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On The Field Theory Limit Of D-Instantons

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Abstract

We study the dilaton/axion configuration near D-instantons in type IIB superstring theory. In the field theory limit, the metric near the instantons becomes flat in the string frame as well as in the Einstein frame. In the large N limit, the string coupling constant becomes zero except near the origin. The supersymmetry of this configuration is analyzed. An implication of this result to the IIB Matrix Model is discussed.

1 Introduction

According to the AdS/CFT correspondence [1–3], string theory on $(p+2)$-dimensional Anti-de Sitter Space (AdS_{p+2}) times a compact space is equivalent to a $(p+1)$-dimensional conformal field theory. In particular, string theory on $AdS_4 \times S^7$, $AdS_5 \times S^5$ and $AdS_7 \times S^4$ are shown to be equivalent to conformal field theory on M2, D3 and M5 branes, respectively. These correspondences were discovered by studying the near horizon region of p-branes in two different ways. One is to consider strings propagating in the curved background generated by the p-brane. Another is to use the collective coordinates of the p-brane, which in some limit is a $(p+1)$-dimensional conformal field theory. The equivalence of the two descriptions implies the correspondence.

In this paper, we study the region near (-1)-branes, *i.e.* instantons. This case is of particular interest since the corresponding 0-dimensional gauge theory has been conjectured to give a non-perturbative definition of the type IIB superstring [4, 5]. We study the dilaton/axion fields configuration near the instantons. If we take the field theory limit of Sen and Seiberg [6, 7], the metric near the instantons becomes flat in the string frame as well as in the Einstein frame. Moreover, in the large N limit, the string coupling constant becomes zero except near the origin. This seems in support of the conjecture [4, 5]. The situation in the case of finite N is subtle and we will comment on this case toward the end of this paper.

2 Wick Rotation and Supersymmetry

The instanton solution is defined in the Euclidean signature space. Since the Wick rotation of type IIB supergravity is subtle, we would like to start our discussion by stating our prescriptions for the Wick rotation and supersymmetry in the Euclidean signature space.

In the Minkowski signature space, the dilaton and axion fields of type IIB theory parametrize the upper half-plane, or the coset space $SL(2,R)/U(1)$. Following [8], we introduce the frame field,

$$V = \begin{pmatrix} V_-^1 & V_+^1 \\ V_-^2 & V_+^2 \end{pmatrix}, \tag{2.1}$$

and define the local $U(1)$ action as

$$V \to V \begin{pmatrix} e^{-i\Sigma} & 0 \\ 0 & e^{i\Sigma} \end{pmatrix} \tag{2.2}$$

where Σ is a $U(1)$ phase. It is convenient to parametrize the matrix V as

$$V = \frac{1}{\sqrt{-2i\tau_2}} \begin{pmatrix} \bar{\tau}e^{-i\lambda} & \tau e^{i\lambda} \\ e^{-i\lambda} & e^{i\lambda} \end{pmatrix}. \tag{2.3}$$

We can fix the $U(1)$ gauge symmetry by setting the scalar field λ to be a function of τ. For example, λ is set equal to $-Im \log(\tau+i)$ in [9] whereas $\lambda = 0$ is used in [10].

The complex scalar field τ in (2.3) is related to the dilaton ϕ and the axion a as

$$\tau = a + ie^{-\phi}. \tag{2.4}$$

To write the type IIB supergravity equations of motion, we introduce two $SL(2,R)$ singlet currents,

$$\begin{aligned} P_\mu &= -\epsilon_{\alpha\beta} V_+^\alpha \partial_\mu V_+^\beta = \frac{i}{2}\frac{\partial_\mu \tau}{\tau_2} e^{2i\lambda} \\ Q_\mu &= -i\epsilon_{\alpha\beta} V_-^\alpha \partial_\mu V_+^\beta = \partial_\mu \lambda - \frac{1}{2}\frac{\partial_\mu \tau_1}{\tau_2}. \end{aligned} \tag{2.5}$$

Under the $U(1)$ gauge symmetry (2.2), they transform as

$$\begin{aligned} P_\mu &\rightarrow P_\mu e^{2i\Sigma} \\ Q_\mu &\rightarrow Q_\mu + \partial_\mu \Sigma. \end{aligned} \tag{2.6}$$

The equations of motion (in the absence of the p-form fields, $p = 2, 4, 6$) are

$$\begin{aligned} R_{\mu\nu} &= P_\mu P_\nu^* + P_\mu^* P_\nu \\ D^\mu P_\mu &= (\nabla_\mu - 2iQ_\mu) P_\mu = 0, \end{aligned} \tag{2.7}$$

where $R_{\mu\nu}$ is the Ricci tensor. Substituting (2.5) into these, the equations of motion can be expressed in terms of ϕ and a as

$$\begin{aligned} &R_{\mu\nu} = \frac{1}{2}(\partial_\mu\phi\partial_\nu\phi + e^{2\phi}\partial_\mu a \partial_\nu a) \\ &\Delta a + 2\partial^\mu\phi\partial_\mu a = 0 \\ &\Delta\phi - e^{2\phi}(\partial a)^2 = 0. \end{aligned} \tag{2.8}$$

