilon_r^{1/2}/K\}$. The values of $\mathrm{Im}\{\Delta\beta_1/K\}$ and $\mathrm{Im}\{\Delta k_1\epsilon_r^{1/2}/K\}$ tend to peak in the Bragg region.

The second-order and all even-order interactions produce very different dispersion relations from the first-order case for gain/loss coupling as shown in the

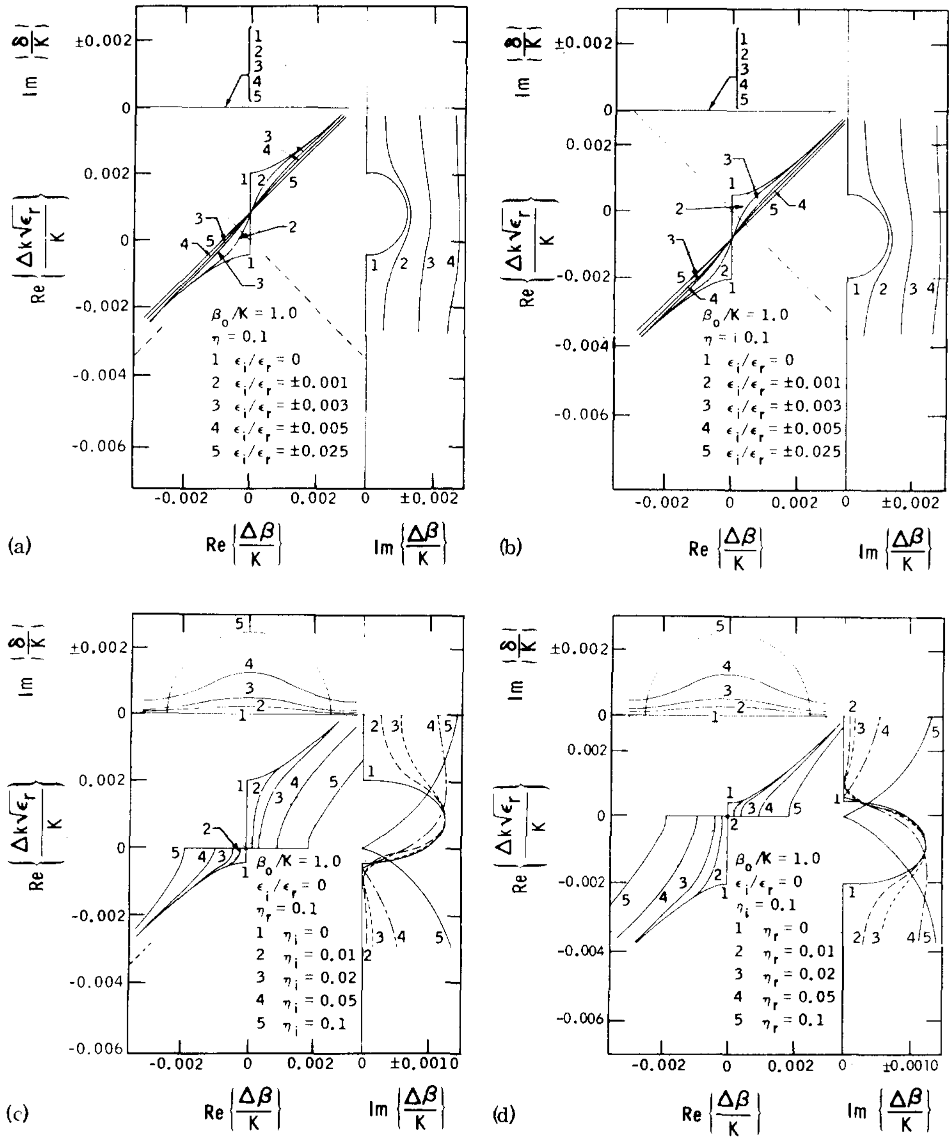

FIG. 3. Brillouin diagram at second Bragg order using: (a) index coupling, (b) gain/loss coupling, and (c) and (d) both couplings in the ECW theory.