These can be derived from the Lagrangian density

$$\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial a)^2. \tag{2.9}$$

The supersymmetry transformations of the dilatino ρ and the gravitino ψ_μ are given by

$$\begin{aligned} \delta\rho &= iP_\mu \gamma^\mu \epsilon^* \\ \delta\psi_\mu &= \left(\nabla_\mu - \frac{i}{2}Q_\mu\right)\epsilon. \end{aligned} \tag{2.10}$$

The instanton is a solution in Euclidean signature space. The equations is this case are obtained from (2.8) by the substitution $a \to \alpha = ia$,

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{2}(\partial_\mu\phi\partial_\nu\phi - e^{2\phi}\partial_\mu\alpha\partial_\nu\alpha) \\ \Delta\alpha &+ 2\partial^\mu\phi\partial_\mu\alpha = 0 \\ \Delta\phi &+ e^{2\phi}(\partial\alpha)^2 = 0. \end{aligned} \tag{2.11}$$

The supersymmetry transformation rules that can be derived analogously [11–13]. We make the substitution $a \to \alpha = ia$, and we treat ϵ and ϵ^* as independent spinors. In addition, we Wick rotate the spinors as in [12, 13]. This yields

$$\begin{aligned} \delta\rho &= iP_\mu\gamma^\mu\epsilon^*, \qquad \delta\rho^* = -iP_\mu^*\gamma^\mu\epsilon \\ \delta\psi^\mu &= (\nabla_\mu - \frac{i}{2}Q_\mu)\epsilon, \qquad \delta\psi^{\mu*} = (\nabla_\mu + \frac{i}{2}Q_\mu)\epsilon^*, \end{aligned} \tag{2.12}$$

where

$$\begin{aligned} P_\mu &= \frac{1}{2}\frac{(\partial_\mu\tau_1 - \partial_\mu\tau_2)}{\tau_2}e^{2i\lambda}, \\ P_\mu^* &= -\frac{1}{2}\frac{(\partial_\mu\tau_1 + \partial_\mu\tau_2)}{\tau_2}e^{-2i\lambda}, \\ Q_\mu &= \partial_\mu\lambda + \frac{i}{2}\frac{\partial_\mu\tau_1}{\tau_2}. \end{aligned} \tag{2.13}$$

The invariance of the Wick-rotated action under these rules directly follows from the invariance of the original action under the transformation rules (2.10). For a more complete discussion of Euclidean spinors and Wick rotation we refer to [11–13].

3 Instanton Solution

As shown in [9], the Euclidean equations of motion (2.11) have a solution where the metric (in the Einstein frame) is flat $g_{\mu\nu} = \delta_{\mu\nu}$ and the dilaton and the axion are related as

$$\partial_\mu\alpha = \pm e^{-\phi}\partial_\mu\phi. \tag{3.1}$$

In the following, we choose the plus sign in the right hand side. The equations (2.11) are then satisfied if the dilaton obeys

$$\Delta e^\phi = 0. \tag{3.2}$$

A spherically symmetric solution to (3.2) with the boundary condition $e^\phi \to g_s$ at infinity $r = \infty$ is given by

$$e^\phi = g_s\left(1 + \frac{c}{r^8}\right) \tag{3.3}$$

for some constant c. The equation (3.1) then determines the axion α as

$$\alpha = \frac{-1}{g_s\left(1+\frac{c}{r^8}\right)} + \text{const.} \tag{3.4}$$

Since α behaves for large r as

$$\alpha \simeq \frac{c}{g_s}\frac{1}{r^8} + \cdots, \tag{3.5}$$

the constant c is related to the instanton charge N as

$$c = c_0 g_s N l_s^8 \tag{3.6}$$

where l_s is the string length and c_0 is a numerical constant related to the volume of the unit 9-sphere.

To summarize, the instanton solution with N unites of charge is given by

$$\begin{aligned} g_{\mu\nu} &= \delta_{\mu\nu} \\ e^{\phi} &= g_s\left(1 + c_0\frac{g_s N l_s^8}{r^8}\right) \\ \alpha &= -g_s^{-1}\left(1 + c_0\frac{g_s N l_s^8}{r^8}\right)^{-1} + \text{const.} \end{aligned} \tag{3.7}$$

4 Field Theory Limit of D-Instantons

It was shown in [9] that the solution (3.7) preserves half of the maximal supersymmetry. Here we will study the field theory limit [1, 6, 7]

$$u = \frac{r}{l_s^2}, \quad g_{YM}^2 = \frac{g_s}{l_s^4} : \text{fixed}, \qquad l_s \to 0, \tag{4.1}$$

of the solutions*. In this limit, the l_s-dependence of the dilaton/axion fields in (3.7) disappears and we obtain

$$\tau_2 = e^{-\phi} \simeq \frac{N u^8}{c_0 (g_{YM}^2 N)^2}, \quad \tau_1 = -\tau_2 + \text{const.} \tag{4.2}$$