Floquet results of Fig. 1. This is due to the term $(i\eta_i)^{2N}$ in the dispersion relation which takes on different signs depending on the oddness or eveness of N. In this case, the perturbation affects not only the coupling χ_2, but also the phase mismatch δ_2. In fact, η now acts to modify the gain or loss from $i\epsilon_i/2\epsilon_r$ to its effective value of $i(\epsilon_i/2\epsilon_r)(1-\frac{1}{6}\eta^2)$.

The Brillouin diagram for the case of index coupling is shown in Fig. 3(a) for various values of ϵ_i/ϵ_r with $\eta = 0.1$. The result is similar to the first-order case with the exception of the band-gap shift. Again, at the center of the band gap ($\delta_2/K=0$) the effective gain/loss and the perturbation add as the sum of their squares. The spatial gain or loss is greatly enhanced near Bragg resonance when

$$|\epsilon_i/\epsilon_r| \ll |\eta|^2,$$

whereas the temporal gain or loss remains constant. The gain/loss coupling case [Fig. 3(b)] is now a mirror image of the index coupling case [Fig. 3(a)] about the axis $\Delta k_2\epsilon^{1/2}/K=0$. That is, at the second Bragg order, the difference between index and gain/loss coupling is simply a matter of upward or downward band-gap shift. This is easily seen from the expression for the phase mismatch δ_2, which shows that the band-gap shift is dependent upon η^2 and accounts for the phase speeding and slowing effects shown by the Floquet results.

Figures 3(a) and 3(d) are similar mirror images about $\Delta k_2\epsilon^{1/2}/K=0$ for $\epsilon_i/\epsilon_r=0$ and various values of the ratio η_i/η_r. It can be shown that if $\epsilon_i/\epsilon_r=0$, $\text{Im}\{\Delta\beta_2/K\}=0$ at $\Delta k_2\epsilon_r^{1/2}/K=0$ only when $|\eta_i|=|\eta_r|$. This corresponds to curve 5. As the ratio of η_i/η_r changes, the Brillouin diagrams form into a band gap of index coupling type [Fig. 3(c), curve 1] or the gain/loss coupling type [Fig. 3(d), curve 1]. Note that for finite values of η_i/η_r, $\text{Im}\{\Delta\beta_2/K\}$ has a minimum in the region of Bragg resonance. Thus, the perturbation will mask the spatial gain or loss of the dielectric for some frequency near Bragg resonance, while a large gain/loss will occur for frequencies slightly closer to the exact Bragg resonance condition.

The third-order case (not shown) is similar to the first-order case for index or gain/loss coupling except for the presence of a finite band-gap shift in the former case. Differences arise in the case of both index and

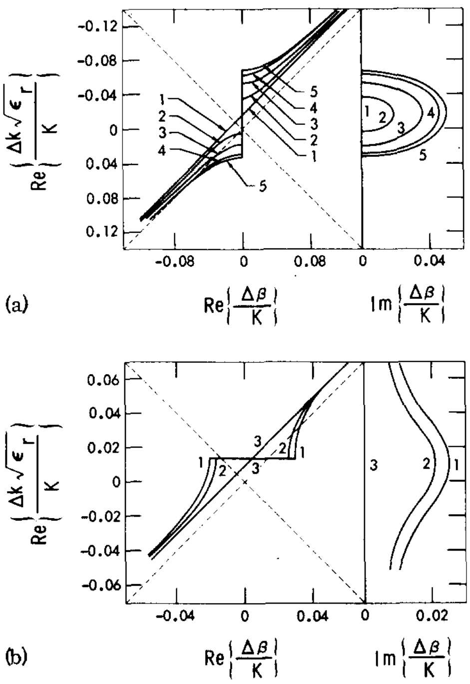

FIG. 4. Brillouin diagram at second Bragg order in lossless media using multiharmonic periodicities in the ECW theory, (a) index coupling used for both periodicities with relative phase θ between the two periodicities, $\eta_1 f_1=(0.2)^{1/2}$, $\eta_2 f_2=0.1$, curve (1) $\theta=0$, curve (2) $\theta=\frac{1}{4}\pi$, curve (3) $\theta=\frac{1}{2}\pi$, curve (4) $\theta=\frac{3}{4}\pi$, curve (5) $\theta=\pi$ (b) index coupling used for the lower-frequency periodicity and gain coupling for the higher-frequency perioidicity, $\eta_1 f_1=(0.2)^{1/2}$, $\eta_2 f_2=i0.1$, curve (1) $\theta=0$, π, curve (2) $\theta=\frac{1}{4}\pi$, $\frac{3}{4}\pi$, curve (3) $\theta=\frac{1}{2}\pi$. Note that $\mathrm{Im}\{\Delta k\epsilon^{1/2}/K\}=0$ for all $\mathrm{Re}\{\Delta\beta/K\}$.

gain/loss coupling which are also due to the finite band-gap shift. However, as $\eta_i/\eta_r\to\infty$ or $\eta_i/\eta_r\to 0$, the Brillouin diagrams assume the appropriate shape as in the first-order case.

IV. MULTIHARMONIC PERIODICITIES

As in the lossless case, interesting possibilities arise when multiharmonic periodicity is present. Here the dielectric constant becomes

$$\epsilon(z)=\epsilon_r+i\epsilon_i+\epsilon_r\sum_p(\eta_r+i\eta_i)_p f_p\cos(pKz+\theta_p)\qquad (p=1,2,\cdots), \quad (8)$$

where we have also allowed phase differences θ_p to exist among the Fourier components f_p.

The ECW theory cannot give explicit dispersion relations for arbitrary Bragg order N since the number of significant f_p's have to be specified. However, the extension is straightforward, although laborious, for any specific case.

Consider the case when only f_1 and f_N are significant Fourier components and $(\eta_1 f_1)^N\sim O(\eta_1 f_N)$,

$$\epsilon(z)\approx\epsilon_r+i\epsilon_i+\epsilon_r[\eta_1 f_1\cos Kz+\eta_N f_N\cos(NKz+\theta)], \quad (9)$$

where

$$\eta_p=(\eta_r+i\eta_i)_p,\quad p=1,N.$$

Then, following the previous derivation, we find

$$\delta_N=\frac{k^2\epsilon\{1-\zeta_N(\frac{1}{2}\eta_1 f_1)^2[N^2/2(N^2-1)]\}-\beta_0^2}{2\beta_0}, \quad (10)$$

$$\chi_N^{\pm}=\frac{k^2\epsilon_r}{2\beta_0}\left(\frac{\eta_N f_N(\cos\theta\pm i\sin\theta)}{2}+\frac{(-1)^{N+1}(\eta_1 f_1)^N}{2^N}\times\frac{1}{\Pi^*[4n(n-N)/N^2]^2}\right), \quad (11)$$

$$\left(\frac{\Delta\beta_N}{K}\right)^2=\left(\frac{\delta_N}{K}\right)^2-\frac{\chi_N^+\chi_N^-}{K^2}, \quad (12)$$

where

$$\beta_0=\tfrac{1}{2}NK$$

and where the ECW equations are slightly modified to

$$F_1'(z)-i\delta_N F_1(z)=i\chi_N^+ B_1(z),$$
$$-B_1'(z)-i\delta_N B_1(z)=i\chi_N^- F_1(z). \quad (13)$$

By varying the phase θ between the Fourier components of the periodicity, χ_N can be varied while δ_N remains constant. Hence, there is now a method of controlling the band-gap width $[=\frac{1}{2}(\chi_N^+\chi_N^-)^{1/2}]$ and coupling without varying the band-gap shift. Figure 4 shows this effect for two cases. In Fig. 4(a), the case of index coupling at the second Bragg order is shown where $\frac{1}{2}(\eta_i f_1)^2/=\eta_2 f_2=0.1$ and $\epsilon_i/\epsilon_r=0$. By varying the phase, the band gap can be made larger or smaller than the value obtained if $\eta_1 f_1$ or $\eta_2 f_2$ acted alone. The band-gap width and absolute value of coupling vary as $\sin(\frac{1}{2}\theta)$. In Fig. 4(b), the case of index coupling of the f_1 component and gain coupling of the f_2 component is shown. Here, $|\frac{1}{2}(\eta_1 f_1)^2|=|\eta_2 f_2|=0.1$, with $\eta_2 f_2=i0.1$ and $\epsilon_i/\epsilon_r=0$. Again, the characteristics of the media can be controlled by varying the phase θ. In both cases, $\mathrm{Im}\{\Delta k\epsilon_r^{1/2}/K\}=0$. Media with small but finite ϵ_i/ϵ_r exhibit similar characteristics.