*The near instanton configuration was also studied in [14] (see also [15] for a related observation). In that paper they considered the strict $r \to 0$ limit instead of the field theory limit (4.1). They showed that the configuration one obtains in this limit preserves the maximal supersymmetry (a similar result holds for all Dp-branes ($p < 7$) [16]). Notice that the final configuration they obtain is singular as the dilaton diverges in the limit. In the field theory limit (4.1), the dilaton and axion remain finite as in (4.2)

Since $\partial_\mu \tau_1 = -\partial_\mu \tau_2$ for the instanton solution (3.7), the composite gauge field Q_μ given by (2.13) is a pure gauge[†]. Therefore it is natural to set $Q_\mu = 0$ by fixing the $U(1)$ gauge symmetry as

$$\lambda = \frac{i}{2}\log\tau_2 \tag{4.3}$$

In this gauge, the gravitino variation in (2.12) becomes simply

$$\delta\psi_\mu = \partial_\mu \epsilon. \tag{4.4}$$

Thus the equation $\delta\psi_\mu = 0$ has the maximal number of solutions.

Let us turn to the dilatino variation in (2.12). Since $\partial_\mu \tau_1 = -\partial_\mu \tau_2$, $P^*_\mu = 0$ and therefore, $\delta\rho^* = 0$. On the other hand,

$$P_\mu = \partial_\mu \frac{1}{\tau_2} = \frac{c_0 (g_{YM}^2 N)^2}{N} \partial_\mu u^{-8} \tag{4.5}$$

which is non-zero in the field theory limit. Thus, only 1/2 of supersymmetry is preserved even in the field theory limit.

5 Comment on the IIB Matrix Model

In the D-brane description of the p-brane [17], the open string dynamics on the brane reduces to the $(p+1)$-dimensional supersymmetric gauge theory [18] in the limit

$$u = \frac{r}{l_s^2}, \quad g_{YM}^2 = \frac{g_s}{l_s^{3-p}} : \text{fixed}, \qquad l_s \to 0. \tag{5.1}$$

Repeating the argument in [1] in the case of the D(-1) brane, it is natural to expect that type IIB string in the flat metric (5.4) and the dilaton background (4.2) is equivalent to the 0-dimensional matrix model given by the action

$$S = -\frac{1}{g_{YM}^2} tr \left(\frac{1}{4} [A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right), \tag{5.2}$$

where A_μ ($\mu = 1, ..., 10$) and ψ are $N \times N$ hermitian matrices. The large N limit of this model has been proposed in [4, 5] as a non-perturbative definition of type IIB string theory. There they found that the string length of the type IIB string is $(g_{YM}^2 N)^{1/4}$ and that the string coupling constant is $(N\epsilon^2)^{-1}$, where ϵ is a cut-off parameter in the matrix integral.

[†]This is the case even before the field theory limit (4.1) is taken.

Let us compare their results with the field theory limit of the instanton studied in this paper. Since the metric for the instanton solution is flat in the Einstein frame, the string frame metric for the instanton solution is given by

$$ds_s^2 = l_s^4 \sqrt{1 + c_0 \frac{g_{YM}^2 N}{l_s^4 u^8}} (du^2 + u^2 d\Omega_9^2). \tag{5.3}$$

Here $d\Omega_9$ is the line element of the unit 9-sphere. In the limit (4.1), this becomes

$$\frac{ds_s^2}{l_s^2} \simeq \sqrt{c_0 g_{YM}^2 N} \left(\frac{du^2}{u^4} + \frac{d\Omega_9^2}{u^2} \right) = \sqrt{c_0 g_{YM}^2 N} (d\tilde{u}^2 + \tilde{u}^2 d\Omega_9^2), \tag{5.4}$$

where $\tilde{u} = 1/u$. Thus the metric in the string frame also becomes flat. Moreover the factor $\sqrt{g_{YM}^2 N}$ is reminiscent of the string length found in [5].

At the same time, the dilaton in the field theory limit is

$$e^\phi \simeq c_0 \frac{(g_{YM}^2 N)^2}{N u^8}. \tag{5.5}$$

This appears to be different from the expression for the string coupling constant found in [5]. The non-constant dilaton (5.5) is also responsible for the breaking of 1/2 of supersymmetry near the instanton, as we saw in (4.5). On the other hand, it was pointed out in [4] that the matrix model (5.2) has $N = 2$ super Poincaré symmetry in ten dimensions.

In the limit $N \to \infty$ with $g_{YM}^2 N$ finite, these two view points are in complete agreement. In the field theory limit, the string coupling given by the dilaton field e^ϕ is small for $u^8 \gg 1/N$. As we take N to be large, the size of this region expands. In the limit $N \to \infty$, one obtains type IIB string in the flat space (5.4) with vanishing string coupling $e^\phi = 0$. From the large N analysis of (5.2), one also finds free type IIB strings [4]. It would be very interesting to clarify the situation at finite N.

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