V. CONCLUSIONS AND DISCUSSION

From the structure of the general dispersion relation, it is apparent that the first Bragg order case is typical of all odd-order cases, and that the second Bragg order case is typical of all even-order cases for index or gain/loss coupling. Other characteristics are explicitly stated as follows:

(1) Index coupling produces noninverting band gaps at all Bragg orders (N) for lossless ($\epsilon_i=0$) media.

(2) Gain/loss coupling produces band gaps in $\mathrm{Re}\{k\epsilon_r^{1/2}\}$ or $\mathrm{Re}\{\beta\}$ at even and odd Bragg orders, respectively, for lossless media. Hence, gain/loss coupling produces dispersion relations similar to index-coupling cases for N even.

(3) Band-gap shift direction is dependent upon the type of coupling. For $\eta_r>\eta_i$, the shift is toward higher frequencies, and for $\eta_r<\eta_i$, the shift is toward lower frequencies. This effect is $O(\eta^2)$ and causes phase speeding or slowing.

(4) The effective spatial gain or loss ($\mathrm{Im}\{\beta\}$) tends to be radically altered in the vicinity of the Bragg reso-

nance. The effective temporal gain or loss ($\mathrm{Im}\{k\epsilon_r^{1/2}\}$) may be constant (e.g., index coupling for all N and gain or loss coupling for $N=2, 4, 6, \cdots$) or may be peaked (e.g., gain/loss coupling for $N=1, 3, 5, \cdots$).

(5) For $|\epsilon_i/\epsilon_r| \gg |\frac{1}{2}\eta|^N$, the periodicity effects are overshadowed by the average gain or loss. For $|\epsilon_i/\epsilon_r| \ll |\frac{1}{2}\eta|^N$ the average gain or loss is negligible.

Note that the dispersion relation shows that a simple scaling rule exists for the ECW results. That is, if η_r, η_i, ϵ_i/ϵ_r, and $\Delta k\epsilon_r^{1/2}/K$ are all scaled by the same factor, then $\Delta\beta/K$ will also be scaled by that factor.

The comparison (not shown) of the Floquet and ECW results shows little or no graphical difference for the first few Bragg orders whenever

$$|\eta_i|, \ |\eta_r|, \ |\epsilon_i/\epsilon_r| \lesssim 0.1.$$

The changing of the band-gap structure with Bragg order and coupling type presents results that are not easy to interpret physically. However, a few tentative observations will be made. First, the increase in effective spatial gain/loss in index coupled media about the Bragg region may be due to multiple reflections within the media. Thus, a ray which enters an index coupled media with average loss will undergo multiple Bragg reflections which increase the effective length of the medium and, hence, will increase the effective loss. In an analogous way, index coupled media with average gain will have an increased effective gain. A similar argument would apply to the case of even-order gain/loss coupling. Second, a problem arises when an identical interpretation is given to odd-order gain/loss coupling since the effective gain/loss is decreased in the Bragg region. In this case, however, we recognize that the inverted band gaps are similar to those of parametric instabilities. Thus, a simple identification of the effective gain or loss which leads to amplification or evanescence may be impossible since the medium may act as a source. A detailed investigation of this phenomenon is saved for future work, since the monochromatic analysis of this paper is insufficient.

The results of the previous analysis have applications to the threshold gain and mode spectrum of DFB lasers. For singly periodic media (i.e., $f_p=0$ for $p \geq 2$) the results of Kogelnik and Shank[1] can be extended to all Bragg orders for index and gain/loss coupling. In particular, we find that increased coupling, and hence decreased threshold gain, can be achieved with multiharmonic periodicities at higher Bragg orders. In addition, the multiharmonic periodicities may be useful for band-gap control or modulation in higher-order DFB lasers and filters. We also expect that the mode spectra will be shifted away from exact Bragg resonance due to the band-gap shift. The detailed calculations for these effects are also left for future work.

The problem of waves in an unbounded complex periodic medium has been studied by the use of the ECW equations and Brillouin diagrams. This allowed dispersion characteristics to be categorized according to Bragg order and type of coupling. Multiharmonic periodicities were discussed and the effect of disappearing band gaps was shown to be an effect of proper phasing of the Fourier components of the periodicities. The methods used here are also applicable to bounded periodic media and to space-time periodic media.

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